Multi Input and Multi Output (MIMO)

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Wireless System

- demand of higher data rates - performance of wireless system
- data rate 1 gigabit/second
- capacity $=$ BW $\log_2(1 + SNR)$ shannon's law
- channel capacity in bits per second
- higher data rates by increasing bandwidth
- higher bandwidth - more data
- boosting power - more range
- increase in SNR marginal gain in channel throughput
- spectrum become a scarce and expensive resource
- limited bandwidth and tranmit power
- goal: high data rate and high link quality

Technology for Higher Demand

- increasing singal bandwidth of channel by increasing symbol rate of a modulated carrier
- increases its suceptibility to multipath fading
- to meet this chanllenge - MIMO
- Multiple antennas and multiple signal streams to boost data rate, range, connection reliability
- higher data rate over longer distances motivations behind MIMO OFDM
- 802.11n, 802.1ac Wi-Fi, WiMAX, LTE (Long Term Evolution) 4G supports MIMO
- supports enhanced data throughput under interference, fading and multipath

Multiple Input Multiple Output (MIMO)

- 1984 Jack Winters at Bell lab wrote a patent on wireless communication using multiple antennas
- 1993 Paulraj and Kailath concept of spatial multiplexing using MIMO
- 2006 802.11n
- boost system capacity and enhance reliability of link uses several diversity schemes
- N_T tranmit antennas and N_R receive antennas - MIMO system model
- $x_1 - T_{x_1}$ $x_2 - T_{x_2} \rightarrow H(N_R \times N_T) \rightarrow R_{x_1} - y_1$ $R_{x_2} - y_2$ ....
- Antennas $T_{x_1}, \ldots, T_{x_{N_T}}$ respectively send signal $x_1, \ldots, x_{N_T}$ to receive antennas $R_{x_1}, \ldots, R_{x_{N_R}}$
- received signals $y_1, \ldots, y_{N_R}$ respectively
- each receive antenna combines incoming signal which coherently add up

Multiple Input Multiple Output (MIMO)

- offers significant increases in data throughput
- link range without additional bandwidth or increased transmit power
- achieves by spreading the same total transmit power over the antennas
- to achieve an array gain that improves the spectral efficiency of bandwidth - more bits per second per hertz
- to improve link reliability - reducing fading by achieving diversity gain

Multiple Input Multiple Output (MIMO)

- time and frequency domain processing are at limits but space is not - MIMO
- MIMO improves BER or data rate using multiple TX/RX
- core scheme of MIMO: space time coding (STC)
- two main functions of STC: diversity and multiplexing
- maximum peformance needs tradeoff between diversity and multiplexing
- N TXs and M RXs; $y(k) = Hx(k) + v(k)$
- multiath propagation, multiple antennas at transmitter and receiver
- establish multiple parallel channels operate on same frequency band at same total radiated power

Shannon's Theorem

- Shannon bound for SISO

$$C = \log_2(1 + SNR) \quad SNR = P_T/\sigma_v^2$$

- each extra bps/Hz requires a doubling of TX power
- 1 bps/Hz to 11 bps/Hz TX power must be increased by 1000 times
- MISO - transmit diversity against fading
- slow logarithmic growth of capacity with no. of antennas

$$C = \log_2(1 + SNR) \text{ for TX1 } C = \log_2(1 + SNR) \text{ for TXN also}$$

- SIMO - receive diversity against fading

$$C = \log_2(1 + SNR) \text{ for RX1 } C = \log_2(1 + SNR) \text{ for RXM}$$

- MIMO - transmit and receive diversity

$$C = \log_2(1 + SNR) \text{ MN channels}$$

- parallel spatial channels

$$C = Q\log_2(1 + SNR) \quad Q = \min(M, N)$$

Wireless System

- quality - minimize probability of error P_e
- minimize complexity, transmission power, and bandwidth
- smart modulation, coding, and multiplexing
- maximum error free transmission rate

$$C = \log_2(1 + SNR) \text{ bits/s/Hz}$$

- R data rate bits/symbol, R_s - symbol rate symbols/second, W - allotted BW Hz
- spectral efficeincy = number of bits transmitted per second per Hz

$$R \times R_s/W \text{ bits/s/Hz}$$

- as a result of filtering / signal reconstruction requirements $R_s \leq W$
- spectral efficiency = R if $R_s = W$
- if transmit data at rate of $R \leq C$ - achieve low P_e

Wireless System

- BPSK 1 b/s/Hz, QPSK 2 b/s/Hz, 16-QAM 4 b/s/Hz, 64-QAM 6 b/s/Hz

$$\mathbf{y} = H\mathbf{s} + \mathbf{n}$$

- h_{ij} complex gaussian random variable that models fading gain between ith transmit and jth receive
- fading is produced by mutliple replicas of transmitted signal at the receiving antenna
- multiple reflections and mutliple communication paths between two radio terminals cause
- fading - amplitude and phase changes
- ellipsoidal fading model - replica by random reflection
- constructive or destructive interference
- worst case 180 degree phase - casue signal cancellation

