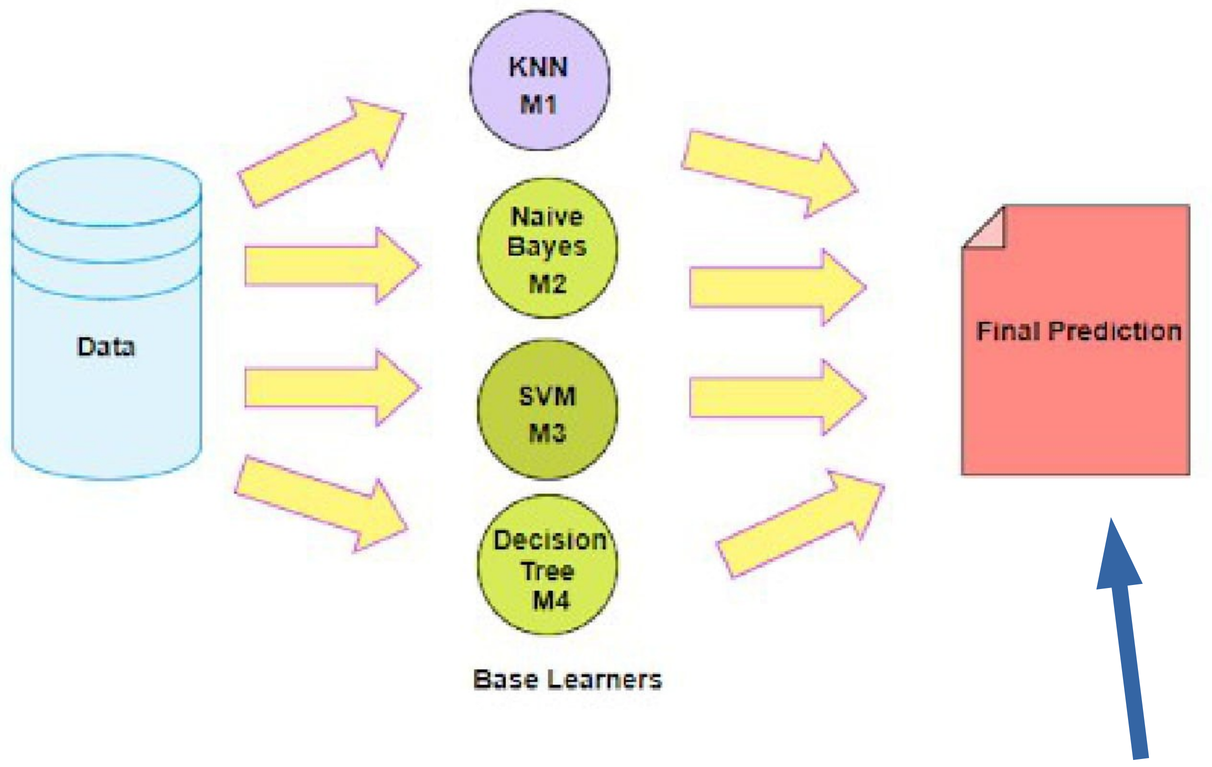
Ensemble Learning



Votes / weighted votes of all members

Ensemble Learning

Consider a set of classifiers h_1, ..., h_L

Idea: construct a classifier $H(\mathbf{x})$ that combines the individual decisions of h_1, ..., h_L

- e.g., could have the member classifiers vote, or
- e.g., could use different members for different regions of the instance space
- Works well if the members each have low error rates

Successful ensembles require **diversity**

- Classifiers should make different mistakes
- Can have different types of base learners

[Based on slide by Léon Bottou]

Practical Application: Netflix Prize

Goal: predict how a user will rate a movie

- Based on the user's ratings for other movies
- and other peoples' ratings
- with no other information about the movies

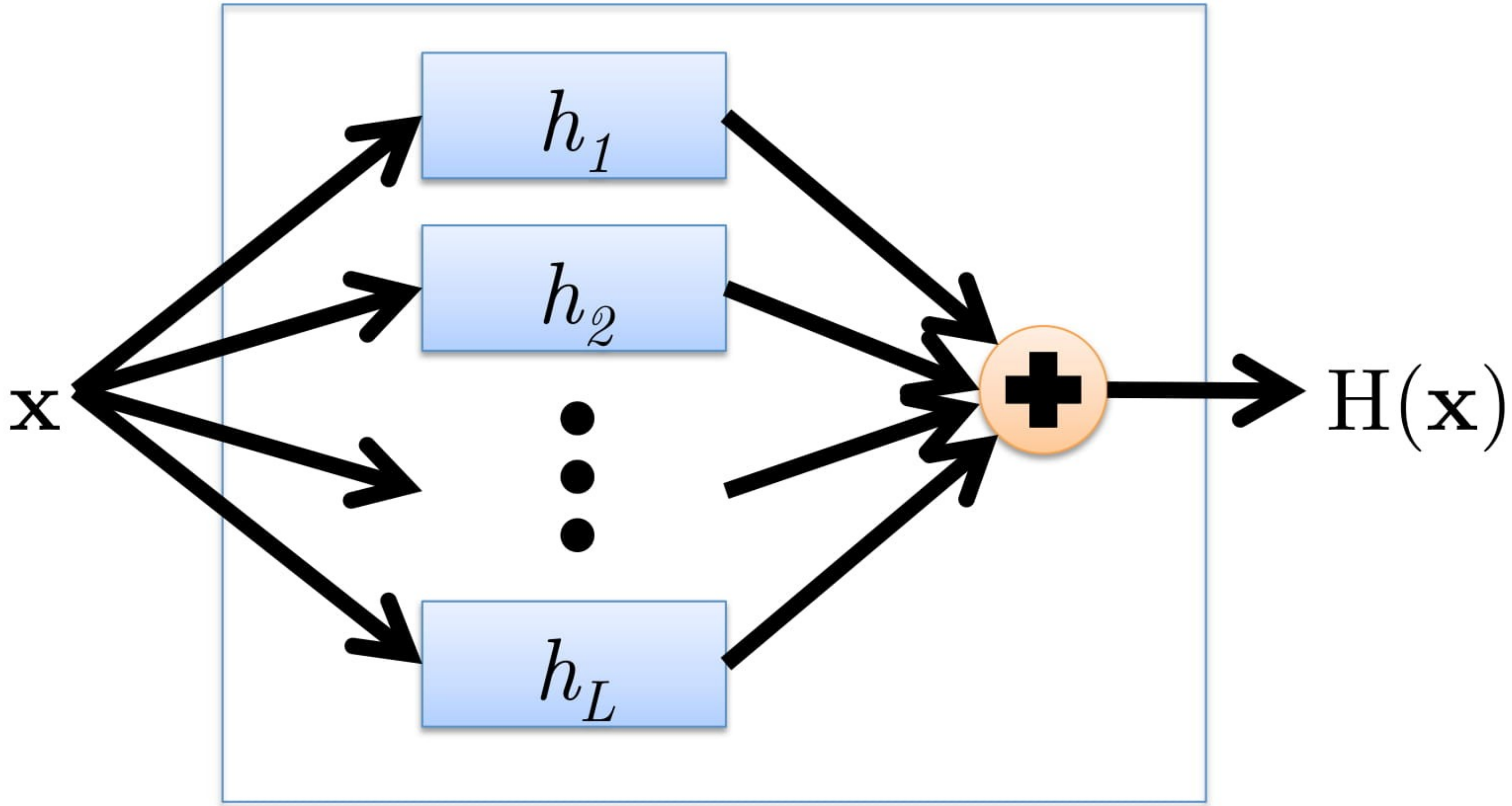
This application is called "collaborative filtering"

Netflix Prize: $1M to the first team to do 10% better then Netflix' system (2007-2009)

