Math 461 Section P1
Quiz 5 Solution

Feb 28, 2013

2. A salesman has scheduled two appointments to sell encyclopedias. His first appointment will lead to a sale with probability 0.3, and his second appointment will lead independently to a sale with probability .6. Any sale is equally likely to be either for the deluxe model, which costs \$ 1000, or the satandard model, which costs \$ 500. Determine the probability mass function of X, the total dollar value of all sales.

Solution: The range of X is $\{0, 500, 1000, 1500, 2000\}$.

If we use the following abbreviations for events:

F: the first appointment leads to a sale;

S: the second appointment leads to a sale;

F_1: getting a sale of 500 at the first appointment;

F_2: getting a sale of 1000 at the first appointment;

S_1: getting a sale of 500 at the second appointment;

S_2: getting a sale of 1000 at the second appointment;

Then

$$P(F^c) = 1 - P(F) = 1 - 0.3 = 0.7,$$

$$P(S^c) = 1 - P(S) = 1 - 0.6 = 0.4,$$

$$F_i \subset F, \quad S_i \subset S \quad (i = 1, 2)$$

and

$$P(F_1 \,|\, F) = P(F_2 \,|\, F) = P(S_1 \,|\, S) = P(S_2 \,|\, S) = 0.5.$$

So for $i = 1, 2$ we have

$$P(F_i) = P(F_i|F)P(F) + P(F_i|F^c)P(F^c) = 0.5 \times 0.3 + 0 = 0.15,$$

and

$$P(S_i) = P(S_i|S)P(S) + P(S_1|S^c)P(S^c) = 0.5 \times 0.6 + 0 = 0.3.$$

Since two appointments are independent of each other,

$$\begin{aligned} P(X=0) &= P(F^c \text{ and } S^c) \\ &= P(F^c)P(S^c) \\ &= (1-.3)(1-.6) \\ &= .28; \end{aligned}$$

$$\begin{aligned} P(X=500) &= P((F_1 \text{ and } S^c) \text{ or } (F^c \text{ and } S_1)) \\ &= P(F_1)P(S^c) + P(F^c)P(S_1) \\ &= 0.15 \times 0.4 + 0.7 \times 0.3 \\ &= .27; \end{aligned}$$

$$\begin{aligned} P(X=1000) &= P((F_2 \text{ and } S^c) \text{ or } (F_1 \text{ and } S_1) \text{ or } (F^c \text{ and } S_2)) \\ &= P(F_2)P(S^c) + P(F_1)P(S_1) + P(F^c)P(S_2) \\ &= 0.15 \times 0.4 + 0.15 \times 0.3 + 0.7 \times 0.3 \\ &= .315; \end{aligned}$$

$$\begin{aligned} P(X=1500) &= P((F_1 \text{ and } S_2) \text{ or } (F_2 \text{ and } S_1)) \\ &= P(F_1)P(S_2) + P(F_2)P(S_1) \\ &= 2 \times 0.15 \times 0.3 \\ &= 0.09 \end{aligned}$$

and

$$\begin{aligned} P(X=2000) &= P(F_2 \text{ and } S_2) \\ &= P(F_2)P(S_2) \\ &= 0.15 \times 0.3 \\ &= .045. \end{aligned}$$