

- demand of higher data rates - performance of wireless system

Multi Input and Multi Output (MIMO)

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- data rate 1 gigabit/second
- capacity = $BW \log_2(1 + SNR)$ shannon's law
- channel capacity in bits per second

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- channel capacity in bits per second
- higher data rates by increasing bandwidth
- higher bandwidth - more data
- boosting power - more range
- increase in SNR marginal gain in channel throughput
- spectrum become a scarce and expensive resource
- limited bandwidth and transmit power
- goal: high data rate and high link quality



Technology for Higher Demand

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- to meet this challenge - MIMO
- Multiple antennas and multiple signal streams to boost data rate, range, connection reliability
- higher data rate over longer distances motivations behind MIMO OFDM
- 802.11n, 802.1ac Wi-Fi, WiMAX, LTE (Long Term Evolution) 4G supports MIMO
- supports enhanced data throughput under interference, fading and multipath



Multiple Input Multiple Output (MIMO)

- 1984 Jack Winters at Bell lab wrote a patent on wireless communication using multiple antennas
- 1993 Paulraj and Kailath concept of spatial multiplexing using MIMO
- 2006 802.11n
- boost system capacity and enhance reliability of link uses several diversity schemes
- N_T transmit antennas and N_R receive antennas - MIMO system model
- $x_1 - T_{x_1} \ x_2 - T_{x_2} \rightarrow H(N_R \times N_T) \rightarrow R_{x_1} - y_1 \ R_{x_2} - y_2 \dots$
- Antennas $T_{x_1}, \dots, T_{x_{N_T}}$ respectively send signal x_1, \dots, x_{N_T} to receive antennas $R_{x_1}, \dots, R_{x_{N_R}}$
- received signals y_1, \dots, y_{N_R} respectively
- each receive antenna combines incoming signal which coherently add up



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- to **improve link reliability** - reducing fading by achieving diversity gain



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- time and frequency domain processing are at limits but space is not - MIMO
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- MIMO improves BER or data rate using multiple TX/RX
- core scheme of MIMO: space time coding (STC)
- two main functions of STC: diversity and multiplexing
- maximum performance needs tradeoff between diversity and multiplexing
- N TXs and M RXs; $y(k) = Hx(k) + v(k)$
- multi-path propagation, multiple antennas at transmitter and receiver
- establish multiple parallel channels operate on same frequency band at same total radiated power



Shannon's Theorem

- Shannon bound for SISO

$$C = \log_2(1 + SNR) \quad SNR = P_T/\sigma_v^2$$

- each extra bps/Hz requires a doubling of TX power
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- MIMO - transmit and receive diversity

$C = \log_2(1 + SNR)$ MN channels

- parallel spatial channels

$C = Q \log_2(1 + SNR) \quad Q = \min(M, N)$



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$$C = \log_2(1 + SNR) \text{ bits/s/Hz}$$

- R data rate bits/symbol, R_s - symbol rate symbols/second, W - allotted BW Hz
- **spectral efficiency** = number of bits transmitted per second per Hz

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- as a result of filtering / signal reconstruction requirements $R_s \leq W$
- spectral efficiency = R if $R_s = W$
- if transmit data at rate of $R \leq C$ - achieve low P_e



Wireless System

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- **multiple reflections and multiple communication paths** between two radio terminals cause
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- **fading - amplitude and phase changes**
- ellipsoidal fading model - replica by random reflection
- **constructive or destructive interference**
- worst case 180 degree phase - cause signal cancellation



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$$H(\tau, t) = \begin{bmatrix} h_{1,1}(\tau, t) & h_{1,2}(\tau, t) & \dots & h_{1,M_T}(\tau, t) \\ h_{2,1}(\tau, t) & h_{2,2}(\tau, t) & \dots & h_{2,M_T}(\tau, t) \\ \vdots & \vdots & \vdots & \vdots \\ h_{M_R,1}(\tau, t) & h_{M_R,2}(\tau, t) & \dots & h_{M_R,M_T}(\tau, t) \end{bmatrix}$$



