In [1]:
```
def mean(a):
  #5100 : row
  #36 : colu
  #a.shape 5100 36
  # a.shape[1] 36
  # a.shape[0] 5100
    result=[]

    a= np.array(a)
    for i in range(a.shape[1]):
        sum = 0
        counter = 0
        for j in range(a.shape[0]):
            sum = sum + a[j][i]
            counter = counter + 1
        mean = sum/counter
        result.append(mean)
    return np.array(result)
```

In [2]:
```
def std(a):
    a = np.array(a)
    result=[]
    mean1 = mean(a)
    var=[]
    for i in range(a.shape[1]):
        sum = 0
        counter = 0
        for j in range(a.shape[0]):
            sum = sum + (a[j][i] - mean1[i]) ** 2
            counter = counter + 1
        answer=sum / counter
        var.append(answer)
    for i in range(len(var)):
        result.append(var[i] ** (1/2))
    return result
```

In [3]:
```
def flip(a):
    answer=a[::-1]
    return np.array(answer)
```

In [4]:
```
def outer(a,b):
    result=[]
    for i in range(len(a)):
        result.append([])
    for i in range(len(a)):
        answer = 0
        for j in range(len(a)):
            answer = a[i]*b[j]
            result[i].append(answer)
    return np.array(result)
```

In [5]:
```
def signfunction(a):
    if a < 0:
        return -1
    elif a == 0:
        return 0
    else:
        return 1
```

In [6]:
```
def normalise(a):
    answer = 0
    for i in range(len(a)):
```

```python
        answer = answer + a[i]**2
    sqrt = answer ** (1/2)
    return sqrt
```

In [7]:
```python
def sumfunction(a):
    sum = 0
    for i in range(len(a)):
        sum = sum + a[i]
    return sum
```

In [8]:
```python
def diag(v):
    res=[]
    for i in range(len(v)):
        for j in range(len(v[i])):
            if i == j:
                res.append(v[i][j])
    return np.array(res)
```

In [9]:
```python
def zeros(n):
    answer = []
    for i in range(n):
        answer.append(0)
    return np.array(answer)
```

In [10]:
```python
def eyes(n):
    answer=[]
    for i in range(n):
        answer.append([])
    for i in range(n):
        for j in range(n):
            if i==j:
                answer[i].append(1.)
            else:
                answer[i].append(0.)
    return np.array(answer)
```

In [11]:
```python
def householder_reflection(a, e):
    '''
    Given a vector a and a unit vector e,
    (where a is non-zero and not collinear with e)
    returns an orthogonal matrix which maps a
    into the line of e.
    '''

    assert a.ndim == 1
    assert np.allclose(1, sumfunction(e**2))

    u = a - signfunction(a[0]) * normalise(a) * e
    v = u / normalise(u)
    H = eyes(len(a)) - 2 * outer(v, v)

    return H
```

In [12]:
```python
def qr_decomposition(A):
    '''
    Given an n x m invertable matrix A, returns the pair:
        Q an orthogonal n x m matrix
        R an upper triangular m x m matrix
    such that QR = A.
    '''
    n, m = A.shape
    assert n >= m
```

```python
    Q = eyes(n)
    R = A.copy()

    for i in range(m - int(n==m)):
        r = R.iloc[i:, i]

        if np.allclose(r[1:], 0):
            continue

        # e is the i-th basis vector of the minor matrix.
        e = zeros(n-i)
        e[0] = 1

        H = eyes(n)
        H[i:, i:] = householder_reflection(r, e)
        Q = H.T
        R = H @ R
        return Q, R
```

In [13]:
```python
def eigen_decomposition(A, max_iter=100):
    A_k = A
    Q_k = eyes(A.shape[1])

    for k in range(max_iter):
        Q, R = qr_decomposition(A_k)
        Q_k = Q_k @ Q
        A_k = R @ Q

    eigenvalues = diag(A_k)
    eigenvectors = Q_k
    return eigenvalues, eigenvectors
```

