Soft Computing: Fuzzy Logic

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- FL is one of the tools used to model a multiinput, multioutput system.

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- FL concepts consist of
  - Fuzzy sets and their properties
  - FL operators
  - Fuzzy proposition and rule-based systems
  - Fuzzy maps and inference engine
  - Defuzzification methods
  - the design of an FL decision system

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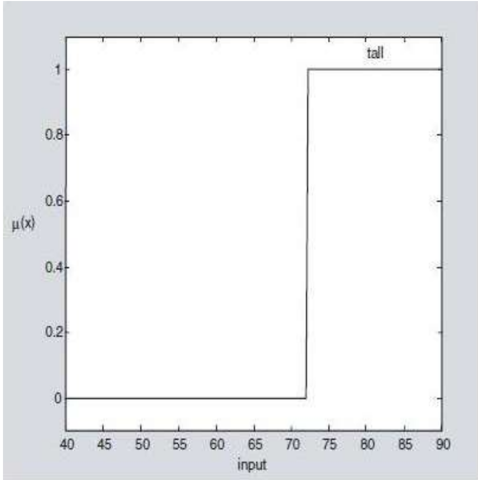
- Fuzzy set theory extends this concept by defining partial membership
- A fuzzy set A on a universe of discourse U is characterized by a membership function $\mu_A(x)$ that takes values in the interval $[0, 1]$

Example: Crisp input



Crisp membership function (src:www)

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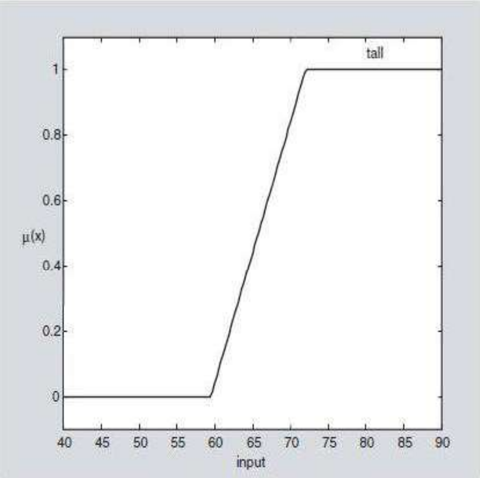
$$A = \{(x, \mu_A(x))\,|x \in U\}$$

x is called support value if $\mu_A(x) > 0$
- $\{(\frac{\mu_A(x_i)}{x_i}), i = 1, 2, \ldots, n\}$ $\frac{\mu_A(x_i)}{x_i}$ fuzzy singleton

Example: Fuzzy input



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- x is height then $T(x)$ may refer to sets such as short, medium or tall
- A membership function is essentially a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1.

Membership functions

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- triangular curves depend on three parameters a, b, and c and are given by

$$f(x; a, b, c) = \begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } a \le x < b \\ \frac{c-x}{c-b} & \text{for } b \le x \le c \\ 0 & \text{for } x > c \end{cases}$$

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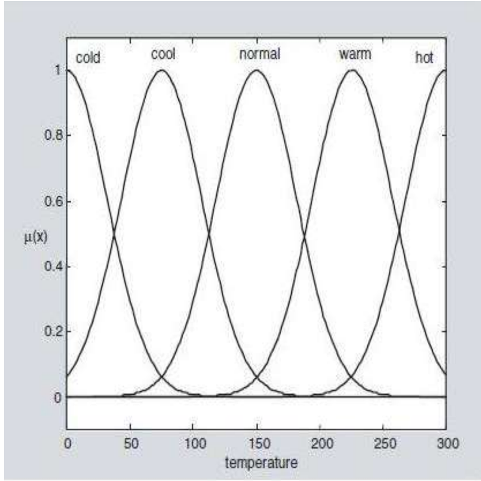
- Gaussian shaped membership function

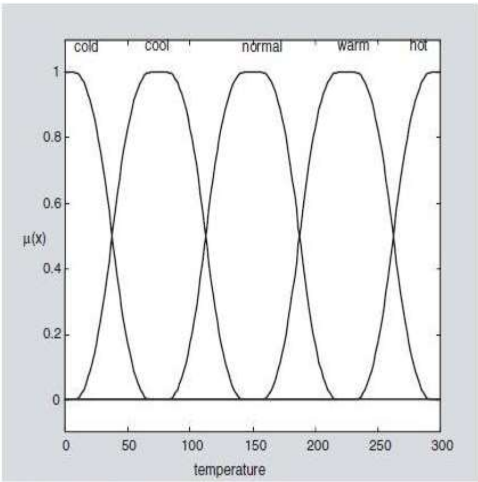
$$f(x;\sigma,c) = \exp\left[\frac{-(x-c)^2}{2\sigma^2}\right]$$

Membership functions



Gaussian membership functions (src:www)

Membership functions



π shaped membership functions (src:www)

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$$f(x;b,c) = \begin{cases} S(x;c-b,c-b/2,c) & \text{for } x \le c \\ 1 - S(x;c,c+b/2,c+b) & \text{for } x > c \end{cases}$$