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- selective fading the case where smaller frequency segments of the signal's spectrum are
- attenuated relative to other remaining frequency segments
- when this occurs, the signal spectrum is distorted and in turn creates communication impairment
- non-selective fading can be countered by providing more signal level margin, or
- using selection diversity techniques to select the best antenna input based on the relative signal strength



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- strongest signal is chosen to make the connection
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- different antenna pick up different signals, filter noise and recombine to generate stronger signal
- basic MIMO - data transmitted scrambled, interleaved and
- divided up into parallel data streams, each which modulates a separate transmitter
- multiple antennas capture different streams, have different phases, travelled different routes and combine into one

Exploiting Multipath

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- each multipath route treated as a separate channel
- separate antennas take advantage of this to transfer more data
- multiplying throughput, range is increased due to antenna diversity
- each receive antenna has a measurement of each transmitted data stream
- maximum data rate per channel grows linearly with the number of different data streams
- that are transmitted in the same channel, providing scalability and reliable link



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- wide bandwidth channel - solution - use a series of narrowband overlapping subcarriers
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- interesting solution to multipath challenge by requiring multiple signal paths
- multiple signal paths to gain knowledge of communication channel



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- lower symbol rates used by narrowband subcarriers **reduces the impact of multipath** signal products
- interesting solution to multipath challenge by requiring multiple signal paths
- **multiple signal paths to gain knowledge of communication channel**
- receiver can recover independent streams from each of transmitter's antennas
- 2 x 2 MIMO produces **two spatial streams** effectively double maximum data rate what achieved in 1 x 1 channel
- **maximum channel capacity of MIMO** - as a function of N spatial streams Capacity = $N \text{ BW} \log_2(1 + \text{SNR})$



Multiple Transmit and Multiple Receive: Generic View

MIMO-OFDM

- for Wi-Fi, OFDM using BPSK, QPSK, 16 phase QAM, depending on data rate
- signals from different antennas combined to reinforce one another, improving SNR
- **smart antenna uses beam forming** to focus transmitted signal energy toward the receiver to strengthen the signal - may provide better range
- beyond 4 by 4 configuration, very little additional gain is achieved
- **2 transmitter and three receivers seems to be the most popular**
- transmitting two or more data streams in the same bandwidth multiplies data rate by the number of streams used
- MIMO 11n allows two 20 MHz channels bonded together into a single 40-MHz channel provide higher data rate



Multiple Transmit and Multiple Receive: Generic View

- in general, m transmit and n receive antennas
- by using the same channel every antenna receives
- not only the direct components intended for it but also
- the indirect components intended for the other antennas
- time dependent narrowband channel is assumed
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- h_{rt} transmission matrix



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- h_{rt} transmission matrix

$$H = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1m} \\ h_{21} & h_{22} & \dots & h_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ h_{n1} & h_{n2} & \dots & h_{nm} \end{bmatrix}$$

$$y = Hx + n$$



MIMO Example: SU MIMO

- **SU - MIMO** - data rate is to be increased for a single UE, called single user MIMO
- base station - two transmit antennas, UE - two receive antennas
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- data to be transmitted is divided into independent data streams
- number of streams M is always less than or equal to the number of antennas
- in the case of asymmetrical ($m \neq n$) antenna constellations,
- it is always smaller or equal the minimum number of antennas
- 4×4 transmit four or fewer streams
- 3×2 transmit two or fewer streams
- theoretically the capacity C increases linearly with the number of streams M

$$C = MB \log_2\left(1 + \frac{S}{N}\right)$$



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- Cyclic delay diversity - CDD
- introduces virtual echoes into OFDM systems
- increases frequency selectivity at the receiver
- in CDD, signals are transmitted by the individual antennas with a time delay introduces additional diversity
- useful as addition to spatial multiplexing

Multi User MIMO

- received signal at antenna R_{x_q}



Multi User MIMO

- **physical models** - channel impulse response evaluated based on experimental measurements for extracting
- **channel propagation parameters** including antenna configuration at transmitter and receiver, antenna polarization, scatters
- propagation parameters such as delay spread, angular spread, spatial correlation and cross polarization discrimination
- **deterministic models** define a channel model according to prediction of propagation signal
- **geometry based** has immediate relation with physical characteristics of propagation channel
- **clusters of scatters** distributed around transmitter and receiver
- scatters result in discrete channel path and can involve **statistical characterization of propagation parameters**