In [14]:
```python
class PCA:
    def __init__(self, n_components=None, whiten=False):
        self.n_components = n_components
        self.whiten = bool(whiten)

    def fit(self, X):
        n, m = X.shape

        # subtract off the mean to center the data.
        self.mu = mean(X)
        X = X - self.mu

        # whiten if necessary
        if self.whiten:
            self.std = std(X)
            X = X / self.std

        # Eigen Decomposition of the covariance matrix
        C = X.T @ X / (n-1)
        self.eigenvalues, self.eigenvectors = eigen_decomposition(C)

        # truncate the number of components if doing dimensionality reduction
        if self.n_components is not None:
            self.eigenvalues = self.eigenvalues[0:self.n_components]
            self.eigenvectors = self.eigenvectors[:, 0:self.n_components]

        # the QR algorithm tends to puts eigenvalues in descending order
        # but is not guarenteed to. To make sure, we use argsort.
        descending_order = flip(np.argsort(self.eigenvalues))
        self.eigenvalues = self.eigenvalues[descending_order]
        self.eigenvectors = self.eigenvectors[:, descending_order]

        return self
```

```
    def transform(self, X):
        X = X - self.mu

        if self.whiten:
            X = X / self.std

        return X @ self.eigenvectors

    @property
    def proportion_variance_explained(self):
        return self.eigenvalues / sumfunction(self.eigenvalues)

        return Q, R
```

In [15]:

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Importing Data file

X = pd.read_csv('DataSet_2.csv')
display(X.head())
```

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	...	V27	V28	V29	V30	V31	V32	V33	V34	V35	V36
0	46	40	119	139	42	30	135	157	42	30	...	115	113	50	46	111	116	44	31	131	142
1	47	37	119	133	44	34	124	143	44	34	...	100	85	50	39	118	132	43	29	133	143
2	80	95	100	74	64	64	104	96	46	36	...	100	81	82	91	92	78	78	83	96	74
3	56	51	72	60	59	54	72	60	59	51	...	71	50	57	55	74	61	57	55	78	65
4	44	34	129	140	44	34	124	136	44	34	...	133	139	43	31	128	135	43	29	128	132

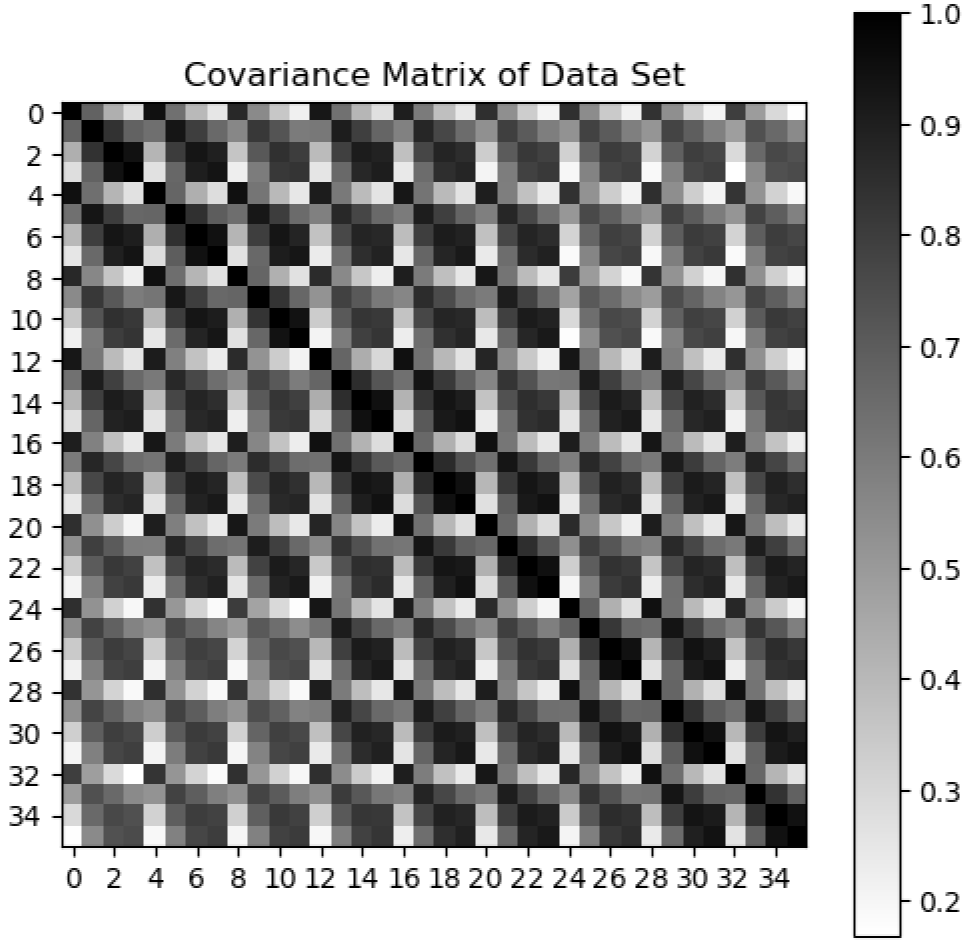
5 rows × 36 columns

In [16]:

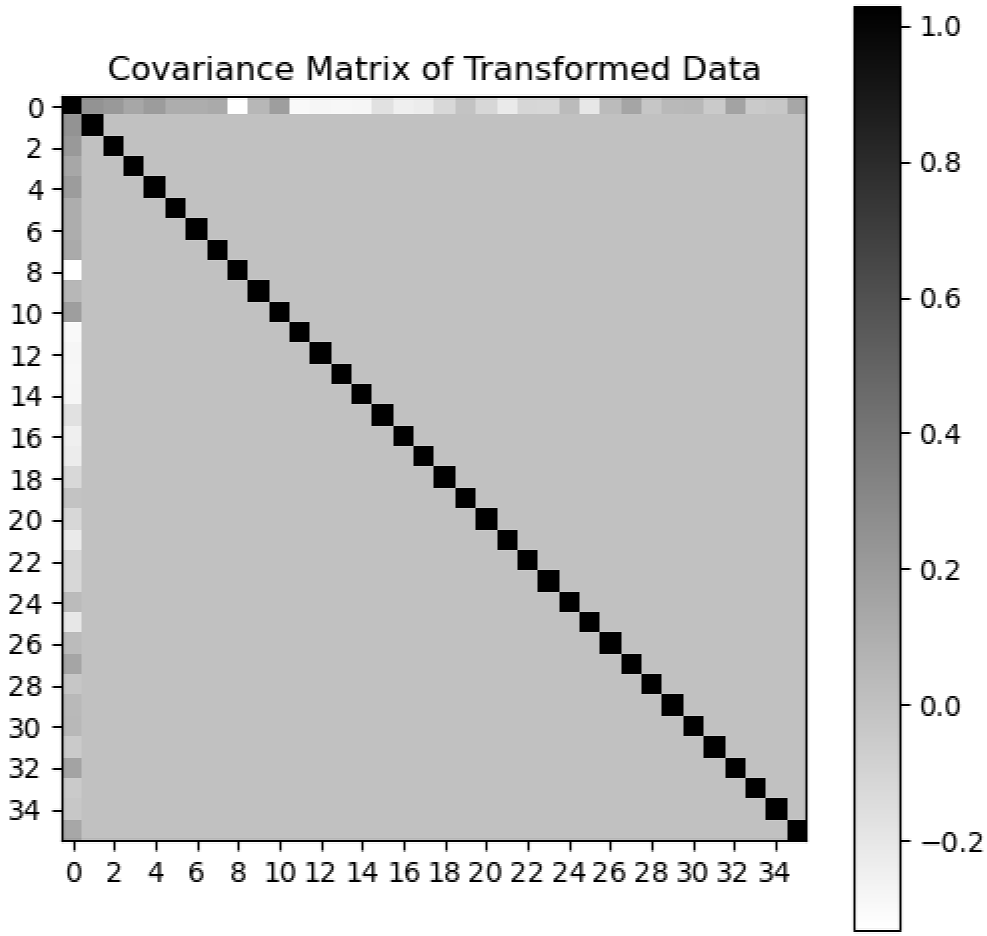
```
# Calculating Co-variance Matrix

