Conditional Probabilities Examples and Questions

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The defintion of conditional probabilities is presented along with examples and their detailed solutions and explanations. More questions are included at the bottom of the page followed by their solutions.

Conditional Probability Definition

We use a simple example to explain conditional probabilities.

Example 1

a) A fair die is rolled, what is the probability that a face with "1", "2" or "3" dots is rolled?

b) A fair die is rolled, what is the probability that a face with "1", "2" or "3" dots is rolled given (or knowing) that the number of dots rolled is odd?

Solution to Example 1

a)

Let S be the sample space (all possible outcomes) when a die is rolled, hence S as a set is given by

$S = \{1,2,3,4,5,6\}$

$n(S) = 6$, number of elements in the set S

Let A be the set representing the event "a "1", "2" or "3" is rolled", hence

$A = \{1,2,3\}$

$n(A) = 3$, number of elements in A

Using the formula of the classical probability, we have

$$P(A) = \dfrac{n(A)}{n(S)} = 3/6 = 1/2$$

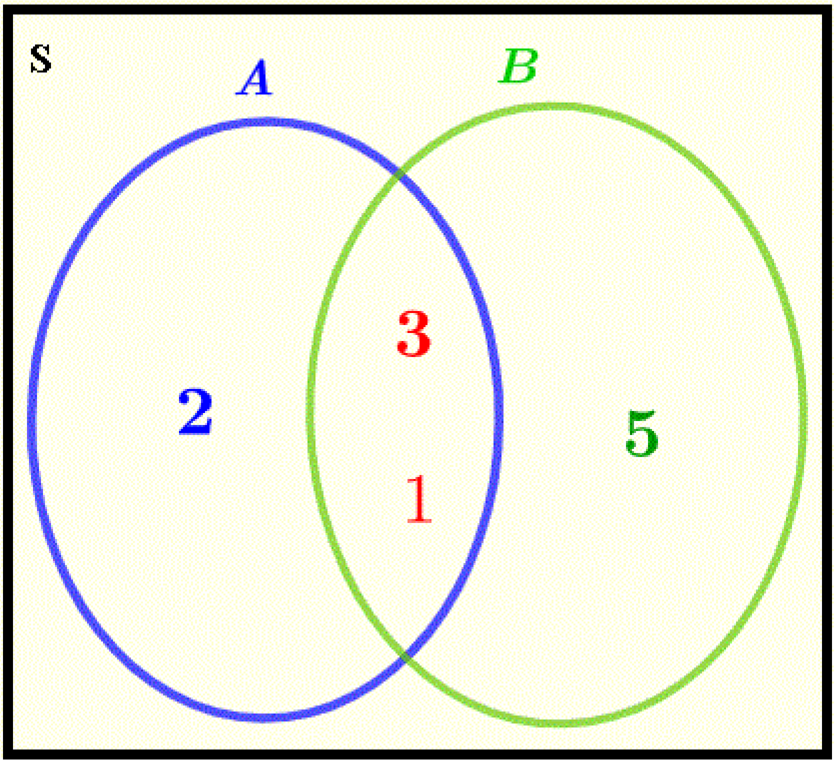
b)

Event A already defined in part a).

Let B be set representing the event "the number of dots rolled is odd", hence

$B = \{1,3,5\}$

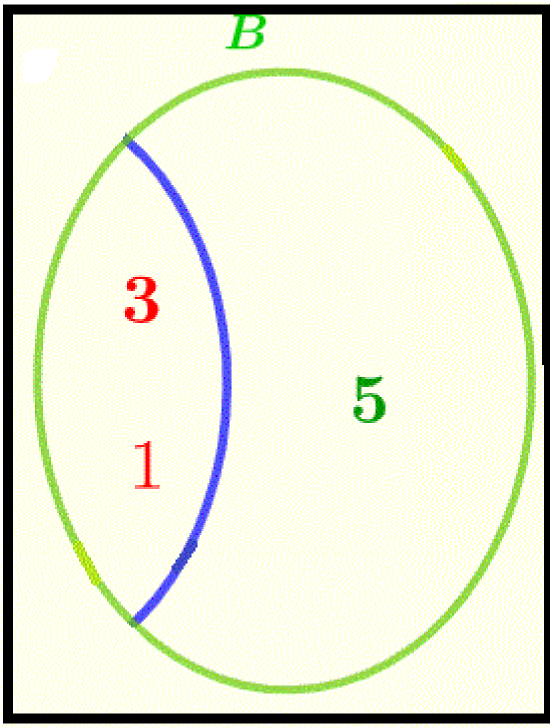
We use the Venn diagram to represent the sets A and B as follows



Because we know that the number rolled is in set B (odd number), part of \(A \) that is not in the intersection could be omitted and we are left with a restricted sample space B (see diagram below).