Multi User MIMO

- **analytical models** - based on statistical properties through measurement
- distribution of direction of departure, distribution of direction of arrival
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- MIMO - improves capacity of communication link without the need of increase the transmitter power
- MIMO system capacity evaluated according to following scenarios
 - when no channel state information (CSI) available at transmitter
 - the power is equally split between N_T transmit antennas
 - instantaneous channel capacity (bits/s/Hz)

$$C(H) = \log_2 \left[\det(I_{N_R} + \frac{\gamma}{N_T} HH^*) \right]$$



- γ denotes SNR, $(\cdot)^*$ stands for conjugate transpose operator

Multi User MIMO

- when CSI is available at receiver, SVD is used to derive MIMO channel capacity

$$C_{SVD}(H) = R \log_2 \left[\det(1 + \frac{\gamma}{N_T} HH^*) \right]$$

$R = \min(N_R, N_T)$ rank of channel matrix H

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- when CSI is available at both transmitter and receiver
- the channel capacity is computed by performing water-filling algorithm

$$C_{WF}(H) = \sum_{p=1}^R \log_2 \left[\left(\frac{\lambda_{H,p}\mu}{\sigma_b^2} \right)^+ \right]$$

- $a^+ = \max(a, 0)$, $\lambda_{H,p}$ p-th singular value of channel matrix H

- μ constant scalar satisfies total power constraint, σ_b^2 noise signal power



MIMO: Diversity

- introducing higher modulation types or providing larger bandwidth - achieved by MIMO
- MIMO refers to channel - transmitter is channel input and the receiver channel output
- different diversity modes**
- time diversity** - different timeslots and channel coding
- frequency diversity** - different channels, spread spectrum and OFDM
- spatial diversity** - use of multiple antennas at the transmitter or receiver end
- MIMO increase data rate - spatial multiplexity**



MIMO: Antenna Diversity

- the effect of attenuation can be compensated by
- receiving the signal at two different receiver antenna placement location and
- selecting the best antenna based on the signal strength** or some other receiver performance measure



MIMO: Antenna Diversity

- the effect of attenuation can be compensated by
- receiving the signal at two different receiver antenna placement location and
- selecting the best antenna based on the signal strength** or some other receiver performance measure
- the signal strengths associated with the antennas are generally not correlated in time and/or space and
- when one signal is in a null, the other can be found near a maximum and selected for communication
- the selection of one antenna over the other based on signal quality** or
- selection diversity** is technique that works well for non-selective fading in that
- the antenna selection essentially is a signal strength restoration technique**



MIMO: Antenna Diversity

- selection diversity and other signal strength restoration techniques are not enough to
- equalize the impairments imposed by the multipath channel due to selective fading
- selective fading occurs when the signal bandwidth exceeds the coherence bandwidth of channel
- the coherence bandwidth of the channel is approximately the reciprocal of delay spread of the channel



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- the delay spread is the rms average of delay times of the complex impulse response of the channel h
- h is the received complex envelope at a particular point in space assuming that a carrier signal is modulated by dirac impulse
- the arrival of various signal components can be modeled by FIR filter



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- a flat fading process (constant attenuation versus frequency) and a single fixed time delay (linear phase)
- actually, frequency dependent fading of amplitudes and nonlinear phase response
- if the sidebands of a complex modulated signal where distributed across the transfer function,
- there would be signal distortion
- to compensate these effects the transfer function encountered via communication channel
- must be equalized at the receiver



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- operate on time domain representation of the baseband signal **to achieve equalization**
- many **rely on training sequences and training periods**
- that creates communication overhead



OFDM and Channel Equalization

- a means of spectrally efficient communications,
- also a means of **channel equalization in the frequency domain**
- OFDM spectrum **48 data subcarriers and 4 pilot subcarriers**



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- OFDM spectrum **48 data subcarriers and 4 pilot subcarriers**
- **pilot subcarriers** are not information bearing but are used
- **to help maintain the carrier and timing tracking** in low SNR since there are always BPSK modulated
- OFDM provides a means of **sampling the magnitude and phase of the channel**
- **at any or all of the subcarrier frequencies**,



