Application of SVD for Classifying Handwritten Digits

- Kunal Panjwani - 202018001
- Vanditha Vinod - 202018003
- Nihar Shah - 202018014
- Yagn Purohit - 202018035

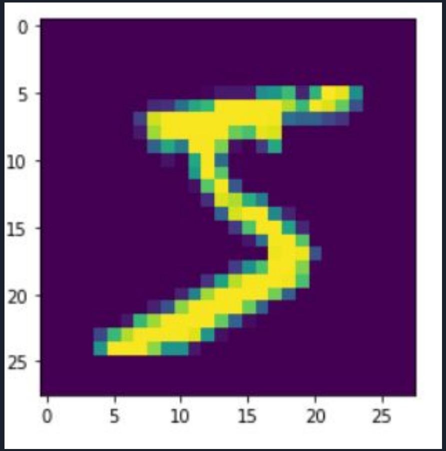
Introduction:

In the last 50 years, there has been a rapid growth of advanced technology. Transcribing an audio/video, recognizing handwritten texts are a very few of the many things we can now do digitally. Our project, recognizing handwritten digits, is a classification problem. However, we have performed dimension reduction on images of handwritten digits using SVD Approximation Theorem. We have further used Least Square Method to classify these images in the respective labels (i.e. digits).

Dataset:

We have used the MNIST Dataset that consists of 70,000 images of handwritten digits out of which 60,000 images are for training and 10,000 are for testing. Each image is of 28x28 pixels.

There are labels corresponding to each image (i.e. 0-9). We need to predict these labels for the testing images.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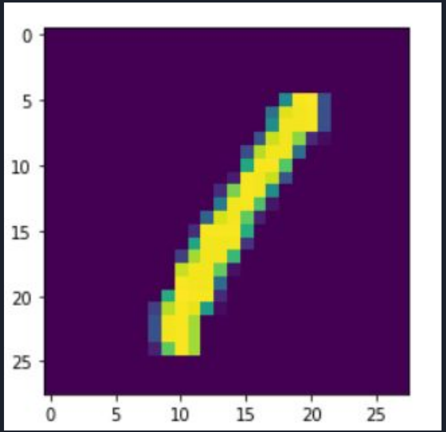
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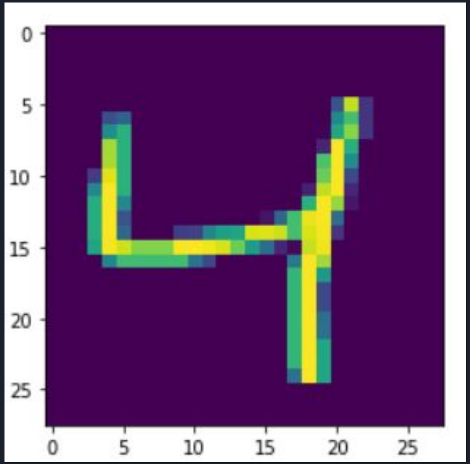
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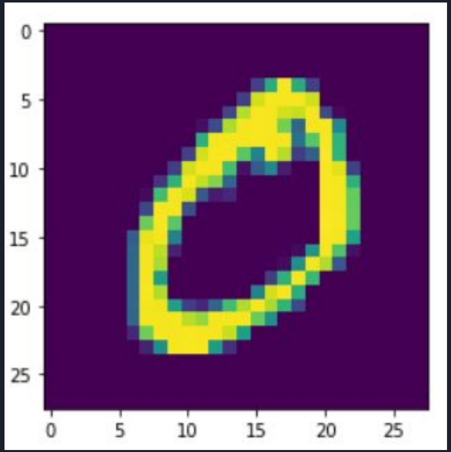
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Important Definitions & Theorems

1. Singular Value Decomposition (SVD)

Singular value decomposition is a method of decomposing a matrix into three other matrices:

$$A = USV^T$$

Where:

- A is an $m \times n$ matrix
- U is an $m \times n$ *unitary* matrix
- S is an $n \times n$ *diagonal matrix*
- V is an $n \times n$ *unitary* matrix

2. SVD Approximation Theorem

"Let $A \in R^{m \times n}$ be a non-zero matrix with rank r. Let $\sigma_1, \ldots, \sigma_r$ be the singular values of A, with associated left and right singular vectors $u_1, \ldots, u_r$ and $v_1, \ldots, v_r$ respectively, and let $k \leq r$. Then $A = U\Sigma V^T = \Sigma^r_{j=1} \sigma_j u_j v_j^T$, and $A_k = \Sigma^k_{j=1} \sigma_j u_j v_j^T$ is the best rank k approximation for A under the 2-norm."

Corollary-1:

"Let $A \in R^{m \times n}$ be a nonzero matrix of rank r with singular value decomposition $A = U\Sigma V^T$. Then the first $k < r$ columns $u_1, \ldots, u_k$ of U form an orthogonal basis for the column space of A_k. Furthermore, $U_k = [u_1\, u_2 \ldots u_k]$ implies $U_k^T U_k = I$."

Corollary-2:

“Let $A \in R^{m\times n}$ *be a nonzero matrix of rank r with a rank k approximation* A_k*. The least squares problem* $\min_x ||U_k x - d||_2^2$ *has the solution* $x = U_k^T d$ *with residual* $||U_k U_k^T d - d||_2^2$*.”*

Methods:

- We first converted each image into a 28x28 matrix. We then reshaped this matrix into a 784x1 column vector.
- We then classified all the training vectors, with respect to their labels, into 10 different matrices Di (i = 0, 1, ..., 9).
- We performed SVD on all these matrices.

- Then we applied SVD approximation theorem to reduce the dimension of each image.
- On obtaining k value for each Di, we verified corollary-1, i.e., $U_k^T U_k = I$.
- Now, for each test image (taken as d), $qi = ||U_{ik} U_{ik}^T d - d||_2^2$ is calculated for all i=0,1,...,9 and $\min_i(q_i)$ is computed and the test image d is classified as i.

Results and conclusions:

- Unfortunately the results we obtained were not as per our expectations. Only 4 and 9 were correctly classified.
- The accuracy obtained in our project is 10.73%.
- The future scope of our project is to improve the accuracy.

References:

- Algorithms for Handwritten Digit Recognition by Michael J. M. Mazack and Adviser: Dr. Tjalling Ypma.
- Understanding Singular Value Decomposition and its Application in Data Science - (Towards Data Science).
- MNIST Dataset.

Link to the code.

Thank You!