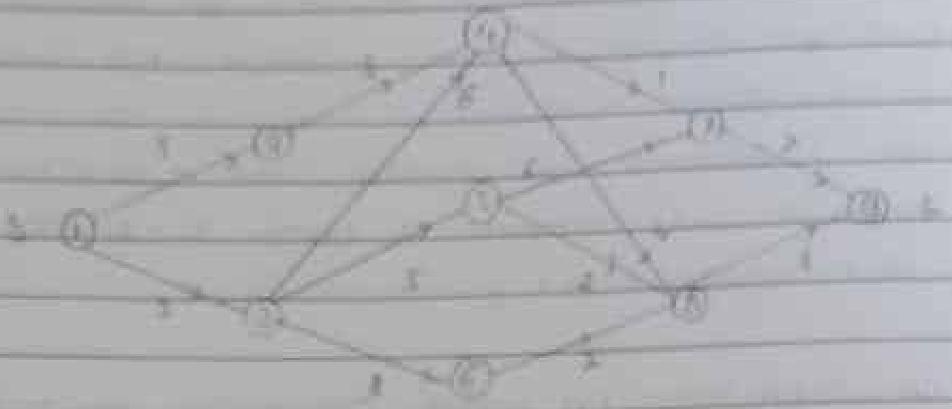
Assignment No. 6

Q.1. Find minimum cost path from s to t in multistage graph given below



$$b\,cost(i,j) = \min_{\substack{l \in V_{i-1} \\ \langle l,j \rangle \in E}} \{ b\,cost(i-1, l) + c(l,j) \}$$

1) Stage 1

$b\,cost(1,1) = 0$

2) Stage 2

i) $b\,cost(2,2) = \min\{b\,cost(1,1) + (1,2)\}$
$= \min\{0+5\} = 5.$

ii) $b\,cost(2,3) = \min\{b\,cost(1,1) + (1,3)\}$
$= \min\{0+2\} = 2.$

3) Stage 3

i) $b\,cost(3,4) = \min\{b\,cost[2,2] + (2,4)\}$
$\{b\,cost(2,3) + (3,4)\}$
$= \min\left\{\begin{matrix} 5+3 \\ 2+6 \end{matrix}\right\} = 8$

ii) b cost (3,5) = min {b cost + (2,3) + (3,5)}
= min {2+5} = 7

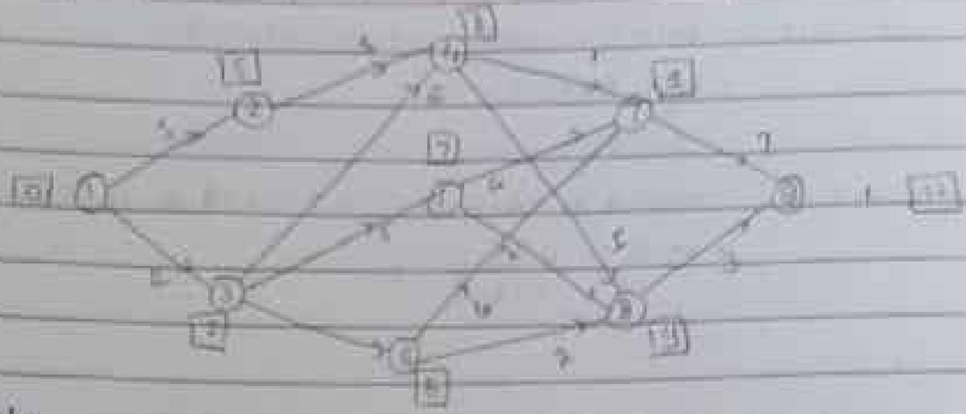
iii) b cost (3,6) = min {b cost (2,2) + (2,6)} b cost (2,3) + (3,6)}
= min $\begin{Bmatrix} 5+3 \\ 2+8 \end{Bmatrix}$ = 8

4) stage 4
1) b cost (4,7) = min {b cost (3,4) + (4,7)
b cost (3,5) + (5,7)
b cost (3,6) + (6,7)}
min {8+1
7+6 = 9
8+6}

ii) b cost (4,8) = min {b cost (3,4) + (4,8)
b cost (3,5) + (5,8)
b cost (3,6) + (6,8)}
= min = {8+4
7+2 = 9
8+2}

5) Stage 5
b cost (5,9) = min {b cost (4,7) + c(7,9)
b cost (4,8) + c (8,9)}
= min $\begin{Bmatrix} 9+7 \\ 9+3 \end{Bmatrix}$ = 12

①



graph

Q2 Explain reliability design problem with suitable example

The reliability design problem is the designing of a system composed of several devices connected in series or parallel. Reliability means the probability to get the sucess of the device.

example

construct a model using reliability design for $c = 135$ the $c_1 = 30$, $c_2 = 25$, $c_3 = 40$ & it having the reliability $r_1 = 0.7$, $r_2 = 0.8$, $r_3 = 0.9$.