OFDM and Channel Equalization

- a means of spectrally efficient communications,
- also a means of **channel equalization in the frequency domain**
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- **at any or all of the subcarrier frequencies**,
- since the carrier phase tracked at the receiver and **all of the subcarrier phases are coherent to the main carrier**
- in practice, performing continuous subcarrier phase tracking on each of subcarrier is not executed, rather,



OFDM and Channel Equalization

- short and long training symbols with known patterns are transmitted with each packet
- such that the receiver can determine the channel transfer function, invert it and then
- apply the inverse to the received spectrum in order to equalize the channel on a packet by packet basis

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- apply the inverse to the received spectrum in order to equalize the channel on a packet by packet basis
- how the channel transfer function due to selective fading weights the received spectrum at the receiver
- the magnitude and phase of each of subcarriers provide an estimate of channel transfer function $H_c()$
- equalization transfer function $H_e()$ can be applied to the channel weighted spectrum
- to re-normalize the spectrum such that the effects of the channel can be reduced or eliminated

OFDM and Channel Equalization

$$S_{RX}(\omega) = H_c(\omega)S_{TX}(\omega)$$

$$H_c(\omega) = \frac{S_{RX}(\omega)}{S_{TX}(\omega)}$$



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$$H_c(\omega) = \frac{S_{RX}(\omega)}{S_{TX}(\omega)}$$

$$H_r(\omega) = H_c(\omega)H_e(\omega)$$

$$H_e(\omega) = H_c^{-1}(\omega)$$

$$H_r(\omega) = H_c(\omega)H_c^{-1}(\omega) = 1$$

- using MIMO - additional paths can be used to advantage - by increasing data capacity
- two main formats for MIMO
- spatial diversity - refers to transmit and receive diversity



Exploiting Diversity

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- receivers with selection diversity schemes or linear maximal ratio combining scheme

Exploiting Diversity

- MISO - provides transmitter diversity



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- it couples to the channel at a different point in space such that
- the links will not have the same fading characteristics though to the receive antenna and
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- transmitter signal design can be such that the combining at the single receiver
- can be made in an optimal fashion as is done in space-time coding techniques



MIMO: Spatial Multiplexing

- MIMO systems de-multiplex the source data stream into multiple independent channel streams
- provides both redundancy and channel capacity improvement
- MIMO referred to as **Spatial Division Multiplexing**

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- a single source data stream is multiplexed between two spatial streams
- **direct links and cross links** between two transmitters and two receivers, for example,
- there are four different communication channels that connect the two terminals,
- the channel characterization has a higher complexity



MIMO: Spatial Multiplexing

- **combine data streams** arriving from different paths and at different times to effectively
- **increase receiver signal capturing power**
- if **there are more antennas than spatial streams**; the additional antennas can add receiver diversity and increase range
- increase the capacity of a given channel
- **increasing the number of receiver and transmit antennas**
- it is possible to linearly increase the throughput of the channel with every pair of antennas added to the system
- **SISO there is no diversity and no additional processing required**
- performance limited by interference and fading
- throughput depends upon the channel bandwidth and SNR



MIMO: Spatial Multiplexing

- another method to minimize the effects of a multipath communication channel through its redundancy and channel equalization properties
- MIMO - to **transfer more data at the same time**
- **take advantage of a radio wave phenomenon called multipath** where transmitted information bounces off walls, ceilings and other objects
- reaching the receiving antenna multiple times via different angle and at slightly different times
- **allow multiple antennas to send and receive multiple spatial streams at the same time**



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- **SIMO - receive diversity**
- receives signals from a number of independent sources to combat the effects of fading
- **processing is required in the receiver**
- receiver located in mobile device, limited by size, cost and battery drain



Exploiting Diversity

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- receiver located in mobile device, limited by size, cost and battery drain
- switched diversity SIMO - looks for strongest signal and switches to that antenna
- maximum ratio combining SIMO - takes both signals and sums them to give a combination
- signals from both antennas contribute to overall signal