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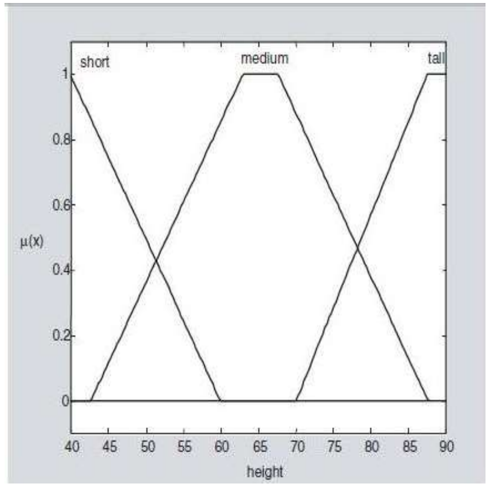
$S(x;a,b,c)$ represents a membership function defined as

$$S(x;a,b,c) = \begin{cases} 0 & \text{for } x < a \\ \frac{2(x-a)^2}{(c-a)^2} & \text{for } a \le x < b \\ 1 - \frac{2(x-c)^2}{(c-a)^2} & \text{for } b \le x \le c \\ 1 & \text{for } x > c \end{cases}$$

The parameter b is the half width of the curve at the crossover point.

Membership functions



Trapezoidal membership functions (src:www)

Membership functions

- Trapezoidal curves depend on four parameters and are given by

$$f(x;a,b,c,d) = \begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } a \le x < b \\ 1 & \text{for } b \le x < c \\ \frac{d-x}{d-c} & \text{for } c \le x < d \\ 0 & \text{for } d \le x \end{cases}$$

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- In designing a fuzzy inference system, membership functions are associated with term sets that appear in the antecedent or consequent of rules.

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- crisp set operations: union, intersection, and complement
- correspond to OR, AND, and NOT operators
- A and B be two subsets of U

$$
\begin{aligned}
\mu_{A\cup B}(x) &= 1 \text{ if } x \in A \text{ or } x \in B \\
\mu_{A\cap B}(x) &= 1 \text{ if } x \in A \text{ and } x \in B \\
\mu_{\bar{A}}(x) &= 1 \text{ if } x \notin A
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$$\mu_{INT(A)}(x) = \begin{cases} 2[\mu_A(x)]^2 & \text{for } 0 \leq \mu_A(x) \leq 0.5 \\ 1 - 2[1 - \mu_A(x)]^2 & \text{otherwise} \end{cases}$$

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- fuzzy modifiers not, very, and more or less
- $\mu_{not}(x) = 1 - \mu(x)$ $\mu_{very}(x) = (\mu(x))^2$ $\mu_{notvery}(x) = 1 - (\mu(x))^2$ $\mu_{moreorless} = \mu(x)^{0.5}$

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- $I(A) = minimum$ iff $\mu_A(x_i) = 0$ *or* $1 \forall i$
- $I(A) = maximum$ iff $\mu_A(x_i) = 0.5 \forall i$
- $I(A) \geq I(A^*)$ A^* sharpened version of A

$$\mu_{A^*}(x_i) \geq \mu_A(x_i) \text{ if } \mu_A(x_i) \geq 0.5$$
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- $I(A) = I(A^c)$
- index of fuzziness of a fuzzy set A having n supporting points is defined $\nu(A) = \frac{2}{n^k} d(A, \underline{A})$

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\nu_l(A) &= \frac{2}{n}\sum_{i=1}^{n} |\mu_A(x_i) - \mu_{\underline{A}}(x_i)| \\
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$$

$$
\nu_q(A) = \frac{2}{\sqrt{n}}\sqrt{\left[\sum_{i=1}^{n} \{\mu_A(x_i) - \mu_{\underline{A}}(x_i)\}^2\right]}
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\begin{aligned}
H(A) &= \frac{1}{n\ln(2)}\sum_{i=1}^{n}\{S_n(\mu_A(x_i))\} \\
S_n(\mu_A(x_i)) &= -\mu_A(x_i)\ln(\mu_A(x_i)) - \{1-\mu_A(x_i)\}\ln\{1-\mu_A(x_i)\}
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H(A) &= \frac{1}{n(\sqrt{e}-1)}\sum_{i=1}^{n}\{S_n(\mu_A(x_i))-1\} \\
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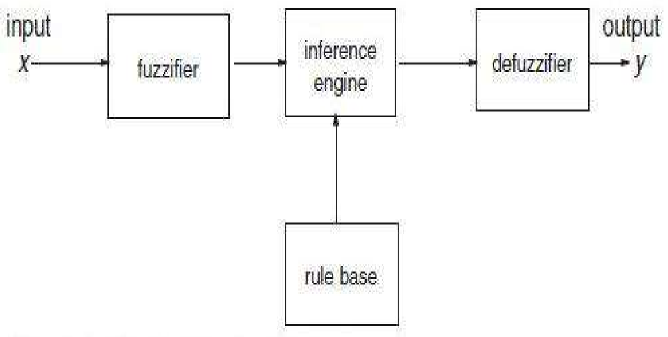
- The rule base contains linguistic rules that are provided by experts.

