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CO602 - Wireless Network & Mobile Computing

Question :- Considering a maximum distance of r from base station to furthest device for four geometries (a) circle (b) rectangle (c) triangle (d) hexagon evaluate the coverage area, overlapping area & deadzone area. Which cell shape is preferred as a part of conclusion?

Circle

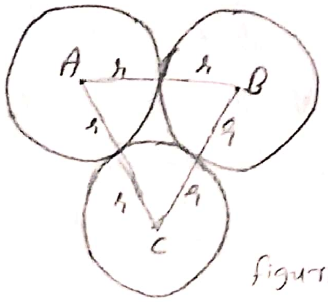

figure 1

Coverage area for one circle will be πr^2 if radius of circle is r.

Dead zone : As shown in figure 1 assume 3 circles with centre A, B, C. now triangle connecting this 3 circle is assume ΔABC. Sides of this circle $\overline{AB} = BC = AC = 2r$.

So, we can say that ΔABC is equilateral tringle & $\angle A = \angle B = \angle C = 60^\circ$

so, for Area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$

$\therefore$ for $\angle A$ Area of Sector $= \frac{60}{360} \times \pi r^2 = \frac{\pi r^2}{6}$

now, area of $\Delta ABC = \frac{\sqrt{3}}{4}(2r)^2 = \sqrt{3} r^2$

So, area of dead zone = area of ΔABC - 3 $\times$ area of sectors

$= \sqrt{3} r^2 - \frac{\pi}{2} r^2$

$= r^2 \left(\sqrt{3} - \frac{\pi}{2}\right)$

rectangle

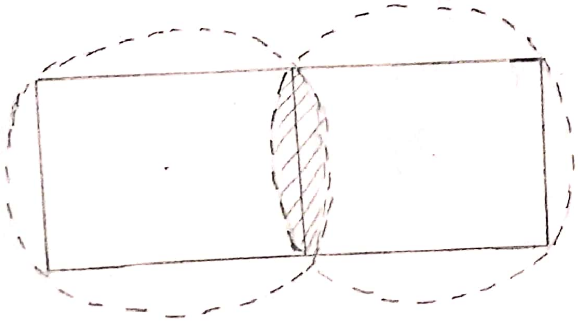

figure 2.1

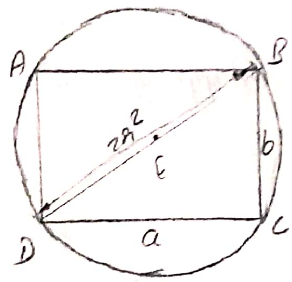

figure 2.2

as shown in figure 2.2
Assume rectangle $\square ABCD$ in the circle with center E. Sides of rectangle are a, b, a & b So area of rectangle will be

Area of $\square ABCD$ = $a \times b$ — (i)

now, ~~area of for~~ ~~Δ BCD will~~

now, for ΔBCD we can say that

$$a^2+b^2 = (2r)^2$$
$$a^2+b^2 = 4r^2 \quad \text{— (ii)}$$

& area of $\square ABCD$ will be from (i) & (ii) $A = a\sqrt{4r^2-a^2}$ — (iii)

now, we have to maximize area of $\square ABCD$ into circle so we differentiate (iii) withe respecte to a

$$\frac{dA}{da} = \frac{-2a^2}{2\sqrt{4r^2-a^2}} + \sqrt{4r^2-a^2}$$

now, taking $\frac{dA}{da} = 0$

$$0 = -\frac{a^2}{\sqrt{4r^2-a^2}} + \sqrt{4r^2-a^2}$$

$$a^2 = 4r^2-a^2$$
$$a^2 = 2r^2$$
$$\therefore a = \sqrt{2}r$$

from (ii) putting $a=\sqrt{2}r$

$$b^2 = 2r^2$$
$$\therefore b = \sqrt{2}r$$

so, area of rectangle $\square ABCD = 2r^2$

Coverage area of two rectange = 2 X area of rect

$= 2 \times 2r^2 = 4r^2$

overlapping area for single side of rectangle $= \frac{1}{4}$ (area of circle - area of rectangle)

$$= \frac{1}{4}(\pi r^2 - 2r^2)$$

$$= \frac{r^2}{4}(\pi - 2)$$

So, final overlapping area = 2X overlapping area

$$= 2 \times \frac{r^2}{4}(\pi-2) = \frac{\pi r^2}{2} - r^2$$

triangle

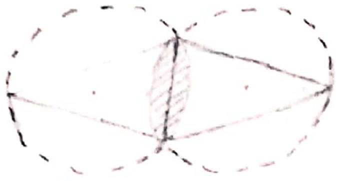

figure 3.1

figure 3.2

As shown in figure 3.2 assume ΔPQR in circle with center S. maximum area covered by inscribe triangle in circle is equilateral triangle.

$\angle R = 60^\circ$, $\angle T = 90^\circ$

so, in triangle ΔSTR $\angle S = 60^\circ$, $\angle T = 90^\circ$, $\angle R = 30^\circ$

assume $\overline{QR} = x$

so, $\cos C = \frac{RT}{RS}$

$\cos 30 = \frac{x}{2r}$

$\therefore x = \sqrt{3}r$

so area of $\Delta PQR = \frac{\sqrt{3}}{4} \times x^2$

$= \frac{\sqrt{3}}{4} (\sqrt{3}r)^2$

$= \frac{3\sqrt{3}}{4} r^2$

Coverage area $= 2 \times \frac{3\sqrt{3}}{4} r^2$ (as shown is figure 3.1)

$= \frac{3\sqrt{3}}{2} r^2$

overlapping area over one side of triangle =

$= \frac{1}{3}$(area of circle - area of triangle)

$= \frac{1}{3}\left(\pi r^2 - \frac{3\sqrt{3}}{2} r^2\right) = \frac{r^2}{3}\left(\pi - \frac{3\sqrt{3}}{2}\right)$ — (1)

Total overlapping area $= 2 \times$ (1)

$= \frac{2r^2}{3}\left(\pi - \frac{3\sqrt{3}}{2}\right)$

Hexagon

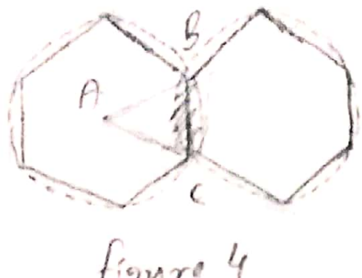

figure 4

As shown in figure 4 assume triangle ΔABC. ΔABC is equilateral triangle.

area of ΔABC $A_1 = \frac{\sqrt{3}}{4} r^2$ - (i)

area of Hexagon = 6× area of triangle

$= 6 \times A = 6 \frac{3\sqrt{3}}{2} r^2$ - (ii)

Coverage area for figure 2 $= 2 \times \frac{3\sqrt{3}}{2} r^2 = 3\sqrt{3} r^2$

Overlapping area for one side of Hexagon $= \frac{1}{6}$ (area of Circle - area of Hexagon)

$= \frac{1}{6} \left(\pi r^2 - \frac{3\sqrt{3}}{2} r^2 \right)$

$= \frac{1}{6} \left(\pi r^2 - \frac{3\sqrt{3}}{2} r^2 \right)$ - (iii)

final overlapping area = 2× overlapping area (iii)

$= 2 \times \frac{1}{6} \left(\pi r^2 - \frac{3\sqrt{3}}{2} r^2 \right)$

$= \frac{1}{3} \left(\pi r^2 - \frac{3\sqrt{3}}{2} r^2 \right)$

$= \frac{\pi r^2}{3} - \frac{\sqrt{3}}{2} r^2$