Chapter 9
Morphological image processing

Morphological image processing

- Morphology

  A branch of biology that deals with the form and structure of animals and plants.

- Mathematical morphology

  A tool to extract image components for representing and describing region shapes

  E.g.: boundary, skeleton, convex hull…

Application

- Preprocessing
  - Filtering
  - Shape simplification
- Segmentation using object shape
  - Object quantification area, perimeter etc.
- Enhancing object structure
  - Skeletonization
  - Thining
  - Thicking
  - Convex hull
  - Object marking

Morphological image processing

Definitions

Let *A be a set in* Z^2. *If* $a = (a_1, a_2)$ *is an element of A, then we write* $a \in A$.

If *a is not an element of A we write* $a \notin A$

A set with no elements is called the *null or empty set and is denoted by* the symbol $\varnothing$

A set is specified by the contents of two braces: $\{\cdot\}$

For binary images, the elements of the sets are the *coordinates of pixels* representing objects

Morphological transformation

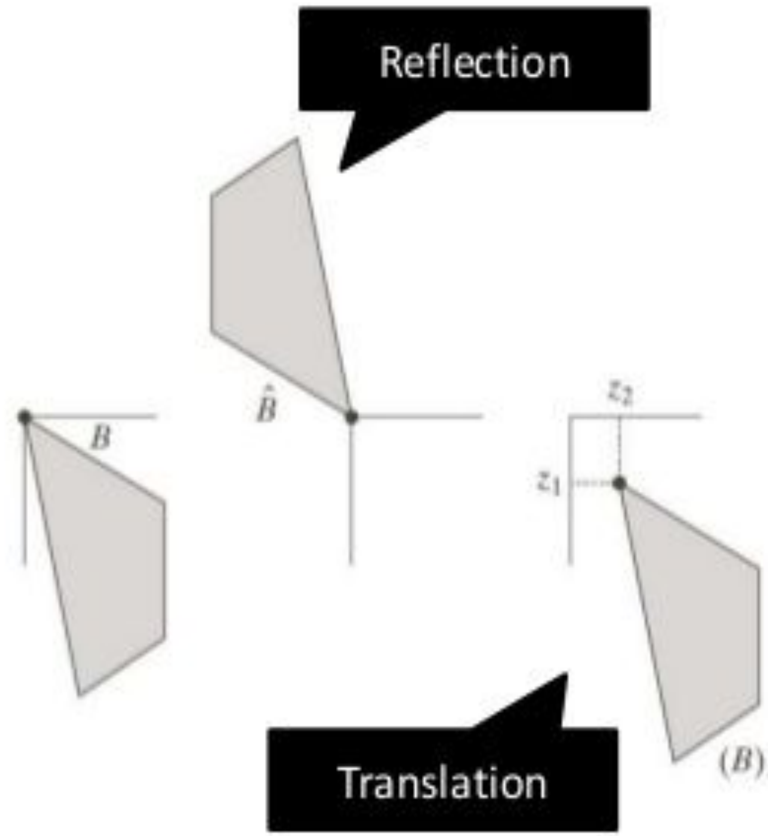
- ➔ Complement of A, $A^c = \{w \mid w \notin A\}$
- ➔ Union of A and B: $A \cup B$
- ➔ Intersection of A and B: $A \cap B$
- ➔ Difference of A and B: $A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$

Reflection of B: $\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$

Translation of A by point $z = (z_1, z_2)$: $(A)_z = \{c \mid c = a + z, \text{ for } a \in A\}$

Introduction (Cont…)

- The concept of set reflection and translation are extensively used in mathematical morphology.
- In reflection the set of points in ***B*** whose (*x, y*) coordinates have been replaced by *(-x,-y)*.
- In translation the set of points in ***B*** whose (*x, y*) coordinates have been replaced by (*x+z1, y+z2*).



Introduction (Cont...)

- **Structure elements (SE)**
  - Small sets or sub-images used to probe an image under study for properties of interest.
  - Structuring elements can be any size and make any shape.
  - However, for simplicity we will use rectangular structuring elements with their origin at the middle pixel.

Structuring elements

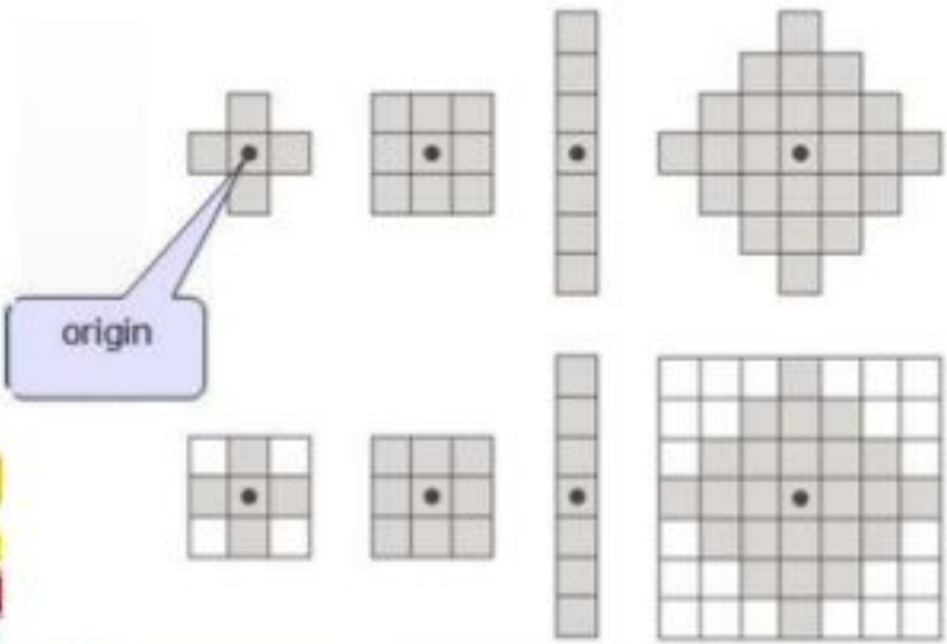
- Two-dimensional, or flat, structuring elements consist of a matrix of 0's and 1's, typically much smaller than the image being processed.
- The center pixel of the structuring element, called the origin, identifies the pixel of interest--the pixel being processed.
- The pixels in the structuring element containing 1's define the neighborhood of the structuring element.

Introduction (Cont…)

1	1	1
1	1	1
1	1	1

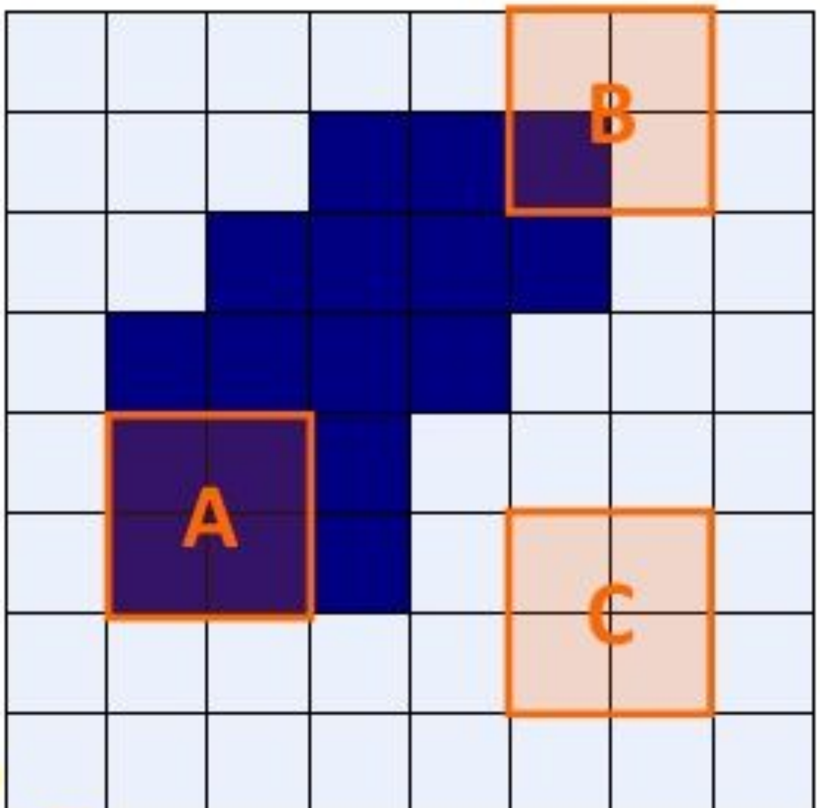
0	1	0
1	1	1
0	1	0

0	0	1	0	0
0	1	1	1	0
1	1	1	1	1
0	1	1	1	0
0	0	1	0	0



First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

Introduction (Cont...)



Structuring Elements, Hits & Fits:

Fit: All *on pixels* in the structuring element cover *on pixels* in the image

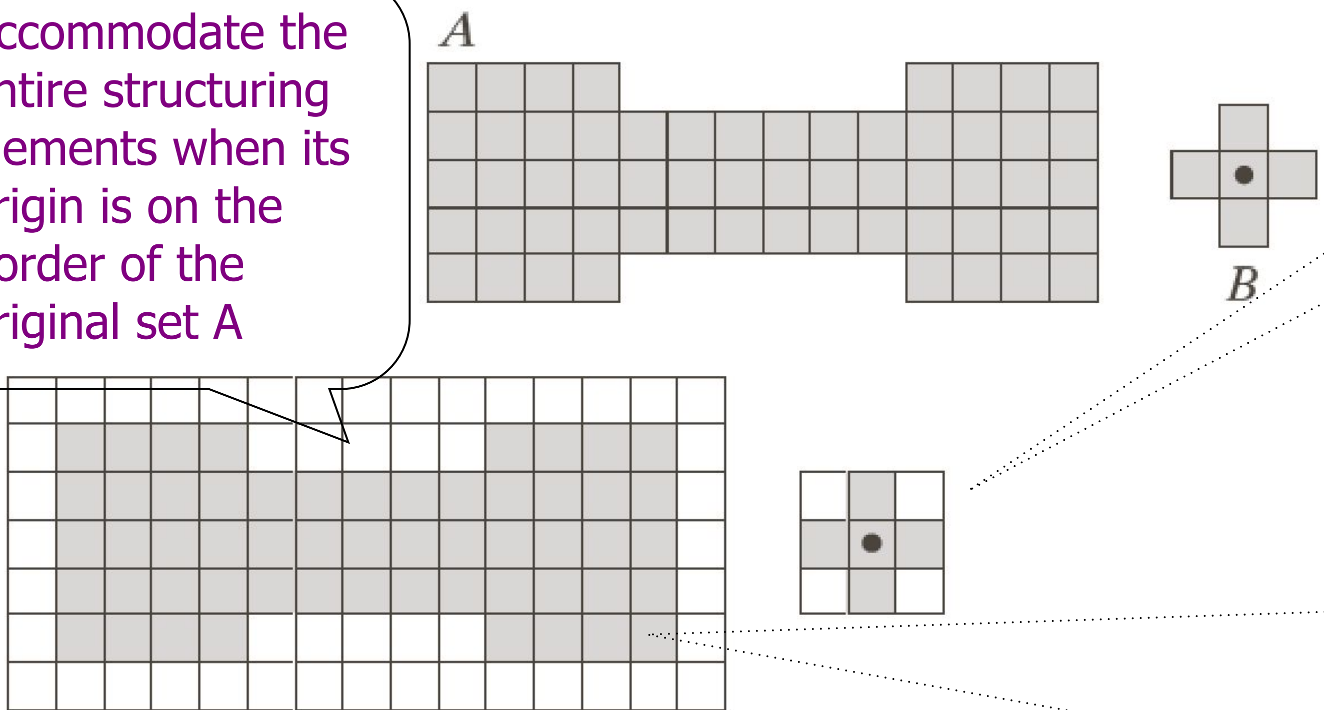
Hit: Any *on pixel* in the structuring element covers an *on pixel* in the image