X_white = (X - mean(X))/std(X)
C = X_white.T @ X_white / (X_white.shape[0] - 1)

# Visualizing Co-variance Matrix

plt.figure(figsize=(6,6))
plt.imshow(C, cmap='binary')
plt.title("Covariance Matrix of Data Set")
plt.xticks(np.arange(0, 36, 2))
plt.yticks(np.arange(0, 36, 2))
plt.colorbar()
```

Out[16]:

```
<matplotlib.colorbar.Colorbar at 0x2a7b80234d0>
```

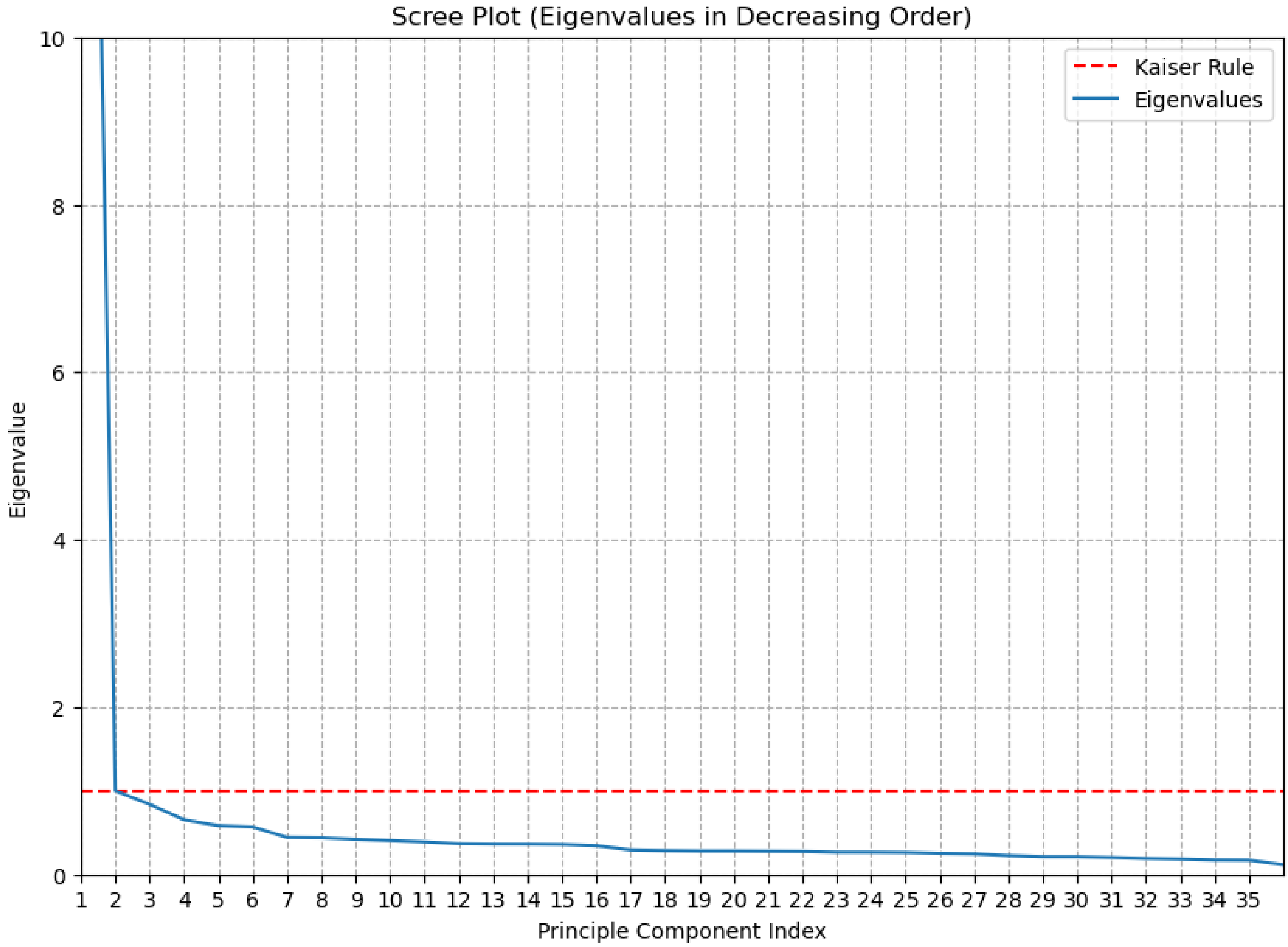


In [17]:
```
# Calculating

pca = PCA(whiten=True)
pca.fit(X)
X_prime = pca.transform(X)
pca.eigenvalues
C_prime = pca.eigenvectors
X_white = (C_prime - mean(C_prime))/std(C_prime)
C_prime = X_white.T @ X_white / (X_white.shape[0] - 1)