The Venn diagram with the restricted sample space (see diagram below) makes the calculation of the probability of A given B defined as follows

\(P(A|B) = \dfrac{\text{number of elements of A remaining in B }}{n(B)} = \dfrac{n(A \cap B)}{n(B)} = \dfrac{2}{3} \)



Divide the numerator and denominator by \(n(S) \) the sample space of a rolled die we obtain

\(P(A|B) = \dfrac{ \dfrac{n(A \cap B)}{n(S)} } {\dfrac{n(B)}{n(S)}} = \dfrac{P(A \; and \; B)}{P(B)} \)

By definition, the probability of event \(A \) occurring given that \(B \) has occurred is called conditional probability is written as \(P(A|B) \) and given by

\[P(A|B) = \dfrac{P(A \; and \; B)}{P(B)} \] or using the set notation

$$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$$

The above formulas are valid for $P(B) \ne 0$

NOTE Whenever possible in the examples below we use the definition as a formula and also the restricted sample space to solve conditional probability questions. This helps in a deeper understanding of the concept of conditional probabilities.

More Examples with Detailed Solutions

Example 2

In a group of kids, if one is selected at random the probability that he/she likes oranges is 0.6, the probability that he/she likes oranges AND apples is 0.3. If a kid, who likes oranges, is selected at random, what is the probability that he/she also likes apples?

Solution to Example 2

Let event O: kid likes oranges , event A: kid likes apples

Given $P(O) = 0.6$

Given $P(A \; and \; O) = 0.3$

We are asked to find the conditional probability $P(A|O)$

that the kid likes apples given that he likes oranges.

$P(A|O) = \dfrac{P(A \; and \; O)}{P(O)} = \dfrac{0.3}{0.6} = 0.5$

Example 3

The results of a survey of a group of 100 people who bought either a mobile phone or a tablet from any of two brands A and B is shown in the table below.

	Mobile Phone	Tablet	Total
A	20	10	30
B	30	40	70
Total	50	50	100

If a person is selected at random from the group, what is the

probability that he/she

a) bought brand B?

b) bought a mobile phone from brand B?

c) bought a mobile phone given that she/he bought brand B?

Solution to Example 3

Let

event Mp: bought a mobile phone

event T: bought a tablet

event A: bought brand A

event B: bought brand B

a)

A total of 70 people out of the total of 100 bought brand B;

hence

$P(B) = 70/100 = 0.7$

b)

A total of 30 people out of 100 bought a mobile from brand B

$P(Mp \; and \; B) = 30/100 = 0.3$

c)

$P(Mp|B) = \dfrac{P(Mp \; and \; B)}{P(B)} = 0.3/0.7 = 3/7$

The conditional probability $P(Mp|B)$ may also be found by restricting the sample space to brand B.

The are 30 mobile phones out of a total of 70 brand B; hence

$P(Mp|B) = \dfrac{30}{70} = 3/7$

Example 4

A single card is drawn from a deck. A card is selected at random, find the probability of selecting a

a) King

b) red card

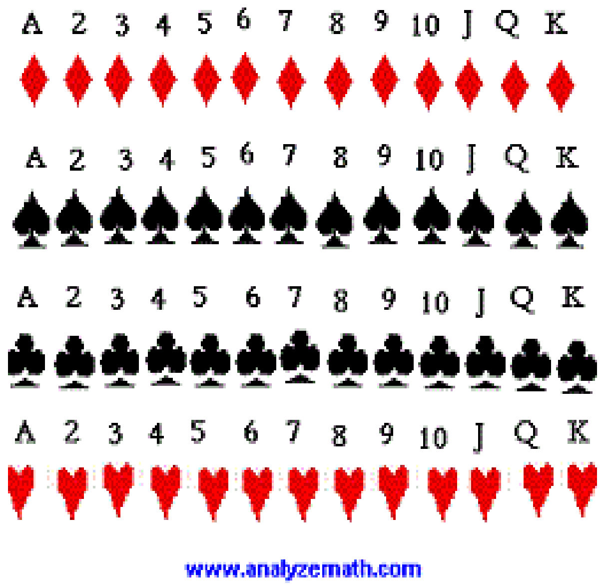
c) King of red card

d) King given that it is a red card

e) red card given that it is a King

f) Queen given that it is a Heart

Solution to Example 4



Outcomes are all equally likely to occur.