- All morphological processing operations are based on these simple ideas

Examples: Structuring Elements (2)

Accommodate the entire structuring elements when its origin is on the border of the original set A

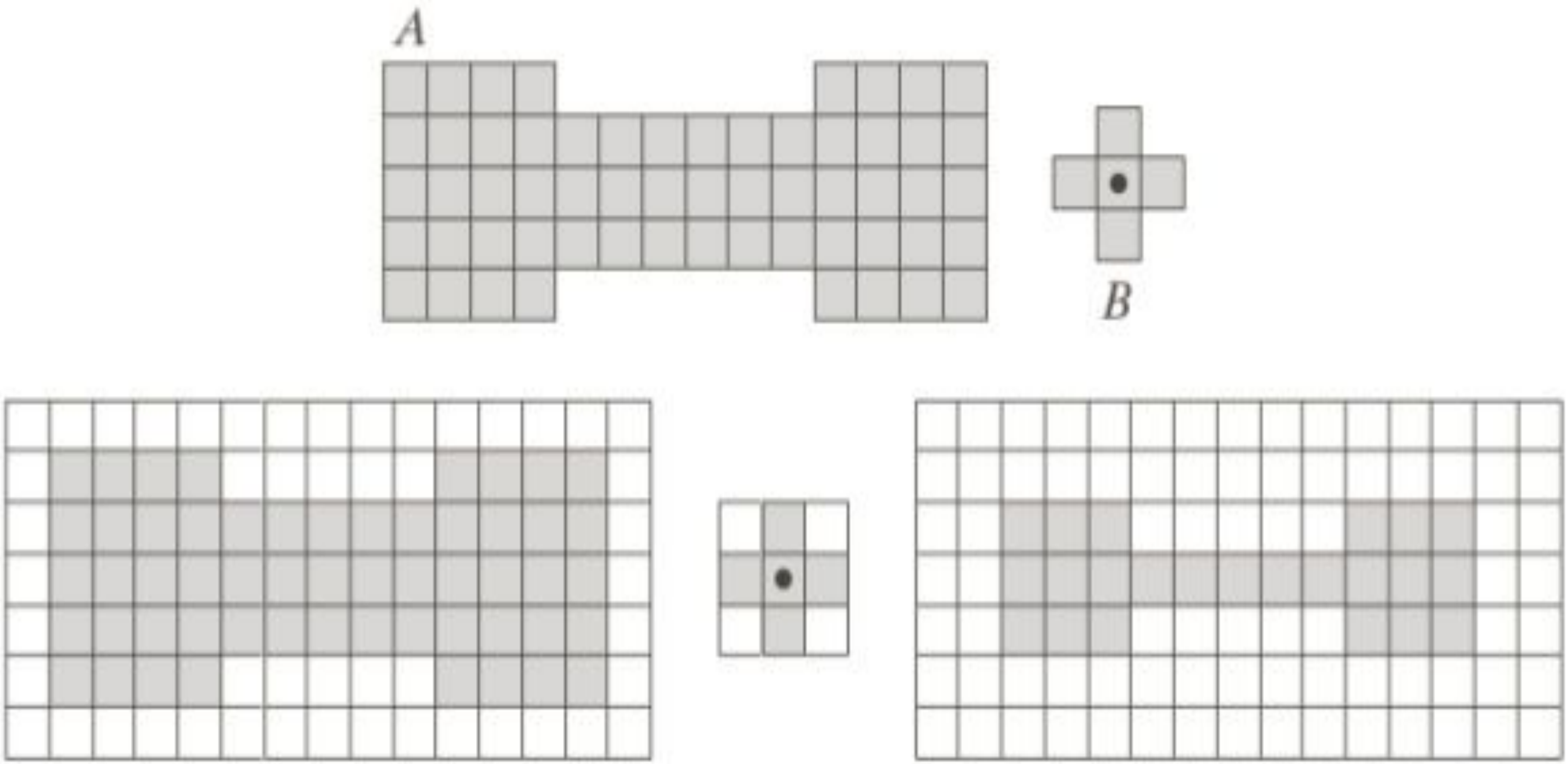
Origin of B visits every element of A



At each location of the origin of B, if B is completely contained in A, then the location is a member of the new set, otherwise it is not a member of the new set.

a b
c d e

FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.


A
B

Although Arrays A and B contains both shaded and non-shaded elements,

only the shaded elements of both sets are considered in determining whether or not B is contained in A!!

Erosion and Dilation

- **Erosion**
  - Erosion of image f by structuring element s is given by $f \ominus s$.
  - The structuring element ***s*** is positioned with its origin at (x, y) and the new pixel value is determined using the rule:

$$g(x, y) = \begin{cases} 1 \text{ if } s \text{ fits } f \\ 0 \text{ otherwise} \end{cases}$$

- Erosion:
  - With A and B as sets in Z^2 , the erosion of A by B is defined as

$$A \ominus B = \{ z \mid (B)_Z \subseteq A \}$$

  - Erosion of A by B is the set of all points z such that B, translated by z, is contained in A.

$$A \ominus B = \{ z \mid (B)_Z \cap A^c = \varnothing \}$$

  - B has to be contained in A is equivalent to B not sharing any common elements with the background

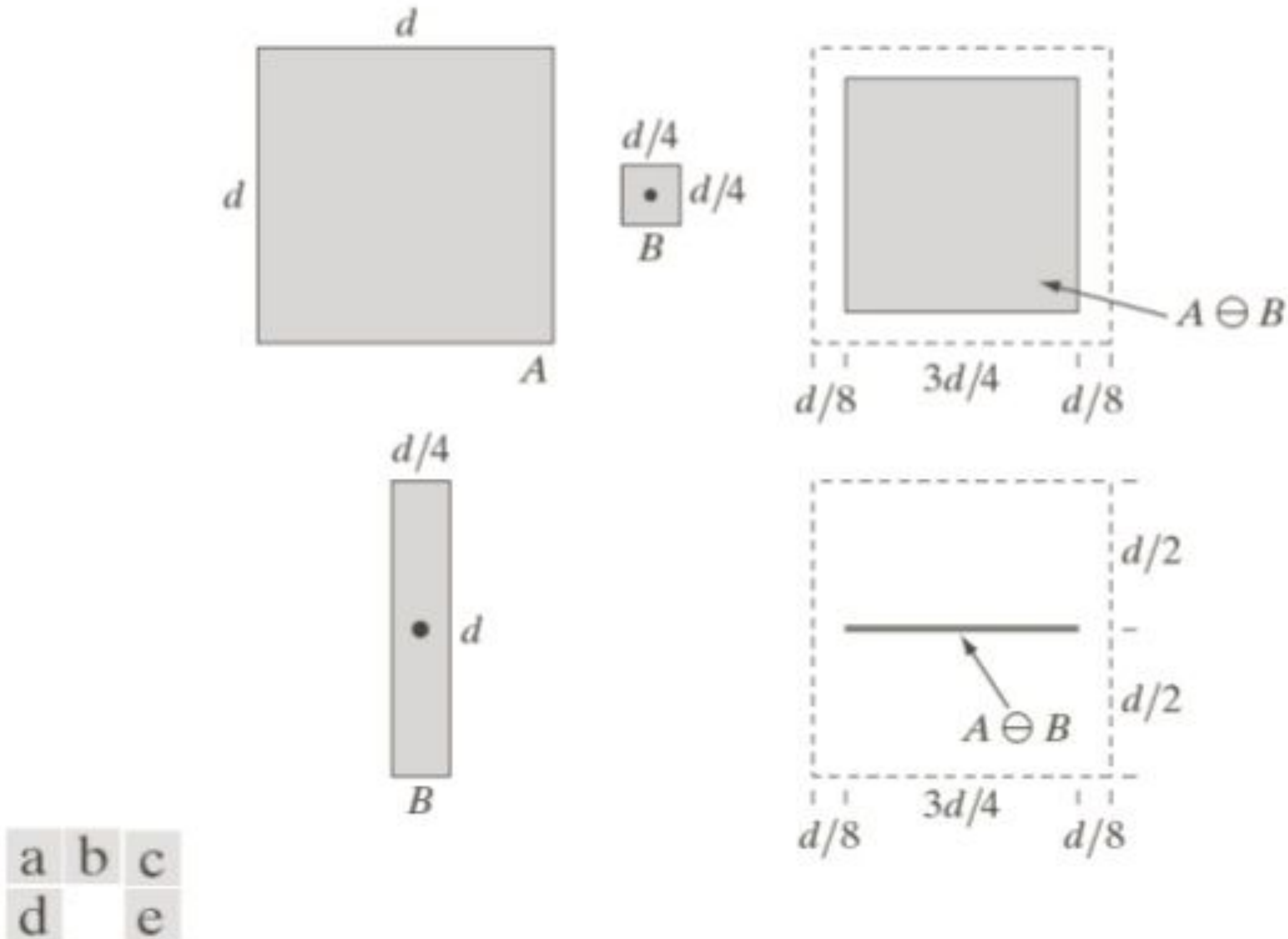
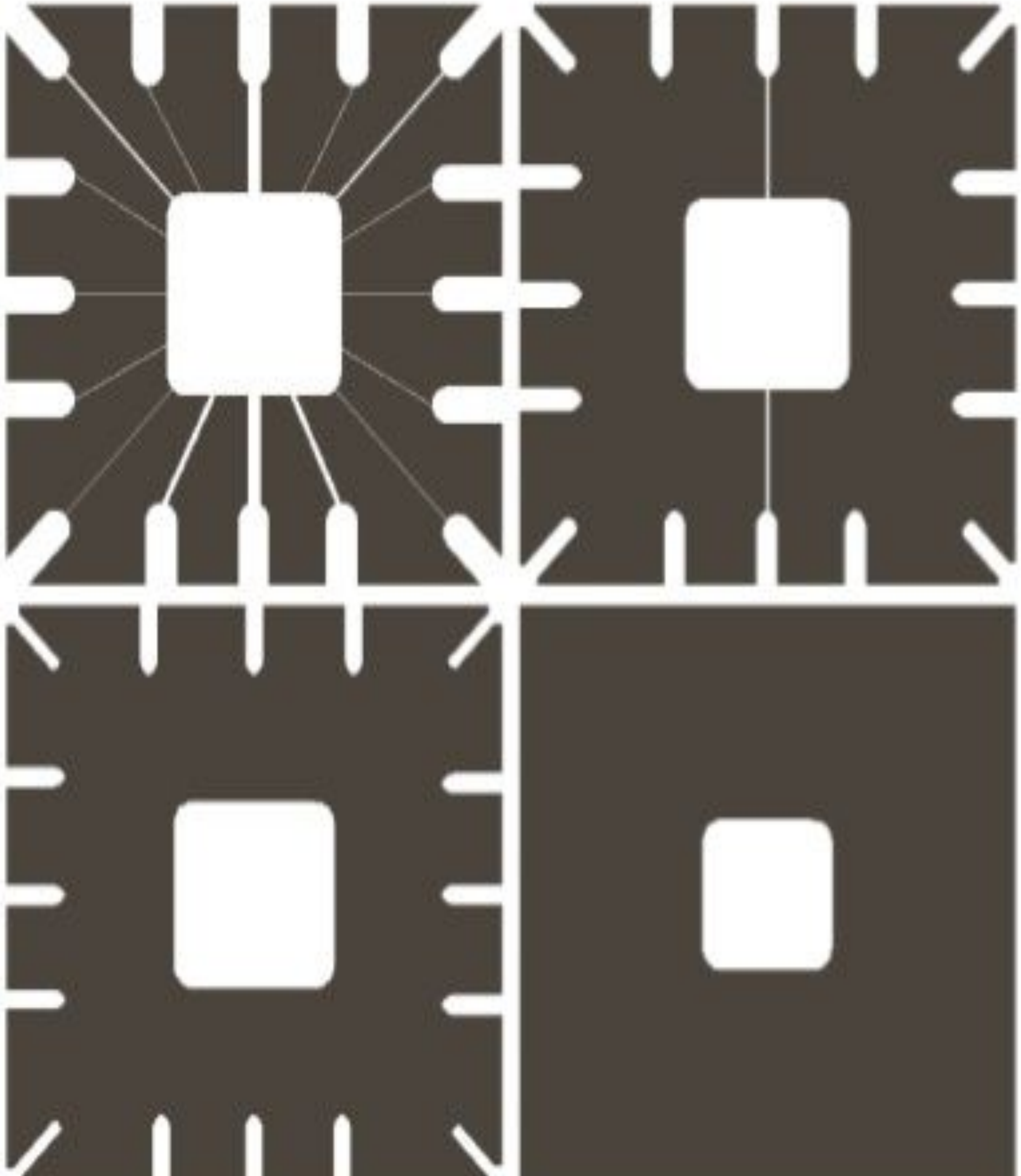


