Multi User Detection

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Single User System

- transmitting a sequence $\{b[0], b[1], \ldots, b[M-1]\}$
- $(b[i] = \pm 1)$ or a finite alphabet of complex numbers
- linear modulation using a signaling waveform

$$x(t) = \sum_{i=0}^{M-1} b[i] w_i(t)$$

$w_i(\cdot)$ modulation waveform associated with the i^{th} symbol, for example

$$w_i(t) = Ap(t - iT)e^{j(w_c + \phi)}$$

$A > 0$ $\phi \in (- = pi, \pi)$ and $p(\cdot)$ baseband pulse shape

$$p(t) = p_T(t) \triangleq \begin{cases} \frac{1}{\sqrt{T}} & 0 \leq t < T \\ 0 & \text{otherwise} \end{cases}$$

Single User System

- or spreading waveform for DSSP system

$$p(t) = \sum_{j=0}^{N-1} c_j \psi(t - jT_c)$$

N is the spreading gain and

- $c_0, c_1, \ldots, c_{N-1}$ is a pseudorandom spreading code ($c_j \in \{+1, -1\}$),
- $\psi(\cdot)$ chip waveform and $T_c \triangleq T/N$ chip interval
- chip waveform may be a unit-energy rectangular pulse of duration T_c: $\psi(t) = p_{T_c}(t)$
- repeat the same spreading code in every symbol interval
- system with long spreading codes, the periodicity is much longer than a single symbol interval and varies spreading code from symbol to symbol

$$p_i(t) = \sum_{j=0}^{N-1} c_j^i \psi(t - jT_c)$$

Single User System

- spread spectrum modulation can take the form of frequency hopping
- carrier frequency is changed over time according to a pseudorandom pattern
- carrier frequency changes at a rate much slower than the symbol rate - slow frequency hopping
- fast hopping - the carrier changes within a symbol interval
- multicarrier system by choosing $\{w_i(\cdot)\}$ with different frequencies

$$w_i(t) = Ap(t)e^{j(w_i t + \phi_i)}$$

- individual carrier can also be direct-spread - multicarrier CDMA
- OFDM: baseband pulse shape is a unit pulse p_T,
- intercarrier spacing is $1/T$ cycles per second, and the phases are orthogonal at this spacing

Multiple-Access Techniques

- radio resources shared among multiple users
- FDMA - frequency band available is divided into subbands and
- allocated to individual user, users do not transmit signals within other subbands
- TDMA - time is divided into equal-length intervals,
- each user is allowed to transmit throughout the entire allocated frequency band during a given slot
- FDMA allows each user to use part of the spectrum all of the time
- TDMA allows each user to use all of the spectrum part of the time
- FDMA and TDMA systems are intended to assign orthogonal channels to all active users by giving each, for exclusive use, a slice of the available frequency band or transmission time
- channels are said to be orthogonal because interference between user does not

Multiple-Access Techniques

- code-division multiple access (CDMA) assigns channels in a way that
- allows all users to use all of the available time and frequency resources simultaneously,
- through the assignment of a pattern or code to each user
- that specifies the way in which these resources will be used by that user
- spread spectrum modulation - pattern is the pseudorandom code
- that determines the spreading sequence in the case of direct sequence or the hopping pattern in the case of frequency hopping
- channel is defined by a particular pseudorandom code, and
- each user is assigned a channel by being assigned a pseudorandom code
- for a system of K users

$$x_k(t) = \sum_{i=0}^{M-1} b_k[i] w_{i,k}(t) \qquad k = 1, 2, \ldots, K$$

$w_{i,k}(\cdot)$ represents ith modulation waveform of user k

Multi User System

- each user in a multiple-access system can be modeled in the same way as in a single-user system
- if the waveforms $\{w_{i,k}(\cdot)\}$ are of the form of sinusoidal with different carrier frequencies $\{w_k\}$ - FDMA
- if they are with time-slotted amplitude pulses $\{p_k(\cdot)\}$ - TDMA
- if they are spread-spectrum signals of this form but with different pseudorandom spreading codes or hopping patterns - CDMA
- another aspect of wireless network
  - ambient noise, propagation losses, multipath, interference
  - properties arising from the use of multiple antennas
- ambient noise - thermal motion of electrons on the antenna and the receiver electronics and from background radiation sources
- modeled as a very wide bandwidth and no particular deterministic structure (e.g. AWGN)

