Design and Analysis of Algorithms, MTech-I (1st semester)

Chapter 6: NP Theory - I

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Broad Contents of the talks

- Talk1: Complexity Classes of Problems and a few "Hard" problems.

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- Talk3: Non-determinism, Working with NPHard, NPComplete.

Topics to be discussed

A. **Talk1: Complexity Classes of Problems and a few "Hard" problems.**
   1. What does solving problems algorithmically, mean?
   2. Classifying problems
   3. A Motivating Example to illustrate hardness
   4. Some Hard Problems

Topics to be discussed...

- **B** **Talk2: Relating Problem Hardness & Polynomial Reductions**
  1. Reductions
  2. How can we relate hardness of two problems ?
  3. How can we relate solvability of two problems ?
  4. Polynomial Reduction of one problem to the other
  5. Polynomial Equivalence of one problem to the other
  6. Three methods of reductions: illustrations

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  5. Proofs associated
  6. Summarizing

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  - Intuitive. Then, has to be iterative.

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- Polynomial Time complexity ? How to define it ?
- Explore the ways to improve?

Polynomial or Intractable ?

	n	$n \lg n$	N^2	N^3	1.5^n	2^n	$n!$
n=10	< 1 sec	< 1 sec	< 1 sec	< 1 sec	< 1 sec	< 1 sec	< 4 sec
n=30	< 1 sec	< 1 sec	< 1 sec	< 1 sec	< 1 sec	18 min	10^{25} yrs
n=50	< 1 sec	< 1 sec	< 1 sec	< 1 sec	11 min	36 yrs	very long
n=100	< 1 sec	< 1 sec	< 1 sec	1 sec	12.89 yrs	10^{17} yrs	< 4 sec
n=1000	< 1 sec	< 1 sec	1 sec	18 min sec	very long	very long	very long
n=10K	< 1 sec	< 1 sec	2 min	12 days	very long	very long	very long
n=100K	< 1 sec	2 sec	3 hrs	32 yrs	very long	very long	very long
n=1M	1 sec	20 sec	12 days	31.71 yrs	very long	very long	very long

Figure: Complexity Orders related to the time of execution

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- Searching ($O\ lg\ n$), Sorting ($O(n\ lg\ n)$), Polynomial evaluation ($O(n)$. . . . . . .

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- Though this may not be true always. . . . .

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- If the input size is n, typical time taken is 2^n.

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- In 1936, Alan Turing proved that the halting problem - the question of whether or not a Turing machine halts on a given program - is undecidable. This result was later generalized by Rice's theorem.

Should we waste our time designing an algorithm to a problem that is globally and over time believed to be undecidable OR intractable ?

A Motivating Example

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Machine scheduling to minimize the average job completion time

- Given a set of m processes $j_1, j_2, j_3, \ldots j_m$ with running times $t_1, t_2, t_3 \ldots\ldots t_t$ to be scheduled on specified n no of machines $m_1, m_2, m_3, \ldots\ldots m_n$ such that

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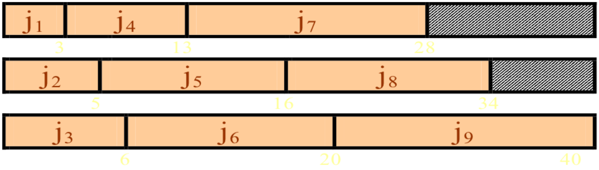
  - there is non-preemptive scheduling
  - the average job completion time is minimized.

A Motivating Example...

Goal: Minimizing the average job completion time.....



M_1	M_2	M_3	M_1	M_2	M_3	M_1	M_2	M_3
j_1	j_2	j_3	j_4	j_5	j_6	j_7	j_8	j_9
3	5	6	10	11	14	15	18	20
3	5	6	13	16	20	28	34	40

Figure: What is the Average job completion time?

A Motivating Example...

Goal: To prove that the SJF (or SRTN, if premptive scheduling) scheduling indeed ensures optimal i.e. minimal average job completion time.....
Proof:

A Motivating Example...

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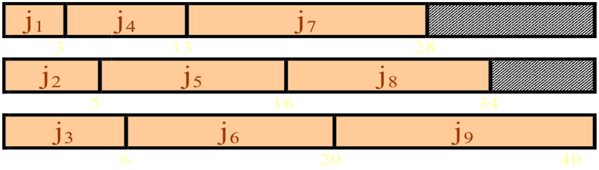
A Motivating Example...

Machine scheduling to minimize the final completion time of machines used.

