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Q.1 (a) Define the following terms using necessary variable, equations and diagrams:

a) Scene Irradiance:

Scene Irradiace define as the amount of the light is falling on a surface (object).

b) Scene Radiance:

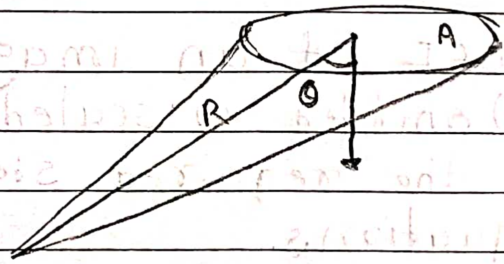
Scene radiance is define as the amount of light radiated from a surface (object)

c) Image Irradiance:

Image irradiance is define as the amount of light falling on the image plane.

d) Solid angle:

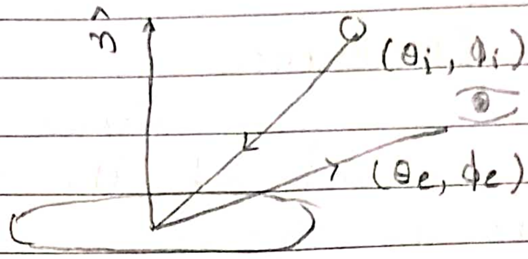
Solid angle of a cone of directions is defined as the area cut out by the cone on the unit sphere.



A small planar patch of area A at distance R from the origin subdends on a solid angle.

$$\Omega = \frac{A\cos\theta}{R^2}$$

e) Bidirectional Reflectance Distribution Function



BRDF $f(\theta_i, \phi_i, \theta_e, \phi_e)$ tells us how bright a surface appears when viewed from one direction while light falls on light falls on it from another.

the amount of light falling on the surface from the direction (θ_i, ϕ_i) the raddiance be $\delta E(\theta_i, \phi_i)$

the brightness of the surface as seen from the direction (θ_e, ϕ_e) the radiance be $\delta L(\theta_e, \phi_e)$.

BRDF is the ratio of radiance to Irradiance.

$$f(\theta_i, \phi_i, \theta_e, \phi_e) = \frac{\delta L(\theta_e, \phi_e)}{\delta E(\theta_i, \phi_i)}$$

Q.:-1 (b) What happens to DFT of an image if image is a) rotated b) shifted c) scaled? Illustrate through the required steps of derivation and necessary equations.

a) rotated
Using the polar coordinates
$x = r\cos\theta$ $y = r\sin\theta$ $u = \omega\cos\varphi$ $v = \omega\sin\varphi$

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$$f(r, \theta) \Leftrightarrow F(\omega, \phi)$$

Property says that

$$f(r, \theta + \theta_0) \Leftrightarrow F(\omega, \phi + \theta_0)$$

which indicates that rotating $f(x,y)$ by an angle θ_0 rotates $F(u,v)$ by the same angle. Conversly, rotating $F(u,v)$ rotates $f(x,y)$ by the same angle.

b) shifted

$$f(x,y)\, e^{j2\pi(u_0 x/M + v_0 y/N)} \Leftrightarrow F(u-u_0, v-v_0)$$

$$f(x-x_0, y-y_0) \Leftrightarrow F(u,v)\, e^{-j2\pi(x_0 u/M + y_0 v/N)}$$

which indicates that multiplying $f(x,y)$ by the exponential shown shifts the origin of the DFT to (u_0, v_0) and conversly, multiplying $F(u,v)$ by the negative of that exponential shifts the origin of $f(x,y)$ to (x_0, y_0)

c) scaled.

$$DFT\{f(am, bn)\} = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} f(am, bn)\, e^{-j\frac{2\pi}{N}mk} \cdot e^{-j\frac{2\pi}{N}nk}$$

By multiplying and dividing the power of the exponential term $e^{-j\frac{2\pi}{N}mk}$ with a and $e^{-j\frac{2\pi}{N}nk}$ with b