Fuzzy Inference System



Block diagram of a fuzzy inference system (src:www)

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- Let U and V be two universes of discourse
- A fuzzy relation $R(U, V)$ is a set in the product space $U \times V$ and is characterized by the membership function $\mu_R(x, y)$ where, $x \in U$ and $y \in V$ $\mu_R(x, y) \in [0, 1]$

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- The first step is to evaluate the antecedent, which involves fuzzifying the input
- The second step is implication, or applying the result of the antecedent to the consequent, which essentially evaluates the membership function $\mu_{A \to B}(x, y)$

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- It determines the degree to which the antecedent is satisfied for each rule.
- If the antecedent of a given rule has more than one clause, fuzzy operators are applied to obtain one number that represents the result of the antecedent for that rule.

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- In an FIS, the order in which rules are fired does not affect the output.

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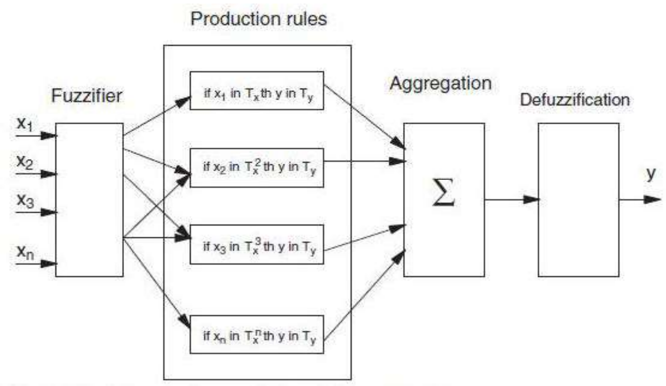
- The most popular defuzzification method is the centroid, which calculates and returns the center of gravity of the aggregated fuzzy set.

Fuzzy Inference System



Schematic diagram of a fuzzy inference system (src:www)

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- Let $\mathbf{x} = (\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_n})^{\mathbf{T}}$ be the input vector and $\mathbf{y} = (\mathbf{y_1}, \mathbf{y_2}, \ldots, \mathbf{y_m})^{\mathbf{T}}$ be the output vector.

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- The linguistic variable x_i in the universe of discourse U is characterized by

$$\mathbf{T}(\mathbf{x}) = \left\{\mathbf{T}_{\mathbf{x}}^{\mathbf{1}}, \mathbf{T}_{\mathbf{x}}^{\mathbf{2}}, \ldots, \mathbf{T}_{\mathbf{x}}^{\mathbf{k}}\right\} \text{ and } \mu(\mathbf{x}) = \{\mu_{\mathbf{x}}^{\mathbf{1}}, \mu_{\mathbf{x}}^{\mathbf{2}}, \ldots, \mu_{\mathbf{x}}^{\mathbf{k}}\}$$

- $\mathbf{T}(\mathbf{x})$ is a term set of $\mathbf{x}$; that is, it is the set of names of linguistic values of $\mathbf{x}$, with each T_x^i being a fuzzy member and the membership function μ_x^i defined on U.

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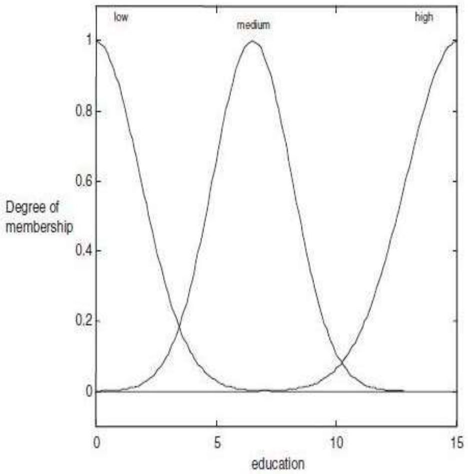
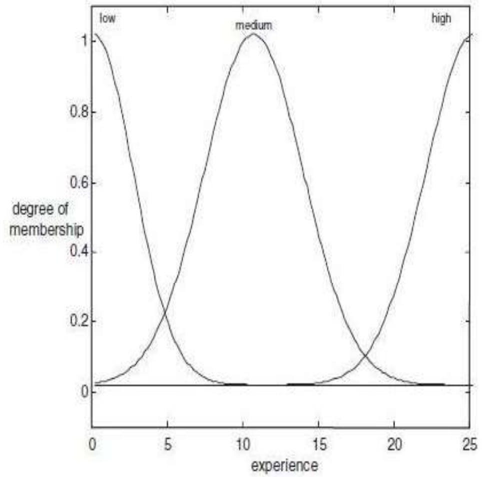
- Let x_2 indicate the number of years of experience, the universe of discourse be $[0 - 30]$, and the corresponding term set be $\{low, medium, high\}$.

FIS Example: Input x_1



Fuzzy membership functions for Education input (src:www)

FIS Example: Input x_2



Fuzzy membership functions for Experience input (src:www)

FIS Example: Output (Salary)

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- Variable y represents salary, then $T(y)$ represents a term set $\{verylow, low, medium, high, veryhigh\}$, and the universe of discourse is $[20 - 200]$, which represents the minimum and maximum in thousands of dollars, that is, 20,000, and 200,000, respectively.

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- In order to map input variables x_1 and x_2 to output y, it is necessary that we first define the corresponding fuzzy sets.