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

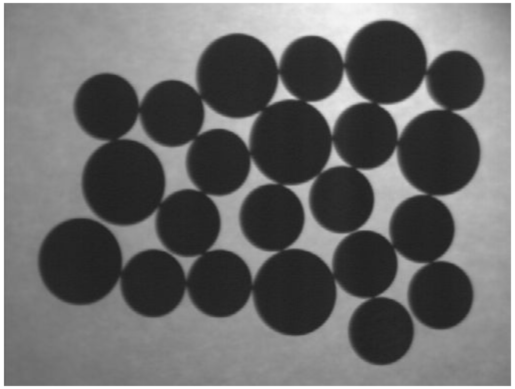
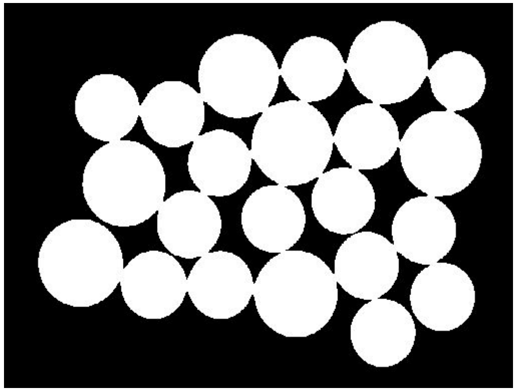
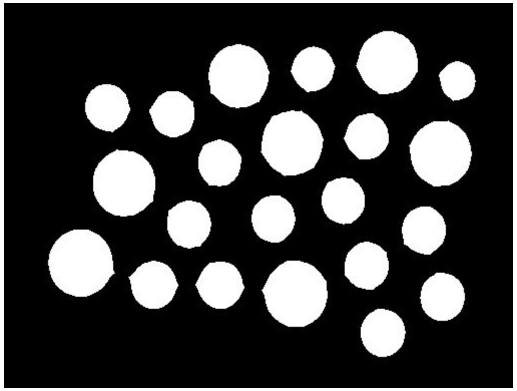


a b
c d

FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11×11, 15×15, and 45×45, respectively. The elements of the SEs were all 1s.

Counting Coins

- Counting coins is difficult because they touch each other!
- Solution: Binarization and Erosion separates them!

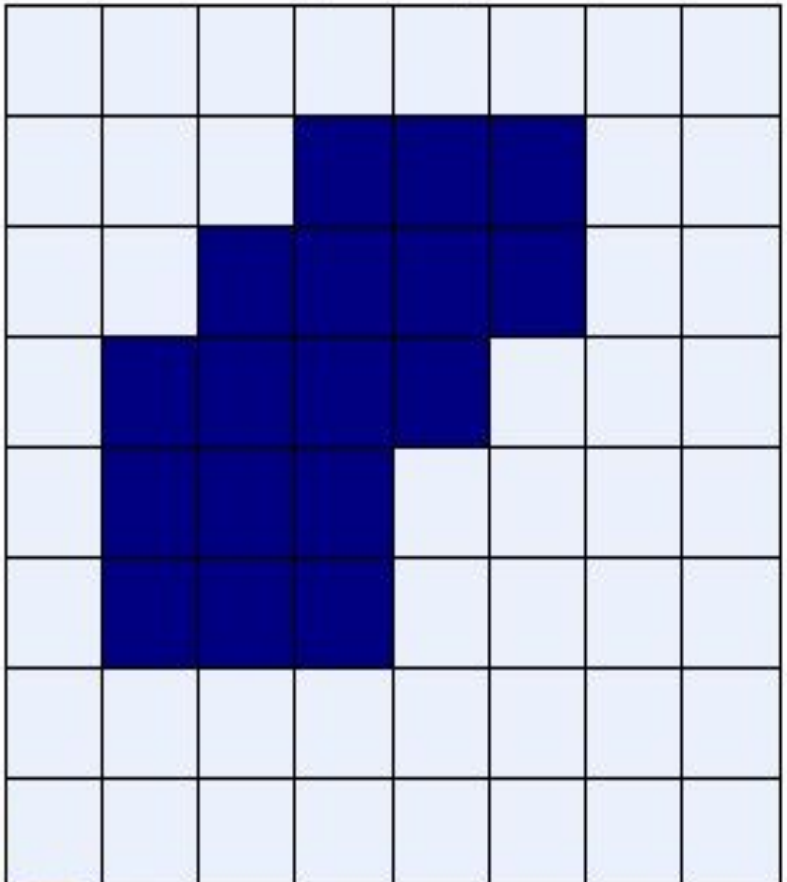




Erosion and Dilation (Cont…)

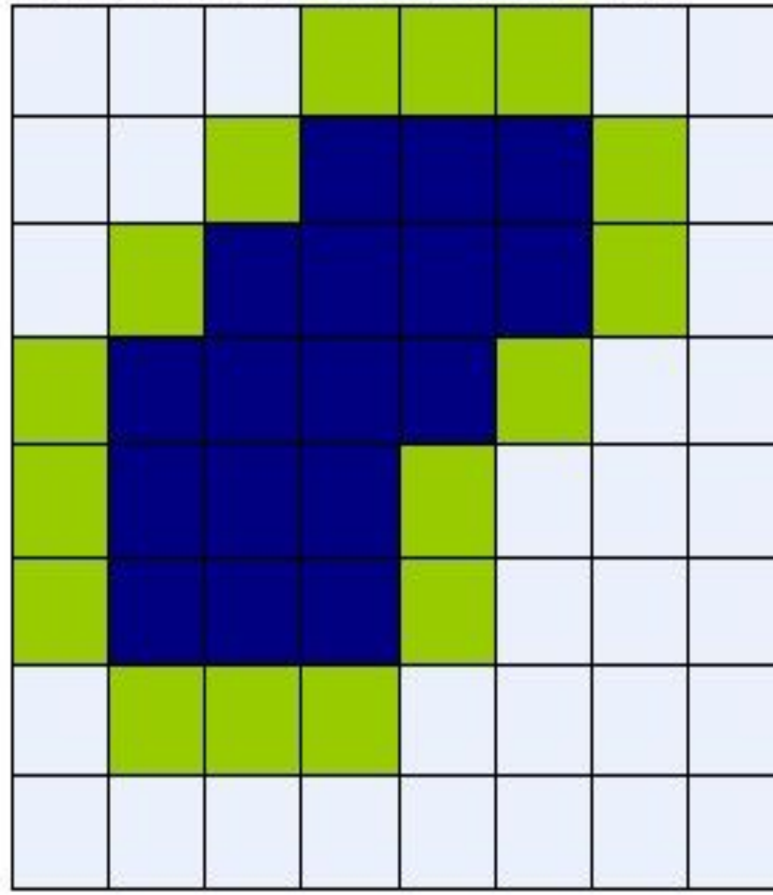
- **Dilation**
  - Dilation of image f by structuring element s is given by $f \oplus s$
  - The structuring element s is positioned with its origin at (x, y) and the new pixel value is determined using the rule:

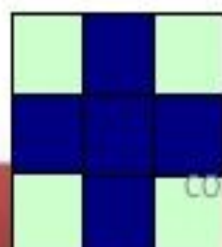
$$g(x, y) = \begin{cases} 1 \text{ if } s \text{ hits } f \\ 0 \text{ otherwise} \end{cases}$$

Original Image



Processed Image With Dilated Pixels

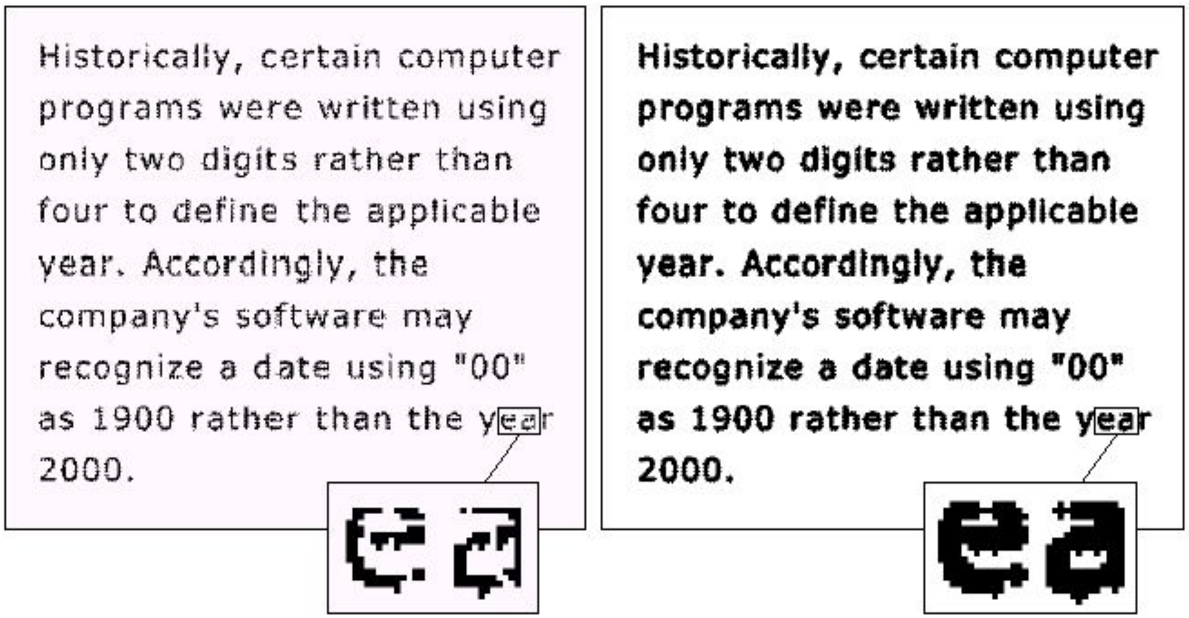




Structuring Element

- Dilation:
  - With A and B as sets in Z^2 , the dilation of A by B is defined as $A \oplus B = \left\{ z \mid \left(\hat{B} \right)_z \cap A \neq \varnothing \right\}$
  - Reflecting B about its origin, and shifting this reflection by z
  - The dilation of A by B then is the set of all displacements, z, such that $\hat{B}$ and A overlap by at least one element $$A \oplus B = \left\{ z \mid \left[\left(\hat{B} \right)_z \cap A \right] \subseteq A \right\}$$

Dilation



a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

0	1	0
1	1	1
0	1	0

- The basic effect of the operator on a binary image is to gradually enlarge the boundaries of regions of foreground pixels (*i.e.* white pixels, typically).