MIMO: Spatial Diversity

- to make the transmission more robust
- no increase in data rate; mode uses redundant data on different paths
- receive diversity - RX diversity
- RX diversity uses more antennas on the receiver side than on the transmitter side
- simplest scenario consists of two RX and TX SIMO 1×2
- special coding methods are not needed, this scenario easy to implement only two RF paths are needed for the receiver
- RX \leftarrow A and RX \leftarrow B; two paths A and B
- switched diversity $C = \max(A, B)$ - uses stronger signal
- maximum ratio combining $C = (A + B)$ - uses the sum signal from two signals



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- maximum ratio combining $C = (A + B)$ - uses the sum signal from two signals
- different transmission paths, the receiver sees two differently faded signals
- by appropriate method in the receiver, SNR can be increased

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- space-time codes combine spatial and temporal signal copies



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- space-time signal processing
- multiple spatially distributed antennas located at different points
- signal can take many paths - as a result of number of objects introduce interference



MIMO: Spatial Multiplexing

- signaling technique combine with properties of OFDM modulation leads to efficient channel estimation and equalization scheme
 - even signals are transmitted on TX0 and odd symbols are transmitted on TX1
 - two receivers RX0 and RX1 receive both the transmitted streams
 - through 4 possible channel transfer functions $h_{00}, h_{01}, h_{10}, h_{11}$ forms a channel matrix



MIMO Channel Equalization

- the received symbols at each of receive inputs can be computed as follows:
$$\begin{aligned} r_0 &= h_{00}s_0 + h_{10}s_1 & \begin{bmatrix} r_0 \\ r_1 \end{bmatrix} &= \begin{bmatrix} h_{00} & h_{10} \\ h_{01} & h_{11} \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} = [R] = [H][S] \\ r_1 &= h_{10}s_0 + h_{11}s_1 \end{aligned}$$
 - for case of pilot subcarriers relative to null subcarriers
 - $r_{00} = h_{00}s_0$ $r_{10} = h_{10}s_1$ $r_{01} = h_{01}s_0$ $r_{11} = h_{11}s_1$
 - the magnitude and phase of pilot subcarriers is known,
 - the the s-terms can be factored out and the elements of the channel matrix are determined
 - given channel matrix the received signals can be equalized and restored



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 - through 4 possible channel transfer functions $h_{00}, h_{01}, h_{10}, h_{11}$ forms a channel matrix
 - in order to equalize and extract the source symbol stream,
 - estimate of channel matrix must be made transmitting both streams using
 - a coordinated system of pilot subcarriers and null carriers



MIMO Channel Equalization

$$[S] = [H]^{-1}[R] = \left[\frac{\text{adj}[H]}{|H|} \right] [R] = \left[\frac{\begin{bmatrix} h_{11} & -h_{01} \\ -h_{10} & h_{00} \end{bmatrix}^T}{h_{00}h_{11} - h_{10}h_{01}} \right] [R]$$

- transmitted signal can be restored perfectly if channel matrix can be estimated perfectly
 - does not work due to singularity in the matrix inversion where all channel elements are equal or correlated



MIMO: Types of space time code

- spatial diversity
 - ▶ ST block code - provides diversity gain, no coding gain
 - ▶ ST trellis code - diversity and coding gain
- spatial multiplexing
 - ▶ layered ST code - coding gain and diversity gain, bandwidth efficiency
- BLAST: Bell labs layered space time

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- vertical BLAST; it is $1 : N$ demux on channel cc_0, cc_1, cc_2, cc_N
- $TX_1 \rightarrow \text{encoder } \alpha \rightarrow \alpha_0 \alpha_1 \dots$
- $TX_2 \rightarrow \text{encoder } \beta \rightarrow \beta_0 \beta_1 \dots$
- $TX_3 \rightarrow \text{encoder } \gamma \rightarrow \gamma_0 \gamma_1 \dots$
- space vs. time
- simple and low complexity, lower capacity than shannon bound
- Data rate $\propto N$

MIMO: Spatial diversity

- diagonal BLAST
- $TX_1 \rightarrow \text{encoder } \alpha \rightarrow \alpha_0 \beta_0 \gamma_0 \dots$
- $TX_2 \rightarrow \text{encoder } \beta \rightarrow \dots \alpha_1 \beta_1 \gamma_1 \dots$
- $TX_3 \rightarrow \text{encoder } \gamma \rightarrow \dots \dots \alpha_2 \dots$
- closer to shannon bound due to spatial switch diversity gain
- high complexity, coding constraints
- Beamforming schemes
- beamformer - spatial filter that combines array inputs
- form a beam in space to receive desired signal while suppressing interference and noise
- transmit beamformer and receive beamformer