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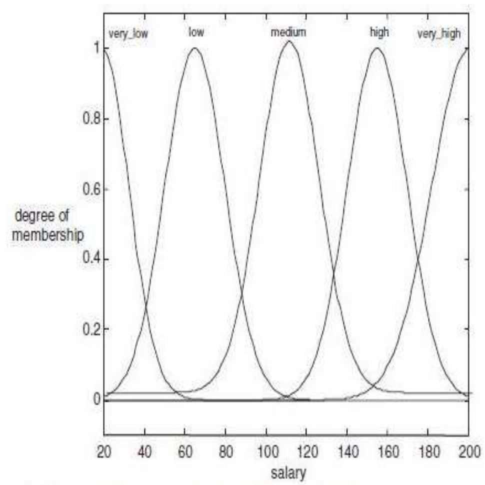

Fuzzy membership function for salary output (src:www)

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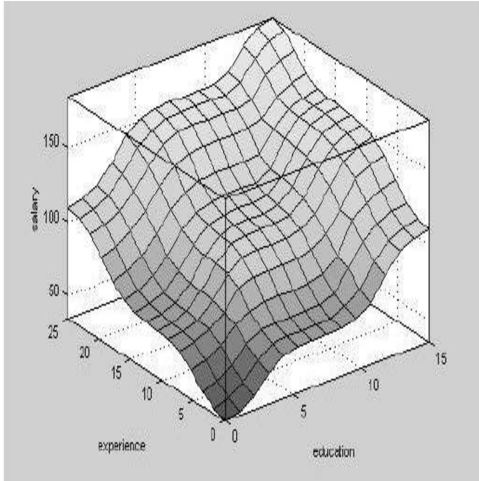
- A fuzzy rule base contains a set of fuzzy rules R. A single if-then rule assumes the form -if x is T_x then y is T_y.
- An example of a rule might be if *education* is *high* and *experience* is *high*, then *salary* is very *high*.

Mapping surface



Example of Mapping surface (salary) (src:www)

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$$\begin{aligned} R &= (R_1, R_2, \ldots, R_n) \\ R_i &= \text{ if } \left(x_1 \text{is } T_{x_1}, \text{and } \ldots, x_p \text{ is } T_{x_p}\right) \text{ then} \\ & \quad \left(y_1 \text{ in } T_{y_1}, \text{ and} \ldots, y_q \text{ in } T_{y_q}\right) \end{aligned}$$

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- If we consider a multiinput, single-output system, then the consequent reduces to $(y_1 \text{ is } T_{y_1})$.

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- R_4: if edu is medium and exp is low, then sal is low
- R_5: if edu is medium and exp is medium, then sal is medium
- R_6: if edu is medium and exp is high, then sal is high
- R_7: if edu is high and exp is low, then sal is medium
- R_8: if edu is high and exp is medium, then sal is high
- R_9: if edu is high and exp is high, then sal is very high

Fuzzy rule

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- For example, consider an ith rule
- R_i: if x_1 is T_{x_i} and x_2 is T_2^i then y is T_y^i

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- The firing strength is then used to shape the output fuzzy set that represents the consequent part of the rule.
- The implication method is defined as the shaping of the consequent (the output fuzzy set), based on the antecedent.

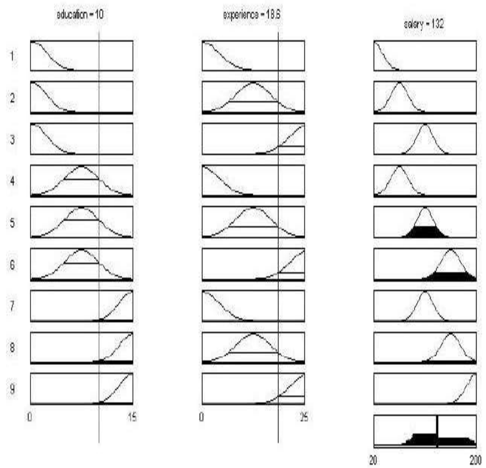
Fuzzy rule

- The input for the implication process is a single number given by the antecedent, and the output is a fuzzy set.
- Two methods are commonly used: the minimum and the product methods, represented, respectively, by

$$
\begin{aligned}
\mu^i_y(w)' &= \min\left(\alpha_i, \mu^i_y(w)\right) \\
\mu^i_y(w)' &= \alpha_i \mu^i_y(w)
\end{aligned}
$$

  w is the variable that represents the support value of the membership function.
- For the given example, if we assume that education equals 10 years and experience equals 18.6 years, then the rules R_5 and R_6 fire.