- Duality

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

$$(A \oplus B)^c = A^c \ominus \hat{B}$$

- Proof:

$$(A \ominus B)^c = \left\{ z \middle| (B)_z \cap A^c = \varnothing \right\}^c$$

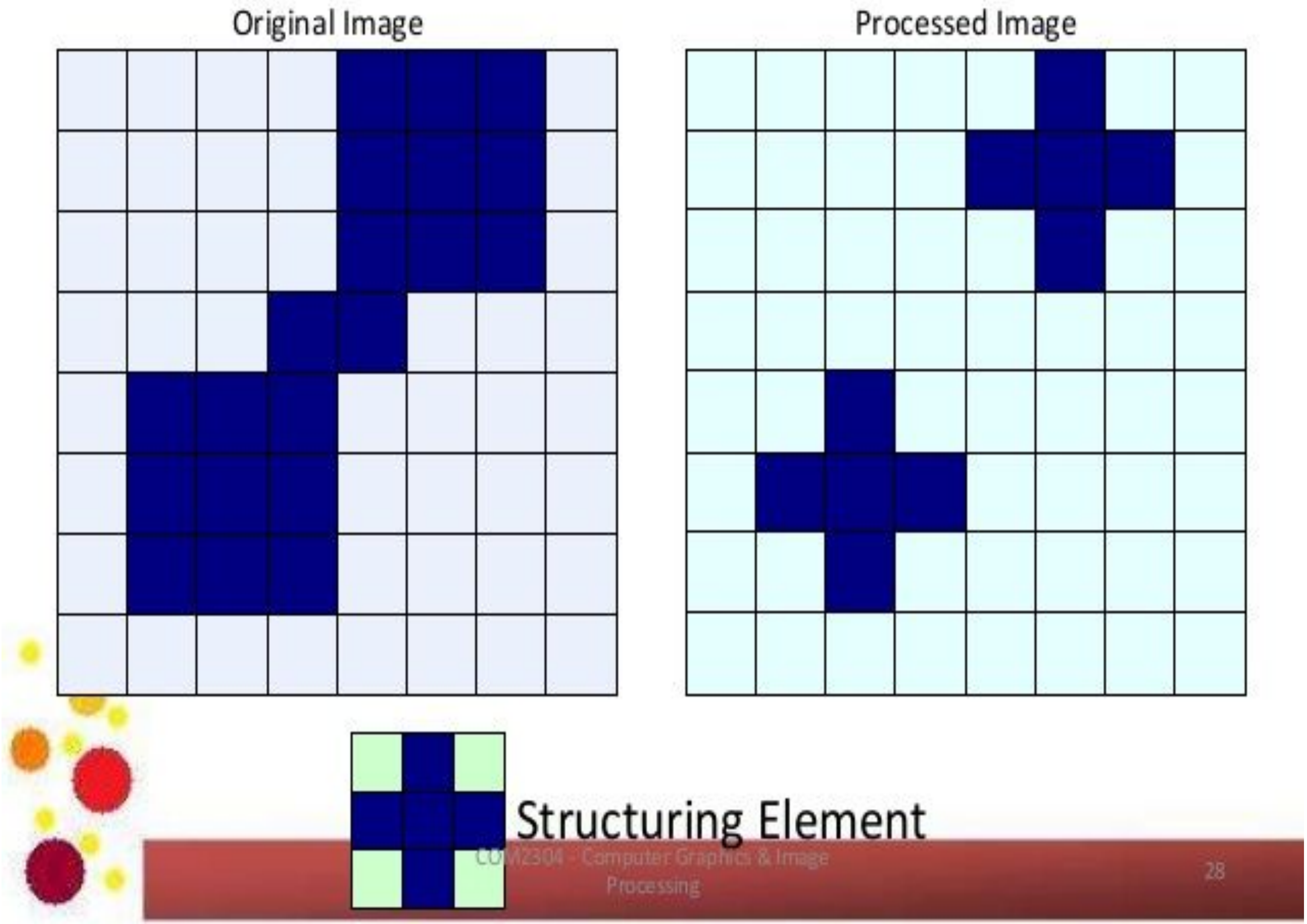
$$(A \ominus B)^c = \left\{ z \middle| (B)_z \cap A^c \neq \varnothing \right\}$$

$$= A^c \oplus \hat{B}$$

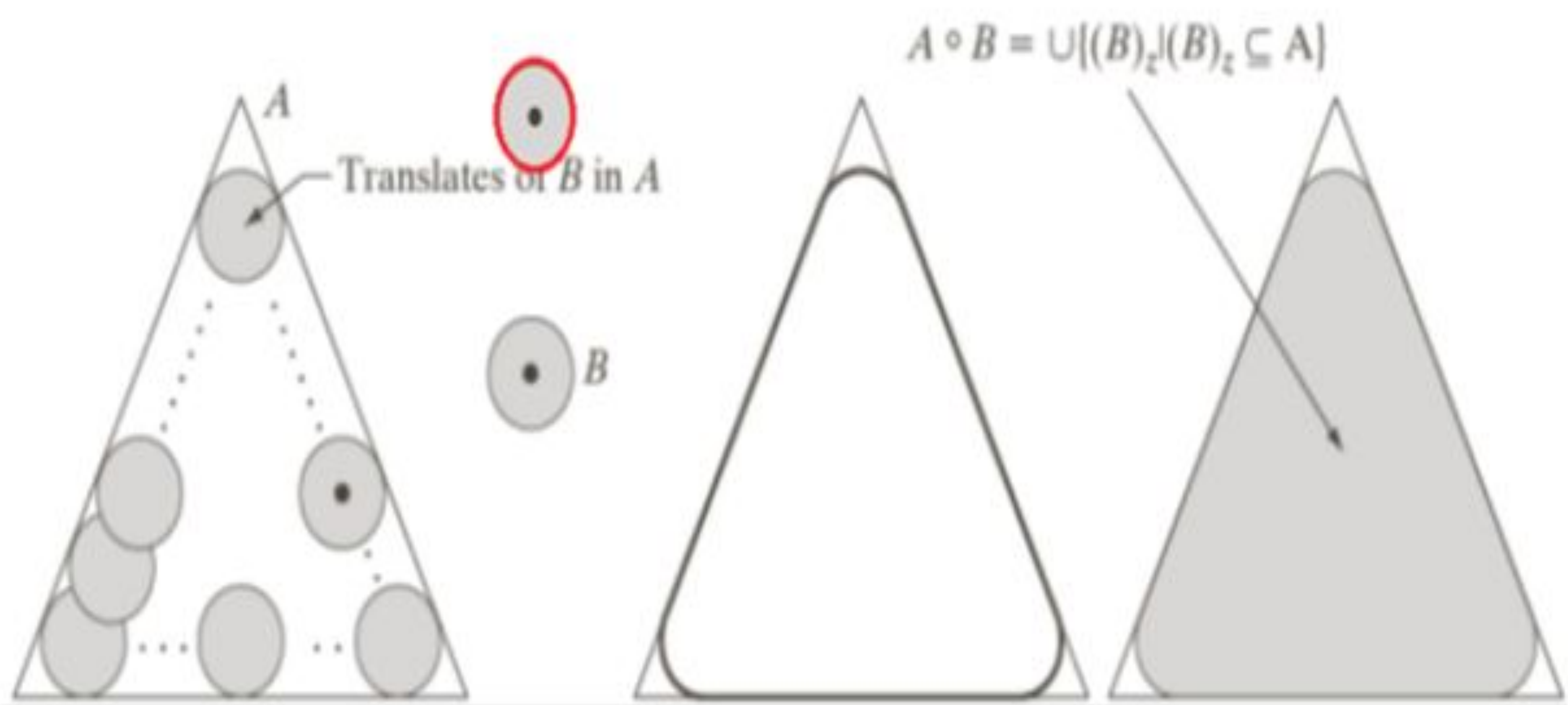
$$A \oplus B = \left\{ z \middle| (\hat{B})_z \cap A \neq \varnothing \right\}$$

Opening and Closing

- Opening
  - A single erosion followed by a single dilation by the same operator
  - $R' = (R \ominus A) \oplus A$
- Closing
  - A single dilation followed by a single erosion by the same operator
  - $R' = (R \oplus A) \ominus A$


Original Image
Processed Image
Structuring Element

Opening: $A \circ B = \bigcup\{(B)_z | (B)_z \subseteq A\}$



a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

Examples: Closing

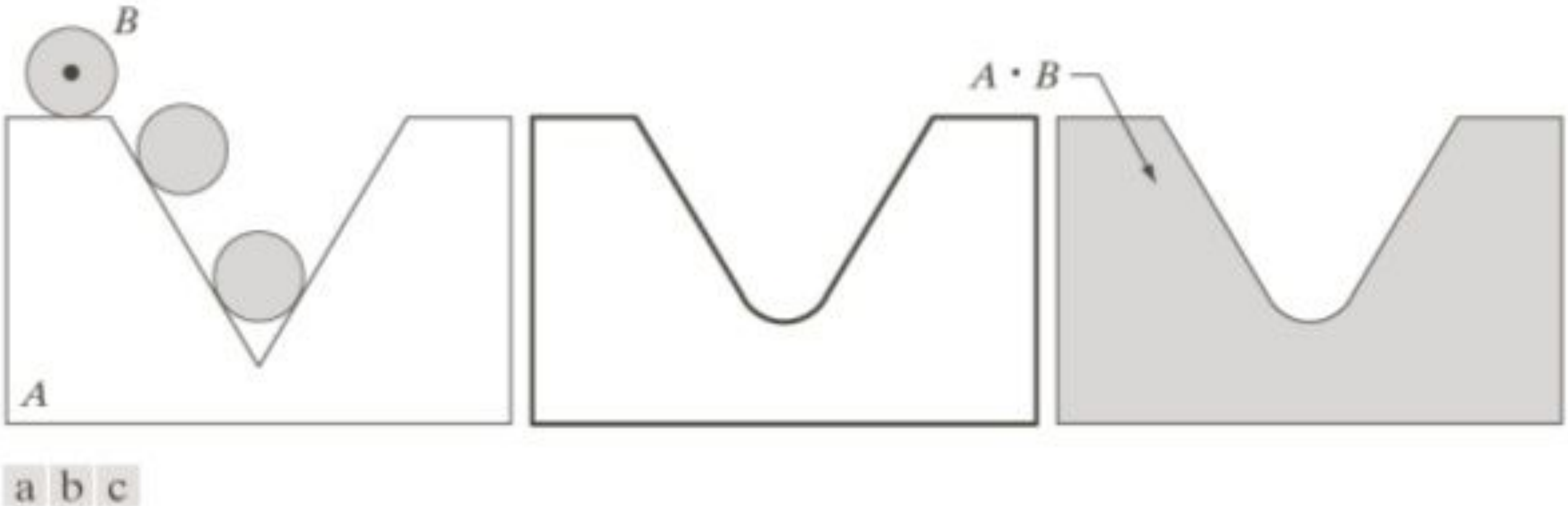
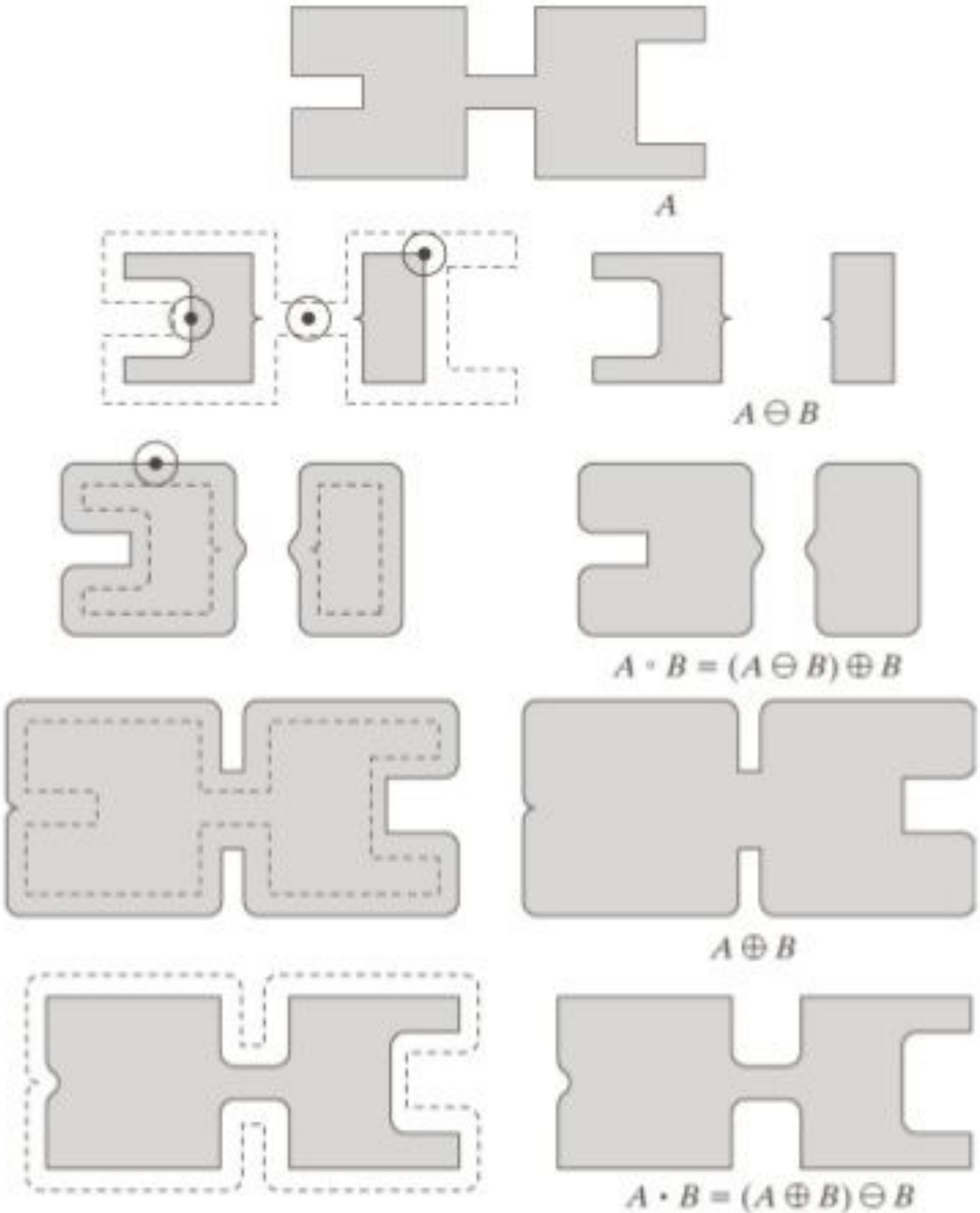


FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.



a	
b	c
d	e
f	g
h	i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

Opening and Closing

a b c d e f

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of *A*.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

$$(A \bullet B)^c = (A^c \circ \hat{B})$$

$$(A \circ B)^c = (A^c \bullet \hat{B})$$

(a) A o B is a subset (subimage) of A.
(b) If C is a subset of D, then C o B is a subset of D o B.
(c) (A o B) o B = A o B.

(a) A is a subset (subimage) of A • B.
(b) If C is a subset of D, then C • B is a subset of D • B.
(c) (A•B)•B=A•B.

8 * 8

1	1	1	1	1	1	1	
			1	1	1	1	
			1	1	1	1	
		1	1	1	1	1	
			1	1	1	1	
		1	1				

a) Binary image B

1	1	1
1	**1**	1
1	1	1

b) Structuring Element S

8 * 8

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
	1	1	1	1	1	1	1
	1	1	1	1	1	1	1
	1	1	1	1	1	1	1
	1	1	1	1	1	1	1
	1	1	1	1			

c) Dilation $B \oplus S$

				1	1		
				1	1		
				1	1		

d) Erosion $B \ominus S$

	1	1	1	1	1	1	
		1	1	1	1	1	
		1	1	1	1	1	
		1	1	1	1	1	
		1	1	1	1	1	
		1	1				

e) Closing $B \bullet S$

			1	1	1	1	
			1	1	1	1	
			1	1	1	1	
			1	1	1	1	
			1	1	1	1	

f) Opening $B \circ S$

Gray level erosion /dilation

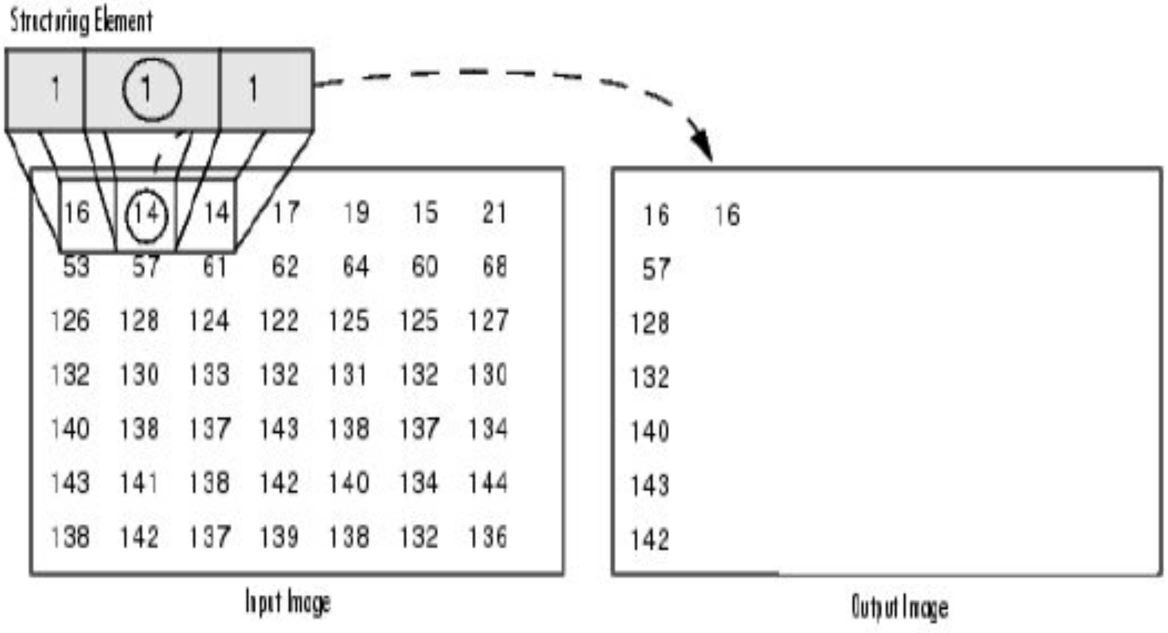
- **Erosion:**
  - Place the structure element with origo at pixel (x,y)
  - Chose the local **minimum** grey level in the region defined by the structure element
  - Assign this value to the output pixel (x,y)
  - Results in darker images and light details are removed

$$[f \ominus b](x,y) = \min_{(s,t)\in B}\{f(x+s, y+t)\}$$

- **Dilation:**
  - Chose the local **maximum** over the region defined by the structure element
  - Let pixel (x,y) in the outimage have this maximum value.
  - Results in brighter images and dark details are removed

$$[f \oplus b](x,y) = \max_{(s,t)\in B}\{f(x-s, y-t)\}$$

Gray level morphology



Morphological Dilation of a Grayscale Image

Hit-and-Miss Transform

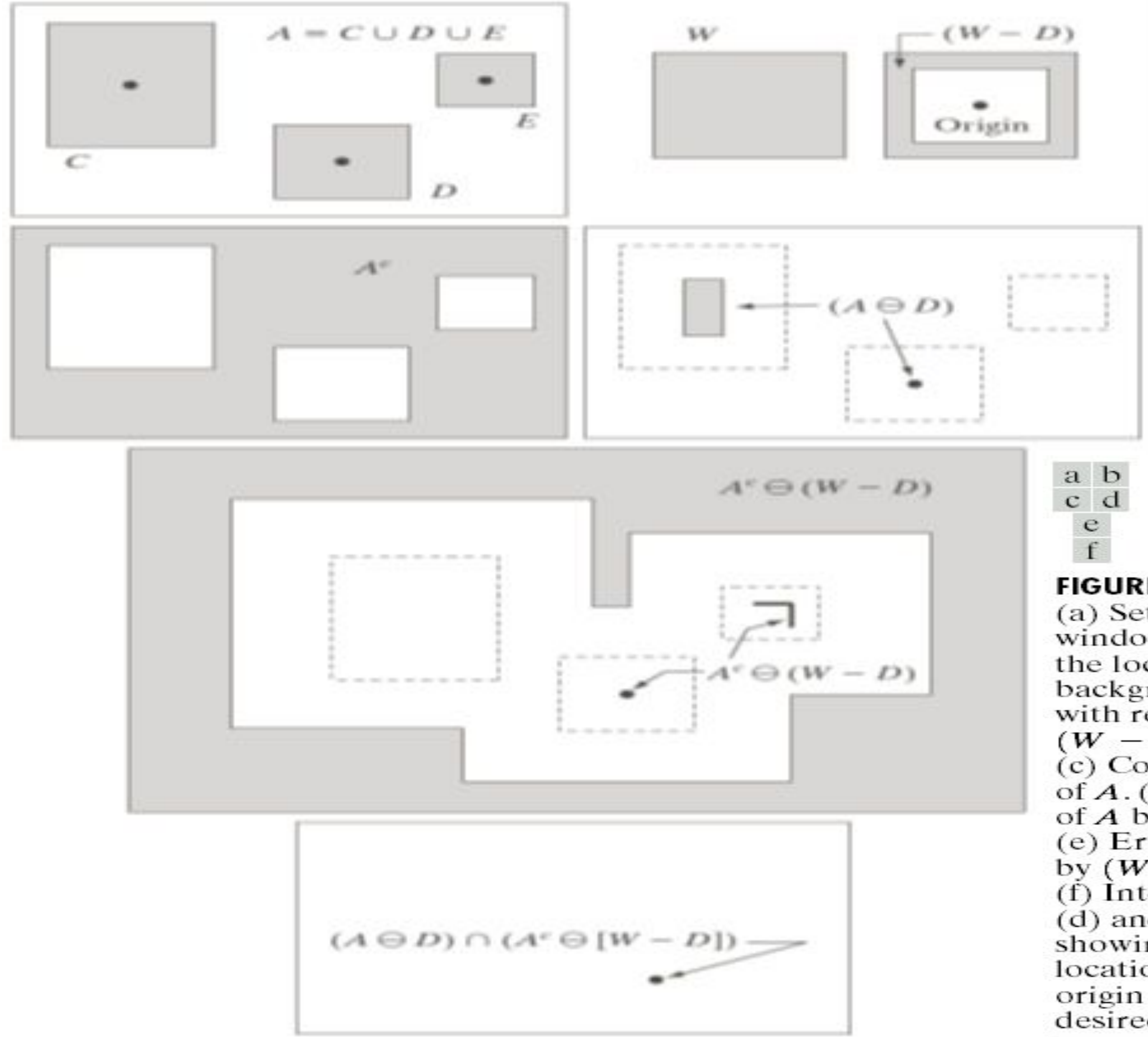
The hit-and-miss transform is a general binary morphological operation that can be used to look for particular patterns of foreground and background pixels in an image.