- Given a set of m processes $j_1, j_2, j_3, \ldots j_m$ with running times $t_1, t_2, t_3 \ldots\ldots t_t$ to be scheduled on specified n no of machines $m_1, m_2, m_3, \ldots\ldots m_n$ such that
  - no machine processes more than one process at a time
  - no process is executed by more than one machine
  - there is non-preemptive scheduling
  - the final completion time is minimized.

A Motivating Example...

Goal: Minimizing the Final Completion time of processors.....



M_1	M_2	M_3	M_1	M_2	M_3	M_1	M_2	M_3
j_1	j_2	j_3	j_4	j_5	j_6	j_7	j_8	j_9
3	5	6	10	11	14	15	18	20
3	5	6	13	16	20	28	34	40

Figure: What is the Final Completion time in this schedule?

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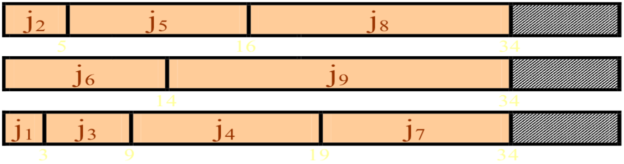

M_3	M_1	M_3	M_3	M_1	M_2	M_3	M_1	M_2
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Figure: What is the Final Completion time in this schedule?

Motivating Example: Attempt#1

- Given a set of m processes $j_1, j_2, j_3, \dots \dots \dots \dots j_m$ with running times $t_1, t_2, t_3 \dots \dots \dots t_t$ to be scheduled on say 2 no of machines m_1, m_2 such as to minimize the final completion time that is to ensure that last job finishes earliest.

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- How do we schedule these jobs on P_1 and P_2 ?
- Say we schedule all the ODD jobs on P_1 and the even jobs on P_2, then, how are the first two jobs scheduled ?
- Then, the last job is ______ & finishes at time ______.
- We wish to improve upon this finish time of whatever is the last job to execute i.e. we wish to evenly distribute the jobs across the two processors s.t. . . . .

Motivating Example: Attempt#2

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- What would be the schedule in the previous example with this approach i.e. for the job mix with the jobs j_1, j_2, j_3 ... ... j_7 with running times 2,100,2,100,2,100,2 ?

Motivating Example: Attempt#2...

- If we put the first job on P_1, second job on P_2 for the job mix with the jobs j_1, j_2, j_3 ... ... j_7 with running times 2,100,2,100,2,100,2, what is the schedule ?

P_1	P_2	Comment
$j_1(2)$	$j_2(100)$	
$j_3(2)$		$t_1 < t_2$
$j_4(100)$		$t_1 + t_3 < t_2$
	$j_5(2)$	$t_2 < t_1 + t_3 + t_4$
	$j_6(100)$	$t_2 + t_5 < t_1 + t_3 + t_4$
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P_1	P_2	Comment
$j_1(2)$	$j_2(100)$	
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- What is the earliest finish time now?

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- What is the earliest finish time now?
- Is it an improvement over the previous attempt?

Motivating Example: Attempt#2...

- However, will this work always ?

Motivating Example: Attempt#2...

- However, will this work always ?
- Think of a counterexample that shows it doesn't work. . . .

Motivating Example: Attempt#3

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    - What is the schedule that we obtain for the previous example with this approach ? What is the finish time of the last process to complete?

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Figure: Optimal Schedule

A Motivating Example: One more schedule

- Jobs $j_1, j_2, j_3.........j_n$ with running times $t_1, t_2, t_3,........t_n$ to be scheduled on say three processors such as to minimize the final completion time, i.e. last job finishes the earliest...for the schedule give below

j_1	j_2	j_3	j_4	j_5	j_6	j_7	j_8	j_9
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Figure: Minimize the Final Completion time with three processors

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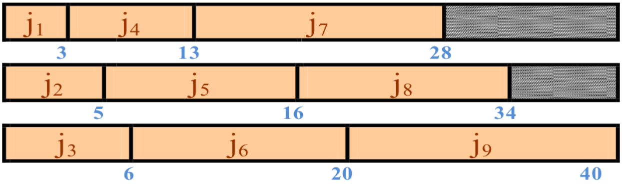
- What is the final completion time ?



