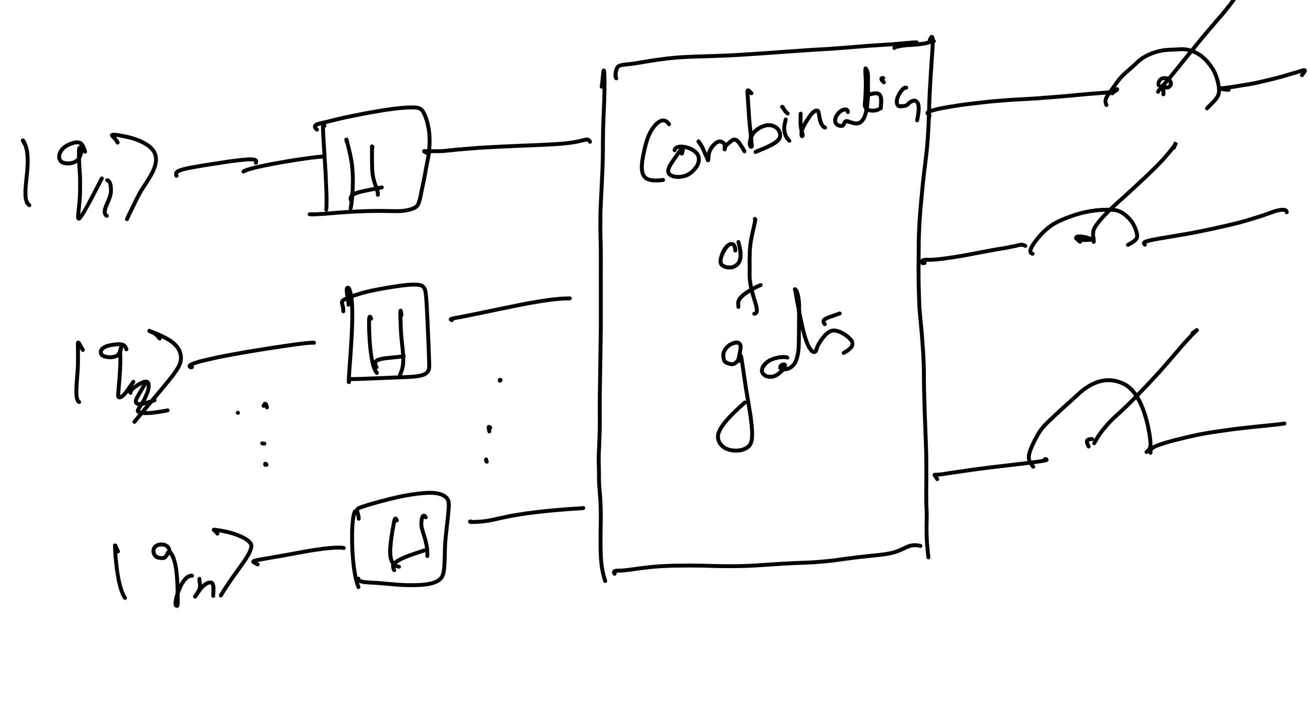
Key ideas:

Quantum Algo construction:

Unique properties of quantum mechanics : Superposition, entanglement, measurements



Perform transformation of superposed states and use entanglement / interference effects to bias correct answers and diminish incorrect ones!