MIMO: Beamforming

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- smart antennas belong to adaptive antenna arrays but differ in their smart direction of arrival estimation
- smart antennas can form a user specific beam



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- smart antennas belong to adaptive antenna arrays but differ in their smart direction of arrival estimation
- smart antennas can form a user specific beam
- optional feedback can reduce complexity of array system
- beamforming is the method used to create the radiation pattern of an antenna array applied to MIMO system



MIMO: Beamforming

- smart antennas divided into two groups
- phased array systems (switched beamforming) with a finite number of fixed predefined patterns



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- smart antennas divided into two groups
- **phased array systems** (switched beamforming) with a finite number of fixed predefined patterns
- **adaptive array systems** (adaptive beamforming) with an infinite number of patterns adjusted to the scenario in realtime
- **switched beamformers** electrically calculate the DoA and switch on the fixed beam
- the user only has the optimum signal strength along the center of the beam
- **the adaptive beamformer deals** with that problem and adjusts the beam in realtime to the moving UE
- complexity and cost higher than switched beamformer
- various mobile radio and network standards use MIMO
- **all standards use TX diversity and spatial multiplexing**



MIMO: Alamouti coding

- signals S_0 and S_1 are multiplexed in two data chains
- after that a **signal replication** is added to create the **alamouti space-time block code**

$$[S_0 \ S_1] \rightarrow \begin{bmatrix} S_0 \\ S_1 \end{bmatrix} \rightarrow \begin{bmatrix} S_0 & -S_1^* \\ S_1 & S_0^* \end{bmatrix}$$

- -TX1 and -TX2 space and time
- coding can also be handled in frequency domain; called **space-frequency coding**



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- -TX1 and -TX2 space and time
- coding can also be handled in frequency domain; called **space-frequency coding**
- **spatial multiplexing**
- not intended to make transmission more robust; rather it increases data rate data is divided into separate streams
- **the streams are transmitted independently via separate antennas**



MIMO Modelling

- received signal vector \mathbf{y} channel matrix \mathbf{H}

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n}$$

$$\mathbf{y} = [y_1 \ y_2 \ \dots \ y_N]^T$$

received signal vector

$$\mathbf{X} = [x_1 \ x_2 \ \dots \ x_M]^T$$

transmitted signal vector

$$\mathbf{n} = [n_1 \ n_2 \ \dots \ n_M]^T$$

noise vector

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1M} \\ h_{21} & h_{22} & \dots & h_{2M} \\ \vdots & \vdots & \vdots & \vdots \\ h_{N1} & h_{N2} & \dots & h_{NM} \end{bmatrix}$$

$$\text{transmitted power } P_T = \mathbf{X}^H \mathbf{X} = |x_1|^2 + |x_2|^2 + \dots + |x_M|^2 \leq C$$



MIMO Modelling

- covariance matrices

$$R_{xx} = E \{ \mathbf{XX}^H \} \quad R_{yy} = E \{ \mathbf{yy}^H \} = E \{ \mathbf{HXX}^H \mathbf{H}^H \} + E \{ \mathbf{nn}^H \}$$

- traces of R_{xx} and R_{yy} give total powers of transmitted and received signals

- offdiagonal elements of R_{xx} and R_{yy} give correlations between the signals at different antennas elements

- consider a symbol period of time T_s for transmitted signals such that $R_{xx} = E \{ \mathbf{XX}^H \} = I_M$ and within T_s

$$R_{yy} = E \{ \mathbf{HXX}^H \mathbf{H}^H \} + E \{ \mathbf{nn}^H \}$$

$$= H E \{ \mathbf{XX}^H \} \mathbf{H}^H + R_{nn} = \mathbf{HH}^H + R_{nn}$$

R_{nn} is noise covariance

- over a longer period of time ($> T_s$) average received signal covariance matrix is

$$R_{yy} = E \{ \mathbf{HH}^H \} + R_{nn}$$



Equivalent MIMO

- select H to optimize the channel output SNR so as the capacity of MIMO
- for MIMO calculation of the capacity is more complicated due to determination of SNR