$c = 135$

$c_1 = 30$ $\quad r_1 = 0.7$

$c_2 = 25$ $\quad r_2 = 0.8$

$c_3 = 40$ $\quad r_3 = 0.9$

$$u_i = \frac{c + c_i - \sum_{j=1}^{n} c_i}{c_i}$$

$$u_1 = \frac{135 + 30 - \sum_{j=1} (30 + 25 + 40)}{30}$$

$$= \frac{135 + 30 - 95}{30} = \frac{70}{30}$$

$u_1 = \lfloor 2.33 \rfloor$

$u_1 = 2$

$$u_2 = \frac{135 + 25 - 95}{25}$$

$$= \frac{65}{25} = 2.6$$

$= \lfloor 2.6 \rfloor$

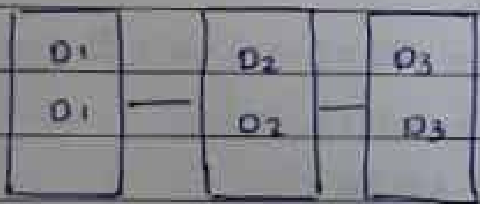
$= 2$

$$u_3 = \frac{135 + 40 - 95}{40}$$

$$= \frac{80}{40} = 2$$

$u_3 = 2$

$u_1 = 2$	30	0.7	$c = 135$
$u_2 = 2$	25	0.8	
$u_3 = 2$	40	0.9	



exceeds the capacity

1) $\left.\begin{aligned} D_1 &= 1 \\ D_2 &= 1 \\ D_3 &= 1 \end{aligned}\right\} 95$

2) $\left.\begin{aligned} D_1 &= 1 \\ D_2 &= 2 \\ D_3 &= 1 \end{aligned}\right\} 120$

3) $\left.\begin{aligned} D_1 &= 1 \\ D_2 &= 1 \\ D_3 &= 2 \end{aligned}\right\} 135$

4) $\left.\begin{aligned} D_1 &= 2 \\ D_2 &= 1 \\ D_3 &= 1 \end{aligned}\right\} 125$

5) $\left.\begin{aligned} D_1 &= 2 \\ D_2 &= 2 \\ D_3 &= 1 \end{aligned}\right\} 150$

6) $\left.\begin{aligned} D_1 &= 2 \\ D_2 &= 1 \\ D_3 &= 2 \end{aligned}\right\} 165$

7) $\left.\begin{aligned} D_1 &= 1 \\ D_2 &= 2 \\ D_3 &= 2 \end{aligned}\right\} 160$

8) $\left.\begin{aligned} D_1 &= 2 \\ D_2 &= 2 \\ D_3 &= 2 \end{aligned}\right\} 190$

$$R = (1-(1-r_i)^{u_i})$$

1) $(D_1, D_2, D_3) = (1,1,1)$

$= (1-(1-0.7)^1) \times (1-(1-0.8)^1) \times (1-(1-0.9)^1)$

$= 0.7 \times 0.8 \times 0.9$

$= 0.504$

$= 50.4\%$

2) $(D_1, D_2, D_3) = (1,2,1)$

$= (1-(1-0.7)^1) \times (1-(1-0.8)^2) \times (1-(1-0.9)^1)$

$= 0.7 \times 0.90 \times 0.9$

$= 0.63 \times 0.96$

$= 0.608 = 60.8\%$

3) $(D_1, D_2, D_3) = (1,1,2)$

$= (1-(1-0.7)^1) \times (1-(1-0.8)^1) \times (1-(1-0.9)^2)$

$= 0.56 \times 0.99$

$= 0.554$

$= 55.4\%$

4) $(D_1, D_2, D_3) = (2,1,1)$

$= (1-(1-0.7)^2) \times (1-(1-0.8)^1) \times (1-(1-0.9)^1)$

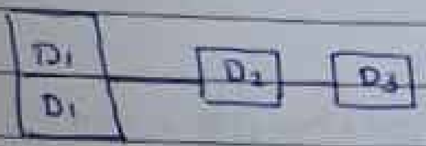
$= 0.91 \times 0.8 \times 0.9$

$= 0.655$

$= 65.5\%$

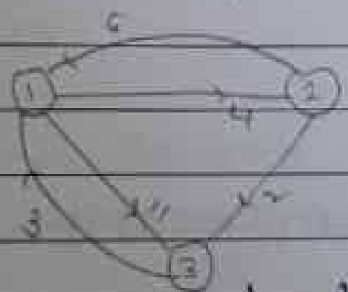
final result

$(D_1, D_2, D_3) = (2,1,1)$



Total capacity = 125

Q find all pair shortest path for following graph



$$D = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 4 & 11 \\ 2 & 6 & 0 & 2 \\ 3 & 3 & \infty & 0 \end{array}$$

D = L

1) k=1 , i=1 , j=1

$D_1[1][1] = \min\{D[1][1],\ D[1][1] + D[1][1]\}$
$= \min\{0, 0+0\}$
$D_1[1][1] = \min\{0,0\} = 0$