Winner: BellKor's Pragmatic Chaos – an ensemble of more than 800 rating systems

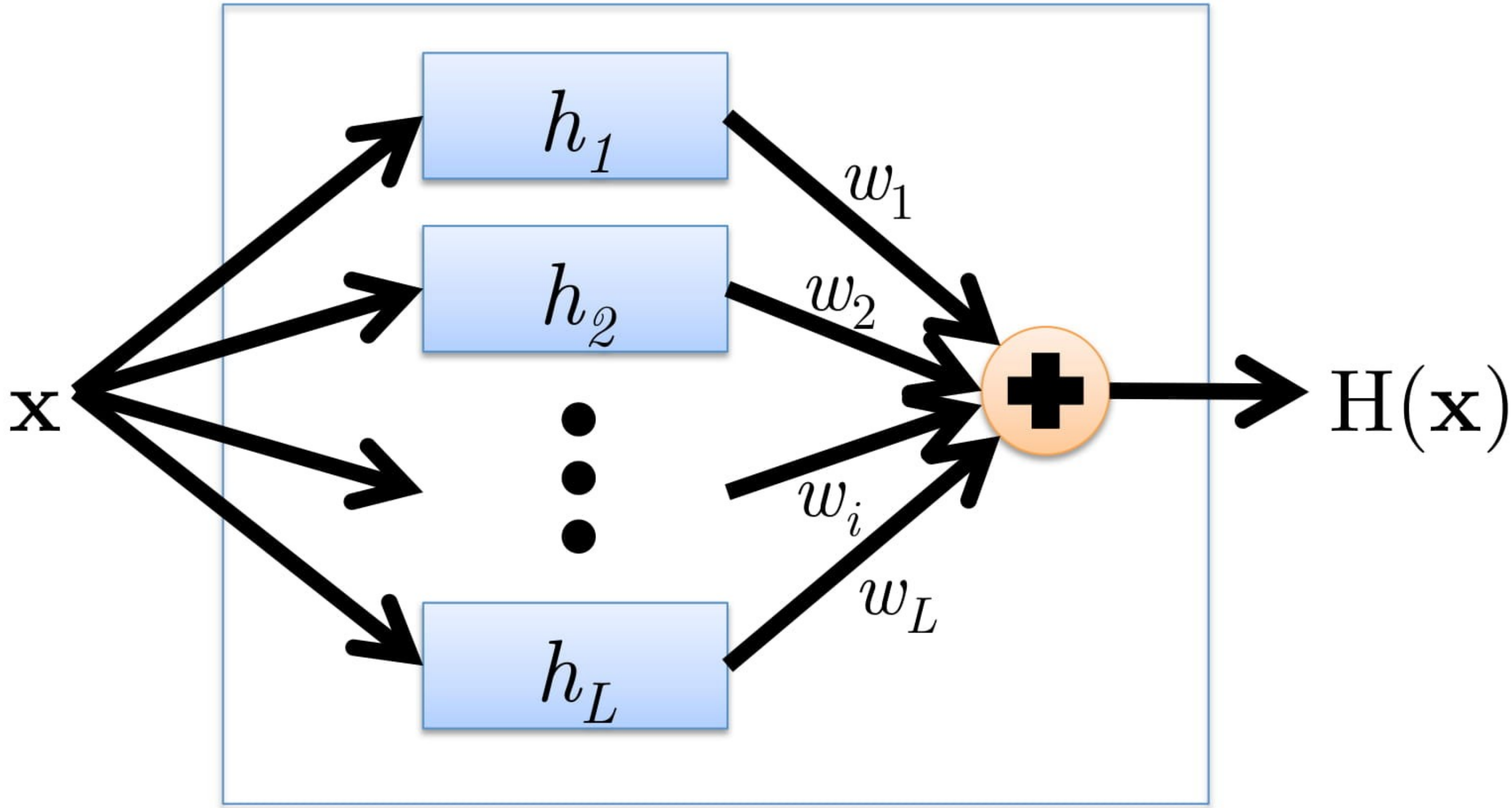
[Based on slide by Léon Bottou]

Combining Classifiers: Averaging



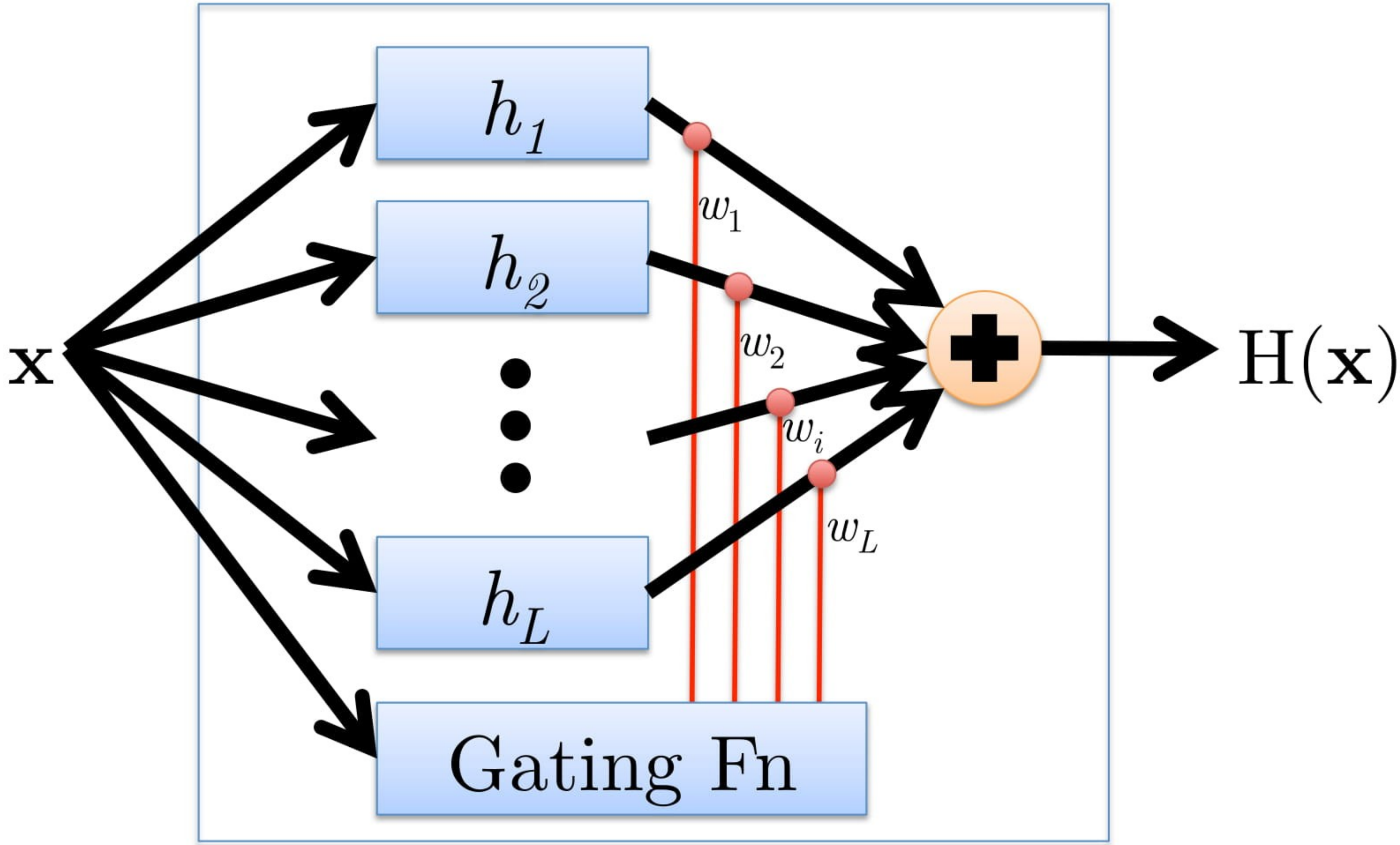
- Final hypothesis is a simple vote of the members