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- select H to optimize the channel output SNR so as the capacity of MIMO
- for MIMO calculation of the capacity is more complicated due to determination of SNR
- channel matrix H ($N \times M$) $y = \mathbf{Hx} + \mathbf{n}$
- using SVD singular value decomposition $H = UDV^H$
- D is $N \times M$ a diagonal matrix with non-negative elements
- U is $N \times N$ unitary matrix; V is $M \times M$ unitary matrix
- $UU^H = U^H U = I_N$ and $VV^H = V^H V = I_M$
- diagonal elements of D are called singular values of H



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- $UU^H = U^H U = I_N$ and $VV^H = V^H V = I_M$
- diagonal elements of D are called singular values of H
- there are non negative square roots of the eigenvalues λ of the following equation

$$(HH^H)X = \lambda X \quad \text{if } N < M$$

$$(H^H H)X = \lambda X \quad \text{if } N \geq M$$



Equivalent MIMO

- x is $N \times 1$ eigenvector associated with λ

$$y = UDV^H X + n$$

- consider transformations $y' = U^H y$ $X' = V^H X$ $n' = U^H n$

$$U^H y = U^H UDV^H X + U^H n$$

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$$R_{y'y'} = E \left\{ y'y'^H \right\} = E \left\{ U^H y y^H U \right\} = U^H R_{yy} U$$

$$R_{x'x'} = E \left\{ x'x'^H \right\} = E \left\{ V^H x x^H V \right\} = V^H R_{xx} V$$

$$R_{n'n'} = E \left\{ n'n'^H \right\} = E \left\{ U^H n n^H U \right\} = U^H R_{nn} U$$

- $tr\{R_{y'y'}\} = tr\{R_{yy}\}$, $tr\{R_{x'x'}\} = tr\{R_{xx}\}$, $tr\{R_{n'n'}\} = tr\{R_{nn}\}$



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 - channel gains given by diagonal elements of $D \rightarrow \sqrt{\lambda_i} i = 1, \dots, N$
 - number of nonzero eigenvalues of matrix HH^H is equal to rank of matrix H denoted by r

$$y'_i = \sqrt{\lambda_i} x'_i + n'_i$$

for $i = 1, 2, \dots, r$

$$y'_i = 0 + n'_i$$

for $i = r, \dots, N$



Equivalent MIMO

- MIMO has more transmitting antennas than receiving antennas
 $M > N$
 - than H is a horizontal matrix with a maximum rank N
 - maximum number of uncoupled equivalent MIMO channel is N
 - remaining $M - N$ transmitting antennas will become redundant with no receiving antennas
 - if $M < N$ - more receiving antennas than transmitting antennas
 - H is vertical matrix with maximum rank M
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 - **maximum number of uncoupled equivalent channels is $\min(N, M)$**



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$$C = B \sum_{i=1}^r \log_2 \left[1 + \frac{P_{y_i}}{\sigma^2} \right]$$

- B channel bandwidth, P_{y_i} power received at the i th receiving antenna σ^2 noise power at receiving antenna
- r rank of H
- classify MIMO according availability of channel knowledge to transmitter or receiver



MIMO Channel Capacity

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- transmitter does not know CSI, the best strategy transmit power equally from all its transmitting antennas
- equivalent MIMO - making all elements of X' have same power
- received power $P_{y_i} = \lambda_i \frac{P}{M}$



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- received power $P_{y_i} = \lambda_i \frac{P}{M}$
- P total transmitting power

$$C = B \sum_{i=1}^r \log_2 \left(1 + \lambda_i \frac{P}{M\sigma^2} \right) = B \log_2 \prod_{i=1}^r \left(1 + \lambda_i \frac{P}{M\sigma^2} \right)$$

- eigenvalue λ_i can be expressed in terms of matrix HH^H or $H^H H$

$$C = \begin{cases} B \log_2 \det \left(I_N + \frac{P}{M\sigma^2} HH^H \right) & N < M \\ B \log_2 \det \left(I_M + \frac{P}{M\sigma^2} H^H H \right) & N \geq M \end{cases}$$