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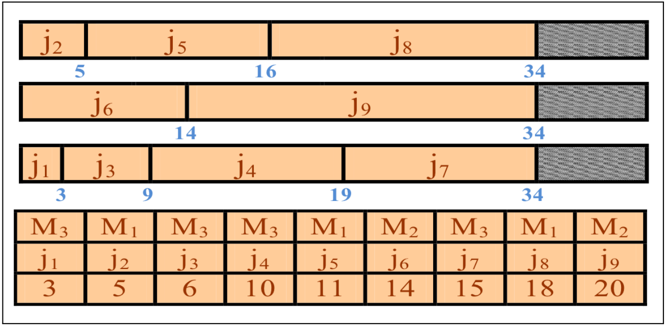

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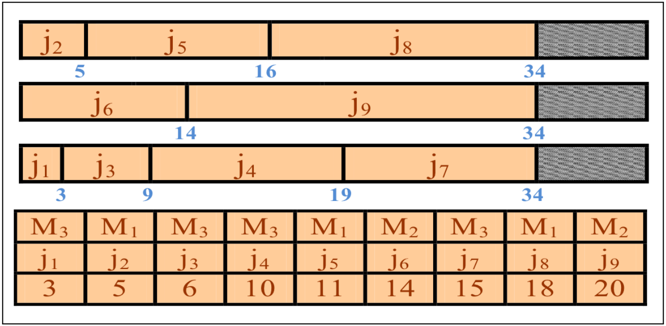
- How could have this schedule been achieved ?

A Motivating Example: One more schedule...

- Applying the brute force......



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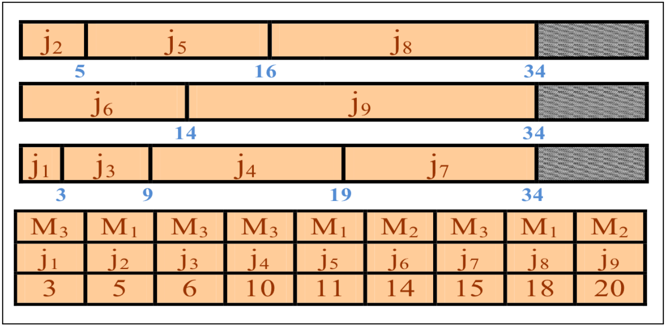

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Why study NP Theory?

- So, now what is the rationale behind exploring the NP theory ?
- The rationale is. . . . .
- Well, the rationale is to understand that there are problems that are going to take infeasible amount of time on a deterministic machine and so one may not waste energy . . . . . . trying to solve them. . . . . . .

Tutorial Problem:Think of one more such example

Can you think of a similar computational problem ?

Tutorial Problem:Think of one more such example

- Say there are two subjects, four teachers and there are four lectures of 1 hour each everyday.

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Classifying Problems

Tractable problems = Efficiently solvable ????

Intractable problems = Inefficiently solvable ????

Figure: Classifying problems

The Frustrating news

No reasonably fast algorithms have been found

Intractability of these problems cannot be proved

Figure: Our Focus now onwards is on Intractable problems

Why do we need to classify problems ?

- Computation has become pervasive in all walks of life a standard tool in about every academic field
  - whole subfields of Chemistry, Biology, Physics, Economics OR
  - others devoted to large-scale computational modelling, simulations and problem solving.
- We need to understand therefore, the limitations of computational power. . . .

Why do we need to classify problems ?...

- Study of P, NP theory
  - helps understand, handle various topics in allied sciences
  - helps what can be feasibly solved and what cannot be also
  - enables one to EXPLOIT the advantage due to HARDNESS of various computational problems
  - e.g. . . . .

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  - Possible answers
    - yes
    - no
  - because it can be proved that all algorithms take exponential time
  - because it can be proved that no algorithm exists at all to solve this problem
  - don't know
  - don't know, but if such algorithm were to be found, then it would provide a means of solving many other problems in polynomial time

Some Hard Problems

Other interesting hard problems

- A large group of students to be grouped to work on projects so as to ensure compatibility
- Matching students in pairs . . . . . . .solvable
- Hard problems
  - Making a group of three so that each pair in each trio is compatible to each other. . . . . . ???

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  - Finding as large group of students as possible so that each pair therein is compatible to each other. . . . . . .???
  - Wanting the student to sit across a table so that no incompatible students sit next to each other . . . .???
  - Putting students into three groups so that each student is in the same group with his/her compatible partner. . . .???

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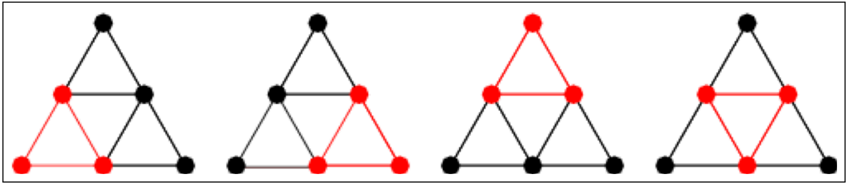
- What is a clique in a graph G(V,E) ?