FIS Example: Fuzzy rule



Fuzzy rule (input: education and experience and output: salary) (src:www)

Fuzzy Inference System: Aggregation

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- If we have two rules with output fuzzy sets represented by two fuzzy sets $\mu_y^1(w)$ and $\mu_y^2(w)$, then, combining the two sets, we obtain the output decision
- $\mu_y(w) = \max\left(\mu_y^1(w)\mu_y^2(w)\right)$
- it is a membership curve

Defuzzification

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- Centroid defuzzification method: the defuzzifier determines the center of gravity (centroid) y_i' of B and uses that value as the output of the FLS.
- For a continuous aggregated fuzzy set, the centroid is given by

$$y' = \frac{\int_s y_i \mu_B(y) dy}{\int_s \mu_B(y) dy}$$

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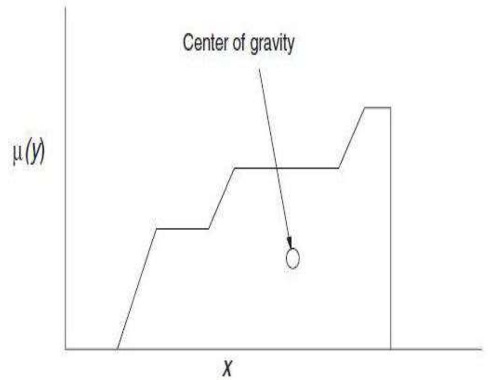
- The centroid defuzzification method finds the 'balance' point of the solution fuzzy region by calculating the weighted mean of the output fuzzy region.

Example: Defuzzification



Centroid defuzzification method (src:www)

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- Maximum-decomposition method: the defuzzifier examines the aggregated fuzzy set and chooses that output y for which $\mu_B(y)$ is the maximum.
- Unlike the centroid method, the maximum-decomposition method has some properties that are applicable to a narrower class of problems.
- The output value for this method is sensitive to a single rule that dominates the fuzzy rule set.
- Also, the output value tends to jump from one frame to the next as the shape of the fuzzy region changes.

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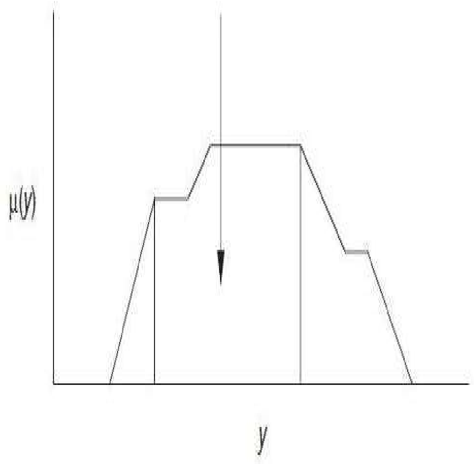
- Output y_h in this case is given by

$$y_h = \frac{\sum_{i=1}^{m} y_i' \mu_B(y_i)}{\sum_{i=1}^{m} \mu_B(y_i)}$$

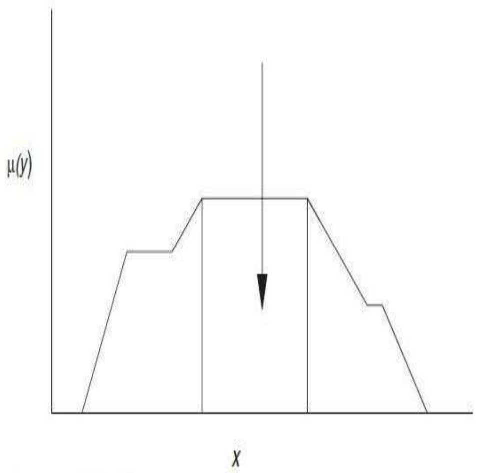
- m represents the number of output fuzzy sets obtained after implication and y_i represents the centroid of fuzzy region i.
- This technique is easy to use because the centers of gravity of commonly used membership functions are known ahead of time.

Example: Defuzzification



Defuzzification (average of maximums) (src:www)

Example: Defuzzification



Defuzzification (maximum-decomposition method) (src:www)

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    - Fuzzify inputs
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    - Apply implication method
    - Aggregate all output fuzzy sets
    - Defuzzify
- Step 1: Fuzzy Inputs The first step is to take inputs and determine the degree to which they belong to each of the appropriate fuzzy sets via membership functions.

Summary: FIS

- Step 2: Apply Fuzzy Operators Once the inputs have been fuzzified, knowing the degree to which each part of the antecedent has been satisfied for each rule.
- If a given rule has more than one part, the fuzzy logical operators are applied to evaluate the composite firing strength of the rule.

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- If a given rule has more than one part, the fuzzy logical operators are applied to evaluate the composite firing strength of the rule.
- Step 3: Apply the Implication Method The implication method is defined as the shaping of the output membership functions on the basis of the firing strength of the rule.
- The input for the implication process is a single number given by the antecedent, and the output is a fuzzy set. Two commonly used methods of implication are the minimum and the product.

Summary: FIS

- Step 4: Aggregate all Outputs Aggregation is a process whereby the outputs of each rule are unified.
- Aggregation occurs only once for each output variable. The input to the aggregation process is the truncated output fuzzy sets returned by the implication process for each rule.
- The output of the aggregation process is the combined output fuzzy set.

