(I) Noisy quantum Devices

Qubits → Relaxation and dephasing on timescales
* T_1 and T_2 (Scale of 10^2 to 10^3 μs) Superconducting qubits)

Single qubit gates

duration $T_{1q} \simeq 20ns$

Two qubit operations and measurements have duration $T_{2q} \simeq 200ns$

$T_m \simeq 700ns$

* Qubit coherence:
Defines timescales over which quantum information is lost

* Gate times and errors define the duration and accuracy of a computation

* Duration of computation should not exceed qubit coherence time

* Number of gates should be such that accumulation of error does not prevent algorithms from

yielding accurate results.

* Width and depth are often used to characterize the cost of a quantum circuit.

→ Width : Number of qubits that comprise the circuit. (N_q)

→ Depth : Maximal length d of a path from the input (qubit initialization) to the output (measurement operation)

* Important computational bottleneck when the physical duration of a circuit, roughly is comparable with coherence time of a qubit.

<u>Problem</u> <u>at</u> <u>hand</u> :-

$$\left[-\frac{1}{2}\nabla^2 + V(x)\right]\psi_i = \varepsilon_i \psi_i$$

$$H|\psi_i\rangle = \varepsilon_i|\psi_i\rangle$$

Finite difference
⇓
Sparse matrix

Quantum Subspace methods

* Construct a variational subspace spanned by a set of states $\{|v_\alpha\rangle\}_{\alpha=0}^{n-1}$

* $S_{\alpha\beta} = \langle v_\alpha | v_\beta \rangle$

  $H_{\alpha\beta} = \langle v_\alpha | \hat{H} | v_\beta \rangle$

$$HC = SC\tilde{E} \quad -(1)$$

* Quantum Device computes $H_{\alpha\beta}$ and $S_{\alpha\beta}$
* classical computer solves (1) to obtain approximate eigen pairs $\{\tilde{E}_\mu^{(n)}, |\tilde{\Psi}_\mu^{(n)}\rangle\}$

* QSE does not require nonlinear parameter optimization like what we have VQE

* Each update of quantum circuit parameters requires a new call of the quantum computer in VQE

* In QSE all circuits required to measure $H_{\alpha\beta}$ and $S_{\alpha\beta}$ can be sent to quantum computer in one call.

* QSE circuits can be naturally parallelised over multiple quantum computers (groups of qubits within a quantum computer)

* Eigenstats are never stored on a quantum devices.

Choice of basis $\{|v_\alpha\rangle\}_{\alpha=0}^{n-1}$

(a) Chebyshev quantum Krylov:-

* Quantum implementation of classical Krylov space.
* No algorithmic error in the construction of Krylov Subspace (No Trotterization and other approximate time evolution algos)
* Generating Powers of Hamiltonian is mathematically equivalent to Subspace generated by any basis for polynomials of Hamiltonian $|V_\alpha\rangle = T_\alpha(H)|V_0\rangle$
* Block encoding of Hamiltonian $\hat{H}$ is key idea

* This requires the use of ancillas and deep quantum circuits!

(b) <u>Gaussian-power quantum Krylov:</u>

* Subspace is constructed using conventional power function acting on the Hamiltonian. Starting state $|V_0\rangle$ is replaced with

$$e^{-\frac{1}{2}(H-E_0)^2\tau^2}|V_0\rangle$$

$$|V_\alpha\rangle = (\hat{H}-E_0)^\alpha e^{-\frac{1}{2}(H-E_0)^2 \tau^2}|V_0\rangle$$

$$\alpha = 0 \ldots n-1$$

(Realized by Linear combination of unitaries LCU)

Time evolution methods

1. Quantum Filter Diagonalization
2. Quantum Lanczos based on imaginary time evolution