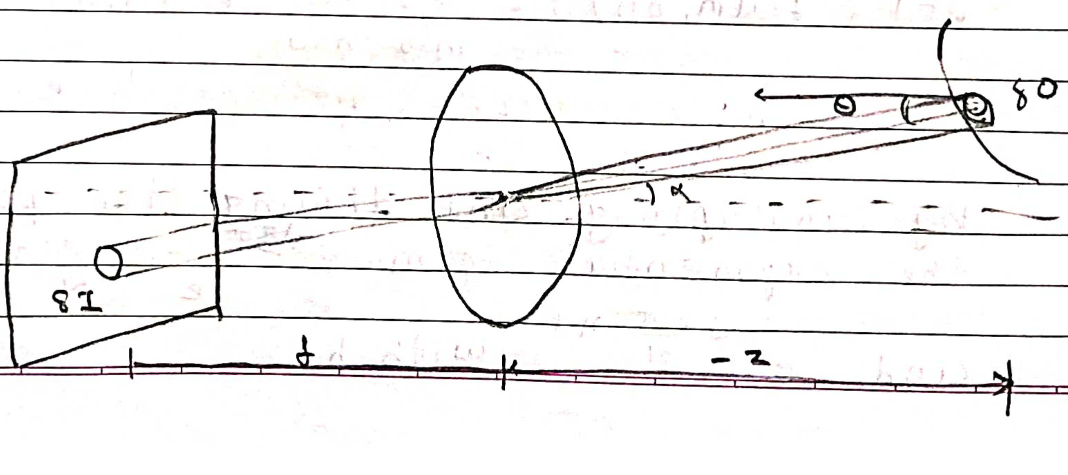
$$= \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} f(am, bn)\, e^{-j \frac{2\pi}{N} mk\left(\frac{a}{a}\right)} \, e^{-j \frac{2\pi}{N} mk\left(\frac{b}{b}\right)}$$

$$= \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} f(am, bn)\, e^{-j \frac{2\pi}{N} ma\left(\frac{k}{a}\right)} \, e^{-j \frac{2\pi}{N} nb\left(\frac{k}{b}\right)}$$

So, that

$$DFT\{f(am, bn)\} = \frac{1}{|ab|} F(k/a, l/b)$$

Q:2 Derive an equation for the mapping of object patch to image patch with necessary diagram and equations. Also show that image irradiance E is proportional to scene radiance L. Assume diameter of lens is d and the focal length is f. The distance between the object patch and in 3D and lens is z.



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- the solid angle of the cone of rays leading to the patch on the object is equal to the solid angle of the cone of rays leading to image patch.

- apparent of the image patch as seen from the center of the lens is $\delta I \cos\alpha$.

- the distance from center of the lens is $f / \cos\alpha$

Solid angle substended by the image patch

$$\frac{\delta I \cos\alpha}{(f / \cos\alpha)^2}$$

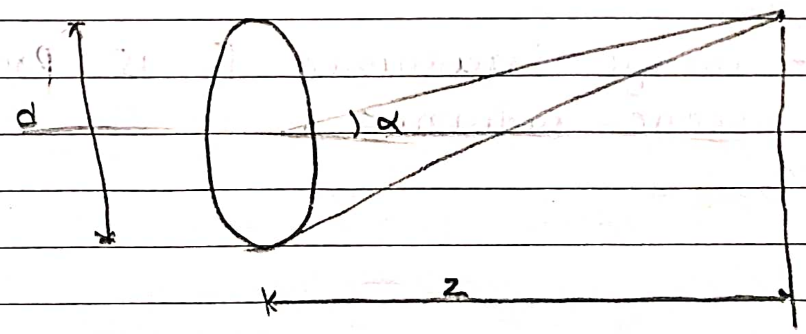
- the solid angle of the object patch as seen from the center of the lens is

$$\frac{\delta O \cos\theta}{(z / \cos\alpha)^2}$$

- with two solid angles are to be equal

$$\frac{\delta O}{\delta I} = \frac{\cos\alpha}{\cos\theta}\left(\frac{z}{f}\right)^2$$

- how much of the light emmited by the surface makes its ways through the lens.



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- Solid angle subsdended by the lens as seen from the object patch.

$$\Omega = \frac{\pi}{4} \frac{d^2 \cos\alpha}{4(z/\cos\alpha)^2} = \frac{\pi}{4}\left(\frac{d}{z}\right)^2 \cos^3\alpha$$

- the power of the light originating on the patch and passing through the lens is

$$\delta P = L\,\delta O\,\Omega \cos\theta = L\,\delta O \frac{\pi}{4}\left(\frac{d}{z}\right)^2 \cos^3\alpha \cos\theta$$

L is the radiance of the surface in the direction towards the lens. This power is concentranted in image.