Combining Classifiers: Weighted Average



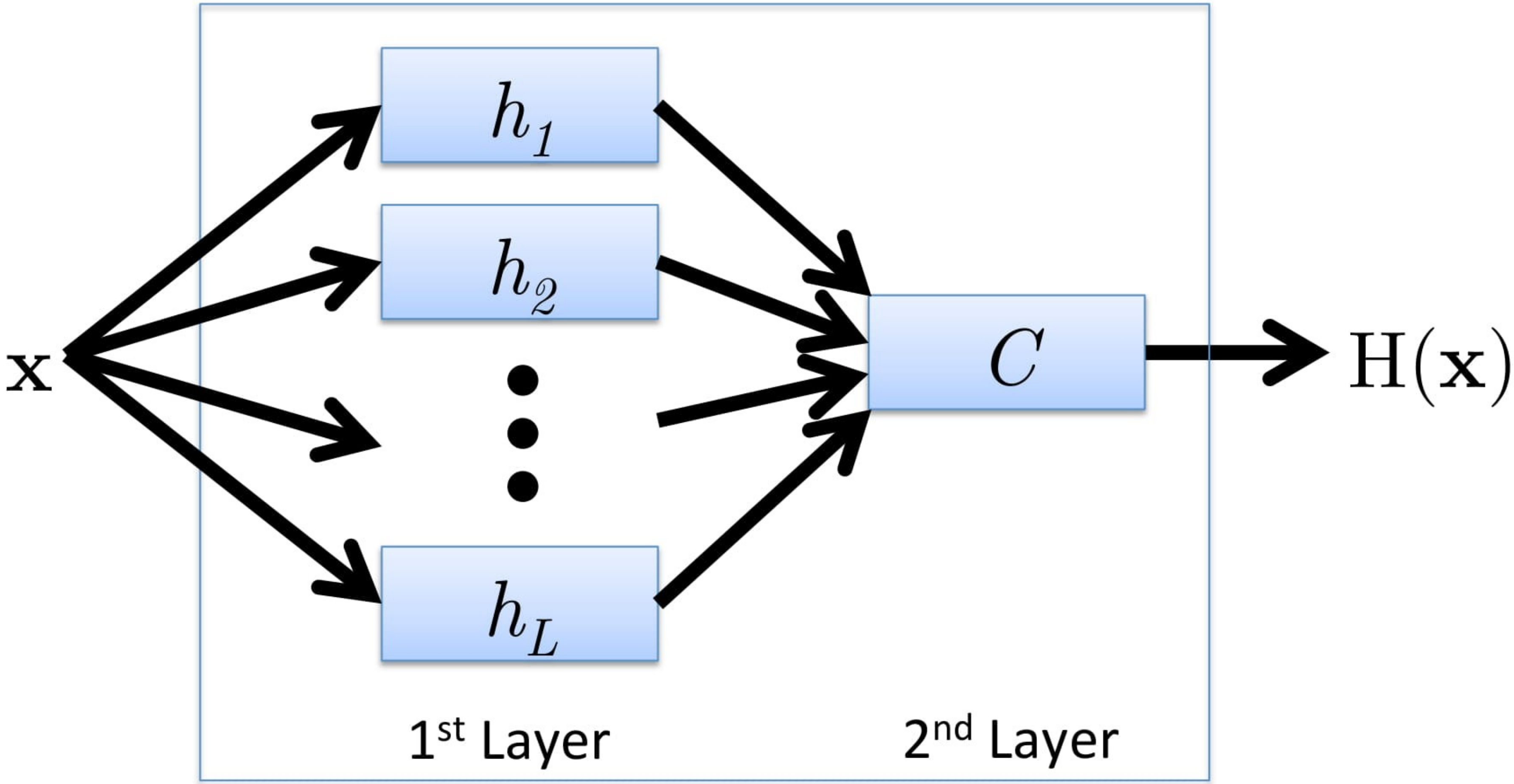
- Coefficients of individual members are trained using a validation set

Combining Classifiers: Gating



- Coefficients of individual members depend on input
- Train gating function via validation set

Combining Classifiers: Stacking



- Predictions of 1st layer used as input to 2nd layer
- Train 2nd layer on validation set

How to Achieve Diversity

Cause of the Mistake	**Diversification Strategy**
Pattern was difficult	Hopeless
Overfitting	Vary the training sets
Some features are noisy	Vary the set of input features

[Based on slide by Léon Bottou]

Manipulating the Training Data

Bootstrap replication:

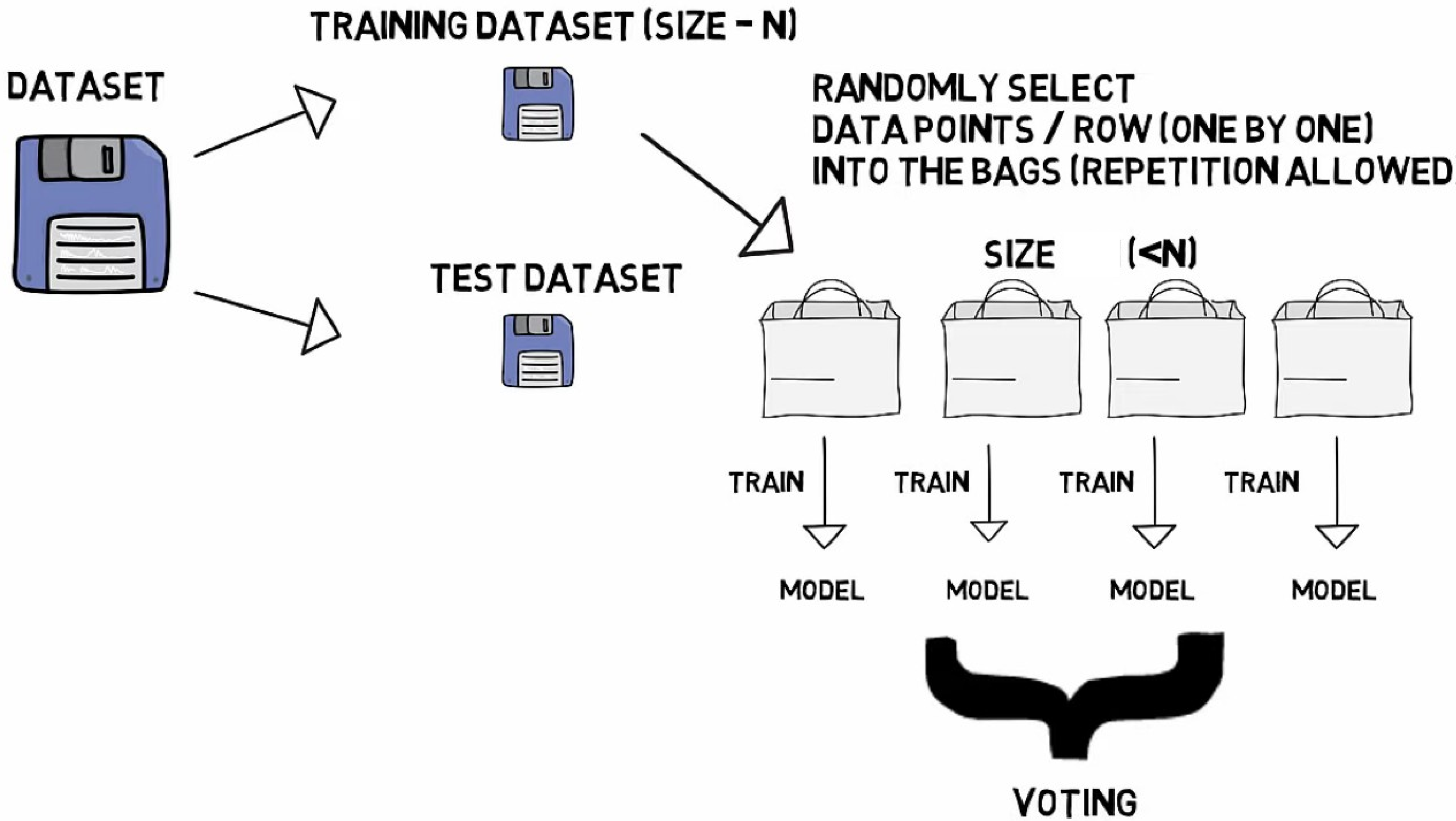
- Given **N** training examples, construct a new training set by sampling n instances with replacement
- Excludes ~30% of the training instances

Bagging:

- Create bootstrap replicates of training set
- Train a classifier (e.g., a decision tree) for each replicate
- Estimate classifier performance using out-of-bootstrap data
- Average output of all classifiers

Boosting: (in just a minute...)