Multi User System

- propagation losses: diffusive losses and shadow fading
- diffusive losses due to open nature of wireless channel, energy decreases with the square of the distance between antenna and source
- shadow fading results from the presence of objects, modeled by an attenuation in signal amplitude that follows a log-normal distribution
- multipath - multiple copies of a transmitted signal are received at the receiver
- multipath is manifested in several ways
  - degree of path difference relative to the wavelength of propagation
  - degree of path difference relative to the signaling rate
  - relative motion between the transmitter and receiver
  - results into Rayleigh fading or frequency-selective fading or time-selective fading

Multi User System

- multipath from scatterers that are spaced very close together will cause a random change in the amplitude of the received signal
- resulting received amplitude is often modeled as being a complex Gaussian random variable
- random amplitude whose envelope has a Rayleigh distribution - termed as Rayleigh fading
- when the scatterers are spaced so that the difference in their corresponding path lengths are significant relative to a wavelength of the carrier and add constructively or destructively
- this is a fading depends on the wavelength of radiation - frequency-selective fading
- when there is relative motion between the transmitter and receiver, this fading depends on time - time-selective fading
- when the difference in path lengths in such that time delay of arrival along different paths is significant relative to a symbol interval, results in dispersion of the transmitted signal and causes ISI

Multi User System

- wideband signaling methods such as spread spectrum - a countermeasure to frequency-selective fading
- dividing a high-rate signal into many parallel lower-rate signal - OFDM mitigates channel dispersion on high-rate signals
- multiple access interference (MAI) - arising from other signals in the same network as
- the signal of interest (if signals received) are not orthogonal to one another
- co-channel interference - due to signals different networks but operating in same frequency band
- the above phenomena can be incorporated into a general analytical model for a wireless multiple-access channel

$$r(t) = \sum_{k=0}^{K} \sum_{i=0}^{M-1} b_k[i] \int_{-\infty}^{\infty} g_k(t,u) w_{i,k}(u) du + i(t) + n(t)$$

Multi User System

- $g_k(t,u)$ impulse response of a linear filter representing the channel between the kth transmitter and the receiver
- $i(\cdot)$ co-channel interference and $n(\cdot)$ ambient noise, in general, all are random processes
- co-channel interference and channel impulse responses are structured and can be parameterized
- pure multipath channel

$$g_k(t,u) = \sum_{l=1}^{L_k} \alpha_{l,k} \delta(t - u - \tau_{l,k})$$

  L_k number of paths between user k and the receiver, α and τ gain and delay (lth path of kth user)
- model includes frequency-selective fading;
- relative delays will cause constructive and destructive interference at the receiver, depending on the wavelength of propagation and
- Rayleigh fading also using path gains

Multi User System

- composite modulation waveform associated with $b_k[i]$:

$$f_{i,k}(t) = \int_{-\infty}^{\infty} g_k(t,u) w_{i,k}(u) du$$

- if these waveforms are not orthogonal for different values of i, ISI will result
- higher-rate transmission are more likely to encounter ISI than are lower-rate transmission
- if the composite waveform for different values of k are not orthogonal, MAI will result
- this can happen in CDMA when pseudorandom code sequences used by different users are not orthogonal,
- this happens in FDMA and TMDA due to the effects of multipath or asynchronous transmission

Multi User System: MIMO

- model can be generalized for multiple antennas at the receiver

$$\mathbf{r}(t) = \sum_{k=1}^{K} b_k[i] \int_{-\infty}^{\infty} \mathbf{g}_k(t,u) w_{i,k}(u) du + \mathbf{i}(t) + \mathbf{n}(t)$$

- pth component of $\mathbf{g}_k(t,u)$ is the impulse response of the channel between user k and the pth element of the receiving array

$$\mathbf{g}_k(t,u) = \sum_{l=1}^{L_k} \alpha_{l,k} \delta(t - u - \tau_{l,k})$$

- multiple antennas at both the transmitter and receiver called multiple-input/multiple-output (MIMO) systems
- channel transfer functions are matrices with the number of rows equal to the number of receiving antennas and the number of columns equal to the number of transmitting antennas at each source