- no light from other areas reaches this image patch

$$E = \frac{\delta P}{\delta I} = L \frac{\delta O}{\delta I} \frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^3\alpha \cos\theta$$

E is the radiance of the image at patch. substituting $\delta O / \delta I$

$$E = L \frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha$$

$$\therefore E \propto L$$

brightness - image irradiance E is propotional to scene radiance L

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Q::1 Show that zooming of a region of an image by a factor of 2 is result of the following operation: given image is interlanced by rows and columns of zero and convolve with

$$h = \begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}$$

state the final result of an image region represented by

$$\begin{matrix} 1 & 4 & 1 \\ 2 & 5 & 3. \end{matrix}$$

⇒ Zoom image:

$$\begin{bmatrix} 1 & 1 & 4 & 4 & 1 & 1 \\ 1 & 1 & 4 & 4 & 1 & 1 \\ 2 & 2 & 5 & 5 & 3 & 3 \\ 2 & 2 & 5 & 5 & 3 & 3 \end{bmatrix}$$

Convolution:

first image is interlanced by rows & colums by zero.

1st 2nd

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 5 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

now convolve with h to

$$\begin{bmatrix} 1 & 1 & 4 & 4 & 1 & 1 \\ 1 & 1 & 4 & 4 & 1 & 1 \\ 2 & 2 & 5 & 5 & 3 & 3 \\ 2 & 2 & 5 & 5 & 3 & 3 \end{bmatrix}$$

which is same as zooming of an image.

Q.1.2 Show that in general the convolution of two arrays of sizes $(M1 \times N1)$ and $(M2 \times N2)$ yields an array of size $(M1 + M2 - 1) \times (N1 + N2 - 1)$ Using a discrete convolution formula state the output of the convolution of two arrays. $x(m, n) = \begin{matrix} 1 & 4 & 1 \\ 2 & 5 & 3 \end{matrix}$

$$h(m, n) = \begin{matrix} 1 & 1 \\ 1 & -1 \end{matrix}$$

For convolution First interchange rows

$$h(m, n) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

Now interchang columns.

$$= \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

→ Pad image and convolve with new h

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 2 & 5 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 5 & 5 & 1 \\ 3 & 10 & 5 & 2 \\ 2 & 3 & -2 & -3 \end{bmatrix}$$

Output matrix size (3×4) which is same as $(2+2-1) \times (2+3-1)$.

Q.4 Describe canny edge detector.

A canny edge detector is a multi step algorithm to detect the edges for any input image. It involves the below-mentioned steps to be followed while detecting edges of an image.

1. Removal of a noise in input image using a Gaussian filter.

2. Compute the gradient magnitudes and angle images.

3. Apply non maxima supression to the gradient magnitude image.

4. Use double thresholding an connectivity analysis to detect and link edges.

Step :- 1

Let $f(x,y)$ denote the input image and $G(x,y)$ denotes the Gaussion function.

We get smooth image by convolving G and f

$$f_s(x,y) = G(x,y) * f(x,y)$$

Step :- 2

$$M(x,y) = \sqrt{g_x^2 + g_y^2}$$

$$\alpha(x,y) = \tan^{-1}\left[\frac{g_y}{g_x}\right]$$

Step :- 3 Non maxima supression.

Let d_1, d_2, d_3 and d_4 denote the four basic directions just discussed for a 3×3 region : horizontal, $-45°$, vertical and $+45°$ respectively. We can formulate the following non maxima suppression scheme for a 3×3 region centered at every point (x,y) in $\alpha(x,y)$

1. Find the direction d_k that is closest to $\alpha(x,y)$
2. If the value of $M(x,y)$ is less than at least one of its two neighbours along d_k, let $g_N(x,y) = 0$ (suppression); otherwise, let $g_N(x,y) = M(x,y)$

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Step: 4 Thresholding.