Symbol Period and Spread

- T_{sym} to be symbol time period
- T_{spread} to be the time at which the last reflection arrives
- frequency selective $T_{spread} > T_{sym}$; $1/T_{sym}$ is longer
- occurs for wideband signal (small T_{sym}); tough to deal it
- frequency flat $T_{spread} < T_{sym}$; $1/T_{sym}$ is less
- occurs for narrowband signal (larger T_{sym})
- easier fading gain is complex gaussian
- multipath not resolvable
- time spread τ - channel time variance t

$$H(\tau, t) = \begin{bmatrix} h_{1,1}(\tau, t) & h_{1,2}(\tau, t) & \dots & h_{1,M_T}(\tau, t) \\ h_{2,1}(\tau, t) & h_{2,2}(\tau, t) & \dots & h_{2,M_T}(\tau, t) \\ \vdots & \vdots & \vdots & \vdots \\ h_{M_R,1}(\tau, t) & h_{M_R,2}(\tau, t) & \dots & h_{M_R,M_T}(\tau, t) \end{bmatrix}$$

Fading

- signal fading, selective and non-selective fading
- non-selective fading where frequency components over the signal bandwidth are
- dynamically attenuated by the same amount and do not create any signal distortion but only temporal signal loss
- selective fading the case where smaller frequency segments of the signal's spectrum are
- attenuated relative to other remaining frequency segments
- when this occurs, the signal specturm is distorted and in turn creates communcation impairment
- non-selective fading can be countered by providing more signal level margin, or
- using selection diversity techniques to select the best antenna input based on the relative signal strength

Exploiting Multipath

- strongest signal is chosen to make the connection
- all other signals filtered out
- multipath reflections from objects, weaker and arrive at different times
- different antenna pick up different signals, filter noise and recombine to generate stronger signal
- basic MIMO - data transmitted scrambled, interleaved and
- divided up into parallel data streams, each which modulates a separate transmitter
- multiple antennas capture different streams, have different phases, travelled different routes and combine into one

Exploiting Multipath

- each multipath route treated as a separate channel
- separate antennas take advantage of this to transfer more data
- multiplying throughput, range is increased due to antenna diversity
- each receive antenna has a measurement of each transmitted data stream
- maximum data rate per channel grows linearly with the number of different data streams
- that are transmitted in the same channel, providing scalability and reliable link

MIMO-OFDM

- wide bandwidth channel - solution - use a series of narrowband overlapping subcarriers
- use of overlapping OFDM subcarriers improve spectral efficiency
- lower symbol rates used by narrowband subcarriers reduces the impact of multipath signal products
- interesting solution to multipath challenge by requiring multiple signal paths
- multiple signal paths to gain knowledge of communication channel
- receiver can recover independent streams from each of transmitter's antennas
- 2 x 2 MIMO produces two spatial streams effectively double maximum data rate what achieved in 1 x 1 channel
- maximum channel capacity of MIMO - as a function of N spatial streams Capacity $= N$ BW $\log_2(1 + SNR)$

MIMO-OFDM

- for Wi-Fi, OFDM using BPSK, QPSK, 16 phase QAM, depending on data rate
- signals from different antennas combined to reinforce one another, improving SNR
- smart antenna uses beam forming to focus transmitted signal energy toward the receiver to strengthen the signal - may provide better range
- beyond 4 by 4 configuration, very little additional gain is achieved
- 2 transmitter and three receivers seems to be the most popular
- transmitting two or more data streams in the same bandwidth multiplies data rate by the number of streams used
- MIMO 11n allows two 20 MHz channels bonded together into a single 40-MHz channel provide higher data rate

Multiple Transmit and Multiple Receive: Generic View

- in general, m transmit and n receive antennas
- by using the same channel every antenna receives
- not only the direct components intended for it but also
- the indirect components intended for the other antennas
- time dependent narrowband channel is assumed
- direct connection from antenna 1 to 1 is h_{11}
- indirect connection from antenna 1 to 2 is h_{21}
- h_{rt} transmission matrix

$$H = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1m} \\ h_{21} & h_{22} & \dots & h_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ h_{n1} & n_{n2} & \dots & h_{nm} \end{bmatrix}$$

$$y = Hx + n$$

MIMO Example: SU MIMO

- SU - MIMO - data rate is to be increased for a single UE, called single user MIMO
- base station - two transmit antennas, UE - two receive antennas
- data to be transmitted is divided into independent data streams
- number of streams M is alwasy less than or equal to the number of antennas
- in the case of asymmetrical $(m \neq n)$ antenna constellations,
- it is always smaller or equal the minimum number of antennas
- 4×4 transmit four or fewer streams
- 3×2 transmit two or fewer streams
- theoretically the capacity C increases linearly with the number of streams M

$$C = MB \log_2(1 + \frac{S}{N})$$

MIMO Example: MU MIMO

- Multi user MIMO MU - MIMO
- individual streams are assigned to various users
- this mode useful in the uplink because the complexity on UE side can be kept at a mimimum by one - only one transmit antenna
- called collaborative MIMO
- base station - two transmit antennas
- two UE -UE1 and UE2 two receive antnnas
- Cyclic delay diversity - CDD
- introduces virtual echoes into OFDM systems
- increases frequency selectivity at the reciever
- in CDD, signals are transmitted by the individual antennas with a time delay introduces additional diversity
- useful as addition to spatial multiplexing