a)

Out of the sample space of 52 cards, there 4 Kings; hence

\(P(King) = \dfrac {4}{52} = 1/13 \)

b)

Out of the sample space of 52 cards, there 26 red; hence

\(P(red) = \dfrac {26}{52} = 1/2 \)

c)

2 cards are King of red out of the 52 cards; hence

\(P(\text{King and red}) = \dfrac{2}{52} = 1/26 \)

d)

Two methods to answer the question.

1) Using Definition of the conditional probability given above

$P(\text{King given that it is a red card}) = P(King|red) = \dfrac{ P(\text{King and red})}{P(red)} = \dfrac{1/26}{1/2} = 1/13$

2) Using the restricted sample space

Out of the 26 red cards (restricted sample space to the red only since this is the condition) there are 2 red; hence $P(King|red) = 2/26 = 1/13$ same as was found above using the definition

e)

Two methods to answer the question.

1) Using Definition of the conditional probability given above

$P(\text{red card given that it is a King}) = P(red|King) = \dfrac{ P(\text{red and King})}{P(King)} = \dfrac{1/26}{1/13} = 1/2$

2) Using the restricted sample space

Out of the 4 Kings cards (restricted sample space to the Kings) there are 2 red; hence

$P(red|King) = 2/4 = 1/2$ same as was found above using the definition

f)

Two methods to answer the question.

1) Using Definition of the conditional probability given above

$P(\text{Queen given that it is Heart}) = P(Queen|heart) = \dfrac{ P(\text{Queen and Heart})}{P(Heart)} = \dfrac{1/52}{13/52} = 1/13$

2) Using the restricted sample space

Out of the 13 hearts (restricted sample space to the hearts) there is 1 King; hence

$P(Queen|heart) = 1/13$ same as was found above using the definition

Example 5

A car dealer has the cars listed in the table below classified by type and color. If a car is selected at random, what is the probability that it is

a) It is black knowing that it is a Van

b) It is an SUV knowing that it is white

c) It is blue knowing that it is a Coupe

	SUV	**Sport Car**	**Van**	**Coupe**	Total
Black	35	10	25	15	85
white	10	15	20	5	50
Blue	15	15	5	30	65
Total	60	40	50	50	200

Solution to Example 5

There are 60 Suv's, 40 Sport cars, 50 Vans and 50 Coupe, a total of 200 cards.

We answer the questions on finding conditional probabilities using two methods: 1) the definition and 2) restriction of the sample space.

a)

Using conditional probability definition

$P(black|Van) = \dfrac{P(\text{black and Van})}{P(Van)} = \dfrac{25/200}{50/200} = 1/2$

Or restrict sample space to the Vans, there are 50 vans out of which 25 are black. Hence

$P(black|Van) = \dfrac{25}{50} = 1/2$

b)

Using conditional probability definition

$P(Suv|white) = \dfrac{P(\text{Suv and white})}{P(white)} = \dfrac{10/200}{50/200} = 1/5$

Restrict sample space to the white cars, there are 50 white out of which 10 are Suv's. Hence

$P(Suv|white) = \dfrac{10}{50} = 1/5$

c)

Using conditional probability definition

$P(blue|Coupe) = \dfrac{P(\text{blue and Coupe)}}{P(Coupe)} = \dfrac{30/200}{50/200} = 3/5$

Restrict sample space to the Coupe, there are 50 Coupe out of which 30 are blue. Hence

$P(blue|Coupe) = \dfrac{30}{50} = 3/5$

Example 6

The results of a survey of a group of 100 people having insurances with a certain company are as follows: 40% have both home and car insurances with the company. The probability that person selected at random from this group, has a car insurance is 0.7. What is the probability that a person selected at random has a home insurance knowing that he has a car insurance?

