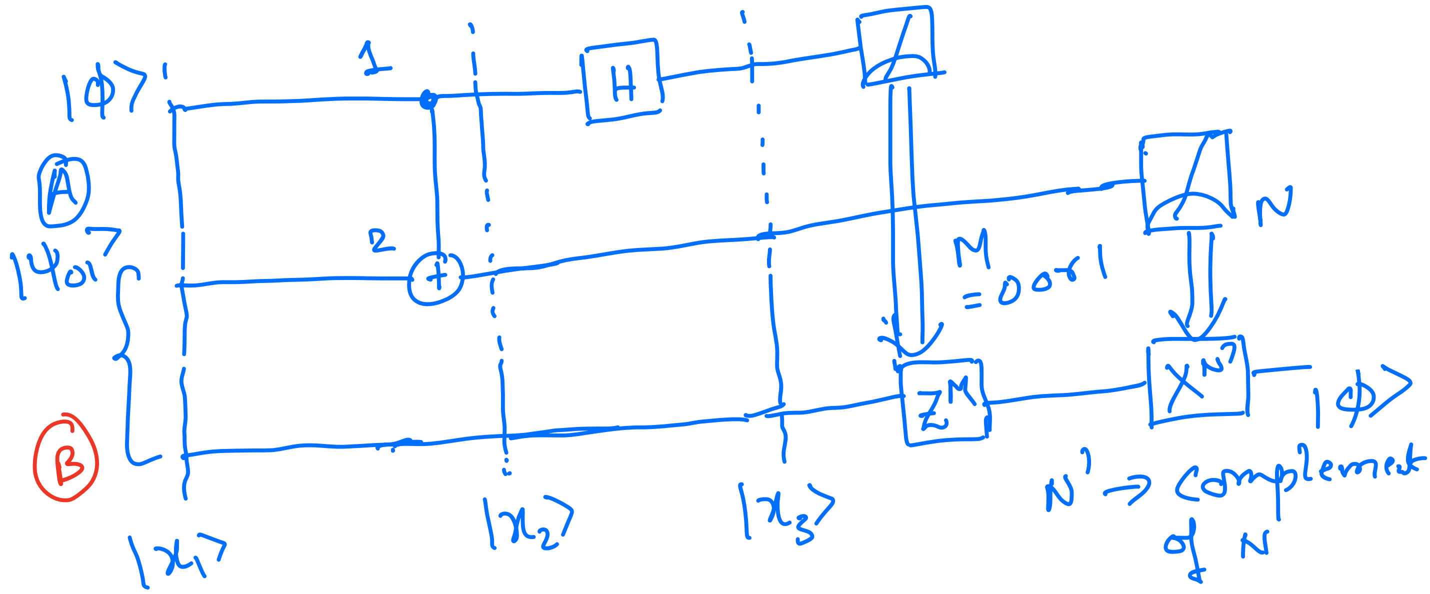
$|\psi_{01}\rangle$, $|\psi_{10}\rangle$ and $|\psi_{11}\rangle$

Exercise?



$$|x_1\rangle = |\phi\rangle \otimes |\psi_{01}\rangle$$

$$= (\alpha|0\rangle + \beta|1\rangle) \otimes \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|x_1\rangle = \frac{1}{\sqrt{2}}\left\{\alpha|0\rangle \otimes (|01\rangle + |10\rangle) + \beta|1\rangle \otimes (|01\rangle + |10\rangle)\right\}$$

$$\Downarrow \text{CNOT}$$

$$|x_2\rangle = \frac{1}{\sqrt{2}}\left\{\alpha|0\rangle \otimes (|01\rangle + |10\rangle) + \beta|1\rangle \otimes (|11\rangle + |00\rangle)\right\}$$

$$\Downarrow H$$

$$|x_3\rangle = \frac{1}{\sqrt{2}}\left\{\alpha|+\rangle[|01\rangle + |10\rangle] + \beta|-\rangle(|11\rangle + |00\rangle)\right\}$$

$$|\chi_3\rangle = \frac{1}{\sqrt{2}}\Big\{\alpha \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)(|01\rangle+|10\rangle) + \beta\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)(|11\rangle+|00\rangle)\Big\}$$

$$= \frac{1}{2}\Big\{\alpha\big[|001\rangle+|010\rangle+|101\rangle+|110\rangle\big] + \beta\big[|011\rangle+|000\rangle-|111\rangle-|100\rangle\big]\Big\}$$