2) k=1 , i=1 , j=2
$D_1[1][2] = \min\{D[1][2],\ D[1][1] + [1][2]\}$
$= \min\{4, 0+4\} = 4$

3) $D_1[2][1] = \min\{D[2][1],\ D[2][1] + D[1][1]\}$
$= \min\{6, 6+0\} = 0$

4) $D_1[2][2] = \min\{D[2][2],\ D[2][1] + D_1[1][2]\}$
$= \min\{0, 6+4\} = 0$

5) $D_1[2][3] = \min\{D[2][3],\ D[2][1] + [1][3]\}$
$= \min\{2, 6+11\}$
$= \min : 2$

6) $D_1[1][3] = \min\{D[1][3],\ D[1][1] + D[1][3]\}$
$= \min\{11, 0+11\}$
$= 11$

7) $D_1[3][1] = \min\{D[3][1],\ D[3][1] + D[1][1]\}$
$= \min\{3, 3+0\}$
$= 3.$

8) $D_1[3][2] = \min\{D[3][2],\ D[3][1] + D[1][2]\}$
$= \min\{\infty, 3+4\} = 7$

9) $D_1[3][3] = \min\{D[3][3],\ D[3][1] + D[1][3]\}$

$= \min\{0, 3+11\} = 0$

$$D_1 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 4 & 11 \\ 2 & 6 & 0 & 2 \\ 3 & 3 & 7 & 0 \end{array}$$

1) $k=2,\ i=1,\ j=1$

$D_2[1][1] = \min\{D_1[1][1],\ D_1[1][2] + D_1[2][1]\}$
$= \min\{0,\ 2+6\} = 0$

2) $D_2[1][2] = \min\{D_1[1][2],\ D_1[1][2] + D_1[2][2]\}$
$= \min\{4,\ 4+0\} = 4$

3) $D_2[1][3] = \min\{D_1[1][3],\ D_1[1][2] + D_1[2][3]\}$
$= \min\{11,\ 4+2\} = 6$

4) $D_2[2][1] = \min\{D_1[2][1],\ D_1[2][2] + D_1[2][1]\}$
$= \min\{6,\ 0+6\} = 6$

5) $D_2[2][2] = \min\{D_1[2][2],\ D_1[2][2] + D_1[2][2]\}$
$= 0$

6) $D_2[3][1] = \min\{D_1[3][1],\ D_1[3][2] + D_1[2][1]\}$
$= \min\{3,\ 7+6\} = 3$

7) $D_2[3][2] = \min\{D_1[3][2],\ D_1[3][2] + D_1[2][2]\}$
$= \min\{7,\ 7+0\} = 7$

8) $D_2[3][3] = \min\{D_1[3][3],\ D_1[3][2] + D_1[2][3]\}$
$= \min\{0,\ 7+0\}$
$= 0$

9) $D_1[3][3] = \min\{D_1[3][3], D_1[3][2] + D_1[2][3]\}$
$= \min\{2, 0+2\} = 2$

$$D_2 : \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 4 & 6 \\ 2 & 6 & 0 & 2 \\ 3 & 3 & 7 & 0 \end{array}$$

1) $k=3$, $i=1$, $j=1$
$D_3[1][1] = \min\{D_2[1][1], D_2[1][3] + D_2[3][1]\}$
$= \min\{0, 6+3\} = 0$

2) $D_3[1][2] = \min\{D_2[1][2], D_2[1][3] + D_2[3][2]\}$
$= \min\{4, 6+7\} = 4$

3) $D_3[1][3] = \min\{D_2[1][3], D_2[1][3] + D_2[3][3]\}$
$= \min\{6, 3+3\} = 6$

4) $D_3[2][1] = \min\{D_2[2][1], D_2[2][3] + D_2[3][1]\}$
$= \min\{6, 2+3\} = 5$

5) $D_3[2][2] = 0$

6) $D_3[2][2] = \min [2][3], D_2[2][3] + D_2[3][3]\}$
$\min\{2, 2+0\} = 2$

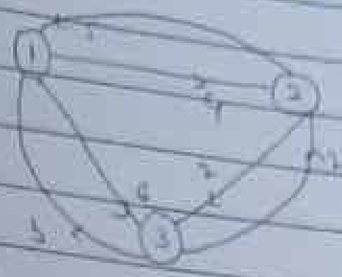
7) $D_3[3][1] = \min\{D_2[3][1], D_2[3][3] + D_2[3][3]\}$
$= \min\{3, 0+3\} = 3$

8) $D_3[3][2] = \min\{D_2\{3][2], D_2[3][3] + D_2[3][2]\}$
$= \min\{7, 0+7\} = 7$

9) $D_3[3][3] = 0$

D3 =

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 4 & 6 \\ 2 & 5 & 0 & 2 \\ 3 & 3 & 7 & 0 \end{array}$$



Hence it is the possible all pair shortest path