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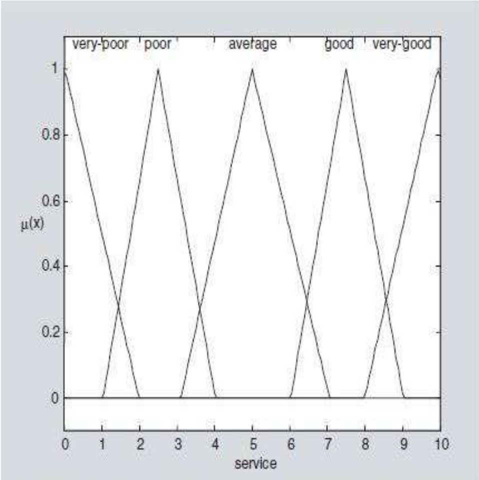
- Step 5: Defuzzify The input for the defuzzification process is a fuzzy set (the aggregated output fuzzy set),
- the output of the defuzzification process is a crisp value obtained by using some defuzzification method such as the centroid, height, or maximum.

Another Example of FIS

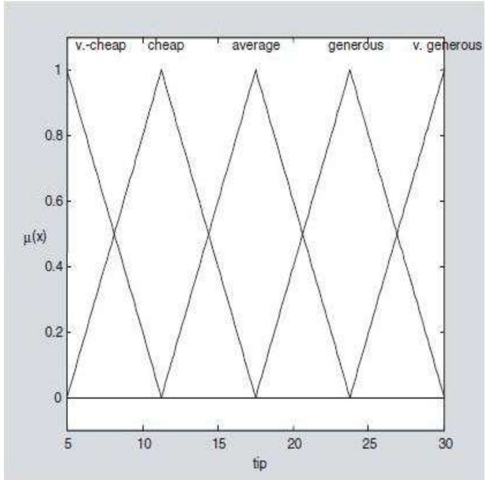
- consider a system that determines dinner in a restaurant on the basis of the service received.
- the input x denotes the quality of the service, which is represented by a number between 0 and 20, where 20 designates *verygood* and 0 *verypoor*.
- The input x is represented by the term set $\{verypoor, poor, average, good, verygood\}$.
- The output y represents the *tip*, which varies between 5 and 30 percent, and is given by the term set $\{verycheap, cheap, average, generous, verygenerous\}$.

Example: Service input



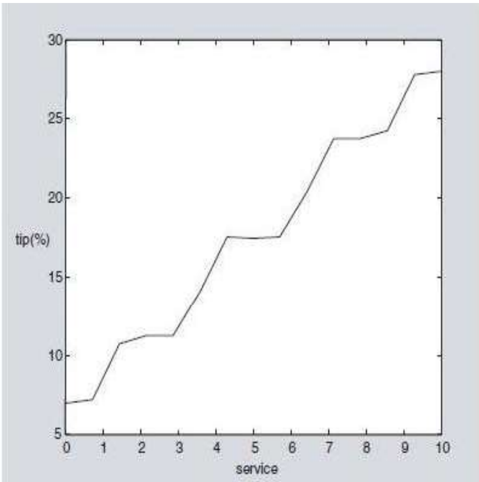
Input fuzzy sets for service (src:www)

Example: Tip output



Output fuzzy sets (tip) (src:www)

Example: Mapping surface



Mapping surface (service) (src:www)

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- R_3: if service is average, then tip is average
- R_4: if service is good, then tip is generous
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- The mapping function $y = f(x)$ depends on the number of input fuzzy sets and their shapes.

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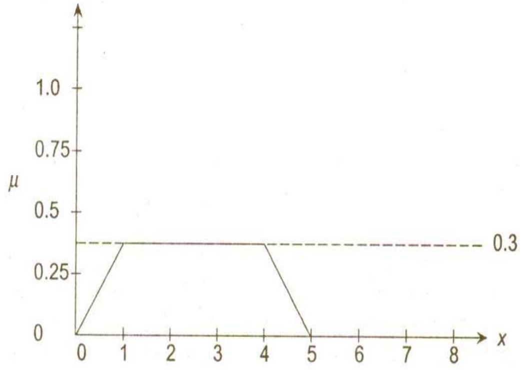
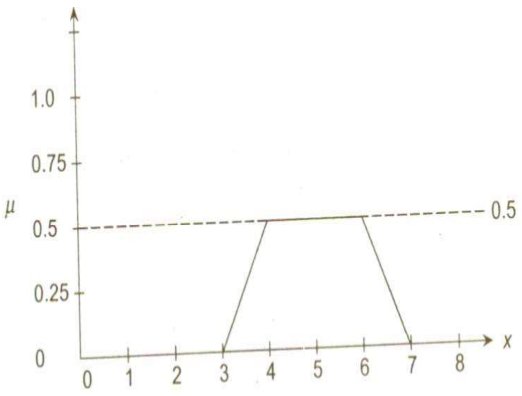
- increase the overlap between the input fuzzy sets, the mapping function becomes smoother.