A B

$$= \frac{1}{2}\Big\{|00\rangle(\alpha|1\rangle+\beta|0\rangle) + |01\rangle(\alpha|0\rangle+\beta|1\rangle) + |10\rangle(\alpha|1\rangle-\beta|0\rangle) + |11\rangle(\alpha|0\rangle-\beta|1\rangle)\Big\}$$

$$|\phi\rangle = \alpha|0\rangle+\beta|1\rangle$$

Alice's Measurement	Bob's state $\lvert\phi'\rangle$
0 0 $\rightarrow$	$\alpha\lvert 1\rangle+\beta\lvert 0\rangle$
0 1 $\rightarrow$	$\alpha\lvert 0\rangle+\beta\lvert 1\rangle$
1 0 $\rightarrow$	$\alpha\lvert 1\rangle-\beta\lvert 0\rangle$
1 1 $\rightarrow$	$\alpha\lvert 0\rangle-\beta\lvert 1\rangle$

0 0 $\rightarrow$ $|\phi'\rangle$ — $\boxed{Z^0}$ — $\boxed{X^1}$ — $|\phi\rangle$

M N

Bit-flip

0 1 $\rightarrow$ $|\phi'\rangle$ — $\boxed{Z^0}$ — $\boxed{X^0}$ — $|\phi\rangle$

M N

Do-nothing

1 0 $\rightarrow$ $|\phi'\rangle$ — $\boxed{Z^1}$ — $\boxed{X^1}$ — $|\phi\rangle$

M N

Bit flip and phase flip

1 1 $\rightarrow$ $|\phi'\rangle$ — $\boxed{Z^1}$ — $\boxed{X^0}$ — $|\phi\rangle$

M N

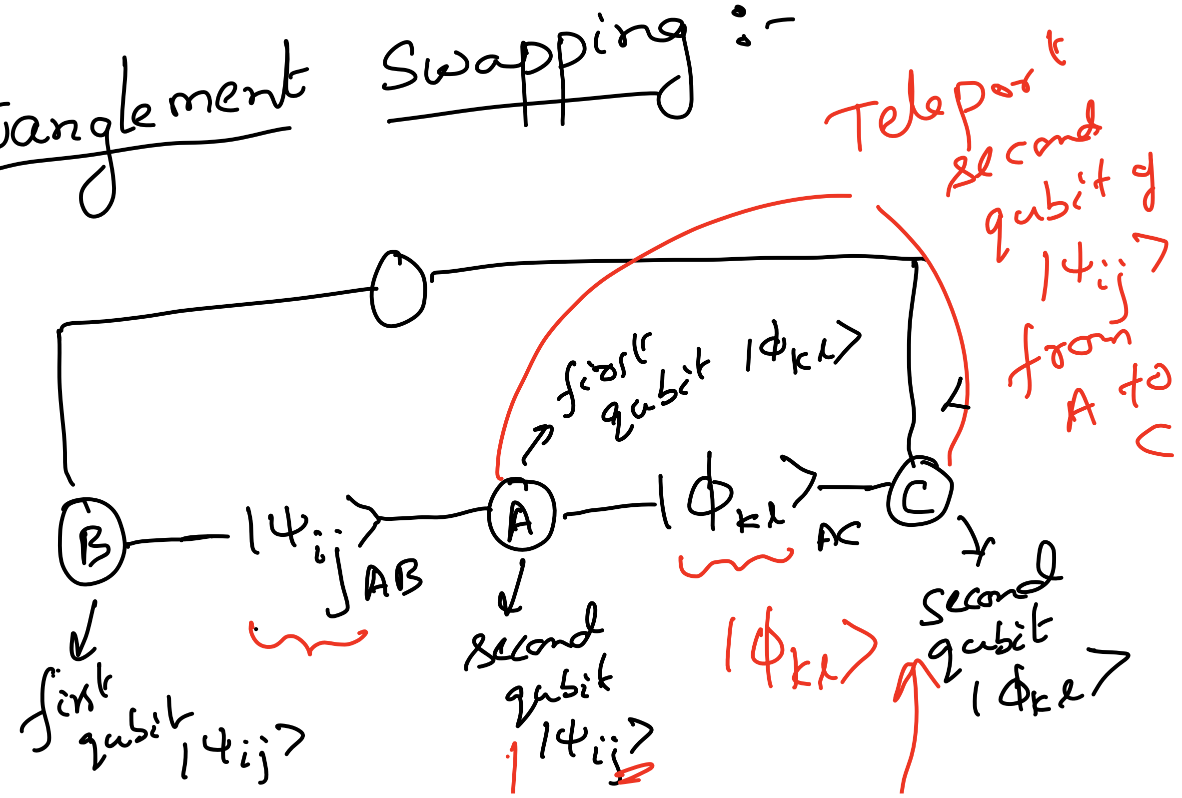
Phase flip

$\boxed{Z^M}$ $\boxed{X^{N'}}$

N' is complement of N

$\rightarrow$ Teleportation protocol has been experimentally realized by chinese where they teleported qubit b/w ground and satellite ($\sim$1400 kms)