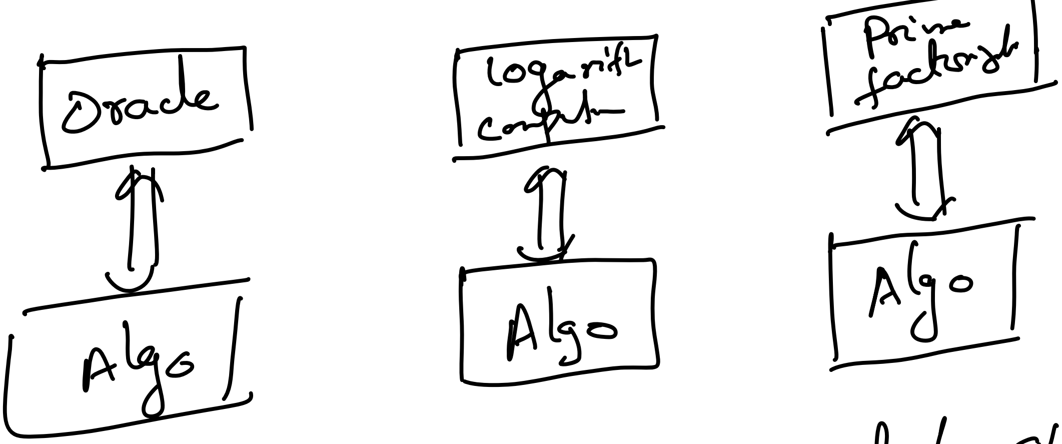
Complexity of Algorithms

* Computational complexity :- Measure computational time taken by the algo

* Space complexity :- Memory space / memory used by algorithm

* Query complexity :-

Oracle :- A black box which provides an answer to the problem of choice



Time taken by main module of algo + number of calls to the oracle. } Time

Examples :-

input $(x_1, x_2, \dots x_N)$

Task of Algo $\rightarrow$ $f(x_1, x_2, \dots x_N)$ with

x_i accessible via queries to black box $\rightarrow$ i $\downarrow$ output x_i

$x_1, x_2, \ldots x_N \in \{0, 1\}$

My task is to determine whether there exist i such that $x_i = 1$

$i \ \boxed{\text{oracle}} \ x_i = 0$ or 1

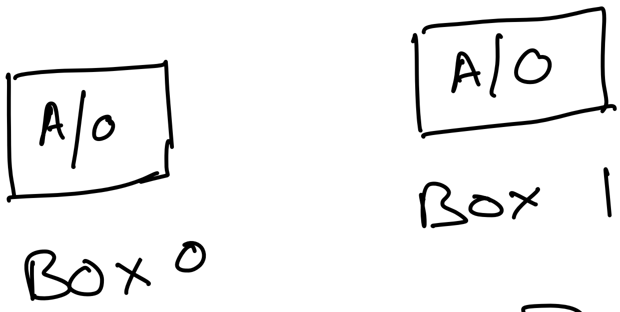
Deutsch and Jozsa Algo

<u>Goal</u>: To create Seperation b/w power of classical and quantum computer

<u>Result</u> :- A problem which for all inputs, quantum computer can solve the problem at hand with certainty in polynomial time but a classical computer takes exponential time to solve with certainty.

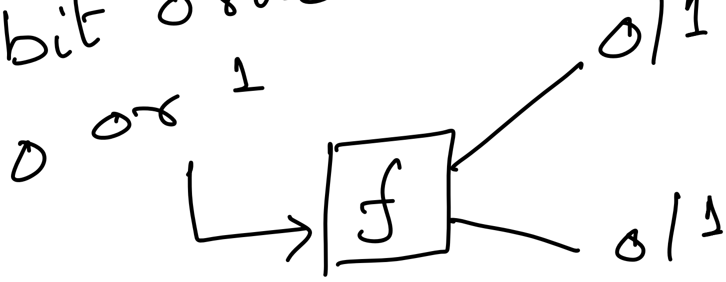
Problem statement :-

A → Apple
O → orange



Task at hand :- Do the two boxes have same kind of fruit or a different kinds of fruits

1 bit oracle



Problem statement :-

* Oracle or black box access to a function f

Input :- 1 bit
output :- single bit either 0 or 1

* $f(x)$ is either constant or balanced

<u>Task</u>: Figure out f is constant or balanced

<u>Constant</u>: Same output for all inputs

<u>balanced</u>: half inputs you get an output 0 and other half output 1

<u>Generalize</u> <u>the</u> <u>above</u>:-

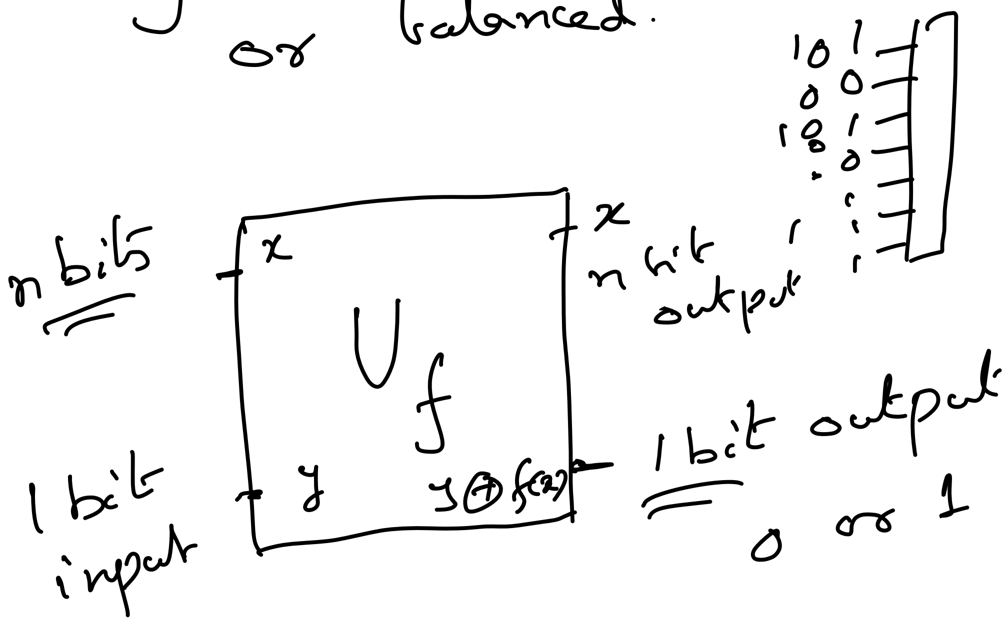
* Oracle or black box access to a function f