Multi User MIMO

- received signal at antenna R_{x_q}

$$y_q = \sum_{p=1}^{N_T} h_{qp} \cdot x_p + b_q$$

  $q = 1, \ldots, N_R$
- flat fading MIMO described by

$$y = H \cdot x + b$$

  H complex channel matrix

$$H = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1N_T} \\ h_{21} & h_{22} & \cdots & h_{2N_T} \\ \vdots & \vdots & \vdots & \vdots \\ h_{N_R1} & h_{N_R2} & \cdots & h_{N_RN_T} \end{pmatrix}$$

  $p = 1, \ldots, N_T$ $q = 1, \ldots, N_R$
- h_{qp} is complex channel gain which links transmit antenna T_{x_p} to receive antenna R_{x_q}

Multi User MIMO

- $x = [x_1, \ldots, x_{N_T}]^T$ is $N_T \times 1$ complex transmitted signal vector
- $y = [y_1, \ldots, y_{N_R}]^T$ is $N_R \times 1$ complex received signal vector
- $b = [b_1, \ldots, b_{N_R}]^T$ is $N_R \times 1$ complex additive noise signal vector
- continuous time delay MIMO channel model of $N_R \times N_T$
- MIMO channel H associated with time delay τ and noise signal $b(t)$

$$y(t) = \int_{\tau} H(t, \tau) x(t - \tau) d\tau + b(t)$$

- $y(t)$ spatio temporal output, $x(t)$ input and $b(t)$ noise signal
- MIMO channel modeling - physical and analytical models
- physical models - deterministic models and geometry based stochastic channel models
- analytical models - correlation based models, statiscal based models, propagation based models

Multi User MIMO

- physical models - channel impulse response evaluted based on experimental measurements for extracting
- channel propagation parameters including antenna configuration at transmitter and receiver, antenna polarization, scatters
- propagation parameters such as delay spread, angular spread, spatial correlation and cross polarization discrimination
- determinstic models define a channel model according to prediction of propagation signal
- geometry based has immediate relation with physical characteristics of propagation channel
- clusters of scatters distributed around transmitter and receiver
- scatters result in discrete channel path and can involve statistical characterization of propagation parameters

Multi User MIMO

- analytical models - based on statistical properties through measurement
- distribution of direction of departure, distribution of direction of arrival
- MIMO - improves capacity of communication link without the need of increase the transmitter power
- MIMO system capacity evaluated according to following scenarios
- when no channel state information (CSI) available at transmitter
- the power is equally split between N_T transmit antennas
- instantaneous channel capacity (bits/s/Hz)

$$C(H) = \log_2 \left[det(I_{N_R} + \frac{\gamma}{N_T} HH^*) \right]$$

- γ denotes SNR, $(\cdot)^*$ stands for conjugate transpose operator

Multi User MIMO

- when CSI is available at receiver, SVD is used to derive MIMO channel capacity

$$C_{SVD}(H) = R \log_2 \left[det(1 + \frac{\gamma}{N_T} HH^*) \right]$$

$R = min(N_R, N_T)$ rank of channel matrix H

- when CSI is available at both transmitter and receiver
- the channel capacity is computed by performing water-filling algorithm

$$C_{WF}(H) = \sum_{p=1}^{R} \log_2 \left[\left(\frac{\lambda_{H,p}\mu}{\sigma_b^2} \right)^+ \right)$$

- $a^+ = max(a, 0)$, $\lambda_{H,p}$ p-th singlular value of channel matrix H
- μ constant scalar satisfies total power constraint, σ_b^2 noise signal power

MIMO: Diversity

- introducing higher modulation types or providing larger bandwidth - achieved by MIMO
- MIMO refers to channel - transmitter is channel input and the receiver channel output
- different diversity modes
- time diversity - differnt timeslots and channel coding
- frequency diversity - different channels, spread spectrum and OFDM
- spatial diversity - use of multiple antennas at the transmitter or receiver end
- MIMO increase data rate - spatial multiplexity

MIMO: Antenna Diversity

- the effect of attenuation can be compensated by
- receiving the signal at two different receiver antenna placement location and
- selecting the best antenna based on the signal strength or some other receiver performance measure
- the signal strengths associated with the antennas are generally not correlated in time and/or space and
- when one signal is in a null, the other can be found near a maximum and selected for communication
- the selection of one antenna over the other based on signal quality or
- selection diversity is technique that works well for non-selective fading in that
- the antenna selection essentially is a signal strength restoration technique

MIMO: Antenna Diversity

- selection diversity and other signal strength restoration techniques are not enough to
- equalize the impairments imposed by the multipath channel due to selective fading
- selective fading occurs when the signal bandwidth exceeds the coherence bandwidth of channel
- the coherence bandwidth of the channel is approximately the reciprocal of delay spread of the channel
- the delay spread is the rms average of delay times of the complex impulse response of the channel h
- h is the received complex envelope at a particular point in space assuming that a carrier signal is modulated by dirac impulse
- the arrival of various signal components can be modeled by FIR filter