MIMO Channel Capacity

- total transmitting power P not known, average received powers P_r at receiving antennas are same

$$P_r = P \times P_{loss}$$

- P_{loss} average path loss form the transmitter to receiver

$$C = \begin{cases} B \log_2 \det \left(I_N + \frac{P_r}{M\sigma^2} \frac{HH^H}{P_{loss}} \right) & N < M \\ B \log_2 \det \left(I_M + \frac{P_r}{M\sigma^2} \frac{H^H H}{P_{loss}} \right) & N \geq M \end{cases}$$



MIMO Channel Capacity

- total transmitting power P not known, average received powers P_r at receiving antennas are same

MIMO Channel Capacity

- CSI known to both transmitter and receiver
- if transmitter knows CSI, channel matrix H
- the best strategy is to **transmit more power** along those channels whose **channel gains are larger** and
- to **transmit less power** or along those channels with a **smaller channel gain**



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- this is called **water-filling principle**
- the transmitting power P_i for the i th channel in equivalent MIMO

$$P_i = \left(\mu - \frac{\sigma^2}{\lambda_i} \right)$$

$i = 1, \dots, r$ $r = \text{rank}(H)$



MIMO Channel Capacity

- if P_i is negative it will be set to zero
- μ is determined by satisfying transmitting power constraint

$$P = \sum_{i=1}^r P_i$$

- received powers

$$P_{y_i} = \lambda_i P_i = (\lambda_i \mu - \sigma^2)$$

- channel capacity is

$$C = B \sum_{i=1}^r \log_2 \left[1 + \frac{1}{\sigma^2} (\lambda_i \mu - \sigma^2) \right]$$



Different Standards and MIMO support

- **3GPP mobile radio standard** - UMTS undergone phases of development
- starting with WCDMA data acceleration methods have been introduced,
- including HSDPA and HSUPA
- newest releases cover **HSPA+ and Long Term Evolution (LTE)**
- HSPA+ (3GPP release 7/8)
- transmit diversity mode introduced in release 99 (WCDMA)
- release 7 of 3GPP specification (HSPA+) expanded this approach to MIMO and increased data rate with respect to release 6 (HSDPA)
- introduction of **64QAM modulation** and MIMO in the downlink makes a peak data rate of 28 Mbps
- MIMO was introduced in the form of double transmit antenna array for **high speed downlink shared channel (HS-DSCH)**



Different Standards and MIMO support

- **LTE (3GPP release 8)** - UMTS LTE introduced in 3GPP release 8
- objective is high data rate, low latency and packet optimized radio access technology
- **LTE is referred as E-UTRA** (Evolved UMTS Terrestrial Radio Access) or
- **E-UTRAN** (Evolved UMTS Terrestrial Radio Access Network)
- the basic concept for LTE is downlink is OFDMA (uplink: SC-FDMA)
- while MIMO is integral part of LTE
- modulation modes are QPSK, 16QAM, and 64QAM
- **peak data rates of up to 300 Mbps (4 X 4 MIMO)** and up to 150 Mbps (2 X 2 MIMO) in the **downlink** and
- upto **75 Mbps in the uplink** are specified

Different Standards and MIMO support

- downlink - transmission modes possible in LTE
 - ▶ single antenna transmission, no MIMO
 - ▶ **transmit diversity**
 - ▶ open-loop **spatial multiplexing**, no UE feedback required
 - ▶ closed-loop spatial multiplexing, UE feedback required
 - ▶ Multi user MIMO (more than one UE is assigned to same resource block)
 - ▶ beamforming
 - ▶ closed-loop precoding for rank=1 (no spatial multiplexing, but precoding is used)
- WiMAX 802.16e-2005
 - ▶ peak data rate of 74 Mbps at a bandwidth of upto 20 MHz
 - ▶ modulation QPSK, 16QAM, 64QAM
- WLAN 802.11n
 - ▶ **peak data rate of up to 600 Mbps at a bandwidth of 40 MHz**
 - ▶ modulation BPSK, QPSK, 16QAM, 64QAM



Thank You