- Input : n bits ✓
* output : single bit either 0 or 1

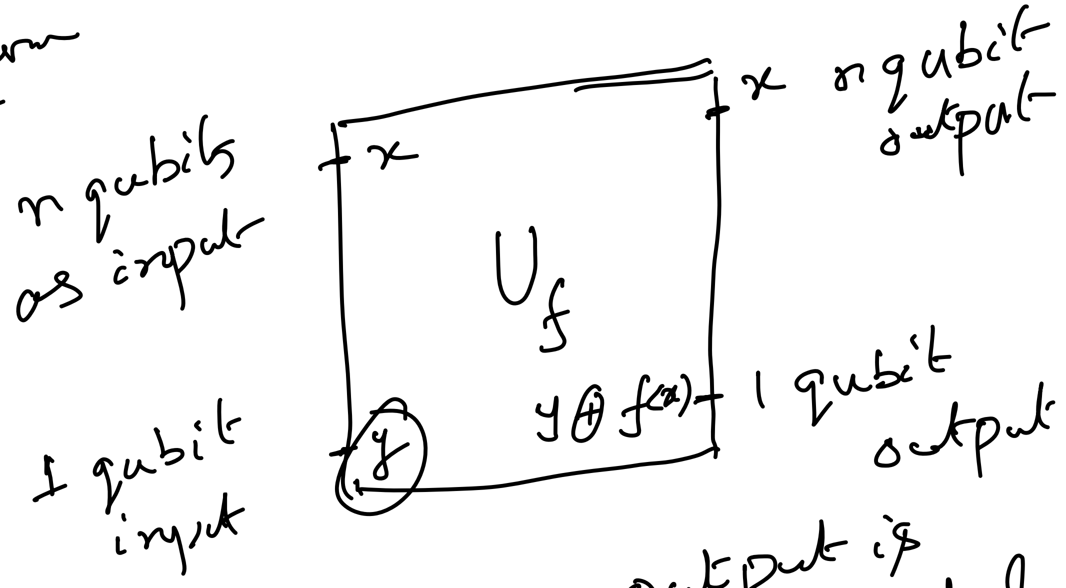
→ $f(x)$ is either constant or balanced

Task:- Figure out f is constant or balanced.

Classical



Quantum



output is embedded

$y = 0$ or 1 $\quad y \oplus f(x) = 1$

$y = 0 \quad f(x) = 1$

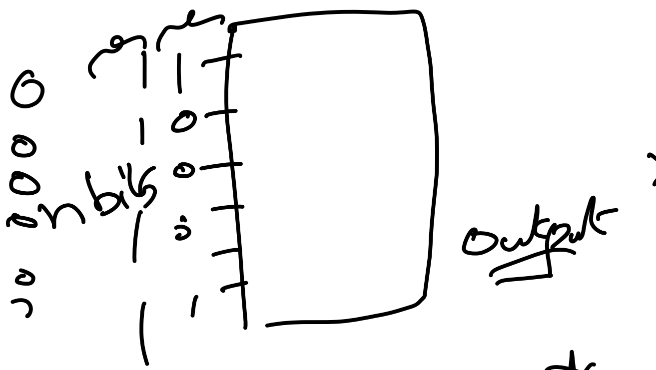
$y \oplus f(x) = 0$

$f(x) = 0$

Classical algorithm

* Input space → 2^n possible inputs

$$\boxed{2^n = N}$$



* How many Queries to my oracle one needs to make?