# Visualizing Co-Variance Matrix of the Transformed Data

plt.figure(figsize=(6,6))
plt.imshow(C_prime, cmap='binary')
plt.title("Covariance Matrix of Transformed Data")
plt.xticks(np.arange(0, 36, 2))
plt.yticks(np.arange(0, 36, 2))
plt.colorbar()
plt.show()
```



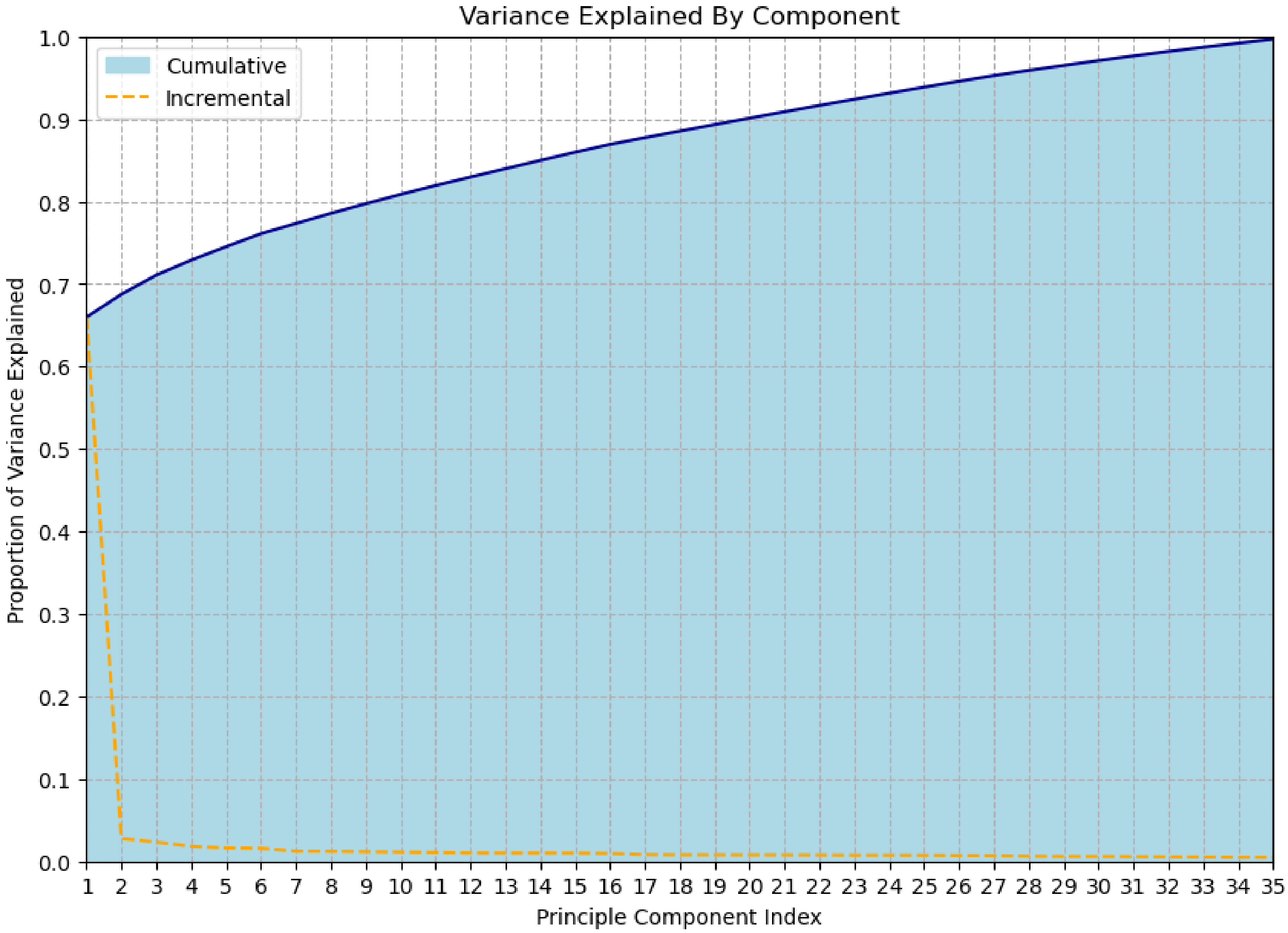
In [18]:

```
# Ploting Eigen Vectors

fig = plt.figure(figsize=(10, 7))
plt.title("Scree Plot (Eigenvalues in Decreasing Order)")
plt.plot([1, 36], [1, 1], color='red', linestyle='--', label="Kaiser Rule")
plt.xticks(np.arange(1, 36, 1))
plt.xlim(1, 36)
plt.ylim(0, 10)
plt.ylabel("Eigenvalue")
plt.xlabel("Principle Component Index")
plt.grid(linestyle='--')
plt.plot(range(1, 37), pca.eigenvalues, label="Eigenvalues")
plt.legend()
plt.show()
```



In [19]:

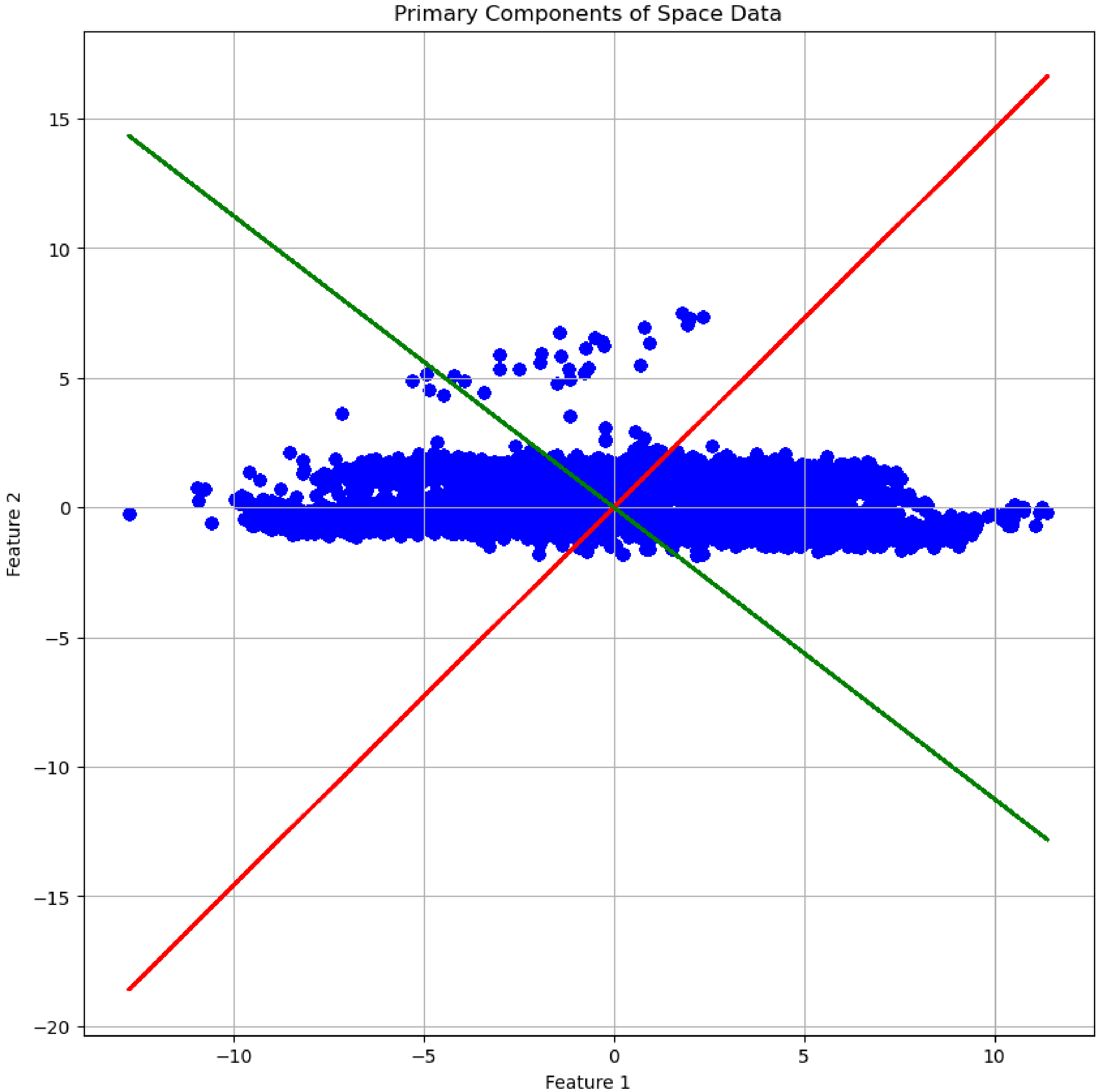
```
# Variance covered by the Eigen Vectors