MIMO: Eqaulizer

- ideally, channel transfer function only adds
- a flat fading process (constant attenuation versus frequency) and a single fixed time delay (linear phase)
- actually, frequency dependent fading of amplitudes and nonlinear phase response
- if the sidebands of a complex modulated signal where distributed across the transfer function,
- there would be signal distortion
- to compensate these effects the transfer function encountered via communication channel
- must be equalized at the receiver

MIMO: Eqaulizer

- ideally the inverse of transfer function applied to signal prior to detection to flatten
- the amplitude response and to linearize phase the signal impairments would be eliminated
- there are several different types of transversal equalizers that
- operate on time domain representation of the baseband signal to achieve equalization
- many rely on training sequences and training periods
- that creates communication overhead

OFDM and Channel Equalization

- a means of spectrally efficient communications,
- also a means of channel equalization in the frequency domain
- OFDM spectrum 48 data subcarriers and 4 pilot subcarriers
- pilot subcarriers are not information bearing but are used
- to help maintain the carrier and timing tracking in low SNR since there are always BPSK modulated
- OFDM provides a means of sampling the magnitude and phase of the channel
- at any or all of the subcarrier frequencies,
- since the carrier phase tracked at the receiver and all of the subcarrier phases are coherent to the main carrier
- in practice, performing continuous subcarrier phase tracking on each of subcarrier is not exectuted, rather,

OFDM and Channel Equalization

- short and long training symbols with known patterns are transmitted with each packet
- such that the receiver can determine the channel transfer function, invert it and then
- apply the inverse to the received spectrum in order to equalize the cahnnel on a packet by packet basis
- how the channel transfer function due to selective fading weights the received spectrum at the receiver
- the magnitude and phase of each of subcarriers provide an estimate of channel transfer funcion $H_c()$
- equalization transfer function $H_e()$ can be applied to the channel weighted spectrum
- to re-normalize the spectrum such that the effects of the channel can be reduced or eliminated

OFDM and Channel Equalization

$$S_{RX}(\omega) = H_c(\omega)S_{TX}(\omega)$$

$$H_c(\omega) = \frac{S_{RX}(\omega)}{S_{TX}(\omega)}$$

$$H_r(\omega) = H_c(\omega)H_e(\omega)$$

$$H_e(\omega) = H_c^{-1}(\omega)$$

$$H_r(\omega) = H_c(\omega)H_c^{-1}(\omega) = 1$$

Exploiting Diversity

- using MIMO - additional paths can be used to advantage - by increasing data capacity
- two main formats for MIMO
- spatial diversity - refers to transmit and receive diversity
- provide improvment in SNR, improving realiability of the system with respect to various forms of fading
- spatial multiplexing - provide additional data capacity by utilizing
- different paths to carry additional traffic, increasing data througput capability
- the maximum amount of data carried by radio channel limited by shannon's law

Exploiting Diversity

- antenna selection diversity is a technique for improving the performance in a non-selective fading channel
- OFDM provides for channel equalization against frequency selective fading
- SISO systmes do not provide any type of robustness or capacity improvment
- SIMO systems provide receive-side diversity and additional robustness but no capacity improvement
- receivers with selection diversity schemes or linear maximal ratio combining scheme

Exploiting Diversity

- MISO - provides transmitter diversity
- it couples to the channel at a different point in space such that
- the links will not have the same fading characterisitcs though to the receive antenna and
- spatial sum of the signals will be dominated by the stronger of the two signals or
- transmitter signal design can be such that the combining at the single receiver
- can be made in an optimal fashion as is done in space-time coding techniques

MIMO: Spatial Multiplexing

- MIMO systems de-multiplex the source data stream into multiple independent channel streams
- provides both redundancy and channel capacity improvement
- MIMO referred to as Spatial Division Multiplexing
- a single source data stream is multiplexed between two spatial streams
- direct links and cross links between two transmitters and two receivers, for example,
- there are four different communication channels that connect the two terminals,
- the channel characterization has a higher complexity

MIMO: Spatial Multiplexing

- another method to minimize the effects of a multipath communication channel through its redundancy and channel equalization properties
- MIMO - to transfer more data at the same time
- take advantage of a radio wave phenomenon called multipath where
- transmitted information bounces off walls, ceilings and other objects
- reaching the receiving antenna multiple times via differnt angle and at slightly differnt times
- allow mutiple antennas to send and receive multiple spatial streams at the same time

MIMO: Spatial Multiplexing

- combine data streams arriving from different paths and at different times to effectively
- increase receiver signal capturing power
- if there are more antennas than spatial streams; the additional antennas can add receiver diversity and increase range
- increase the capacity of a given channel
- increasing the number of receiver and transmit antennas
- it is possible to linearly increase the througput of the channel with every pair of antennas added to the system
- SISO there is no diversity and no additional processing required
- performance limited by interference and fading
- throughput depends upon the channel bandwidth and SNR

Exploiting Diversity

- SIMO - receive diversity
- receives signals from a number of independent sources to combat the effects of fading
- processing is required in the receiver
- receiver located in mobile device, limited by size, cost and battery drain
- switched diversity SIMO - looks for strongest signal and switches to that antenna
- maximum ratio combining SIMO - takes both signals and sums them to give a combination
- signals from both antennas contribute to overall signal