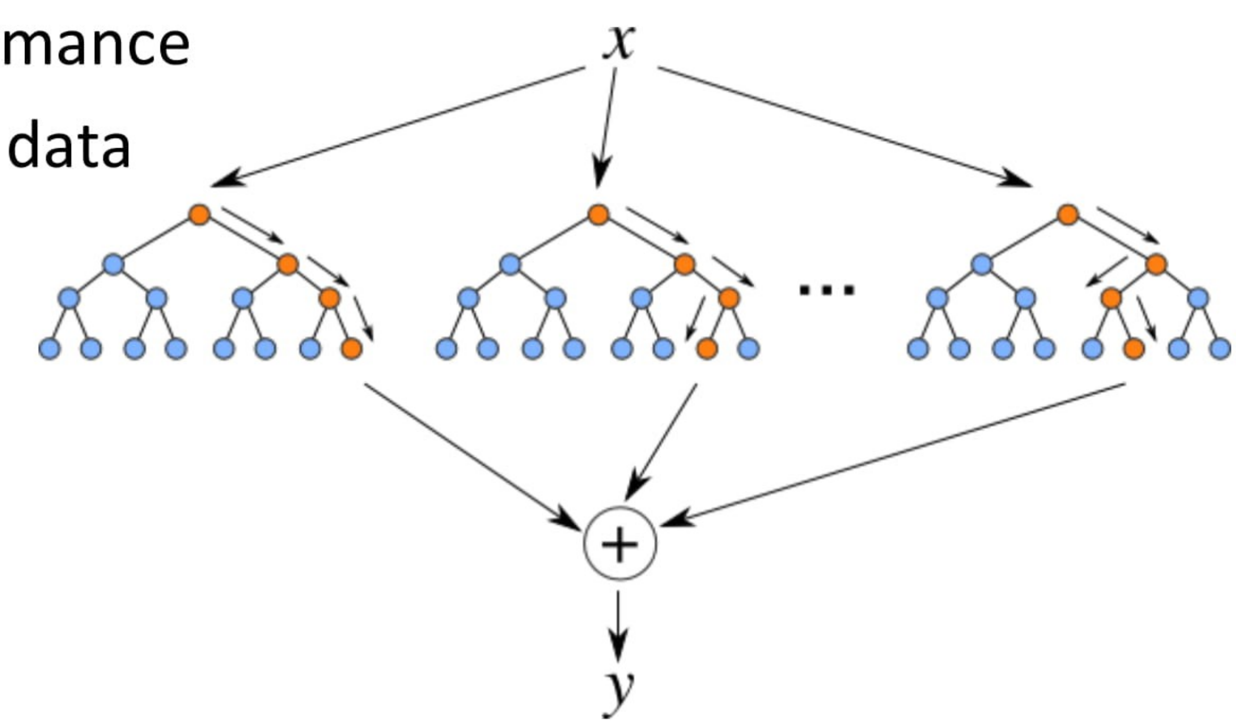
[Based on slide by Léon Bottou]


DATASET
TRAINING DATASET (SIZE - N)
TEST DATASET
RANDOMLY SELECT
DATA POINTS / ROW (ONE BY ONE)
INTO THE BAGS (REPETITION ALLOWED)
SIZE (<N)
TRAIN
TRAIN
TRAIN
TRAIN
MODEL
MODEL
MODEL
MODEL
VOTING

Manipulating the Features

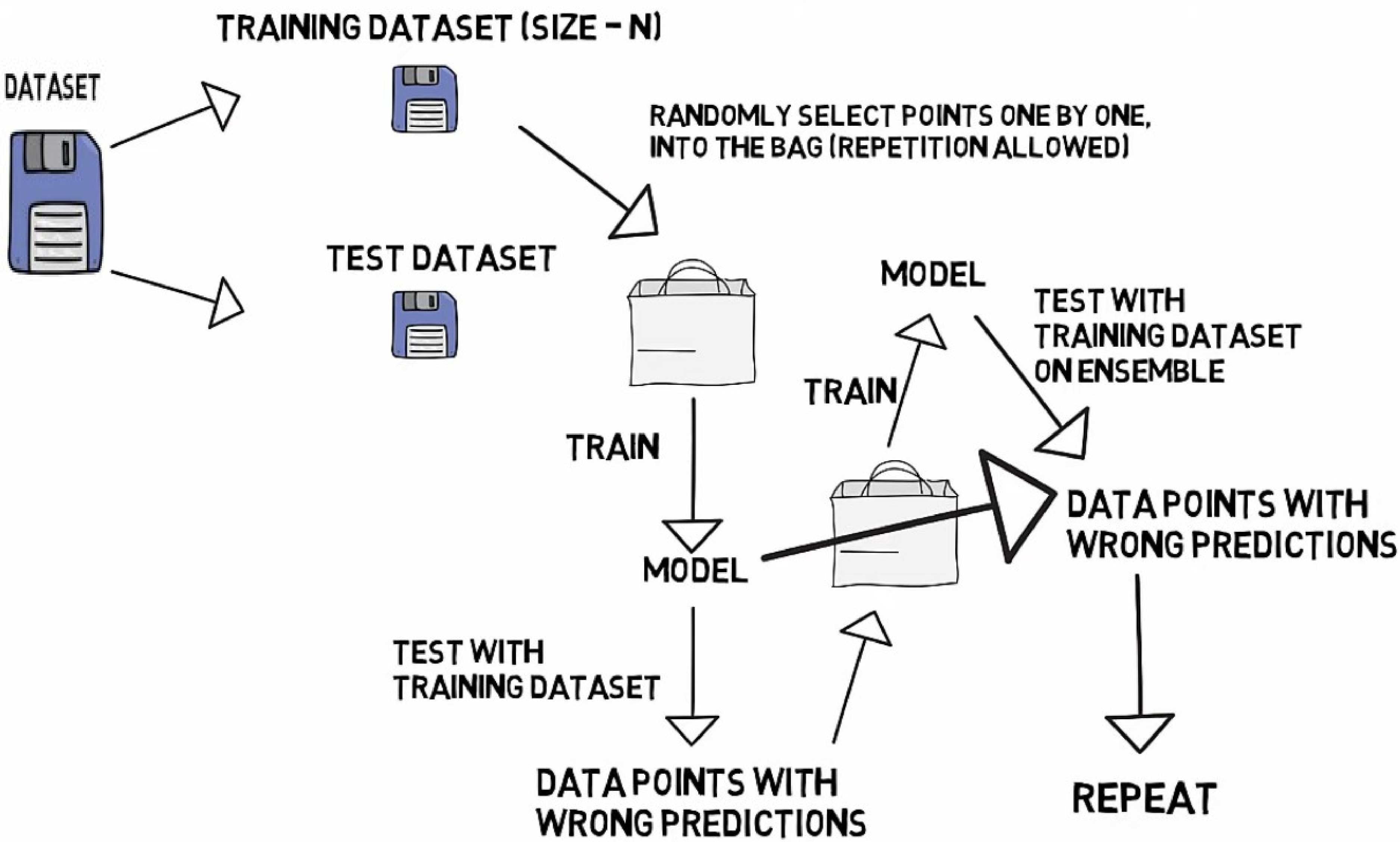
Random Forests

- Construct decision trees on bootstrap replicas
  - Restrict the node decisions to a small subset of features picked randomly for each node
- Do not prune the trees
  - Estimate tree performance on out-of-bootstrap data
- Average the output of all trees



[Based on slide by Léon Bottou]

Boosting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TEST DATASET
RANDOMLY SELECT POINTS ONE BY ONE, INTO THE BAG (REPETITION ALLOWED)
TRAIN
MODEL
TEST WITH TRAINING DATASET
DATA POINTS WITH WRONG PREDICTIONS
TRAIN
MODEL
TEST WITH TRAINING DATASET ON ENSEMBLE
DATA POINTS WITH WRONG PREDICTIONS
REPEAT

AdaBoost

[Freund & Schapire, 1997]

- A meta-learning algorithm with great theoretical and empirical performance

- Turns a base learner (i.e., a “weak hypothesis”) into a high performance classifier

- Creates an ensemble of weak hypotheses by repeatedly emphasizing mispredicted instances