fig = plt.figure(figsize=(10, 7))
plt.title("Variance Explained By Component")
plt.xticks(np.arange(1, 36, 1))
plt.yticks(np.arange(0, 1.0001, 0.1))
plt.xlim(1, 35)
plt.ylim(0, 1)
plt.ylabel("Proportion of Variance Explained")
plt.xlabel("Principle Component Index")
plt.grid(linestyle='--')
plt.fill_between(
    range(1, 37),
    np.cumsum(pca.proportion_variance_explained),
    0,
    color="lightblue",
    label="Cumulative")
plt.plot(
    range(1, 37),
    np.cumsum(pca.proportion_variance_explained),
    0,
    color="darkblue")
plt.plot(
    range(1, 37),
    pca.proportion_variance_explained,
    label="Incremental",
    color="orange",
    linestyle="--")
plt.legend(loc='upper left')
plt.show()
```



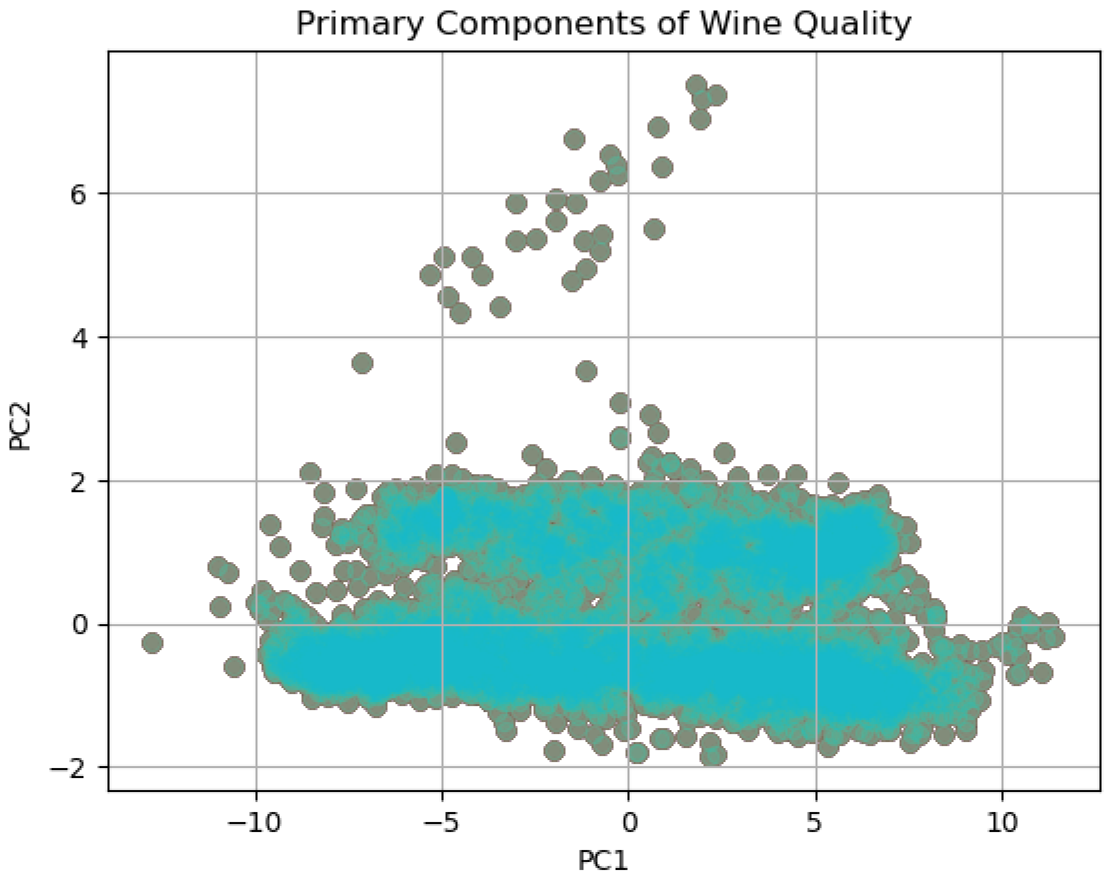
In [20]:

```
# Visualizing Eigen Vectors and Tranformed Data

plt.figure(figsize=(10, 4))
x_data = X.iloc[:,0]
y_data= (pca.eigenvectors[1,0]/pca.eigenvectors[0,0])*X_prime.iloc[:,0]
y_data1= (pca.eigenvectors[1,1]/pca.eigenvectors[0,1])*X_prime.iloc[:,0]
fig = plt.figure(figsize=(10, 10))
for c in np.unique(X):
    X_class = X_prime
    plt.plot(X_prime.iloc[:,0],y_data,color='red')
    plt.plot(X_prime.iloc[:,0],y_data1,color='green')
    plt.scatter(X_class.iloc[:,0],X_class.iloc[:,1],alpha=0.1,color='blue')
plt.title("Primary Components of Space Data")
plt.xlabel("Feature 1")
plt.ylabel("Feature 2")
plt.grid()
plt.show()
```

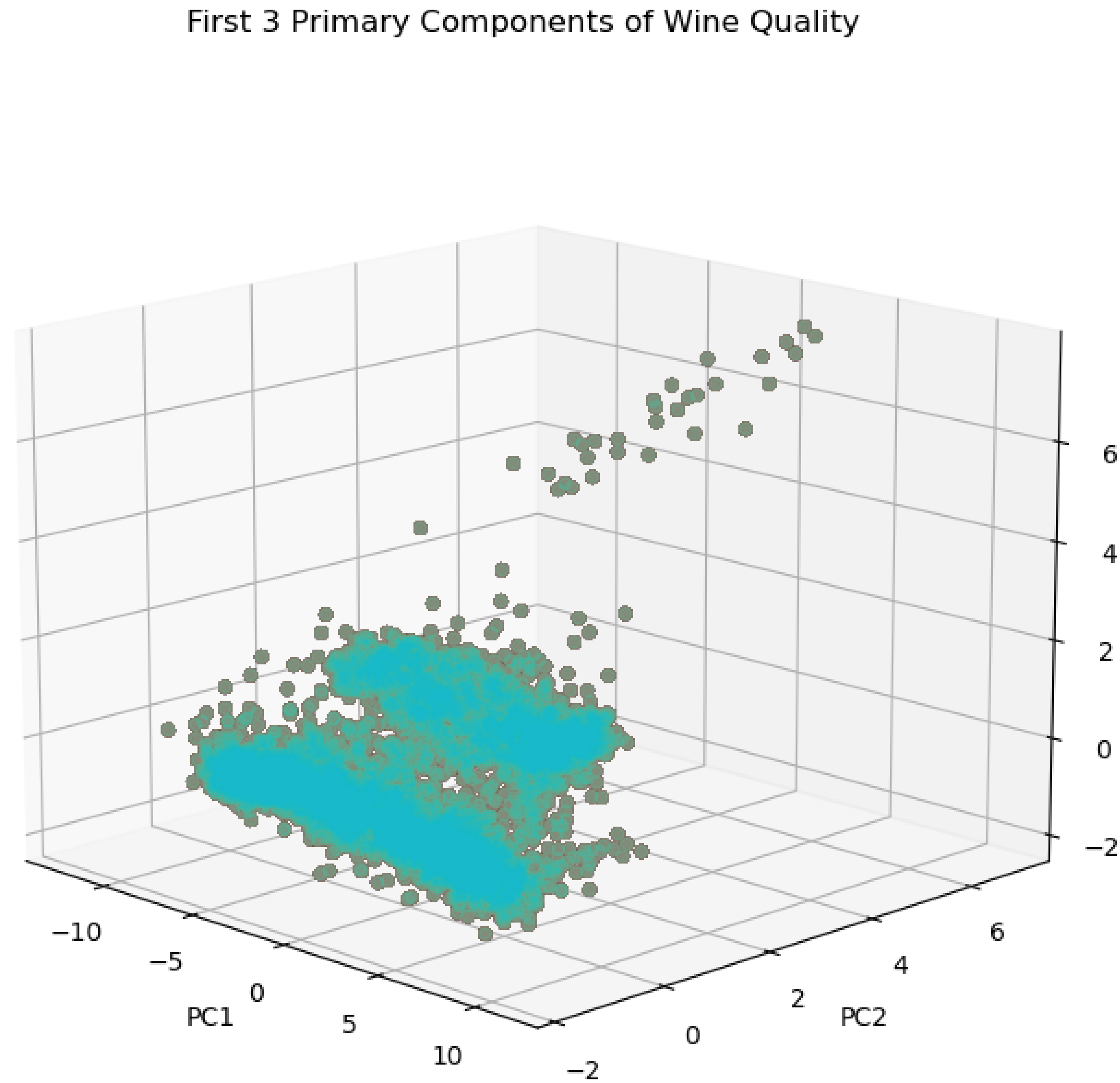
```
<Figure size 1000x400 with 0 Axes>
```



In [21]:
```
# 2D Scatter plot of Tranformed Data

for c in np.unique(X):
    X_class = X_prime
    plt.scatter(X_class.iloc[:, 0], X_class.iloc[:, 1], alpha=0.2)
plt.title("Primary Components of Wine Quality")
plt.xlabel("PC1")
plt.ylabel("PC2")
plt.grid()
plt.show()
```



In [22]:
```
# 3D Scatter plot of Tranformed Data

fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')
ax.view_init(15, -45)
for c in np.unique(X):
    X_class = X_prime
    ax.scatter(X_class.iloc[:, 0], X_class.iloc[:, 2], X_class.iloc[:, 1], alpha=0.2)
plt.title("First 3 Primary Components of Wine Quality")
ax.set_xlabel('PC1')
ax.set_ylabel('PC2')
ax.set_zlabel('PC3')
plt.show()
```


First 3 Primary Components of Wine Quality
PC1
PC2
−10
−5
0
5
10
−2
0
2
4
6
−2
0
2
4
6