Two additionals images are created.

$$g_{NH}(x, y) = g_N(x, y) \geq T_H$$
$$g_{NL}(x, y) = g_N(x, y) \leq T_L$$

$g_{NH}(x,y)$ will have fewer nonzero pixels than $g_{NL}(x,y)$ in general but all the nonzero pixels in $g_{NH}(x,y)$ will be contained in $g_{NL}(x,y)$ because the latter image is formed with a lower threshold.

$$g_{NL}(x, y) = g_{NL}(x, y) - g_{NH}(x, y)$$

The non zero pixels in $g_{NH}(x,y)$ and $g_{NL}(x,y)$ may be viewed as being 'strong' and 'weak' edge pixels respectively.

After the thresholding operations, all strong pixels in $g_{NH}(x,y)$ are assumed to be valid edge pixels and are so marked immediately. Depending on the value of T_H, the edges in $g_{NH}(x,y)$ typically have gaps. Longer edges are formed using the following:

a) Locate the next unvisited edge pixel p, in $g_{NH}(x,y)$.

b) Mark as valid edge pixels all the weak pixels in $g_{NL}(x,y)$ that are connected to p using say 8 connectivity.

c) If all non zero pixels in $g_{NH}(x,y)$ have been visited then go to step d else return to step a

c) ~~I/t /at/ nonzero / pixel~~

d) Set to zero all pixels in $g_{NL}(x, y)$ that were not marked as valid edge points.

After this output image is formed by appending to $g_{NH}(x, y)$ all the non zero pixels from $g_{NL}(x, y)$.

Q :- 5 Derive the filtering mask of size 5x5 for Laplacian of Gaussian.

G is the Gaussian function:

$$G(x, y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$\nabla^2 G(x, y)$ is given as Laplacian of Gaussian

$$\therefore \nabla^2 G(x,y) = \frac{\partial^2 G(x,y)}{\partial x^2} + \frac{\partial^2 G(x,y)}{\partial y^2}$$

$$= \frac{\partial}{\partial x}\left(\frac{-x}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}\right) + \frac{\partial}{\partial y}\left(\frac{-y}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}\right)$$

$$= \left[\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2}\right] e^{-\frac{x^2+y^2}{2\sigma^2}} +$$

$$\left[\frac{y^2}{\sigma^4} - \frac{1}{\sigma^2}\right] e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$= \left[\frac{x^2+y^2-2\sigma^2}{\sigma^4}\right] e^{-\frac{x^2+y^2}{2\sigma^2}}$$

above is LoG

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5x5 LOG

0	0	-1	0	0
0	-1	-2	-1	0
-1	-2	-16	-2	-1
0	-1	-2	-1	0
0	0	-1	0	0

Q.3 In some applications it is useful to model the histogram of input images as Gaussian probability density function modelling image indensity r with m and σ as parameters, namely, the mean and standard deviation of the Gaussian PDF. The approach is to let m and σ be measures of average gray level and contrast of a given image. What is the transformation function you would use for histogram equalization?

Probability density function modeling image intensity r with m and σ are as parameters, namely the mean and standard deviation of the Gaussian PDF. The approach is to let m and σ be measures of average gray level and contrast of a given image. What is the transformation function you would use for histo eqhl

$$P_r(r) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(r-m)^2}{2\sigma^2}}$$

$$S = T(r) = \frac{1}{(L-1)} \int_0^L \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(w-m)^2}{2\sigma^2}} dw$$

$$= \frac{1}{L-1\sqrt{2\pi}\sigma} \int_0^L e^{\frac{-(w-m)^2}{2\sigma^2}} dw$$

$$= \frac{1}{(L-1)\sqrt{2\pi}\sigma} \left[\frac{\sqrt{\pi}\ \mathrm{erf}(w-m)}{4\sigma^2} \right]_0^L$$

$$= \frac{1}{(L-1)\sqrt{2\pi}\sigma} \times \frac{\sqrt{\pi}}{4\sigma^2} \left[\mathrm{erf}(L-m) - \mathrm{erf}(-m) \right]$$

$$= \frac{1}{(L-1)4\sqrt{2}\sigma^2} \left[\mathrm{erf}(L-m) - \mathrm{erf}(-m) \right]$$

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Q:-3 Derive an equation for BRDF for Lumbeortian surface in presence of extended light source using necessary steps and diagram. Consider sphere as an object illuminated by uniform light source and derive the depth equation for it.

BRDF is the ratio of radiance to irradiance

$$f(\theta_i, \phi_i, \theta_e, \phi_e) = \frac{\delta L(\theta_e, \phi_e)}{\delta E(\theta_i, \phi_i)}$$