MIMO: Spatial Diversity

- to make the transmission more roubust
- no increase in data rate; mode uses redundant data on different paths
- receive diversity - RX diversity
- RX diversity uses more antennas on the receiver side than on the transmitter side
- simplest scenario consists of two RX and TX SIMO 1×2
- special coding methods are not needed, this scenario easy to implement only two RF paths are needed for the receiver
- RX $\leftarrow$ A and RX $\leftarrow$ B; two paths A and B
- switched diversity $C = \max(A, B)$ - uses stronger signal
- maximum ratio combining $C = (A + B)$ - uses the sum signal from two signals
- different transmission paths, the receiver sees two differently faded signals
- by appropriate method in the reciver, SNR can be increased

MIMO: TX Diversity

- there are more TX than RX antennas
- two TX and RX MISO 2 by 1
- the same data is transmitted redundantly over two antennas
- advantage that the multiple antennas and redundancy coding is moved from the mobile UE to the base station, where
- these technologies are simpler and cheapter to implement
- to generate a redundant signal, space time codes are used
- Alamouti developed the first codes for two antennas
- space time codes improves performance and make spatial diversity usable
- signal copy is transmitted not only from a different antenna but also at a different time
- delayed transmission is called delayed diversity
- space-time codes combine spatial and temporal signal copies

Exploiting Diversity

- MISO - termed transmit diversity
- the same data is transmitted redundantly from the two transmitter
- receiver is able to receive the optimum signal which it can use to extract required data
- processing is moved from receiver to transmitter
- MIMO - a variety of signal paths to carry the data,
- choosing separate paths for each antenna to enable multiple signal paths to be used
- space-time signal processing
- mutiple spatially distributed antennas located at different points
- signal can take many paths - as a result of number of objects introduce interference

MIMO: Spatial Multiplexing

- signaling technique combine with properties of OFDM modulation leads to efficient channel estimation and equalization scheme
- even signals are transmitted on TX0 and odd symbols are transmitted on TX1
- two receivers RX0 and RX1 receive both the transmitted streams
- through 4 possible channel transfer functions $h_{00}, h_{01}, h_{10}, h_{11}$ forms a channel matrix
- in order to equalize and extract the source symbol stream,
- estimate of channel matrix must be made transmitting both streams using
- a coordinated system of pilot subcarriers and null carriers

MIMO Channel Equalization

- the received symbols at each of receive inputs can be computed as follows:

$$\begin{aligned} r_0 &= h_{00}s_0 + h_{10}s_1 \\ r_1 &= h_{10}s_0 + h_{11}s_1 \end{aligned} \quad \begin{bmatrix} r_0 \\ r_1 \end{bmatrix} = \begin{bmatrix} h_{00} & h_{10} \\ h_{01} & h_{11} \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} = [R] = [H][S]$$

- for case of pilot subcarriers relative to null subcarriers
- $r_{00} = h_{00}s_0 \; r_{10} = h_{10}s_1 \; r_{01} = h_{01}s_0 \; r_{11} = h_{11}s_1$
- the magnitude and phase of pilot subcarriers is known,
- the the s-terms can be factored out and the elements of the channel matrix are determined
- given channel matrix the received signals can be equalized and restored

MIMO Channel Equalization

$$[S] = [H]^{-1}[R] = \left[\frac{adj[H]}{|H|}\right][R] = \left[\frac{\begin{bmatrix} h_{11} & -h_{01} \\ -h_{10} & h_{00} \end{bmatrix}^T}{h_{00}h_{11} - h_{10}h_{01}}\right][R]$$

$$[S] = \left[\frac{1}{h_{00}h_{11} - h_{10}h_{01}}\right] \begin{bmatrix} h_{11} & -h_{10} \\ -h_{01} & h_{00} \end{bmatrix} \begin{bmatrix} r_0 \\ r_1 \end{bmatrix}$$

- transmitted signal can be restored perfectly if channel matrix can be estimated perfectly
- does not work due to singularity in the matrix inversion where all channel elements are equal or correlated

MIMO: Types of space time code

- spatial diversity
  - ST block code - provides diversity gain, no coding gain
  - ST trellis code - diversity and coding gain
- spatial multiplexing
  - layered ST code - coding gain and diversity gain, bandwidth efficiency
- BLAST: Bell labs layered space time
- V-BLAST and D-BLAST
- vertical BLAST; it is 1 : N demux on channel cc0, cc1, cc2, ccN
- TX1 $\longrightarrow$ encoder $\alpha \longrightarrow \alpha_0\ \alpha_1 \dots$
- TX2 $\longrightarrow$ encoder $\beta \longrightarrow \beta_0\ \beta_1 \dots$
- TX3 $\longrightarrow$ encoder $\gamma \longrightarrow \gamma_0\ \gamma_1 \dots$
- space vs. time
- simple and low complexity, lower capacity than shannon bound
- Data rate $\propto N$