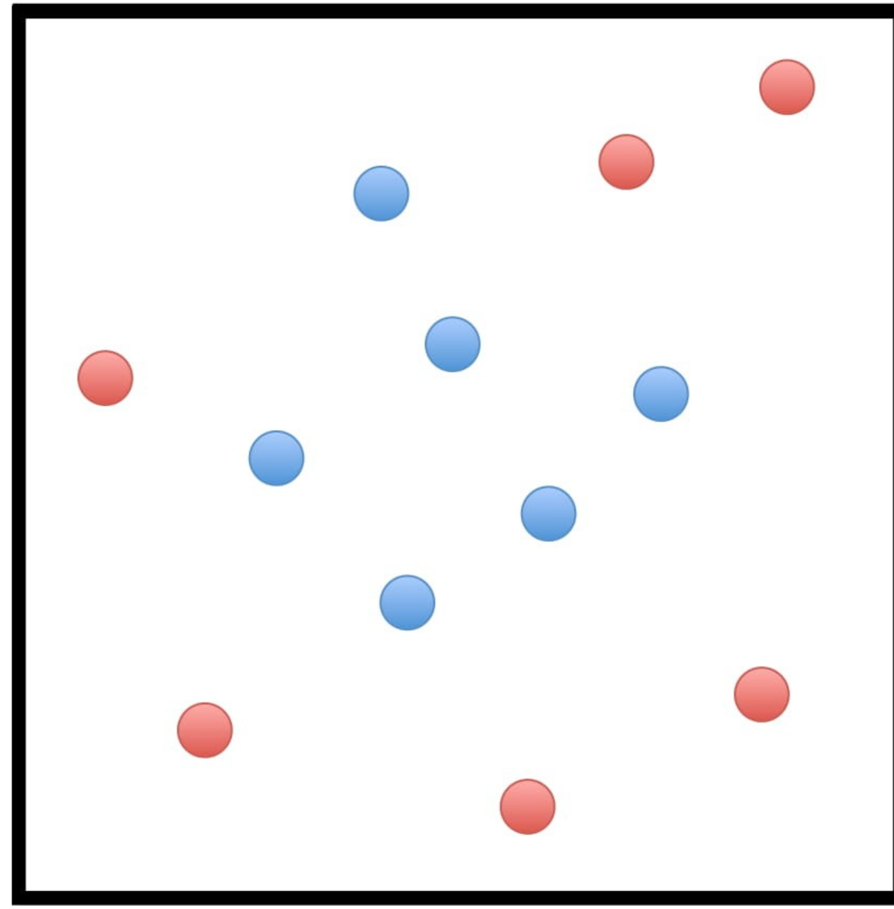
AdaBoost

Initialize a vector of n uniform weights $\mathbf{w}_1$
for $t = 1, \ldots, T$
    Train model h_t on X, y with weights $\mathbf{w}_t$
    Compute the weighted training error of h_t
    Choose $\beta_t = \frac{1}{2} \ln \left(\frac{1-\epsilon_t}{\epsilon_t} \right)$
    Update all instance weights:

$$w_{t+1,i} = w_{t,i} \exp\left(-\beta_t y_i h_t(\mathbf{x}_i)\right)$$

    Normalize $\mathbf{w}_{t+1}$ to be a distribution
end for
Return the hypothesis

$$H(\mathbf{x}) = \operatorname{sign}\left(\sum_{t=1}^{T} \beta_t h_t(\mathbf{x}) \right)$$



- Size of point represents the instance's weight

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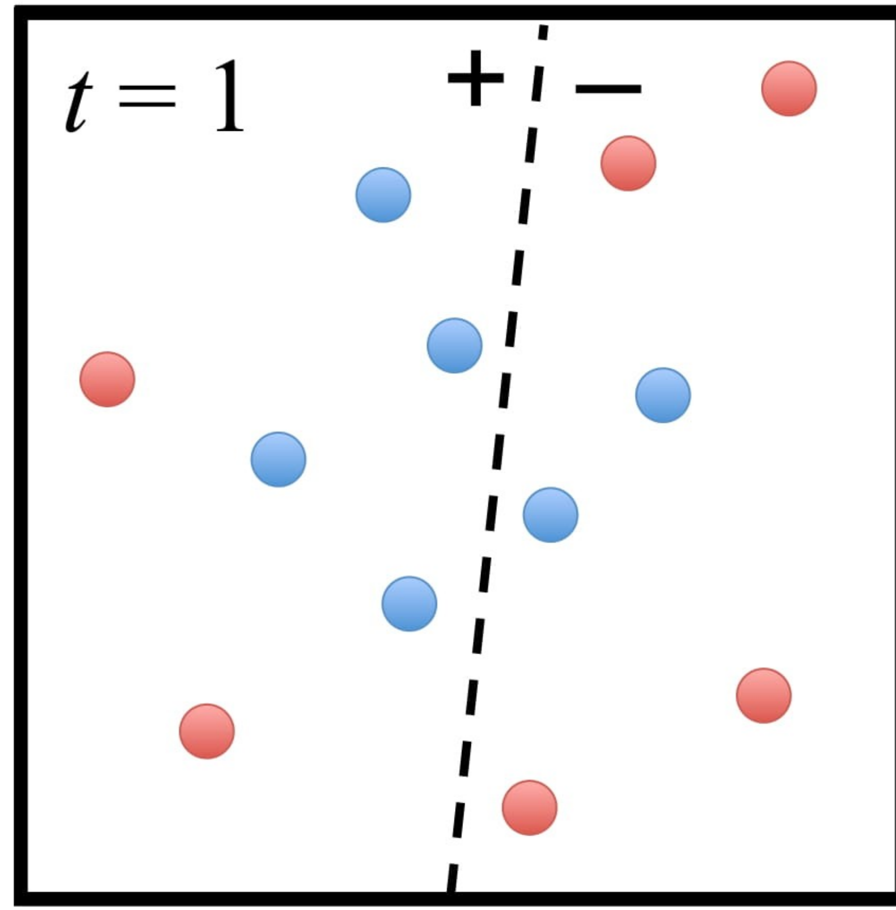

AdaBoost

INPUT: training data $X, y = \{(\mathbf{x}_i, y_i)\}_{i=1}^{n}$,
the number of iterations T

1: Initialize a vector of n uniform weights $\mathbf{w}_1 = \left[\frac{1}{n}, \ldots, \frac{1}{n}\right]$
2: **for** $t = 1, \ldots, T$
3: Train model h_t on X, y with instance weights $\mathbf{w}_t$
4: Compute the weighted training error rate of h_t:

$$\epsilon_t = \sum_{i: y_i \neq h_t(\mathbf{x}_i)} w_{t,i}$$

Error is the sum the weights of all misclassified instances

5: Choose $\beta_t = \frac{1}{2} \ln \left(\frac{1-\epsilon_t}{\epsilon_t} \right)$
6: Update all instance weights:

$$w_{t+1,i} = w_{t,i} \exp\left(-\beta_t y_i h_t(\mathbf{x}_i)\right) \quad \forall i = 1, \ldots, n$$

7: Normalize $\mathbf{w}_{t+1}$ to be a distribution:

$$w_{t+1,i} = \frac{w_{t+1,i}}{\sum_{j=1}^{n} w_{t+1,j}} \quad \forall i = 1, \ldots, n$$

8: **end for**
9: **Return** the hypothesis

$$H(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \beta_t h_t(\mathbf{x}) \right)$$

AdaBoost

INPUT: training data $X, y = \{(\mathbf{x}_i, y_i)\}_{i=1}^n$,
the number of iterations T

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end for
Return the hypothesis

$$H(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^T \beta_t h_t(\mathbf{x}) \right)$$

- β_t measures the importance of h_t
- If $\epsilon_t \leq 0.5$, then $\beta_t \geq 0$
  - Trivial, otherwise flip h_t's predictions
- β_t grows as error h_t's shrinks

AdaBoost

INPUT: traini
the n

Initialize a vecto

for $t = 1, \ldots, T$

Train model

Compute the

$$\epsilon_t = \sum_{i: y_i \neq h_t(\mathbf{x}} \ldots$$

Choose $\beta_t = \frac{1}{2} \ln \left(\ldots \over \epsilon_t \right)$

Update all instance weights:

$$w_{t+1,i} = w_{t,i} \exp\left(-\beta_t y_i h_t(\mathbf{x}_i)\right) \quad \forall i = 1, \ldots, n$$

Normalize $\mathbf{w}_{t+1}$ to be a distribution:

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end for

Return the hypothesis

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This is the same as:

will be ≤ 1

$$w_{t+1,i} = w_{t,i} \times \begin{cases} \boxed{e^{-\beta_t}} & \text{if } h_t(\mathbf{x}_i) = y_i \\ \boxed{e^{\beta_t}} & \text{if } h_t(\mathbf{x}_i) \neq y_i \end{cases}$$

will be ≥ 1

Essentially this emphasizes misclassified instances.