Example of Aggregation



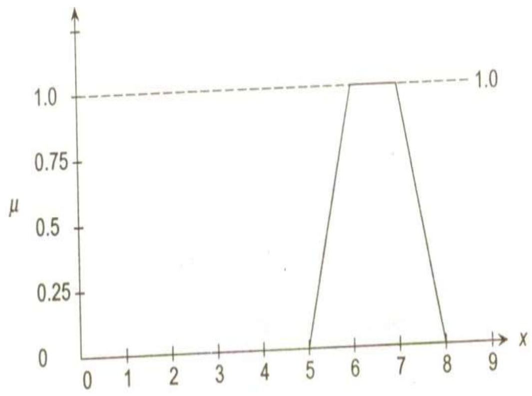
A_1 fuzzy set (src:NFG book)

Example of Aggregation



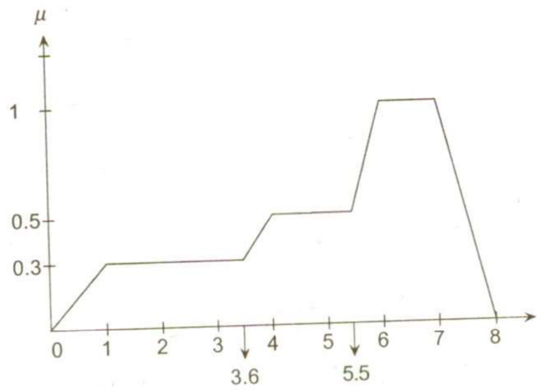
A_2 fuzzy set (src:NFG book)

Example of Aggregation



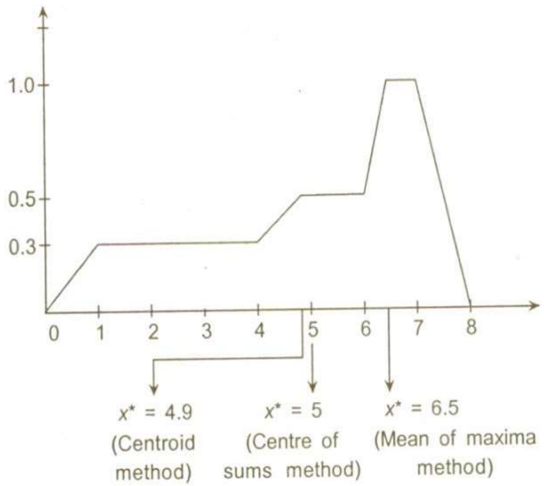
A_3 fuzzy set (src:NFG book)

Example of Aggregation



Aggregation of fuzzy sets A_1, A_2 and A_3 (src:NFG book)

Example of Defuzzification



Defuzzified outputs of the aggregate of A_1, A_2 and A_3 (src:NFG book)

Example of Airconditioner Controller

- dial to control the flow of warm/hot or cool/cold air

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- difference in temperature between room temperature and desired temperature
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- dial set to zero no air is supplied from airconditioner
- determine to what extent the dial should be turned

Example of Airconditioner Controller: Fuzzy rule

- Few terms

approximately zero (ZE)	positively large (PL)
negative large (NL)	negative medium (NM)
positively small (PS)	negatively small (NS)
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- Few terms

approximately zero (ZE)	positively large (PL)
negative large (NL)	negative medium (NM)
positively small (PS)	negatively small (NS)
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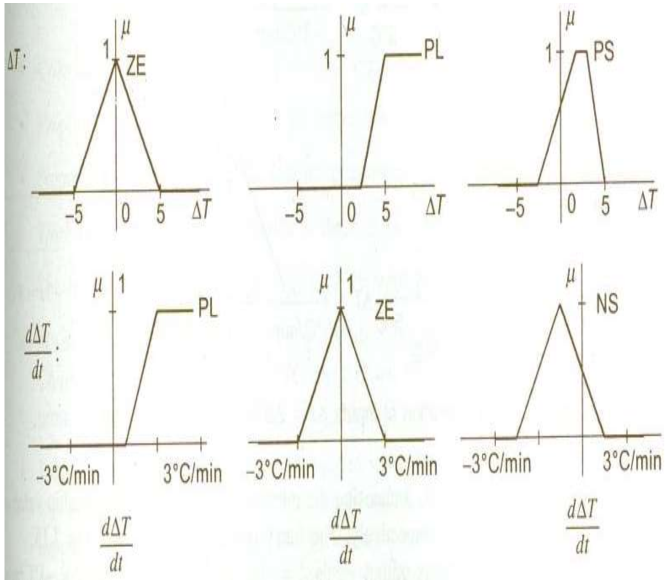
- if $\triangle T$ is ZE and $\frac{d\triangle T}{dt}$ is PL then dial should be NL
- if $\triangle T$ is PL and $\frac{d\triangle T}{dt}$ is ZE then dial should be NM
- if $\triangle T$ is PS and $\frac{d\triangle T}{dt}$ is NS then dial should be ZE

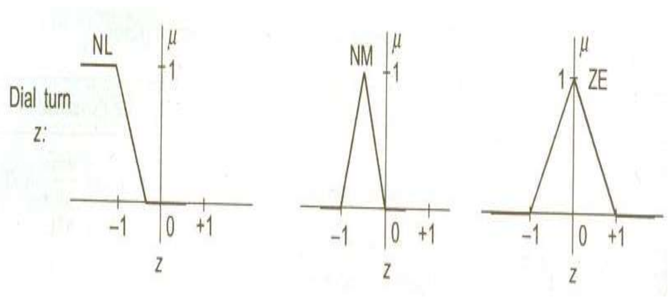
M. A. Zaveri, SVNIT, Surat Soft Computing Techniques 3 April 2011 74 / 157