Multi User Detection

- basic receiver signal processing
- matched filter / RAKE receiver
- say single user $K = 1$ channel impulse $g_1(\cdot,\cdot)$ is known to receiver,
- there CCI $i(\cdot) = 0$, the ambient noise is AWGN with spectral height σ^2

$$r(t) = \sum_{i=0}^{M-1} b_1[i] f_{i,1}(t) + n(t)$$

$$f_{i,1}(t) = \int_{-\infty}^{\infty} g_1(t,u) w_{i,1}(u) du$$

say, there is a single symbol to be transmitted $M = 1$ received waveform

$$r(t) = b_1[0] f_{0,1}(t) + n(t)$$

- optimal inferences about the symbol $b_1[0]$ using likelihood function of observations conditional on the symbol $b_1[0]$

Multi User Detection

$$\mathcal{L}(r(\cdot)|b_1[0]) = \exp\left\{\frac{1}{\sigma^2}\left[2\Re\left\{b_1^*[0]\int_{-\infty}^{\infty} f_{0,1}^*(t)dt\right\} - |b_1[0]|^2 \int_{-\infty}^{\infty} |f_{0,1}|^2 dt\right]\right.$$

asterik - complex conjugation $\Re(\cdot)$ real part of argument

- optimal inferences about $b_1[0]$ can be made using ML or MAP
- using ML that maximizes $\mathcal{L}(r(\cdot)|b_1[0])$ over the symbol alphabet $\mathcal{A}$

$$\hat{b}_1[0] = \arg\left\{\max_{b\in\mathcal{A}} \mathcal{L}(r(\cdot)|b_1[0] = b)\right\}$$

$$= \arg\left\{\max_{b\in\mathcal{A}}\left[2\Re\left\{b^*\int_{-\infty}^{\infty} f_{0,1}^*(t)r(t)dt\right\} - |b|^2\int_{-\infty}^{\infty}|f_{0,1}(t)|^2 dt\right]\right\}$$

- the symboel estimate - the soultion to the problem is $\min_{b\in\mathcal{A}} |b - z|^2$

$$z = \frac{\int_{-\infty}^{\infty} f_{0,1}^*(t) r(t) dt}{\int_{-\infty}^{\infty} |f_{0,1}(t)|^2 dt}$$

Multi User Detection

- ML symbol estimate is the closet point in the symbol alphabet to the observable z
- for BPSK ML symbol estimate is

$$\hat{b}_i[0] = sign\{\Re\{z\}\} = sign\left\{\Re\left\{f_{0,1}^*(t)r(t)dt\right\}\right\}$$

$sign\{\cdot\}$ denotes signum function:

$$sign\{x\} = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ +1 & x > 0 \end{cases}$$

- choices of symbol alphabet are M-ary phase shift keying MPSK and quadrature amplitude modulation QAM

Multi User Detection

- MPSK symbol alphabet is

$$\mathcal{A} = \left\{ e^{j2\pi m/M} | m \in \{0, 1, \ldots, M-1\} \right\}$$

  or some rotation of this set around the unit circle
- QAM symbol alphabet containing $M \times N$ values is

$$\mathcal{A} = \{ b_R + jb_I | b_R \in \mathcal{A}_R and b_I \in \mathcal{A}_I \}$$

  $\mathcal{A}_R$ and $\mathcal{A}_I$ are discrete sets of amplitudes containing M and N points respectively, with $M = N$

$$\mathcal{A}_R = \mathcal{A}_I = \left\{ \pm\frac{1}{2}, \pm\frac{3}{2}, \ldots, \pm\frac{M}{4} \right\}$$

  or a scaled version of this choice
- BPSK $\mathcal{A} = \{-1, +1\}$
- for MPSK ML symbol choice that whose angle is closest to the anlge of complex number z
- for QAM ML symbol estimate are decoupled with $\Re\{b\}$
- being chosen to be the closest element of $\mathcal{A}_R$ to $\Re\{z\}$