Problem k-Clique

- A graph G=(V, E) is a Clique if every two nodes in V are connected by an edge



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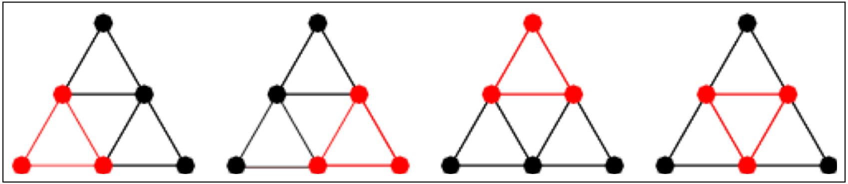

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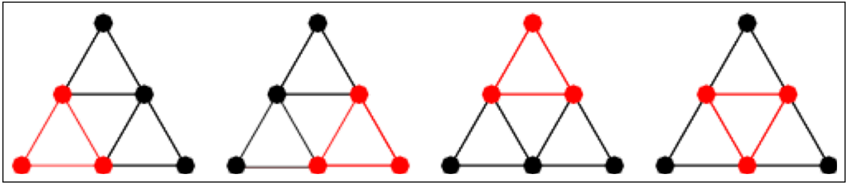

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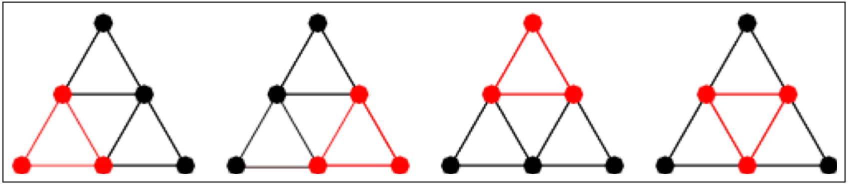

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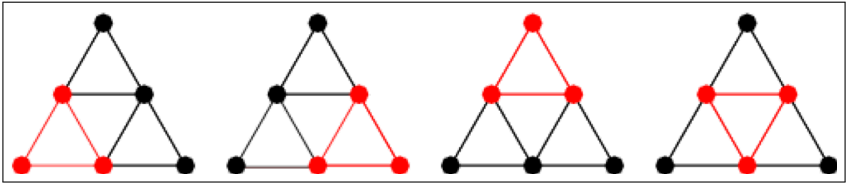

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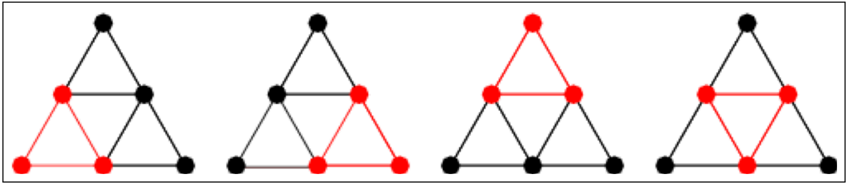

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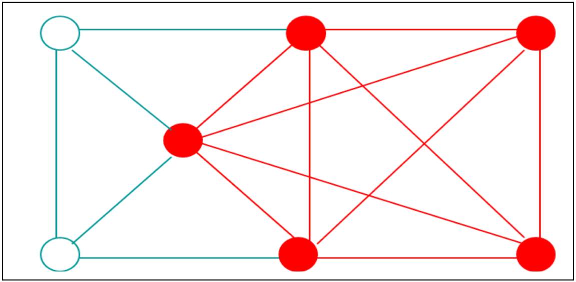

Figure: 5-Clique OR 6-clique?

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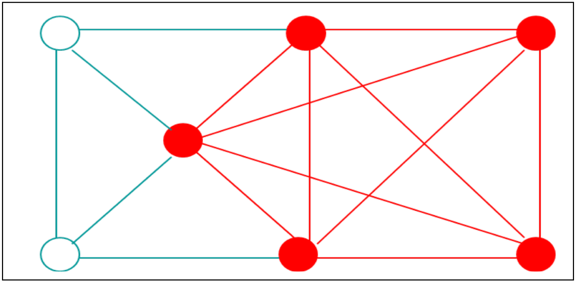

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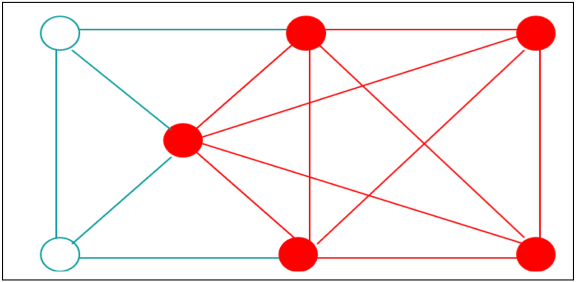

