

Pauli Y-Gate

$|q\rangle \rightarrow$ Y

$$\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\boxed{\sigma_y = i \sigma_x \sigma_z}$$

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↑ ↑

Hadamard Gate :-

$|q\rangle \rightarrow$ H

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$A = A_{00} |0\rangle\langle 0| + A_{01} |0\rangle\langle 1| \\ + A_{10} |1\rangle\langle 0| + A_{11} |1\rangle\langle 1|$$

$$H = \frac{1}{\sqrt{2}} \left[|0\rangle\langle 0| + |0\rangle\langle 1| \right. \\ \left. + |1\rangle\langle 0| - |1\rangle\langle 1| \right]$$

$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \cdot |0\rangle \\ + 1 \cdot |1\rangle \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

$$\boxed{H|0\rangle = |+\rangle}$$

$$H|+\rangle$$

$$\Rightarrow H \left[\frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \right]$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$\boxed{H|+\rangle = |0\rangle}$$

$$H|1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = |-\rangle$$

$$\boxed{H|-\rangle = ?}$$

S-Gate

$$|q\rangle \rightarrow \boxed{S}$$

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

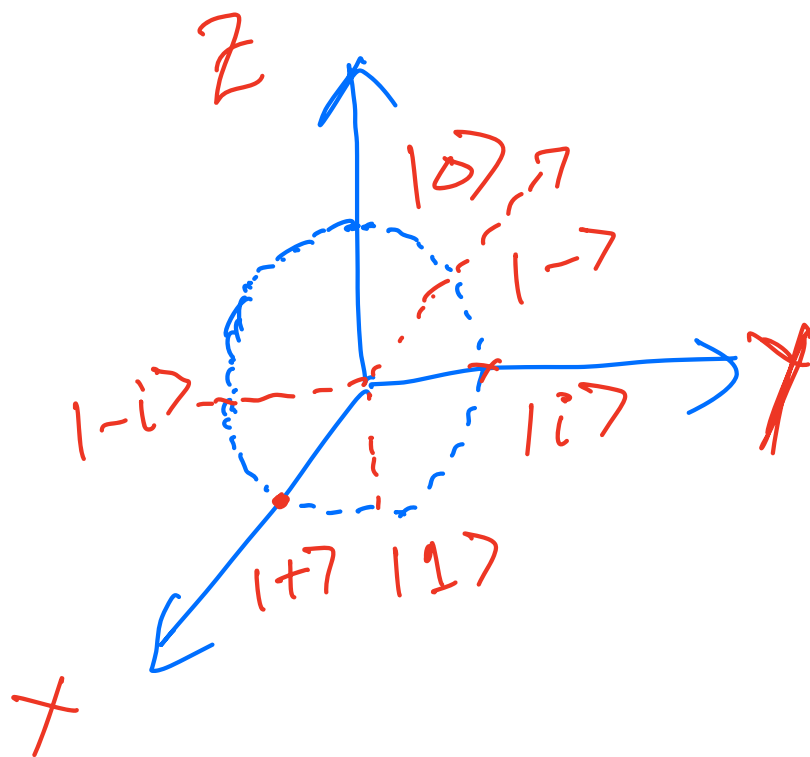
$$S|+\rangle = |+i\rangle$$

$$S|-\rangle = |-i\rangle$$

$$|+i\rangle = \frac{1}{\sqrt{2}} (|0\rangle + i|1\rangle)$$

$$|-i\rangle = \frac{1}{\sqrt{2}} (|0\rangle - i|1\rangle)$$

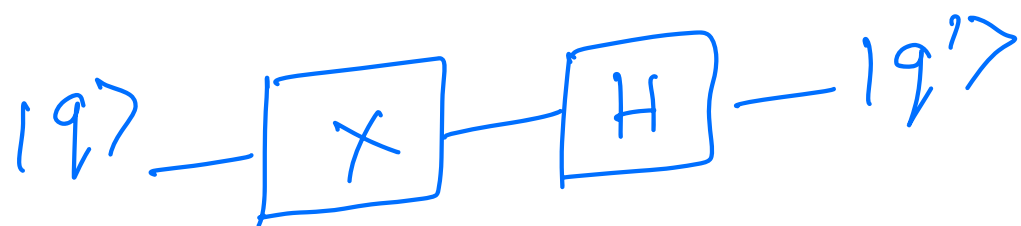
$|+i\rangle$ is a vector along
+ve y -axis intersecting
Bloch Sphere



Rotation
of $\pi/2$

Quantum Circuits

①



$|0\rangle$

$$H \sigma_x |q\rangle = |q'\rangle$$

$$H\sigma_x 10\rangle$$

$$\Rightarrow \sigma_x 10\rangle = 11\rangle$$

$$\Rightarrow H 11\rangle = 1-\rangle$$

$$\boxed{H\sigma_x 10\rangle = 1-\rangle}$$

$$\frac{117}{\sigma_x} \boxed{H} - ?$$

$$H\sigma_x 11\rangle$$

$$\sigma_x 11\rangle = 10\rangle$$

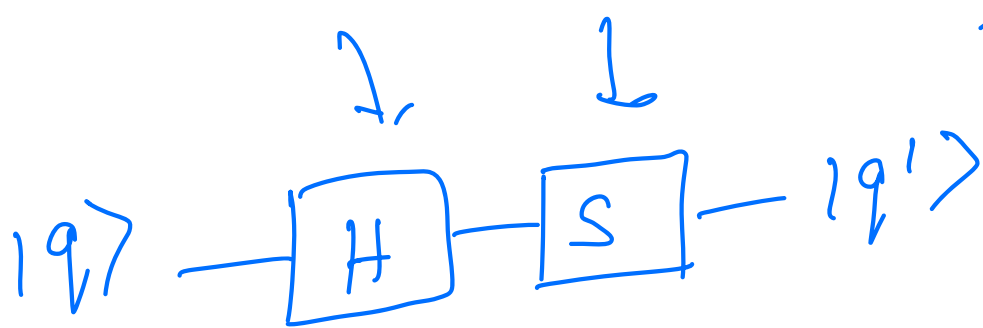
$$H 10\rangle = 1+\rangle$$

$$\boxed{H\sigma_x 11\rangle = 1+\rangle}$$

$$H\sigma_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

(2)



$$SH|q\rangle = |q'\rangle$$

$$|q\rangle = |0\rangle$$

$$SH|0\rangle = ?$$

$$\Rightarrow H|0\rangle = |+\rangle$$

$$S|+\rangle = |++i\rangle$$

$$SH|0\rangle = |++i\rangle$$

$$SH|1\rangle = |--i\rangle$$