Solution to Example 6

Let event H: people with home insurance, event C: people with can insurance

We are given P(C) = 0.8 and P(H and C) = 0.5.

We are asked to find the conditional probability $P(H|C)$ that a person selected at random have a home insurance (H) knowing that this person has a car insurance (C). Hence

$P(H|C) = \dfrac{P(H \; and \; C)}{P(C)} = 0.5 / 0.8 = 0.625$

Example 7

A group of 200 Students were asked whether they played football or basketball. Among the group, 120 said they played football, 50 said they played basketball and 20 said they played both football and basketball.

a) What is the probability that a students selected at random from the group plays football given that he plays basketball?

b) What is the probability that a students selected at random from the group plays basketball given that he plays football?

c) What is the probability that a students selected at random from the group plays football given that he plays one game only.

Solution to Example 7

Let event F: students who play football, event B: students who play basketball

a)

Let us find the following probabilities

$P(F) = 120 / 200 = 0.6$

$P(B) = 50/200 = 0.25$

$P(F \; and \; B) = 20 / 100 = 0.1$

$P(F|B) = \dfrac{P(F \; and \; B)}{P(B)} = 0.1 / 0.25 = 0.4$

b)

$P(B|F) = \dfrac{P(B \; and \; F)}{P(F)} = 0.1 / 0.6 = 0.17$

c)

Let event O: students who play one game only

The number of students who play one game only is

$(120 - 20) + (50 - 20) = 130$

$P(O) = 130/200 = 0.65$

$P(F \; and \; O) = 100 / 200 = 0.5$

$P(F|O) = \dfrac{P(F \; and \; O)}{P(O)} = 0.5 / 0.65 = 0.77$

Questions and their Solutions

Question 1

A die is rolled, find the probability that an even number is obtained knowing the number is greater than 3.

Question 2

A card is drawn at random from a deck of 52 cards. Find the probability of getting a 3 knowing that the card is red.

Question 3

A group 120 people were asked if they own a car or a bicycle. 90 said they owned a car, 40 they owned a bicycle and 10 said they owned neither a car nor a bicycle.

If a person is selected at random from this group of people, what is the probability that this person

a) owns a car and a bicycle?

b) owns a car given that the person selected owns a bicycle?

c) owns a bicycle given that the person owns a car or a bicycle but not both?

Question 4

Two companies A and B are offering 70 and 50 products respectively. Company A is offering 40 software products and 30 hardware products. Company B is offering x hardware products and y software products to be determined.

If a product is selected at random, what is the probability that

a) this product is a hardware product given that it is from company B? (in terms of y)

b) this product is a hardware product given that it is from company A?

c) For what values of y will the probability in part a) be greater than the probability in part b)?

Solution to Question 5

A financial advisor believes that the probability that the stock market will go down is 0.8 given that the economy deteriorates. The advisor also believes that the probability that the economy will deteriorates is 0.5.

Taking the above information into account, what is the probability that the economy will deteriorate and the stock market will go down?

Solutions to the Above Questions

Solution to Question 1

The sample space in throwing a die: $S = \{1,2,3,4,5,6\}$

event E: even number , $E = \{2,4,6\}$

event G number greater than 3: $G = \{4,5,6\}$

$E \cap G = \{4,6\}$

The question is to find the probability that an even number is obtained knowing the number is greater than 3 written as a conditional probability $P(E|G)$.

$P(E|G) = \dfrac{P(E \cap G)}{P(G)} = \dfrac{2/6}{3/6} = 2/3$

This question might be answered as follows

There are 2 elements in $E \cap G$ and 3 elements in G

We restrict the sample space to event G; hence

$P(E|G) = \dfrac{n(E \cap G)}{n(G)} = 2/3$

Solution to Question 2

Let event T: getting a 3 , there are 4 3's in a deck of cards, hence $P(T) = 4/52 = 1/13$.