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- A graph G=(V, E) is a Clique if every two nodes in V are connected by an edge
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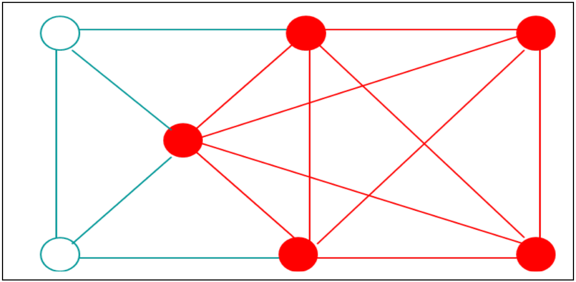

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  - is a clique of the largest possible size in a given graph.

Maximal and Maximum Clique...

- Maximum clique - triangle 1,2,5

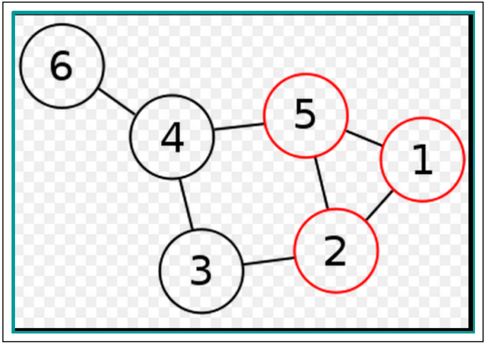
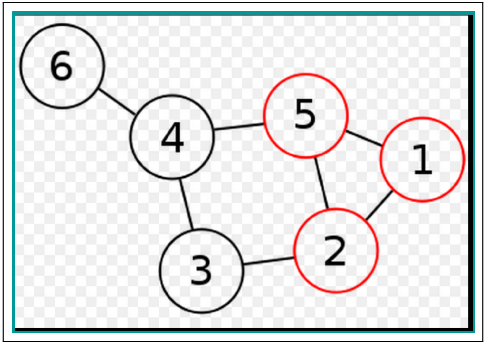


Figure: Maximum/Maximal clique

Maximal and Maximum Clique...

- Maximum clique - triangle 1,2,5
- Four Maximal cliques - pairs of (2,3), (3,4),(4,5), (4,6)



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- Note the clique and independent set problems...... Are they related ?

The clique and independent set problems

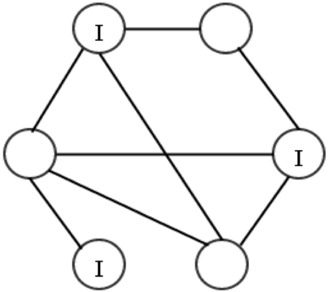

Figure: IS it a clique or an independent set ?

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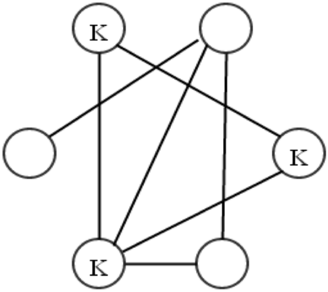

Figure: IS it a clique or an independent set ?

Boolean Satisfiability : Terminologies

- Propositional (boolean) variable - a variable that may be assigned value true or false
- Literal - A Boolean variable or its negation.
- Propositional formula - an expression that is either a propositional variable or a propositional constant or an expression of boolean operator and its operands
- Clause : a disjunction – sequence of literals separated by V
- Conjunctive normal form - A regular form of propositional formula ϕ that is the conjunction of clauses

Boolean Satisfiability ...

- An example propositional CNF formula is
  $\phi = (\bar{x_1} \lor x_2 \lor x_3)\ (x_1 \lor \bar{x_2} \lor x_3)\ (\ x_2 \lor x_3)$
  where x_1, x_2, x_3 are propositional variables
- Truth assignment - a boolean valued function on the set i.e.
  an assignment of values true or false to each propositional variable in the set
- Satisfiability - When does a truth assignment is said to satisfy a formula ?
- SAT - Given a CNF formula ϕ, does it have a satisfying truth assignment?

Boolean Satisfiability Problem

- Input : A boolean formula F in CNF
- Goal:
  - Check if F is satisfiable or not.
  - e.g. if $F = (x_1 + x_2)$ can we assign at least one set of values to the literals of the formula so that $F = 1$.
- How to solve this problem deterministically ?
- What could be the Brute-force approach ?
- Time complexity ??
- $O(2^n|F|)$
- Cannot do any better than that.

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