If B denotes the set composed of D and its background, the match (or sets of matches) of B in A is given by,

$$A \circledast B = (A \ominus D) \cap \left[A^c \ominus (W - D) \right]$$

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

B1: Object related
B2: Background related



a	b
c	d
e	
f	

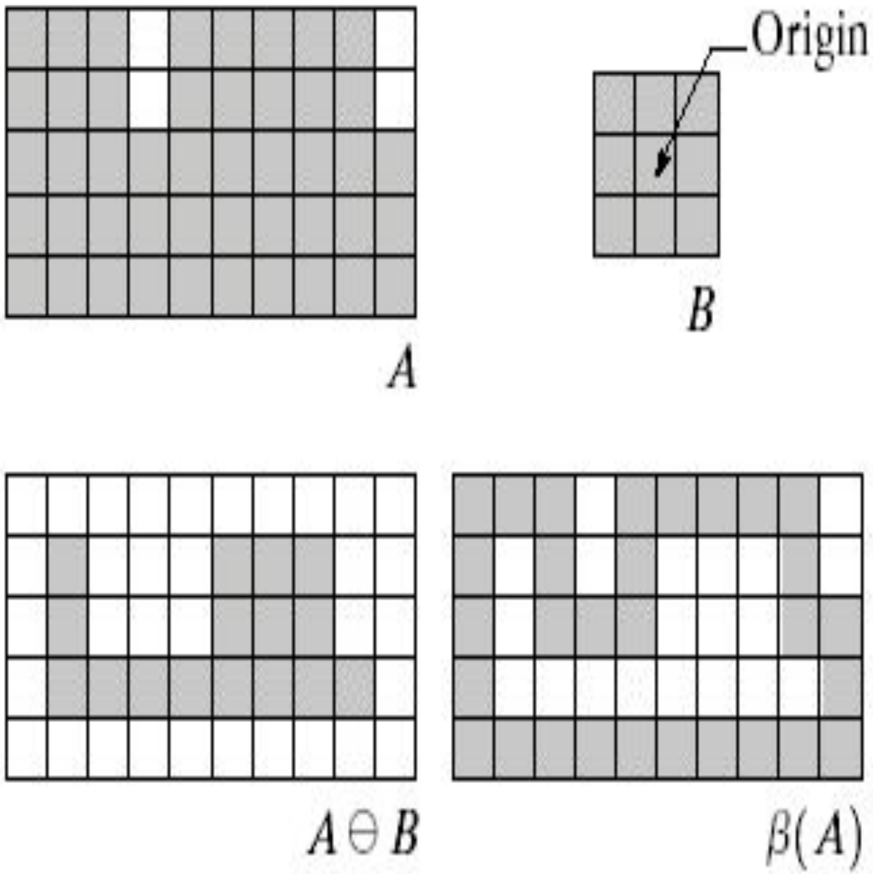
FIGURE 9.12
(a) Set A. (b) A window, W, and the local background of X with respect to W, $(W - X)$.
(c) Complement of A. (d) Erosion of A by X.
(e) Erosion of A^c by $(W - X)$.
(f) Intersection of (d) and (e), showing the location of the origin of X, as desired.

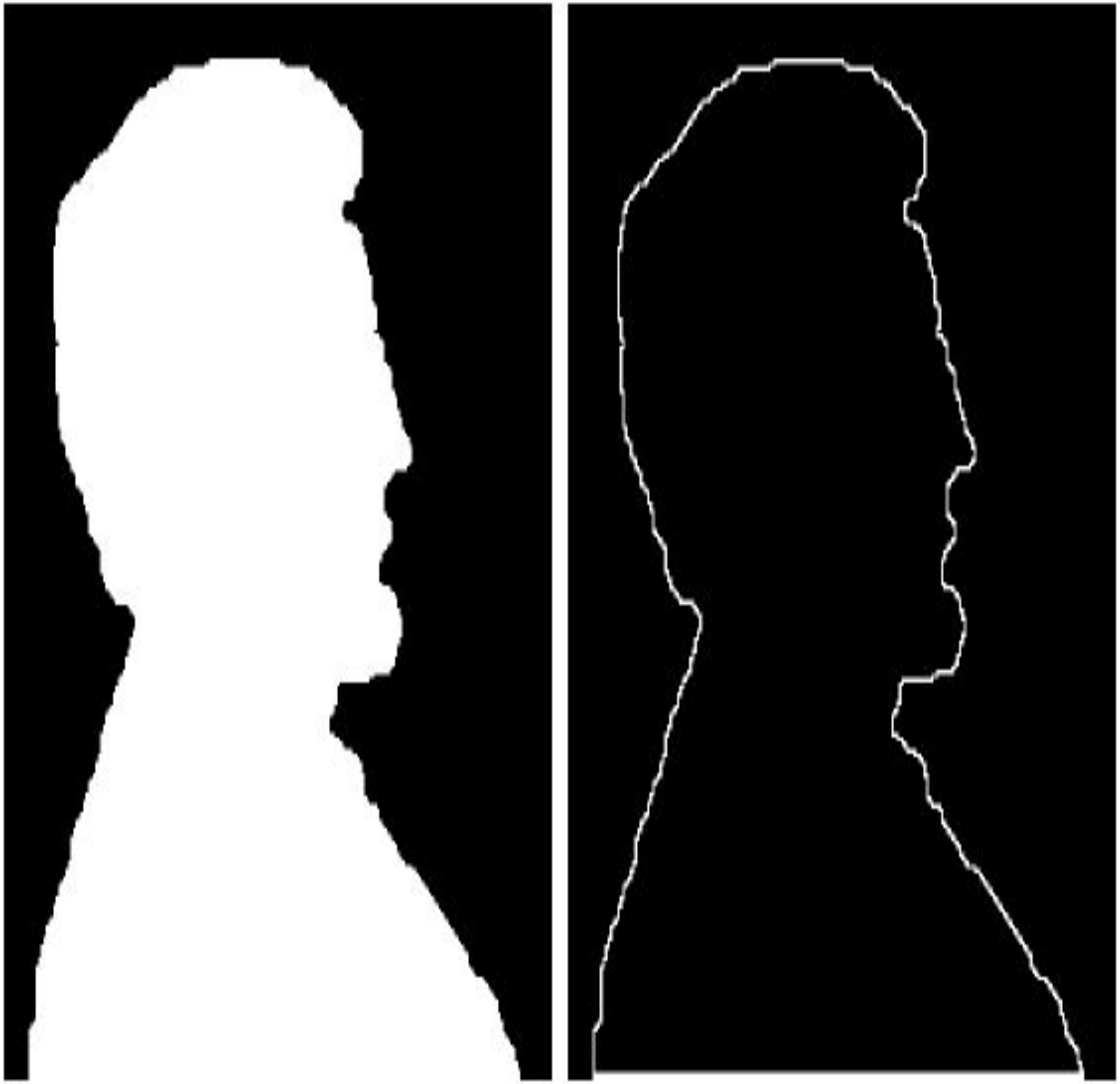
Boundary Extraction

$$\beta(A) = A - (A \ominus B)$$

a	b
c	d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.





a b

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Region Filling/Hole filling

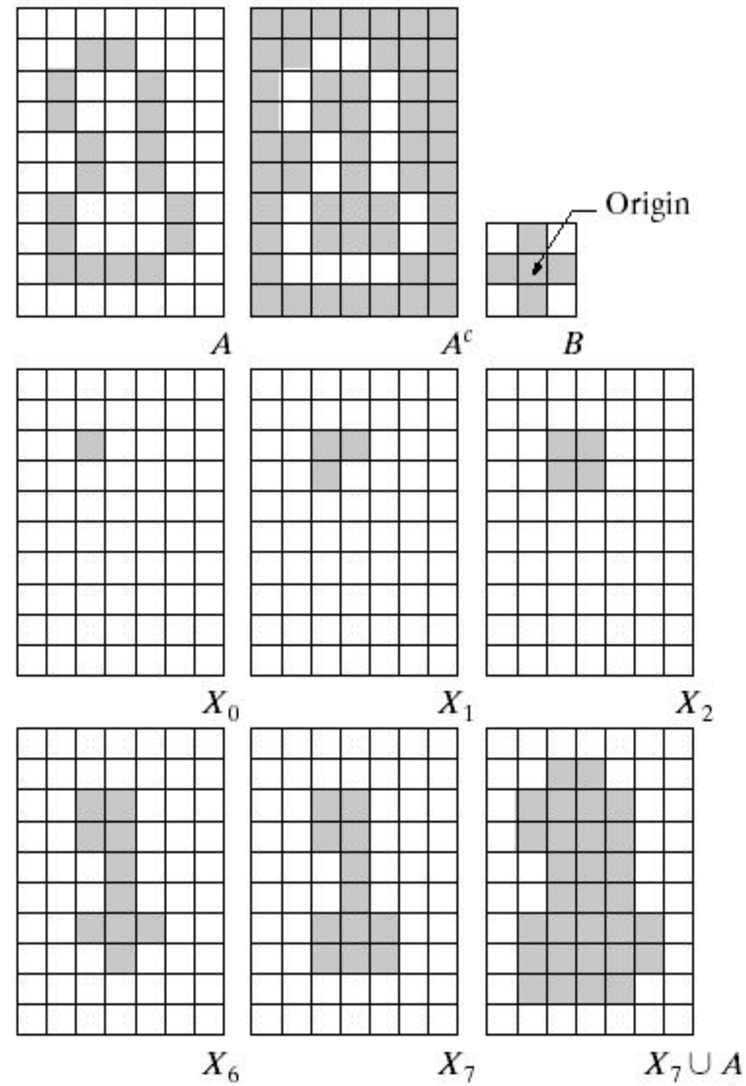
$$X_0 = p$$

$$X_k = (X_k \oplus B) \cap A^c$$

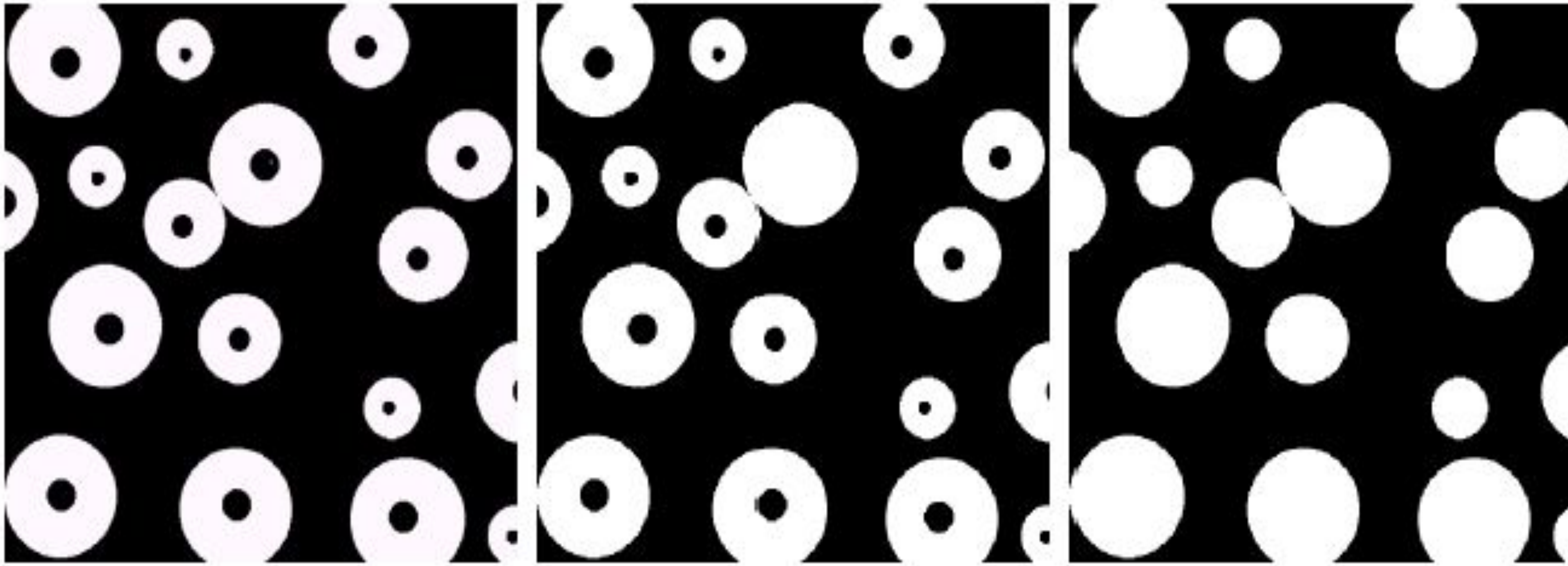
$$\text{Until: } X_{k+1} = X_k$$

a	b	c
d	e	f
g	h	i

FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].



Start from *p* that belongs to the region to be filled.