Let event R: getting a red card , there are 26 red cards in a deck of cards, hence $P(R) = 26/52 = 1/2$

There are 2 3's cards that are red, hence \(P(T \cap R) =

$2/52 = 1/26$

We are asked to find the conditional probability of getting a 3 knowing that the card is red written as $P(T|R)$

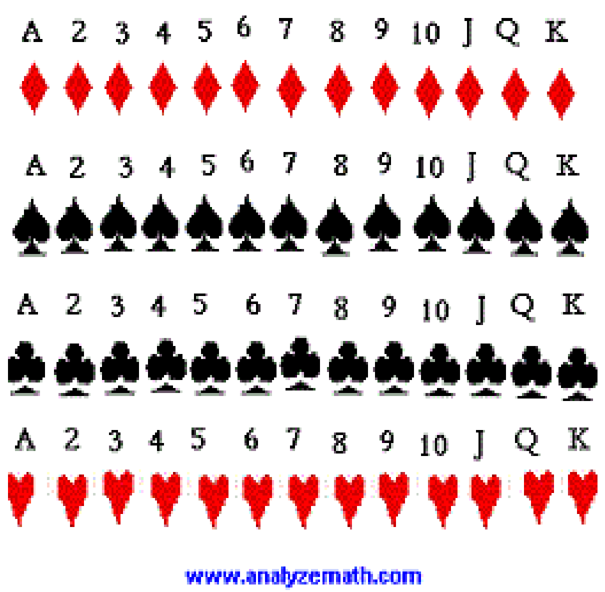
$$P(T|R) = \dfrac{P(T \cap R)}{P(R)} = \dfrac{1/26}{1/2} = 1/13$$

This question might be answered as follows

There are 2 3's cards that are red

We restrict the sample space to red cards which are 26, hence

$P(T|R) = 2/26 = 1/13$



Solution to Question 3

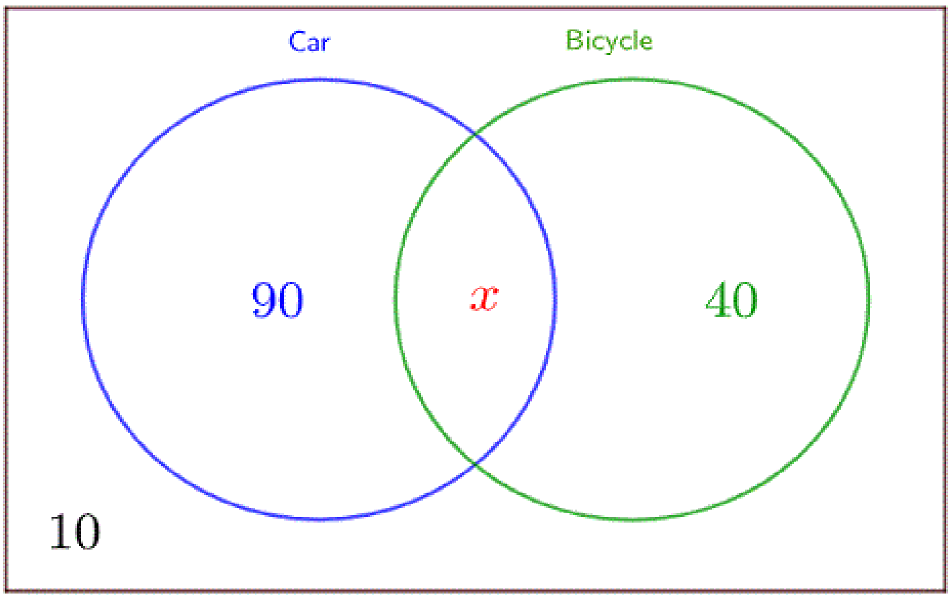
a)

Let C the event: owns a car , Let B the event: owns a bicycle

We are asked to find \(P(C \cap B) \)

Let \(x \) be the number of people who owns a car and a bicycle. Using the Venn diagram, below we have \(90 - x\) own a car only, \(40 - x\) own a bicycle only and \(10 \) own neither and the total is \(120\); hence

\((90 - x) + (40 - x) + x + 10 = 120 \)



Solve for \(x \) to obtain

\(x = 20 \)

\(P(C \cap B) = 20/120 = 1/6\)

b) owns a car given that the person selected owns a bicycle?

We are asked to find the conditional probability $P(C|B)$.

$P(C|B) = \dfrac{P(C \cap B)}{P(B)} = \dfrac{1/6}{40/120} =$

$20/40 = 1/2$

c) owns a bicycle given that the person owns a car or a bicycle but not both?