AdaBoost

INPUT: training data $X, y = \{(\mathbf{x}_i, y_i)\}_{i=1}^{n}$,
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Make $\mathbf{w}_{t+1}$ sum to 1

end for

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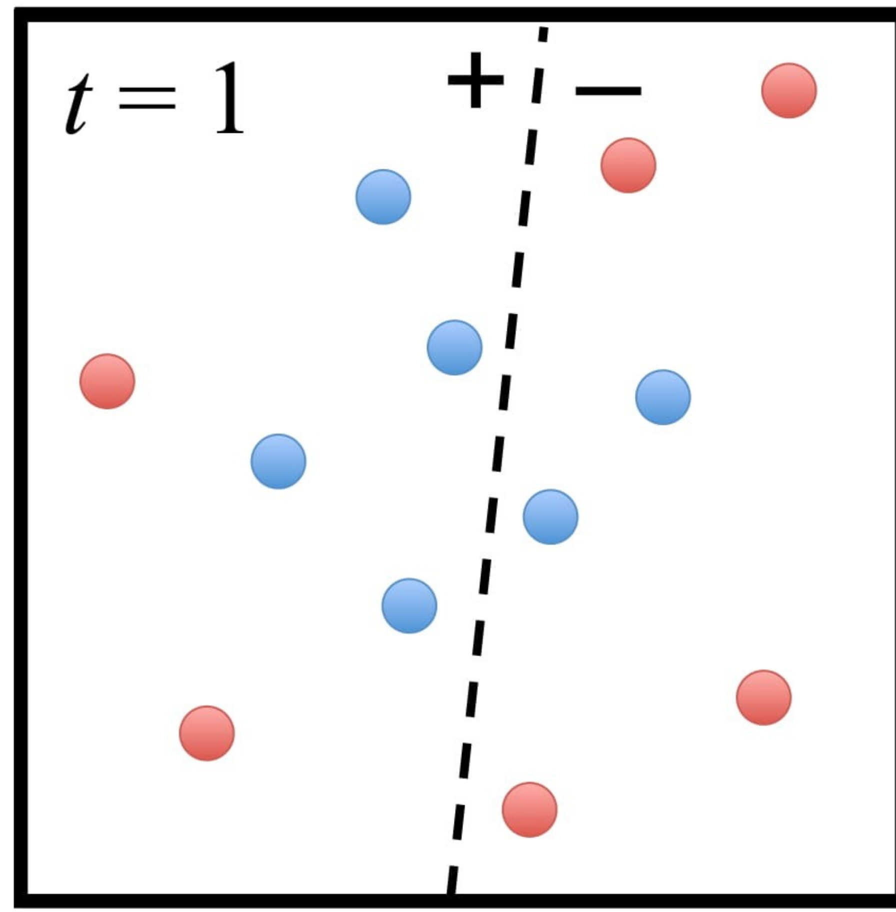

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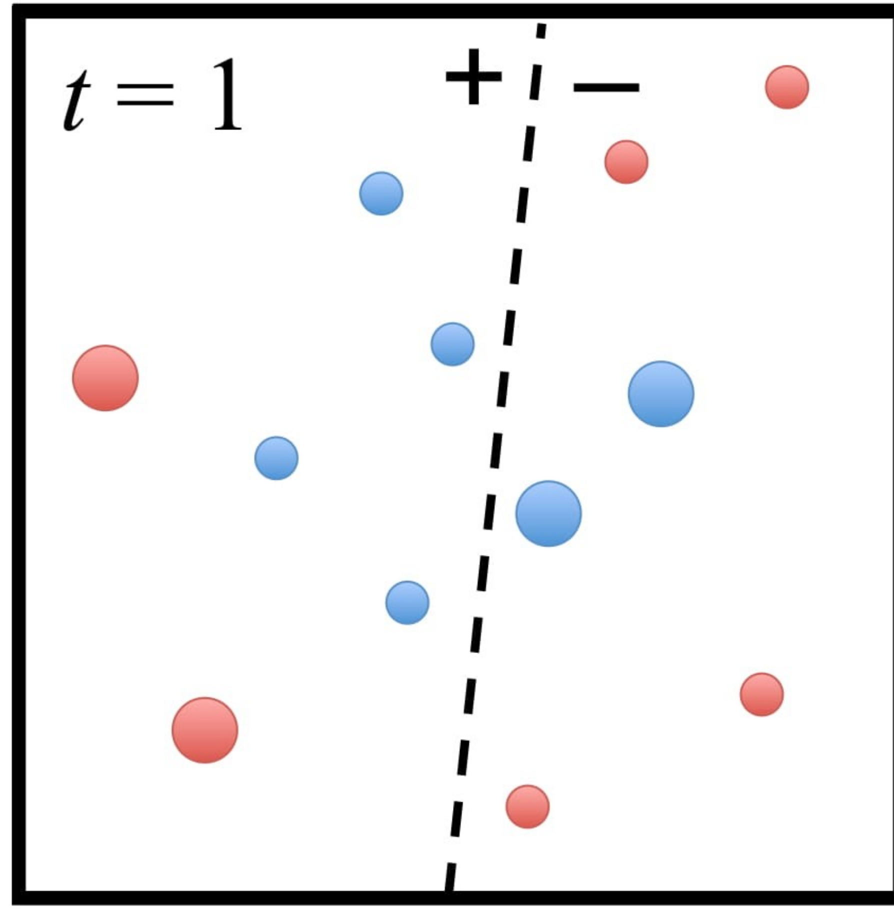

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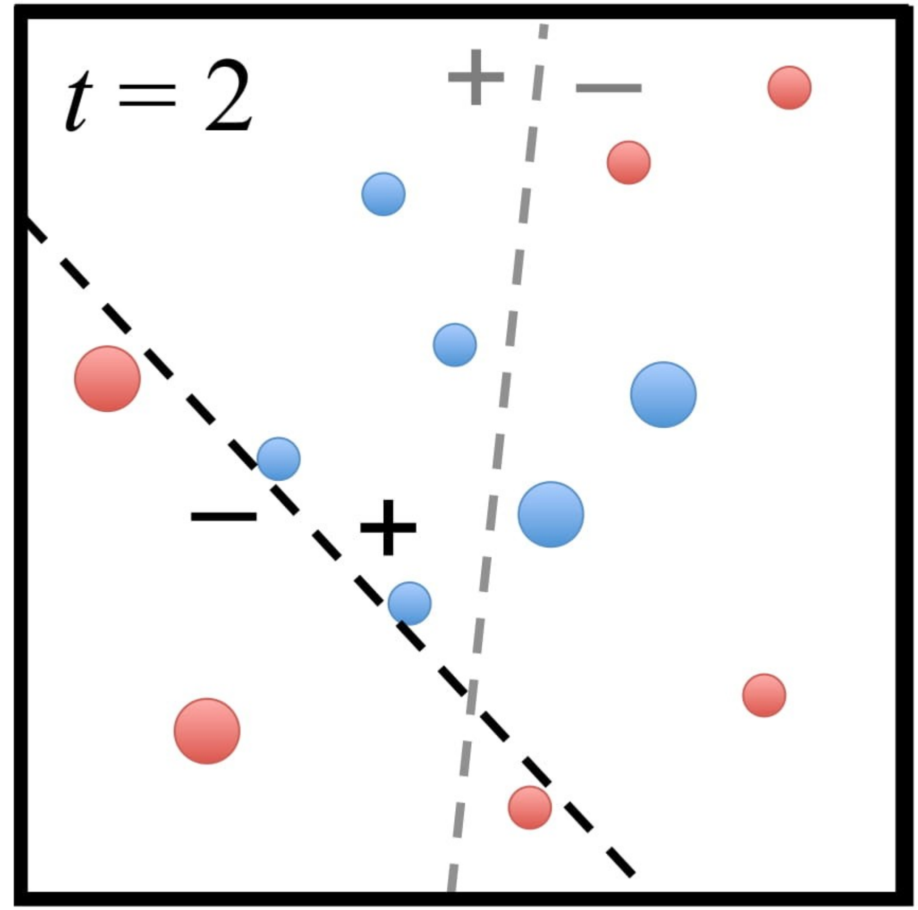