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Connected components Extraction

Start from *p* belong to desired region.

$$X_0 = p$$

$$X_k = (X_k \oplus B) \cap A$$

$$\text{Until: } X_{k+1} = X_k$$

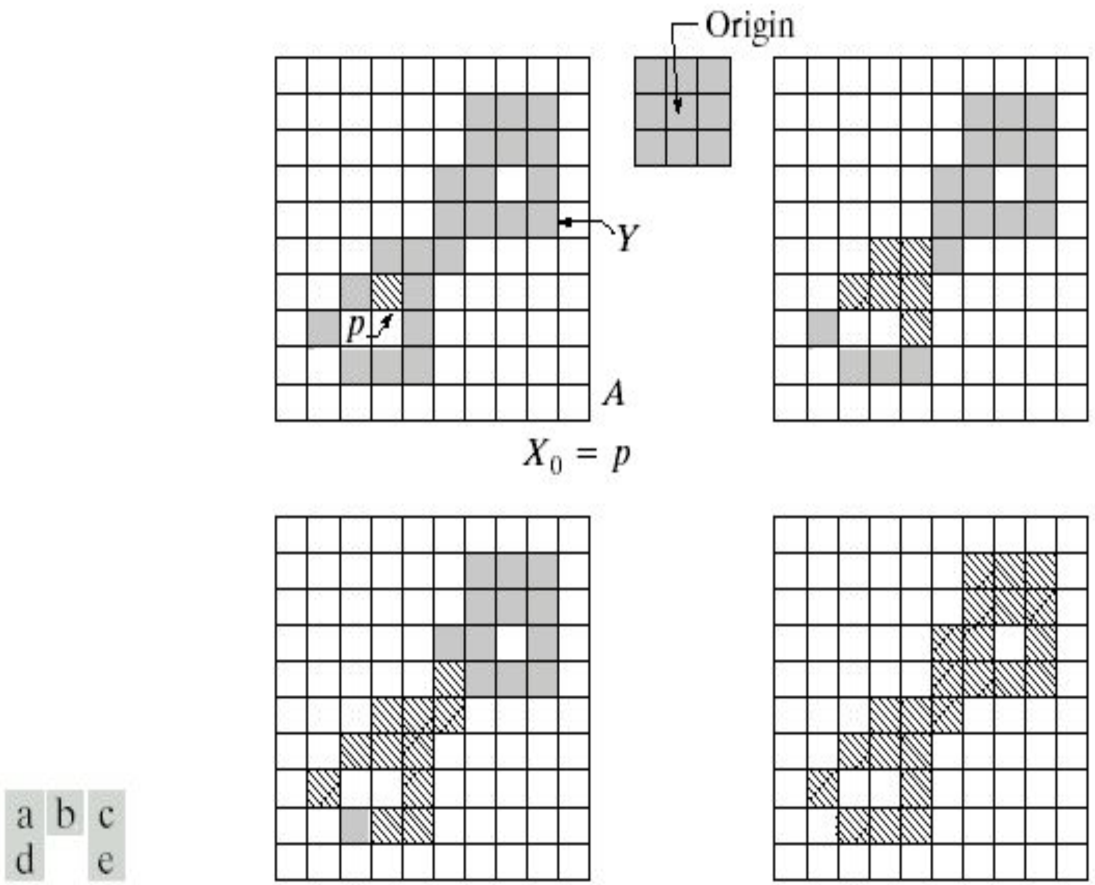
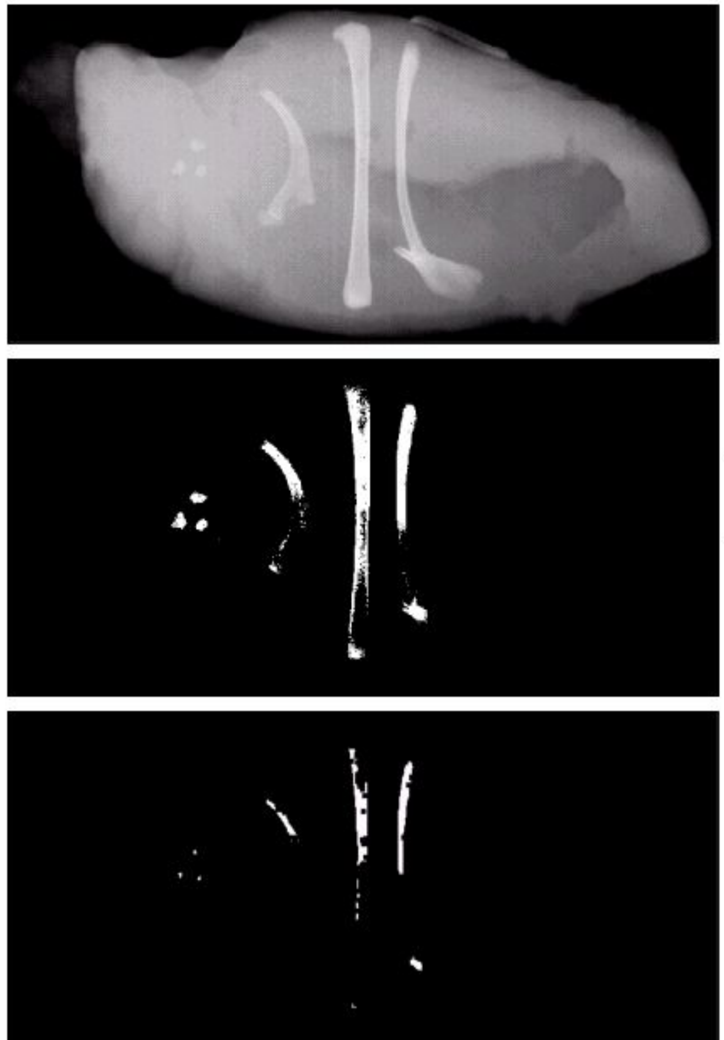


FIGURE 9.17 (a) Set *A* showing initial point *p* (all shaded points are valued 1, but are shown different from *p* to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.



a
b
c d

FIGURE 9.18
(a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Some Basic Morphological Algorithms (4)

- **Convex Hull**

A set A is said to be ***convex*** if the straight line segment joining any two points in A lies entirely within A.

The ***convex hull*** H of an arbitrary set S is the smallest convex set containing S.

Some Basic Morphological Algorithms (4)

- **Convex Hull**

Let B^i, $i = 1,\ 2,\ 3,\ 4$, represent the four structuring elements.

The procedure consists of implementing the equation:

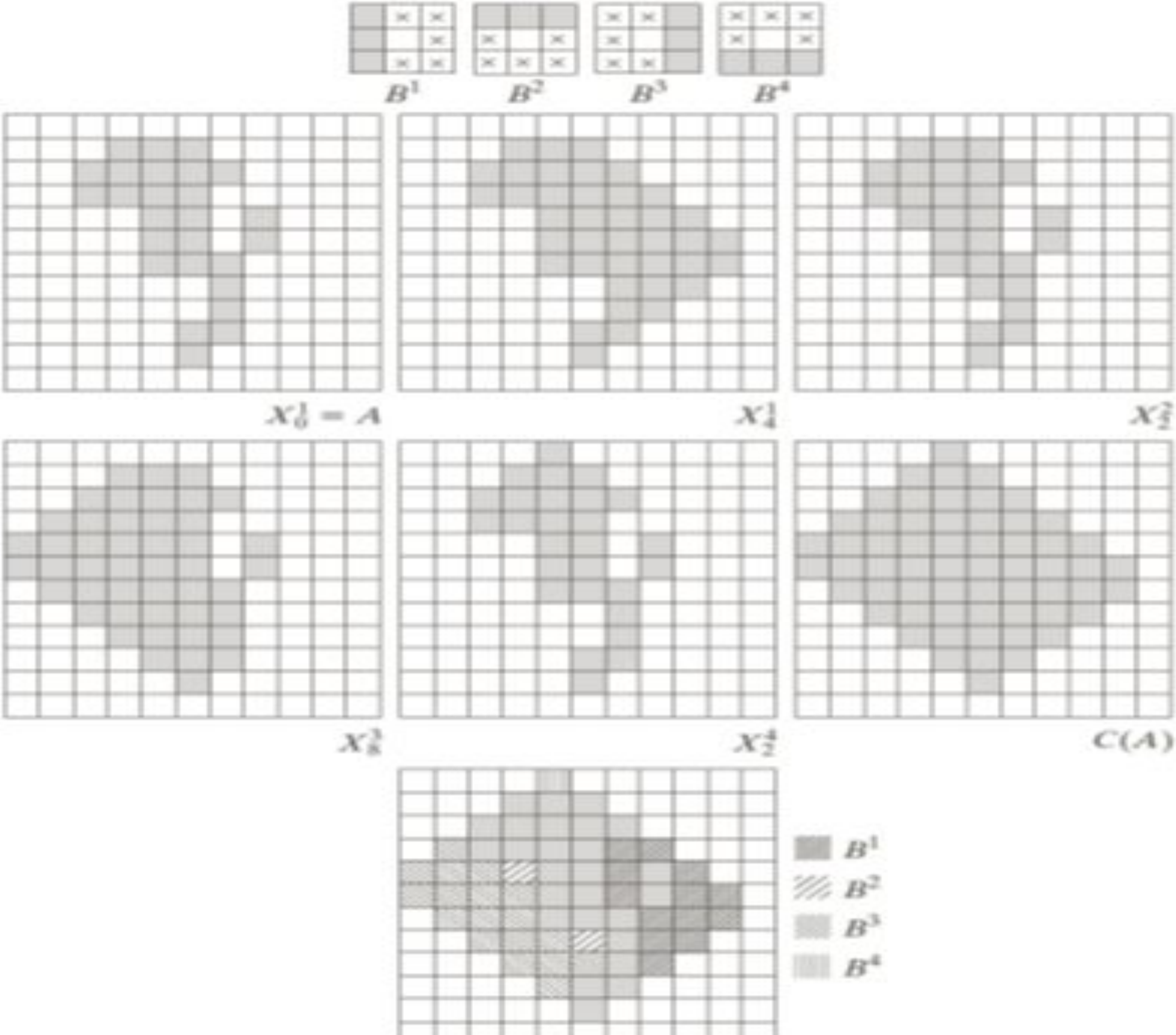
$$X_k^i = (X_{k-1} \circledast B^i) \cup A$$
$$i = 1, 2, 3, 4 \text{ and } k = 1, 2, 3, \ldots$$

with $X_0^i = A$.