Let event COB : own a car or a bicycle but not both

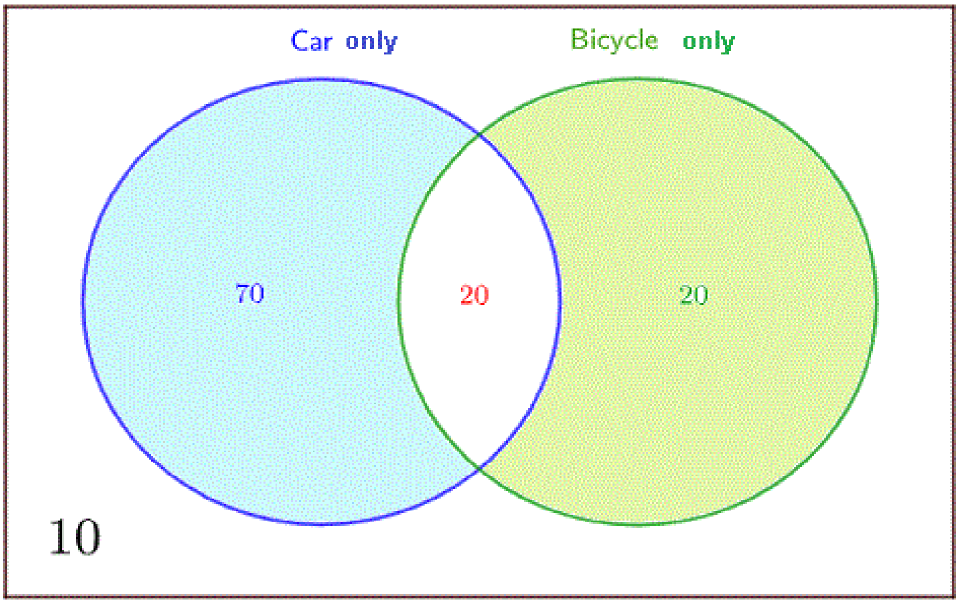
The people who own a car or a bicycle but not both are included in the union without the intersection and their number is

$N = 70 + 20 = 90$

We are asked to find the probability $P(B|COB)$.

$P(B|COB) = \dfrac{P(B \cap COB)}{P(COB)} =$

$\dfrac{20/120}{90/120} = 2/9$



Solution to Question 4

Let us arrange all the given information on a table as follows

	Software	**Hardware**	Total
Company A	40	30	70
Company B	x	y	x + y
Total	40+x	30+y	70+x+y

a)

The total number of products is: $70 + 50 = 120$

We also have

$70+x+y = 120$

which gives

$x + y = 50$

Let event S: product selected is a software product , let event

H: product selected is a hardware product

Let event A: product selected is from company A , let event

B: product selected is from company B

We are asked to find the conditional probability $P(H|B)$.

$P(H|B) = \dfrac{P(H \cap B)}{P(B)} = \dfrac{\dfrac{y}{120}}{\dfrac{x+y}{120}} = \dfrac{y}{x+y} = y / 50$

b)

We are asked to find the conditional probability $P(H|A)$.

$P(H|A) = \dfrac{P(H \cap A)}{P(A)} = \dfrac{\dfrac{30}{120}}{\dfrac{70}{120}} = \dfrac{3}{7}$

c)

We need to solve the inequality

$y / 50 \gt 3 / 7$

Multiply all terms by 50 and simplify

$y \gt 150 / 7$

Simplify

$y \gt 21.42$

y is a positive integer, hence

$y \ge 22$

Solution to Question 5

Let event E: the economy will deteriorate , let event S: the stock market will go down.

We are given the conditional probability $P(S|E) = 0.8$

and the probability $P(E) = 0.5$

We are asked to find $P(S \cap E)$.

The definition of the conditional probability gives

$P(S|E) = \dfrac{P(S \cap E)}{P(E)} = 0.8$

hence

$P(S \cap E) = 0.8 \times P(E) = 0.8 \times 0.5 = 0.4$

More References and links

probability questions

classical formula for probability

Binomial Probabilities Examples and Questions

mutually exclusive events

Introduction to Probabilities

sample space

event

elementary statistics and probabilities.