MIMO: Spatial diversity

- diagonal BLAST
- TX1 $\longrightarrow$ encoder $\alpha \longrightarrow \alpha_0\ \beta_0\ \gamma_0 \dots$
- TX2 $\longrightarrow$ encoder $\beta \longrightarrow \dots\ \alpha_1\ \beta_1\ \gamma_1 \dots$
- TX3 $\longrightarrow$ encoder $\gamma \longrightarrow \dots\ \dots\ \alpha_2 \dots$
- closer to shannon bound due to spatial switch diversity gain
- high complexity, coding constraints
- Beamforming schemes
- beamformer - spatial filter that combines array inputs
- form a beam in space to recive desired signal while suppressing interference and noise
- transmit beamformer and receive beamformer

MIMO: Beamforming

- antenna technologies are the key in increasing network capacity
- started with sectorized antennas
- antennas illuminate 60 or 120 degrees and operate as one cell
- in GSM the capacity can be tripled, by 120 degree antennas
- adaptive antenna arrays intensity spatial multiplexing using narrow beams
- smart antennas belong to adaptive antenna arrays but differ in their smart direction of arrival estimation
- smart antennas can form a user specific beam
- optional feedback can reduce complexity of array system
- beamforming is the method used to create the radiation pattern of an antenna array applied to MIMO system

MIMO: Beamforming

- smart antennas divided into two groups
- phased array systems (switched beamforming) with a finite number of fixed predefined patterns
- adaptive array systems (adaptive beamforming) with an infinite number of patterns adjusted to the scenario in realtime
- switched beamformers electrically calculate the DoA and switch on the fixed beam
- the user only has the optimum signal strength along the center of the beam
- the adaptive beamformer deals with that problem and adjusts the beam in realtime to the moving UE
- complexity and cost higher than switched beamformer
- various mobile radio and network standards use MIMO
- all standards use TX diversity and spatial multiplexing

MIMO: Alamouti coding

- signals S_0 and S_1 are multiplexed in two data chains
- after that a signal replication is added to create the alamouti space-time block code

$$[S_0 \;\; S_1] \rightarrow \begin{bmatrix} S_0 \\ S_1 \end{bmatrix} \rightarrow \begin{bmatrix} S_0 & -S_1^* \\ S_1 & S_0^* \end{bmatrix}$$

- -TX1 and -TX2 space and time
- coding can also be handled in frequency domain; called space-frequency coding
- spatial multiplexing
- not intended to make transmission more robust; rather it increases data rate data is divided into separate streams
- the streams are transmitted independently via separate antennas

MIMO: Alamouti coding

- MIMO transmits via the same channel, transmissions using cross components
- MIMO 2×2 $H = \begin{bmatrix} h_{00} & h_{01} \\ h_{10} & h_{11} \end{bmatrix}$
- if transmission matrix H is known the cross components can be calculated on the receiver
- in open-loop method, the transmission includes the special sections that are also known to the receiver
- the receiver can perform a channel estimation
- in closed-loop method, the receiver reports the channel status to the transmitter via a special feedback channel
- makes it possible to respond to changing circumstances

MIMO: Transmit diversity

- a simple transmit diversity technique for wireless communications
- Alamouti IEEE journal on selected areas in communication Oct 1998
- ST block code

$$[S_0 \;\; S_1] -> \begin{bmatrix} S_0 & -S_1^* \\ S_1 & S_0^* \end{bmatrix}$$

- encoding and tramission sequence: at a given symbol period
- two signals are simultaneously transmitted from the two antennas
- space vs. time; transmit

$$X = \begin{bmatrix} S_0 & -S_1^* \\ S_1 & S_0^* \end{bmatrix}$$

- ST block codeword property

$$X^H X = \begin{bmatrix} |S_0|^2 + |S_1|^2 & 0 \\ 0 & |S_0|^2 + |S_1|^2 \end{bmatrix}$$
$$= \left(|S_0|^2 + |S_1|^2\right) I_2$$

- codeword X that satisfies property is called orthogonal design

MIMO: Transmit diversity

- fading is constant across two consecutive symbols

$$h_0(t) = h_0(t+T) = h_0 = |\alpha_0| e^{j\theta}$$

$$h_1(t) = h_1(t+T) = h_1 = |\alpha_1| e^{j\theta}$$

- T is symbol duration
- received signal can be expressed as

$$r_0 = r(t) = h_0 s_0 + h_1 s_1 + n_0$$

$$r_1 = r(t+T) = -h_0 s_1^* + h_1 s_0^* + n_1$$

$$\mathbf{R} = \begin{bmatrix} r_0 \\ r_1^* \end{bmatrix} = \begin{bmatrix} h_0 & h_1 \\ h_1^* & -h_0^* \end{bmatrix} + \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1^* \end{bmatrix} = \bar{\mathbf{H}}\mathbf{s} + \mathbf{n}$$

MIMO: Transmit diversity

- $\bar{\mathbf{H}}$ satisfies

$$\bar{\mathbf{H}}^H\bar{\mathbf{H}} = \left(|h_0|^2 + |h_1|^2\right)\mathbf{I}_2 = \rho \cdot \mathbf{I}_2$$

- $\rho = \left(|h_0|^2 + |h_1|^2\right)$ is diversity gain
- combining scheme - combiner builds two combined signals that are sent to the ML detector

$$\tilde{\mathbf{S}} = \begin{bmatrix} \tilde{S}_0 \\ \tilde{S}_1 \end{bmatrix} = \bar{\mathbf{H}}^H\mathbf{R} = \bar{\mathbf{H}}^H\bar{\mathbf{H}}\mathbf{S} + \bar{\mathbf{H}}^H\mathbf{n} = \rho\mathbf{S} + \bar{\mathbf{H}}^H\mathbf{n}$$

$$= \rho\begin{bmatrix} S_0 \\ S_1 \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \end{bmatrix}$$