Multi User Detection

- MAP symbol detection $b_1[0]$ is random variable taking values in $\mathcal{A}$ with known probabilities
- MAP estimate via Bayes' formula is

$$P(b_1[0] = b | r(\cdot)) = \frac{\mathcal{L}(r(\cdot)|b_1[0] = b)P(b_1[0] = b)}{\sum_{a \in \mathcal{A}} \mathcal{L}(r(\cdot)|b_1[0] = a)P(b_1[0] = a)}$$

- MAP estimate

$$\hat{b}_1[0] = \arg\left\{ \max_{b \in \mathcal{A}} P(b_1[0] = b | r(\cdot)) \right\}$$

$$= \arg\left\{ \max_{b \in \mathcal{A}} [\mathcal{L}(r(\cdot)|b_1[0] = b)P(b_1[0] = b)] \right\}$$

- if symbol are equiprobabile ML and MAP estimates are same

Multi User Detection

- single user, single symbol, known channel case
- the receiver signal processing task is to compute the term

$$y_1[0] = \int_{-\infty}^{\infty} f_{0,1}^* r(t) dt$$

- this structure is called a correlator because
- it correlates the received signal $r(\cdot)$ with knwon composite signaling waveform $f_{1,0}(\cdot)$
- this structure can be implemented by sampling the output of a time invariant linear filter

$$\int_{-\infty}^{\infty} f_{0,1}^* r(t) dt = (h * r)(0)$$

- convolution between h and r
- h is impulse response of the time invariant linear filter

$$h(t) = f_{0,1}^*(-t)$$

- this structure is called a matched filter,

Multi User Detection

- the impulse response is matched to the composite waveform on which the symbol is received
- the composite signaling waveform has a finite duration so that
- $h(t) = 0$ for $t < -D \leq 0$
- the matched filter receiver can be implemented by sampling at time D
- the output of the causal filter with the impulse response

$$h_D(t) = \begin{cases} f_{0,1}^*(D-1) & t \geq 0 \\ 0 & t < 0 \end{cases}$$

- if signaling waveform $s_{0,1}(t)$ has duration $[0, T]$ and the channel has delay spread τ_d
- the composite signaling waveform will have this property with $D = T + \tau_d$

Multi User Detection

- a special case of correlator - a pure multipath channel in which
- the channel impulse response is

$$g_k(t,u) = \sum_{l=1}^{L_k} \alpha_{l,k}\delta(t-u-\tau_{l,k})$$

- the composite function

$$f_{0,1}(t) = \sum_{l=1}^{L_1} \alpha_{l,1} s_{0,1}(t-\tau_{l,1})$$

- the correlator output

$$y_1[0] = \sum_{l=1}^{L_1} \alpha_{l,1}^* \int_{-\infty}^{\infty} s_{0,1}^*(t-\tau_{l,1}) r(t) dt$$

- a configuration known as RAKE receiver

Multi User Detection

- Equalization
- there is more than one symbol in the frame $M > 1$
- likelihood function of observations $r(\cdot)$ conditioned on the entire frame of symbols $b_1[0], b_1[1], \ldots, b_1[M-1]$
- $\mathcal{L}(r(\cdot)|b_1[0], b_1[1], \ldots, b_1[M-1])$

$$= \exp\left\{\frac{1}{\sigma^2}\left[2\Re\left\{\mathbf{b}_1^H\mathbf{y}_1\right\} - \mathbf{b}_1^H\mathbf{H}_1\mathbf{b}_1\right]\right\}$$

- H conjugate transpose - Hermitian transpose, $\mathbf{b}_1$ column vector whose ith component is $b_1[i]$, $i = 0, 1, \ldots, M-1$
- $\mathbf{y}_1$ its i th component

$$y_1[i] = \int_{-\infty}^{\infty} f_{i,1}^*(t) r(t) dt$$

- $\mathbf{H}_1$ $M \times M$ whose (i,j)th element is cross correlation between $f_{i,1}(t)$ and $f_{j,1}(t)$

Multi User Detection

$$\mathbf{H}_1[i,j] = \int_{-\infty}^{\infty} f_{i,1}^*(t) f_{j,1}(t) dt$$

- likelihood function depends on $r(\cdot)$ through vector $\mathbf{y}_1$ of correlator outputs
- this vector is sufficient statistic for making inferences about the $\mathbf{b}_1$
- maximum likelihood detection

$$\hat{\mathbf{b}}_1 = \arg\left\{\max_{\mathbf{b}\in\mathcal{A}^M}\left[2\Re\left\{\mathbf{b}^H\mathbf{y}_1\right\} - \mathbf{b}^H\mathbf{H}_1\mathbf{b}\right]\right\}$$