Example of Airconditioner control system



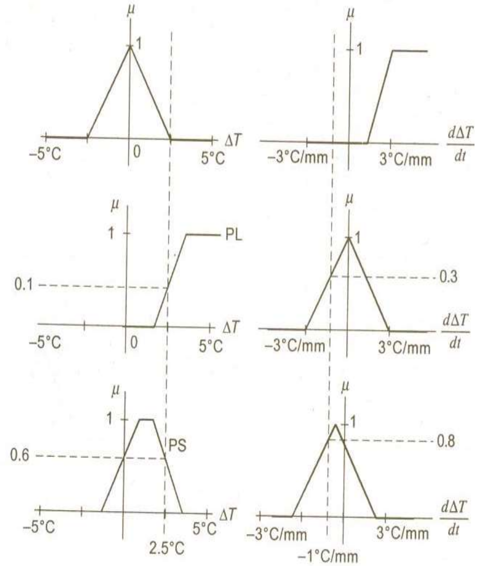
Fuzzy sets for system inputs (src:NFG book)

Example of Airconditioner control system



Fuzzy sets for system output (src:NFG book)

Example of Airconditioner control system

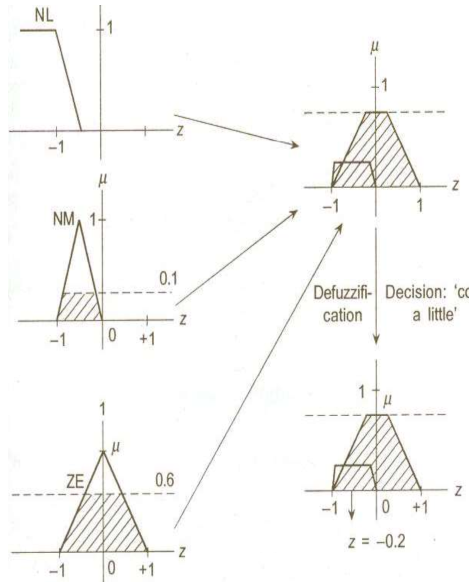


Fuzzification of inputs (src:NFG book)

Example of Airconditioner Controller

- Rules 1, 2 and 3 choosing the minimum of fuzzy membership value of antecedents are 0, 0.1, and 0.6
- Defuzzification of fuzzy output $z = -0.2$ for $\triangle T = 2.5°C$ and $\frac{d\triangle T}{dt} = -1°C/min$
- dial to be turned in negative direction

Example of Airconditioner control system



Defuzzification of fuzzy outputs for z (turn of the dial) (src:NFG book)

Example of Fuzzy cruise controller

- maintain a vehicle at a desired speed

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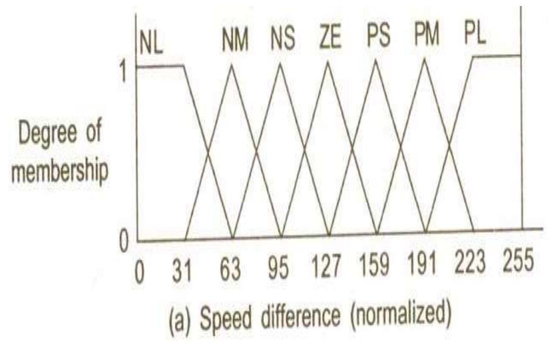
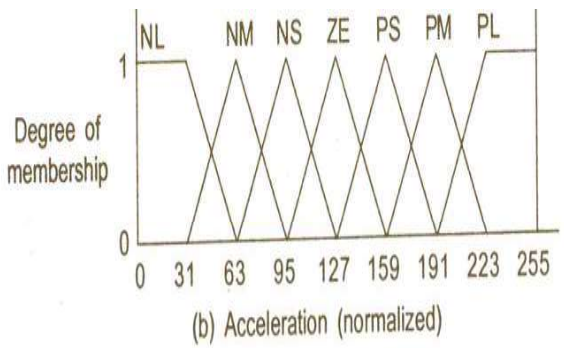
- R_1: if speed difference is NL and acceleration is ZE then throttle control is PL
- R_2: if speed difference is ZE and acceleration is NL then throttle control is PL ....

Example of Fuzzy cruise control



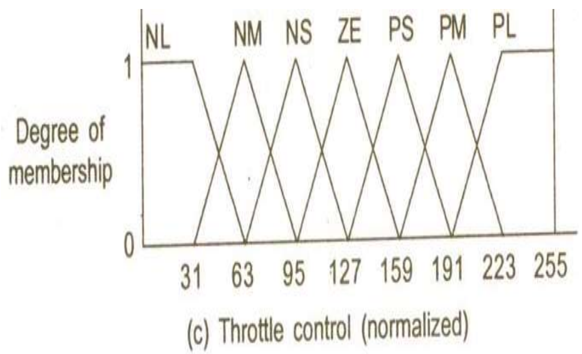
Speed difference: fuzzy input (normalized) (src:NFG book)

Example of Fuzzy cruise control



Acceleration: fuzzy input (normalized) (src:NFG book)

Example of Fuzzy cruise control



Throttle control: fuzzy output (normalized) (src:NFG book)