S_0 and S_1 are detected independently

MIMO Modelling

- received signal vector $\mathbf{y}$ channel matrix $\mathbf{H}$

$$y = Hx + n$$

$$y = [y_1\ y_2\ \dots y_N]^T$$

received signal vector

$$X = [x_1\ x_2\ \dots x_M]^T$$

transmitted signal vector

$$n = [n_1\ n_2\ \dots n_M]^T$$

noise vector

$$H = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1M} \\ h_{21} & h_{22} & \dots & h_{2M} \\ \vdots & \vdots & \vdots & \vdots \\ h_{N1} & h_{N2} & \dots & h_{NM} \end{bmatrix}$$

transmitted power $P_T = X^HX = |x_1|^2 + |x_2|^2 + \dots + |x_M|^2 \leq C$

MIMO Modelling

- covariance matrices

$$R_{xx} = E\left\{XX^H\right\} \quad R_{yy} = E\left\{yy^H\right\} = E\left\{HXX^HH^H\right\} + E\left\{nn^H\right\}$$

- traces of R_{xx} and R_{yy} give total powers of transmitted and received signals
- offdiagonal elements of R_{xx} and R_{yy} give correlations between the signals at different antennas elements
- consider a symbol period of time T_s for transmitted signals such that $R_{xx} = E\left\{XX^H\right\} = I_M$ and within T_s

$$R_{yy} = E\left\{HXX^HH^H\right\} + E\left\{nn^H\right\}$$

$$= HE\left\{XX^H\right\}H^H + R_{nn} = HH^H + R_{nn}$$

R_{nn} is noise covariance

- over a longer period of time ($> T_s$) average received signal covariance matrix is

$$R_{yy} = E\left\{HH^H\right\} + R_{nn}$$

Equivalent MIMO

- select H to optimize the channel output SNR so as the capacity of MIMO
- for MIMO calculation of the capacity is more complicated due to determination of SNR
- channel matrix H ($N \times M$) $y = Hx + n$
- using SVD singular value decomposition $H = UDV^H$
- D is $N \times M$ a diagonal matrix with non-negative elements
- U is $N \times N$ unitary matrix; V is $M \times M$ unitary matrix
- $UU^H = U^HU = I_N$ and $VV^H = V^HV = I_M$
- diagonal elements of D are called singular values of H
- there are non negative square roots of the eigenvalues λ of the following equation

$$\left(HH^H\right)X = \lambda X \quad if \quad N < M$$

$$\left(H^HH\right)X = \lambda X \quad if \quad N \geq M$$

Equivalent MIMO

- x is $N \times 1$ eigenvector associated with λ

$$y = UDV^H X + n$$

- consider transformations $y' = U^H y \quad X' = V^H X \quad n' = U^H n$

$$U^H y = U^H UDV^H X + U^H n$$

$$y' = DV^H X + n' \quad y' = Dx' + n'$$

- called equivalent MIMO system

$$R_{y'y'} = E\left\{y'y'^H\right\} = E\left\{U^H yy^H U\right\} = U^H R_{yy} U$$

$$R_{x'x'} = E\left\{x'x'^H\right\} = E\left\{V^H xx^H V\right\} = V^H R_{xx} V$$

$$R_{n'n'} = E\left\{n'n'^H\right\} = E\left\{U^H nn^H U\right\} = U^H R_{nn} U$$

- $tr\{R_{y'y'}\} = tr\{R_{yy}\}$, $tr\{R_{x'x'}\} = tr\{R_{xx}\}$, $tr\{R_{n'n'}\} = tr\{R_{nn}\}$

Equivalent MIMO

- system has the same total input power, total output power and total noise power as actual MIMO
- the output SNR of equivalent MIMO is same as actual MIMO
- it means channel capacity of equivalent MIMO is same as that of actual MIMO capacity is function of output SNR
- channels all decoupled, N channels are parallel to each other with
- channel gains given by diagonal elements of $D \rightarrow \sqrt{\lambda_i}$ $i = 1, \ldots, N$
- number of nonzero eigenvalues of matrix HH^H is equal to rank of matrix H denoted by r

$$y_i' = \sqrt{\lambda_i} x_i' + n_i'$$

for $i = 1, 2, \ldots, r$

$$y_i' = 0 + n_i'$$

for $i = r, \ldots, N$

Equivalent MIMO

- MIMO has more transmitting antennas than receiving antennas $M > N$
- than H is a horizontal mattrix with a maximum rank N
- maximum number of uncoupled equivalent MIMO channel is N
- remaining $M - N$ transmitting antennas will become redundant with no receiving antennas
- if $M < N$ - more receiving antennas than transmitting antennas
- H is vertical matrix with maximum rank M
- maximum number of uncoupled equivalent MIMO channels is M
- remaining $N - M$ receiving antennas become redundant with no received signals
- maximum number of uncoupled equivalent channels is $\min(N, M)$