- if $\mathbf{H}_1$ is a diagonal matrix (all of its off-diagonal elements are zero) decouples into a set of M independent problems of single symbol type
- the solution in this case

$$\hat{b}_1[i] = \arg\max_{b\in\mathcal{A}} |b - z_1[i]|^2$$

$$z_1[i] = \frac{y_1[i]}{\int_{-\infty}^{\infty} |f_{i,1}(t)|^2 dt}$$

Multi User Detection

- in general case there is intersymbol interference, will not decouple
- the optimization must take place over the entire frame
- a problem known as sequence detection
- MAP estimate very high complexity

$$P(b_1[0] = b|r(\cdot)) = \frac{\sum_{\{\mathbf{a}\in\mathcal{A}^M | a_i = b\}} \mathcal{L}(r(\cdot)|\mathbf{b}_1 = \mathbf{a}) P(\mathbf{b}_1 = \mathbf{a})}{\sum_{\{\mathbf{a}\in\mathcal{A}^M\}} \mathcal{L}(r(\cdot)|\mathbf{b}_1 = \mathbf{a}) P(\mathbf{b}_1 = \mathbf{a})}$$

- the dynamic programming solution knwon as maximum likelihood sequence detector
- a number of lower-complexity algorithms have been devised
- examining sufficient statistic vector $\mathbf{y}_1$, can be written as

$$\mathbf{y}_1 = \mathbf{H}_1\mathbf{b}_1 + \mathbf{n}_1$$

Multi User Detection

- $\mathbf{n}_1$ is complex Guassian random vecotr with indepedent real and imaginary parts having
- identical $\mathcal{N}(\mathbf{0}, \frac{\sigma^2}{2}\mathbf{H}_1)$ distributions
- above equation describes a linear model and the goal of equalization is to fit this model with data vector $\mathbf{b}_1$
- ML and MAP are two ways but exponential complexity with exponent equal to bandwidth of $\mathbf{H}_1$
- the vector $\mathbf{b}_1$ takes on values form a discrete set
- one way is to fit linear model without constraining $\mathbf{b}_1$ to be discrete and then to
- quantize the resulting (continuous) estimate of $\mathbf{b}_1$ into symbol estimates

Multi User Detection

- linear fit $\mathbf{My}_1$ as continuous estimate of $\mathbf{b}_1$
- $\mathbf{M}$ is $M \times M$ matrix
- i the symbol decision is $\hat{b}_1[i] = q([\mathbf{My}_1]_i)$
- $[\mathbf{My}_1]_i$ denotes i the component of $\mathbf{My}_1$
- $q(\cdot)$ dentoes quantizer mapping the complex numbers to the symbol alphabet $\mathcal{A}$
- various choce of $\mathbf{M}$ lead to different linear equalizers
- $\mathbf{M} = \mathbf{I_M}$ $M \times M$ identity matrix
- the resulting linear detector is the common matched filter, which is optimal in the absence of ISI
- matched filter ignores ISI
- if $\mathbf{H}_1$ is invertible choice $\mathbf{M} = \mathbf{h}_1^{-1}$
- forces the ISI to zero

$$\mathbf{H}_1^{-1}\mathbf{y}_1 = \mathbf{b}_1 + \mathbf{H}_1^{-1}\mathbf{n}_1$$

Multi User Detection

- is knwon as zero forcing equalizer ZFE
- it would be optimal, perfect decision in absence of AWGN
- a tradeoff between the extremes is effected by minimum mean square error MMSE linear equalizer
- which chooses $\mathbf{M}$ to give an MMSE fit of the model assuming the symbols are independent of the noise
- this results in choice

$$\mathbf{M} = (\mathbf{H}_1 + \sigma^2 \sum_b^{-1})^{-1}$$

- $\sum_b$ denotes covariance matrix of the symbol $\mathbf{b}_1$ (this will be in the form of a constant time $\mathbf{I_M}$)
- for multi user detection symbols sorted by symbol number and then by user number

Thank You