Query complexity $\frac{2^n}{2} + 1$

$= 2^{n-1} + 1$

is exponential in Length 'n' classically

Quantum Algo :-

Solve this problem with polynomial time.

Problem Simplified

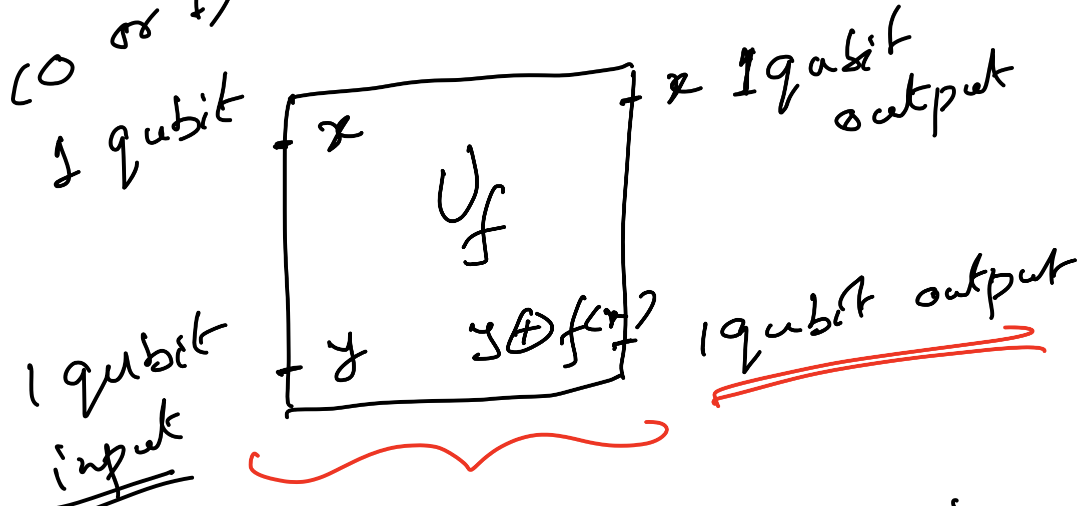
We have two boxes each of them may contain either an apple or an orange.

Task at hand : To answer whether two boxes contain same fruit or not

Input : 1 bit (0 or 1)

output :- Single bit 0 or 1

<u>oracle</u> :- function f is constant if $f(0) = f(1)$ else f is balanced if $f(0) \neq f(1)$



<u>Classical setting</u> : I need 2 queries to the oracle

$$U_f\,|\underline{x}, \underline{\underline{y}}\rangle = \underline{\underline{|x, y \oplus f(x)\rangle}}$$

<u>Classical Algo</u> $\qquad \underline{y = 0}$

$\underline{x = 0 \text{ or } x = 1}$

$x = 0$

$$U_f|0,0\rangle = |0, \underline{0} \oplus f(0)\rangle$$

If $f(0) = 0 \Rightarrow U_f|0,0\rangle = |0,0\rangle$

If $f(0)=1 \Rightarrow U_f|0,0\rangle = |0,1\rangle$

$$\boxed{U_f|0,0\rangle = |0, f(0)\rangle}$$

$x=1$

$$\boxed{\begin{aligned} U_f|1,0\rangle &= |1, 0 \oplus f(1)\rangle \\ &= |1, f(1)\rangle \end{aligned}}$$

2 queries we can measure the outputs $f(0), f(1)$ and compare whether $f(0)=f(1)$ or not

$$U_f|x,y\rangle = |x, y \oplus f(x)\rangle$$

$$|y\rangle = |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$U_f|x,-\rangle = U_f|x, \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\rangle$$

$$\boxed{U_f|x,-\rangle = \frac{1}{\sqrt{2}}\left(U_f|x,0\rangle - U_f|x,1\rangle\right)}$$

$$= \frac{1}{\sqrt{2}}\left(|x, \underbrace{0 \oplus f(x)}\rangle - |x, 1 \oplus f(x)\rangle\right)$$

If $f(x) = 0$

$$U_f|x,-\rangle = \frac{1}{\sqrt{2}}\left(|x,0\rangle - |x,1\rangle\right)$$

If $f(x) = 1$

$$U_f|x,-\rangle = \frac{1}{\sqrt{2}}\left(|x,1\rangle - |x,0\rangle\right)$$

$$\boxed{U_f|x,-\rangle = (-1)^{f(x)}|x,-\rangle}$$