Entanglement swapping :-

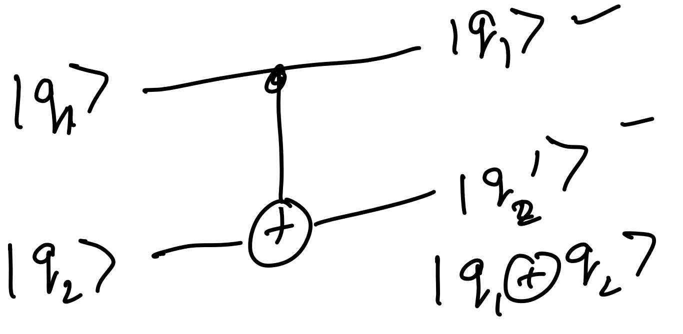


Entanglement Swapping
was demonstrated between two of
Canary Islands (~143 km)

<u>No-cloning theorem</u> :-

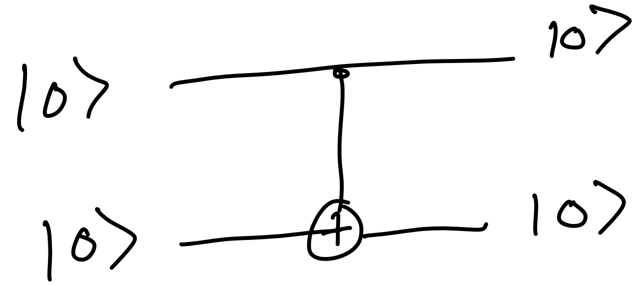
It is not possible to build a quantum circuit to make a copy of an arbitrary qubit
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

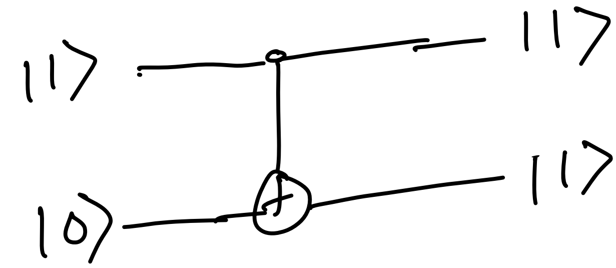
"There is no unitary transformation that can produce two copies of an arbitrary $|\psi\rangle$"



should be able to
$|q_2'\rangle = |q_1\rangle$

$|q_1\rangle$ is $|0\rangle$ or $|1\rangle$





Not possible for any arbitrary $|q_1\rangle$

Quantum Teleportation also does not violate no-cloning theorem.

(This is not a copy but just a transfer)

Eg:-

$$|\psi\rangle \otimes |s\rangle \xrightarrow{U} U(|\psi\rangle \otimes |s\rangle)$$

Let us assume there exists a transformation which violates no-cloning theorem

This means

$$\underbrace{U(|\psi\rangle \otimes |s\rangle)} = \underbrace{|\psi\rangle \otimes |\psi\rangle} \quad -(1)$$

$$\underbrace{U(|\phi\rangle \otimes |s\rangle)} = \underbrace{|\phi\rangle \otimes |\phi\rangle} \quad -(2)$$

$$|\psi\rangle \neq |\phi\rangle$$

Take inner product of L.H.S and R.H.S of (1) and (2)

LHS

$$(\langle\psi| \otimes \langle s|)\underbrace{U^{\dagger}U}_{\to I}(|\phi\rangle \otimes |s\rangle)$$

(Exercise)

$$= \langle\psi|\phi\rangle\langle s|s\rangle$$

$$= \langle\psi|\phi\rangle$$

RHS

$$(\langle\psi| \otimes \langle\psi|)(|\phi\rangle \otimes |\phi\rangle)$$

$$= |\langle\psi|\phi\rangle|^2$$

$$\Rightarrow \quad \langle\psi|\phi\rangle = (\langle\psi|\phi\rangle)^2$$

$x = x^2$

$x = 1$ or $x = 0$

This is true only if

$$\langle \psi | \phi \rangle = 1 \quad \text{or} \quad \langle \psi | \phi \rangle = 0$$

$$\Rightarrow \quad |\psi\rangle = |\phi\rangle \quad (\text{or})$$

$|\psi\rangle$ and $|\phi\rangle$ are orthogonal