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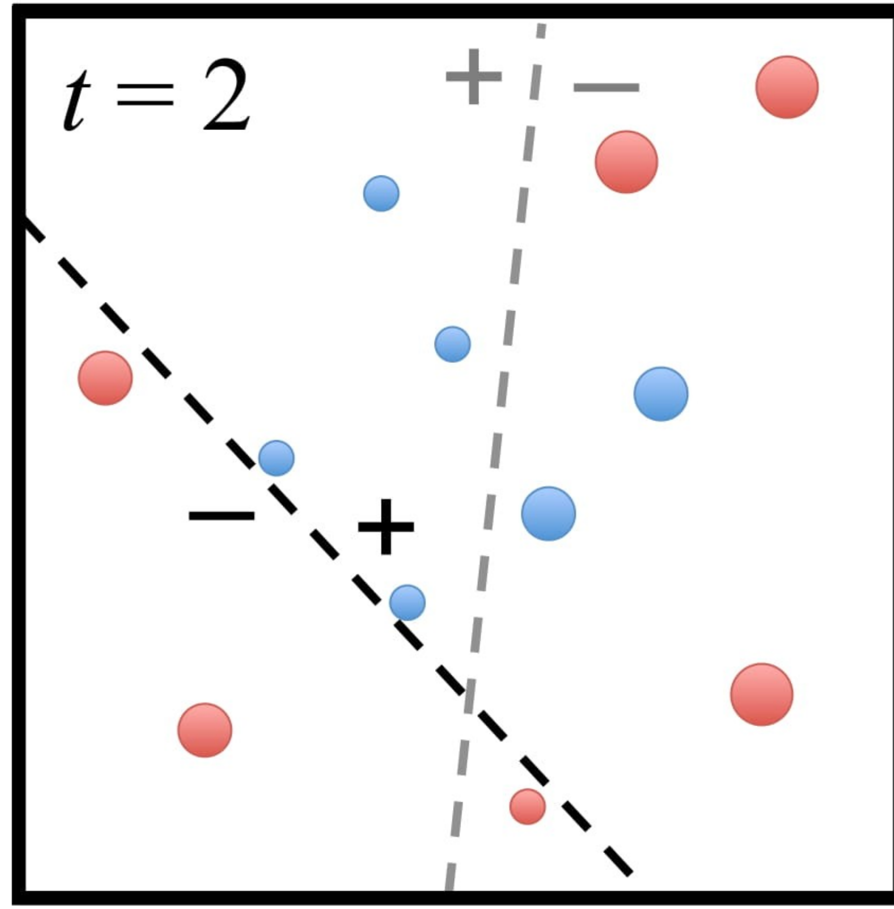

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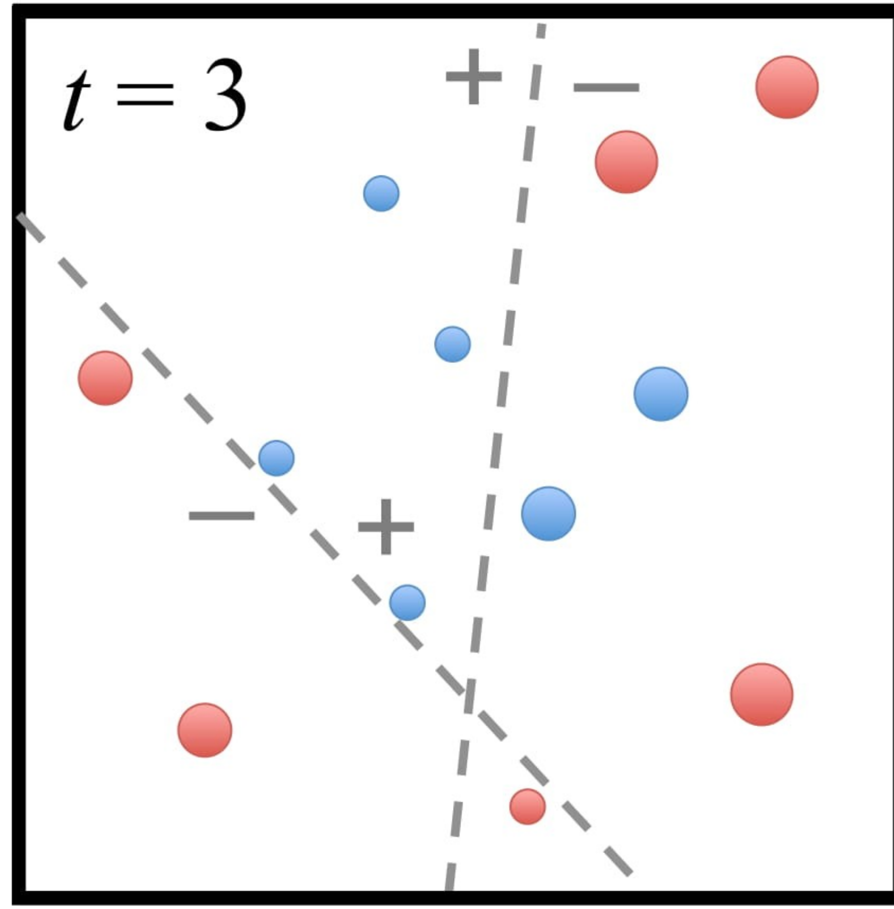

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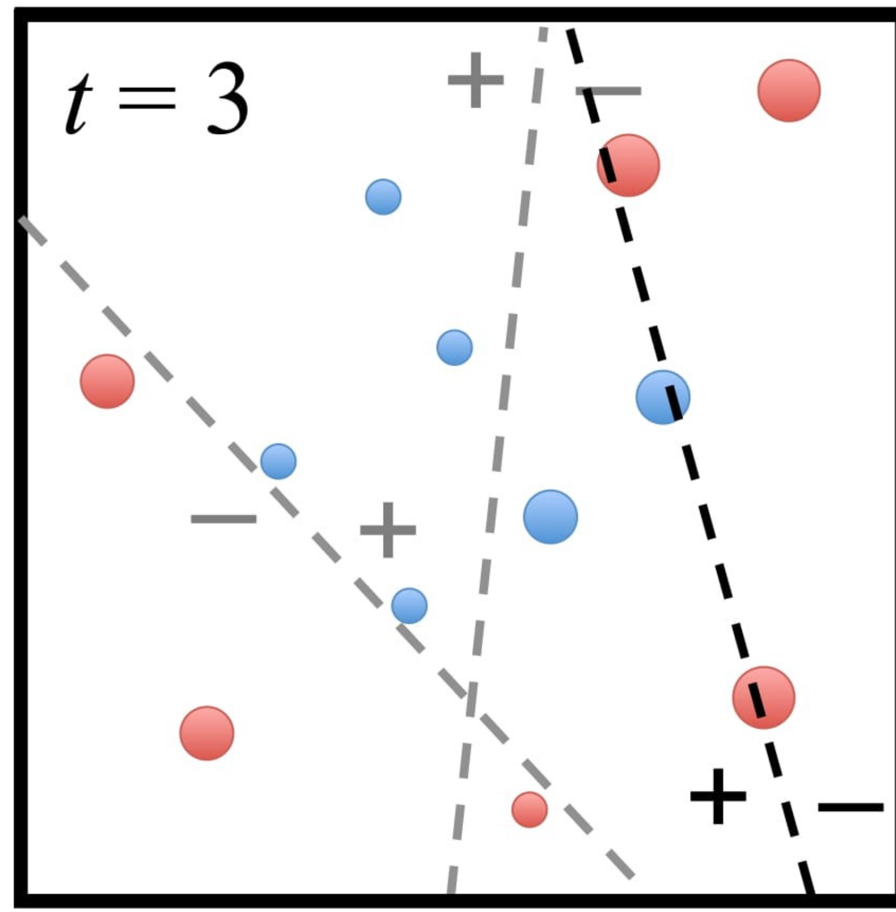