MIMO Channel Capacity

- channels of equivalent MIMO are uncoupled and parallel
- channel capacity can be calculated by a summation of individual capacities of parallel channel

$$C = B \sum_{i=1}^{r} \log_2 \left[1 + \frac{P_{y_i'}}{\sigma^2}\right]$$

- B channel bandwidth, $P_{y_i'}$ power received at the i th receiving antenna σ^2 noise power at receiving antenna
- r rank of H
- classify MIMO according availability of channel knowledge to transmitter or receiver

MIMO Channel Capacity

- channel state information CSI known to receiver only
- transmitter does not know CSI, the best strategy transmit power equally from all its transmitting antennas
- equivalent MIMO - making all elements of X' have same power
- received power $P_{y_i'} = \lambda_i \frac{P}{M}$
- P total transmitting power

$$C = B \sum_{i=1}^{r} \log_2 \left(1 + \lambda_i \frac{P}{M\sigma^2}\right) = B \log_2 \prod_{i=1}^{r} \left(1 + \lambda_i \frac{P}{M\sigma^2}\right)$$

- eigenvalue λ_i can be expressed in terms of matrix HH^H or $H^H H$

$$C = \begin{cases} B \log_2 det \left(I_N + \frac{P}{M\sigma^2} HH^H\right) & N < M \\ B \log_2 det \left(I_M + \frac{P}{M\sigma^2} H^H H\right) & N \geq M \end{cases}$$

MIMO Channel Capacity

- total transmitting power P not known, average received powers P_r at receiving antennas are same

$$P_r = P \times P_{loss}$$

- P_{loss} average path loss form the transmitter to receiver

$$C = \begin{cases} B \log_2 det \left(I_N + \frac{P_r}{M\sigma^2} \frac{HH^H}{P_{loss}}\right) & N < M \\ B \log_2 det \left(I_M + \frac{P_r}{M\sigma^2} \frac{H^H H}{P_{loss}}\right) & N \geq M \end{cases}$$

MIMO Channel Capacity

- CSI known to both transmitter and receiver
- if transmitter knows CSI, channel matrix H
- the best strategy is to transmit more power along those channels whose channel gains are larger and
- to transmit less power or along those channels with a smaller channel gain
- this is called water-filling principle
- the transmitting power P_i for the ith channel in equivalent MIMO

$$P_i = \left(\mu - \frac{\sigma^2}{\lambda_i}\right)$$

$i = 1, \ldots, r$ $r = rank(H)$

MIMO Channel Capacity

- if P_i is negative it will be set to zero
- μ is determined by satisfying transmitting power constraint

$$P = \sum_{i=1}^{r} P_i$$

- received powers

$$P_{y_{i'}} = \lambda_i P_i = (\lambda_i \mu - \sigma^2)$$

- channel capacity is

$$C = B \sum_{i=1}^{r} \log_2 \left[1 + \frac{1}{\sigma^2}(\lambda_i \mu - \sigma^2)\right]$$

Different Standards and MIMO support

- 3GPP mobile radio standard - UMTS undergone phases of development
- starting with WCDMA data acceleration methods have been introduced,
- including HSDPA and HSUPA
- newest releases cover HSPA+ and Long Term Evolution (LTE)
- HSPA+ (3GPP release 7/8)
- transmit diversity mode introduced in release 99 (WCDMA)
- release 7 of 3GPP specification (HSPA+) expanded this approach to MIMO and increased data rate with respect to release 6 (HSDPA)
- introduction of 64QAM modulation and MIMO in the downlink makes a peak data rate of 28 Mbps
- MIMO was introduced in the form of double transmit antenna array for high speed downlink shared channel (HS-DSCH)

Different Standards and MIMO support

- LTE (3GPP release 8) - UMTS LTE introduced in 3GPP release 8
- objective is high data rate, low latency and packet otpimized radio access technology
- LTE is referred as E-UTRA (Evolved UMTS Terrestrial Radio Access) or
- E-UTRAN (Evolved UMTS Terrestrial Radio Access Network)
- the basic concept for LTE is downlink is OFDMA (uplink: SC-FDMA)
- while MIMO is integral part of LTE
- modulation modes are QPSK, 16QAM, and 64QAM
- peak data rates of up to 300 Mbps (4 X 4 MIMO) and up to 150 Mbps (2 X 2 MIMO) in the downlink and
- upto 75 Mbps in the uplink are specified

Different Standards and MIMO support

- downlink - transmission modes possible in LTE
  - single antenna transmission, no MIMO
  - transmit diversity
  - open-loop spatial multiplexing, no UE feedback required
  - closed-loop spatial multiplexing, UE feedback required
  - Multi user MIMO (more than one UE is assigned to same resource block)
  - beamforming
  - closed-loop precoding for rank=1 (no spatial multiplexing, but precoding is used)
- WiMAX 802.16e-2005
  - peak data rate of 74 Mbps at a bandwidth of upto 20 MHz
  - modulation QPSK, 16QAM, 64QAM
- WLAN 802.11n
  - peak data rate of up to 600 Mbps at a bandwidth of 40 MHz
  - modulation BPSK, QPSK, 16QAM, 64QAM

Thank You