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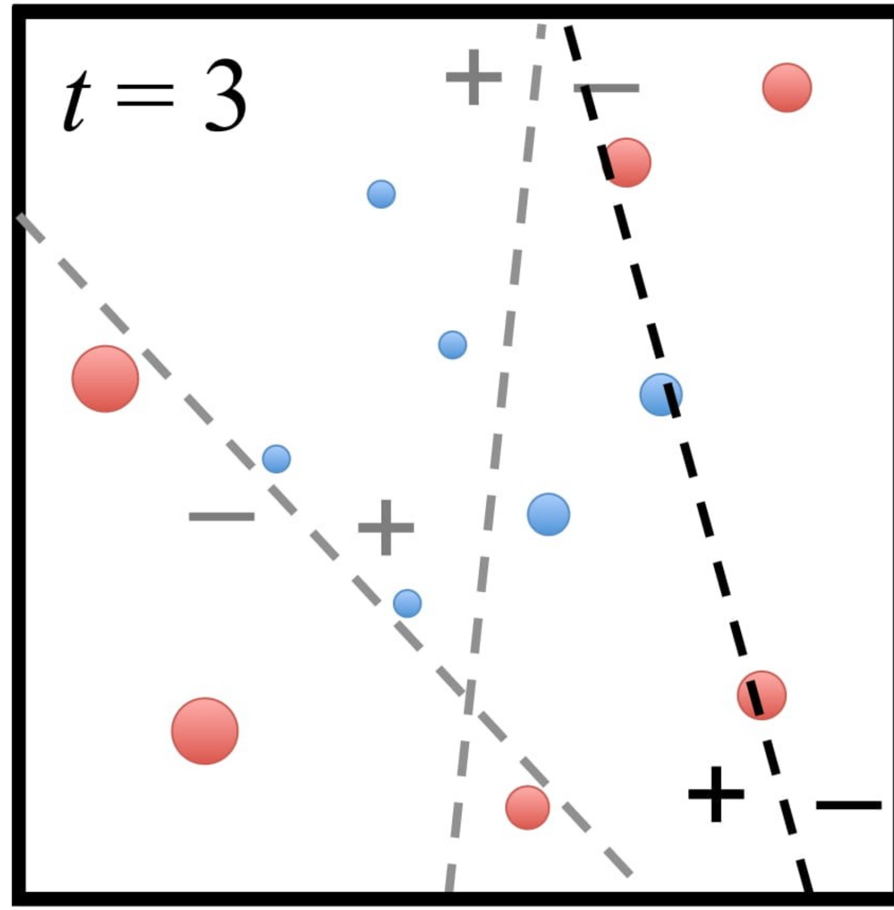

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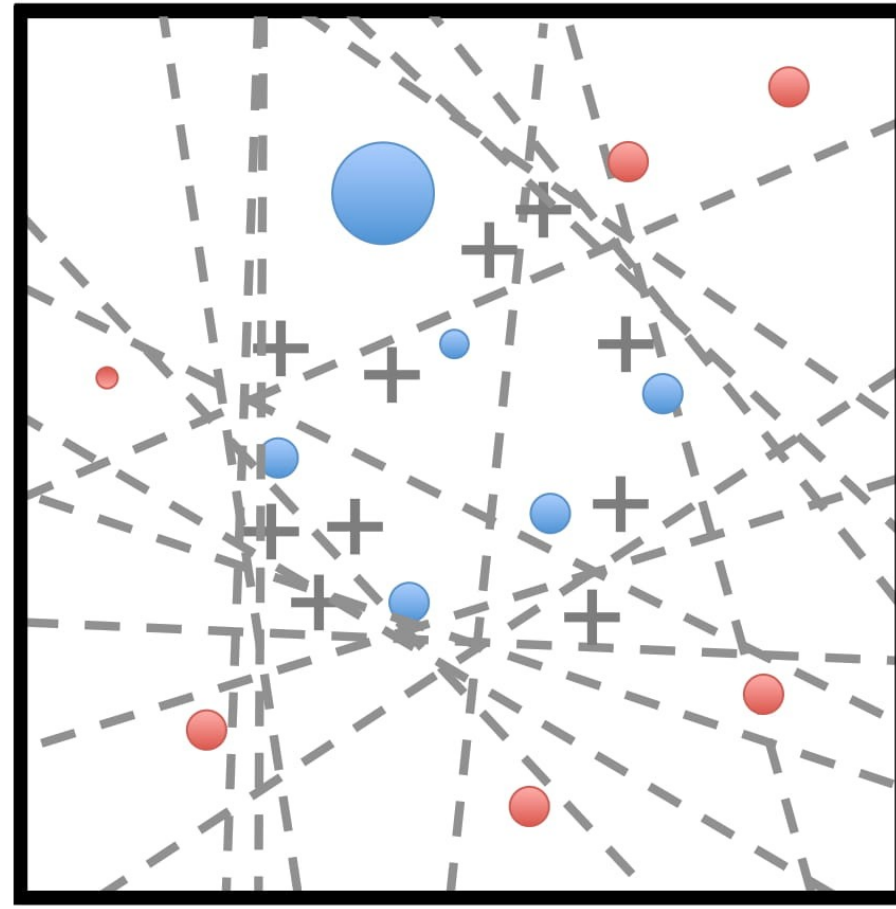
AdaBoost

$t = \mathrm{T}$



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end for
Return the hypothesis

$$H(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \beta_t h_t(\mathbf{x}) \right)$$

Member classifiers with less error are given more weight in the final ensemble hypothesis

Final prediction is a weighted combination of each member's prediction

AdaBoost

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8: **end for**
9: **Return** the hypothesis

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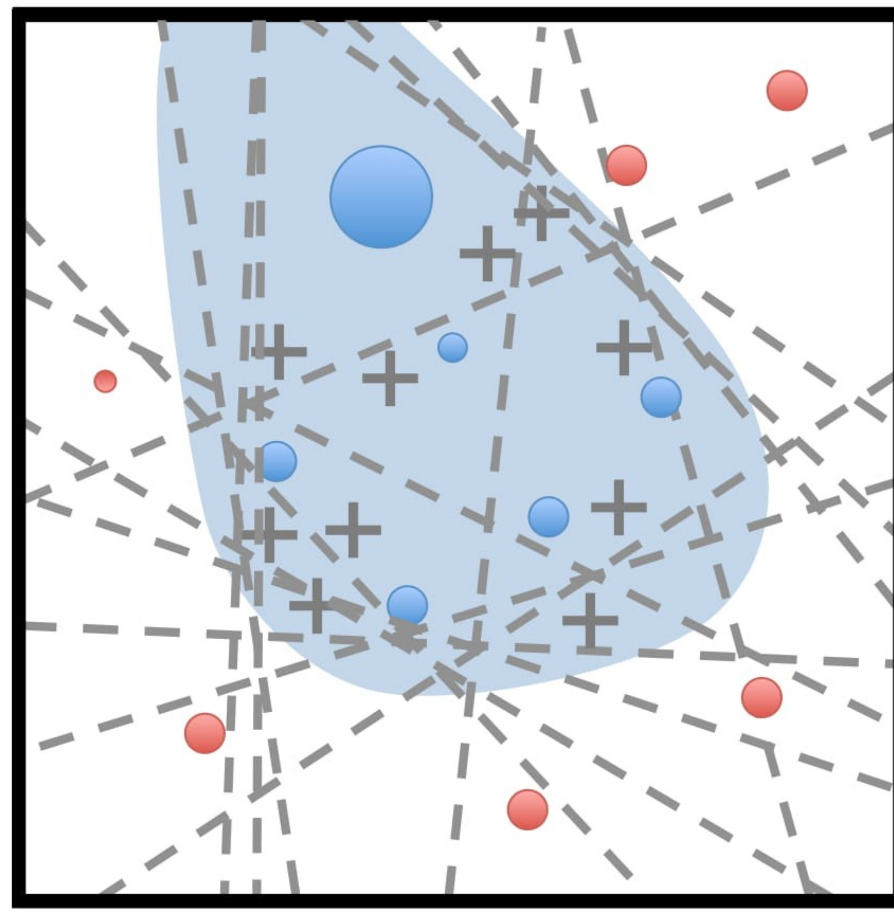
$t = \mathrm{T}$



- Final model is a weighted combination of members
  - Each member weighted by its importance

Base Learner Requirements

- AdaBoost works best with "weak" learners
  - Should not be complex
  - Typically high bias classifiers
  - Works even when weak learner has an error rate just slightly under 0.5 (i.e., just slightly better than random)
    - Can prove training error goes to 0 in O(log n) iterations

- Examples:
  - Decision stumps (1 level decision trees)
  - Depth-limited decision trees
  - Linear classifiers

Dynamic Behavior of AdaBoost

- If a point is repeatedly misclassified...
  - Each time, its weight is increased
  - Eventually it will be emphasized enough to generate a hypothesis that correctly predicts it

- Successive member hypotheses focus on the hardest parts of the instance space
  - Instances with highest weight are often outliers