When the procedure converges, or $X_k^i = X_{k-1}^i$, let $D^i = X_k^i$,

the convex hull of A is

$$C(A) = \bigcup_{i=1}^{4} D^i$$


B^1
B^2
B^3
B^4
$X_0^1 = A$
X_4^1
X_2^2
X_8^3
X_2^4
$C(A)$
B^1
B^2
B^3
B^4

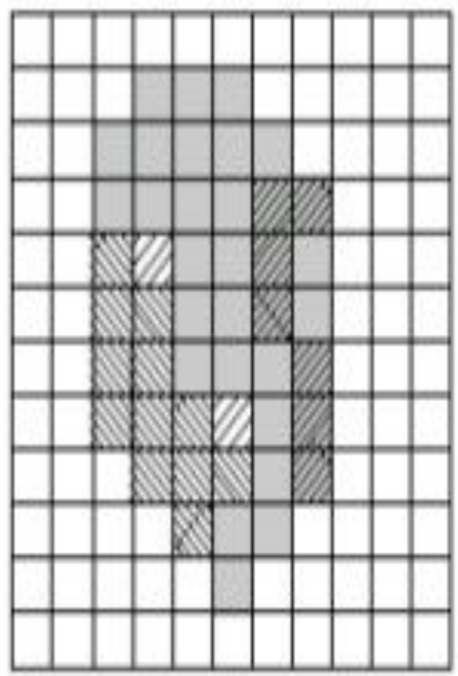


FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

Thinning

– Thinning:

$$A \otimes B = A - \left(A * B\right) = A \bigcap \left(A * B\right)^c$$

Another approach:

$$\{\mathbf{B}\} = \left\{B^1, B^2, \cdots, B^n\right\}$$

$$A \otimes \{\mathbf{B}\} = \left(\left(\cdots\left(\left(A \otimes B^1\right) \otimes B^2\right)\cdots\right) \otimes B^n\right)$$

Repeat until convergence

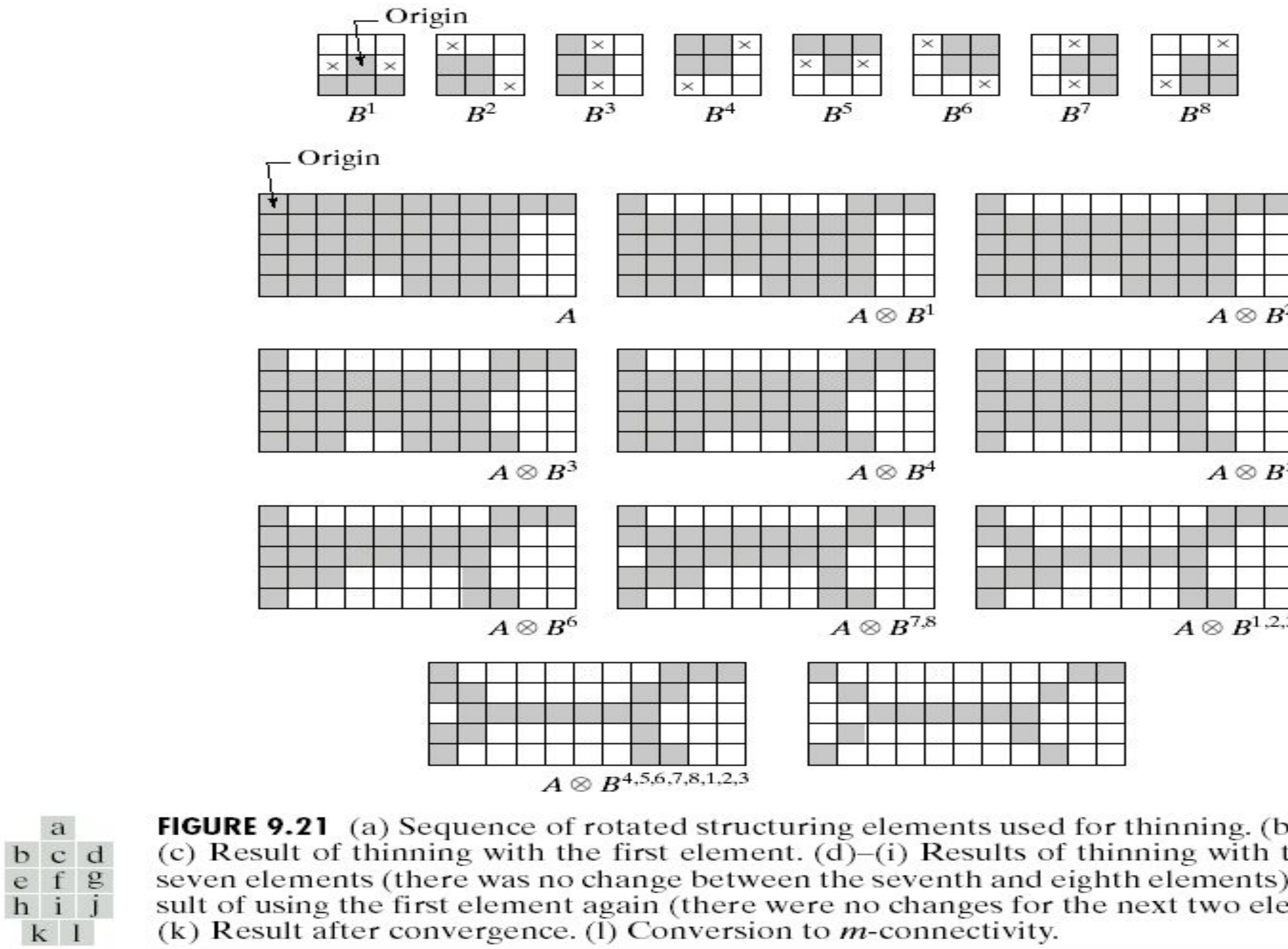


FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to m-connectivity.

Thickening

$$A \odot B = A \bigcup \left(A * B \right)$$

$$A \odot \{\mathbf{B}\} = \left(\left(\cdots \left(\left(A \odot B^1 \right) \odot B^2 \right) \cdots \right) \odot B^n \right)$$

Thickening

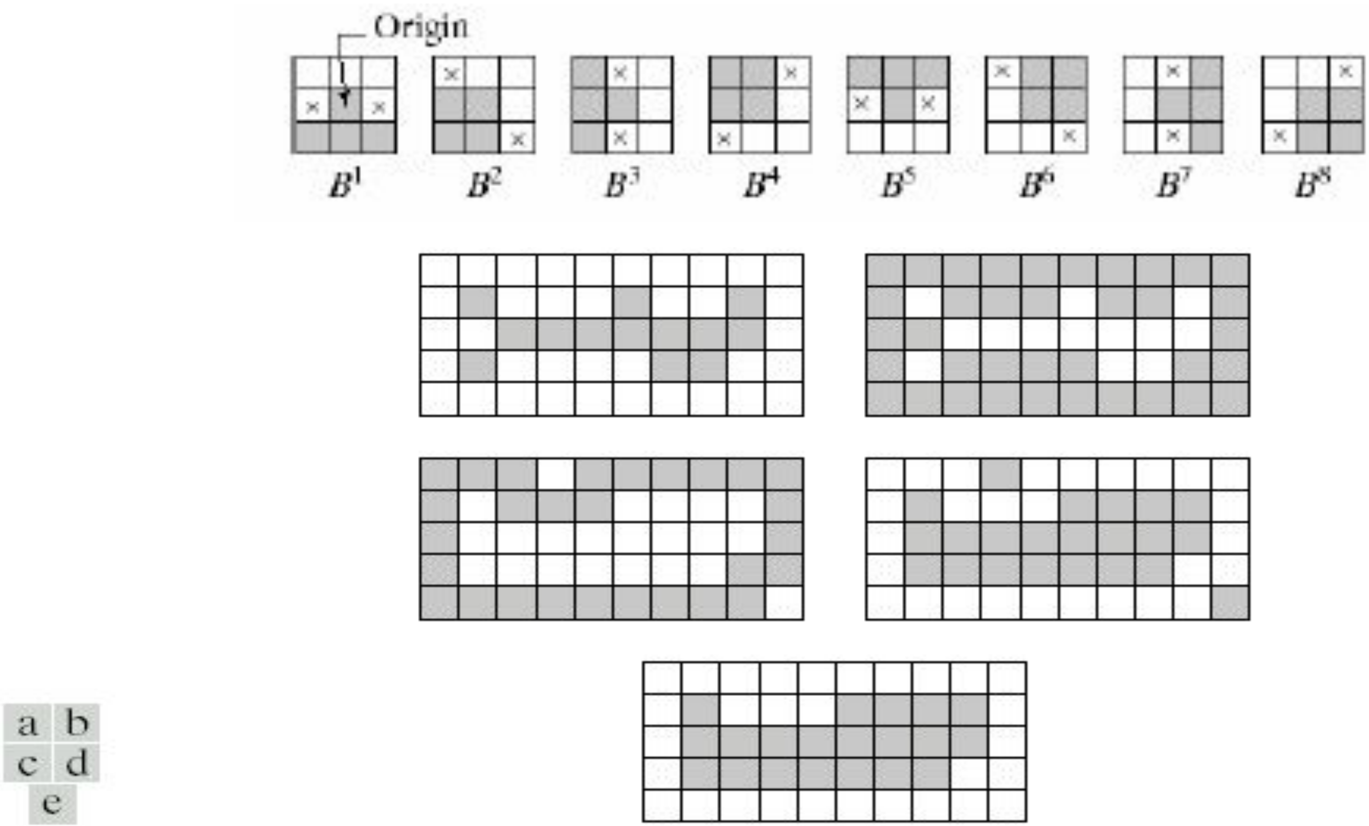
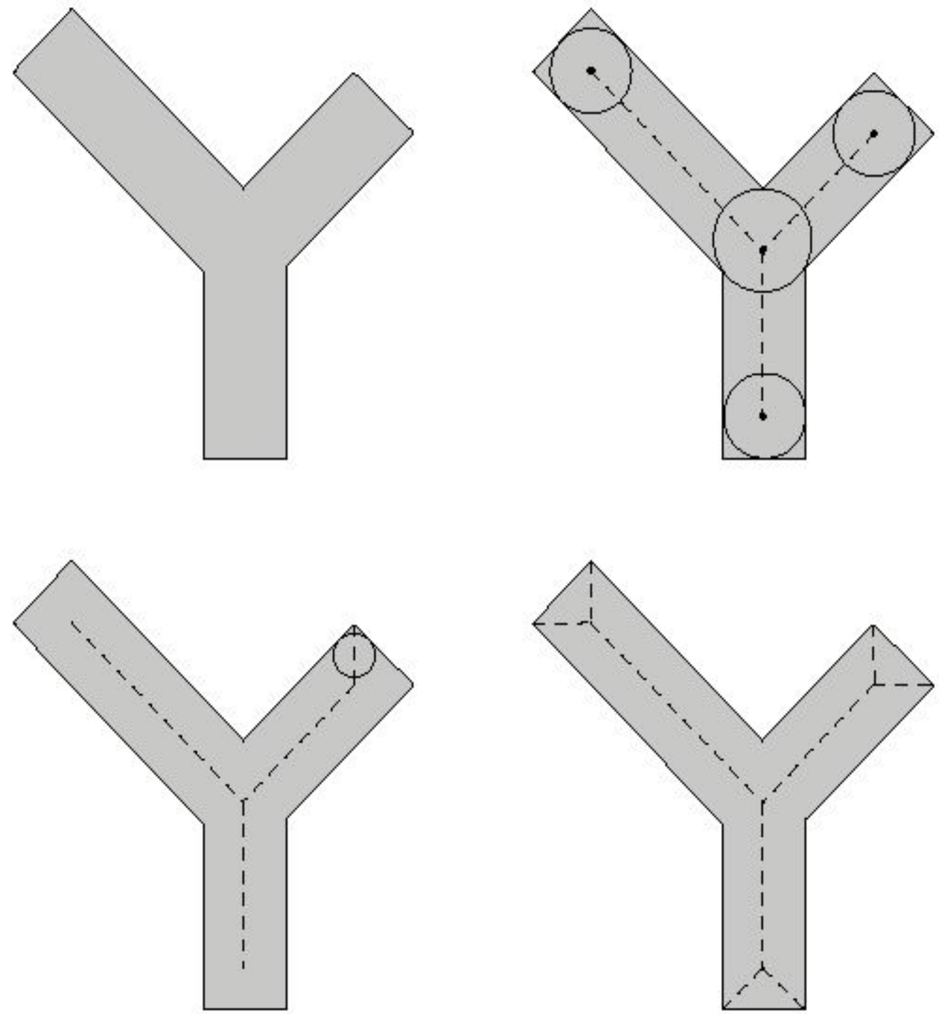


FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Skeletons

- Skeletons A with notation S(A):
  - For z belong to *S*(*A*) and (*D*)*z*, the largest disk
  - centered at z and contained in A, one can not find a
  - larger disk containing (*D*)*z* and included in A.
  - Disk (*D*)*z* touches the boundary of A at two or
  - more different points.



a b
c d

FIGURE 9.23
(a) Set A.
(b) Various positions of maximum disks with centers on the skeleton of A.
(c) Another maximum disk on a different segment of the skeleton of A.
(d) Complete skeleton.

❑Skeletons A with notation S(A):

$$S(A)=\bigcup_{k=0}^{K} S_k(A)$$

$$S_k(A)=(A\Theta kB)-(A\Theta kB)\circ B, \quad \circ: \text{Opening}$$

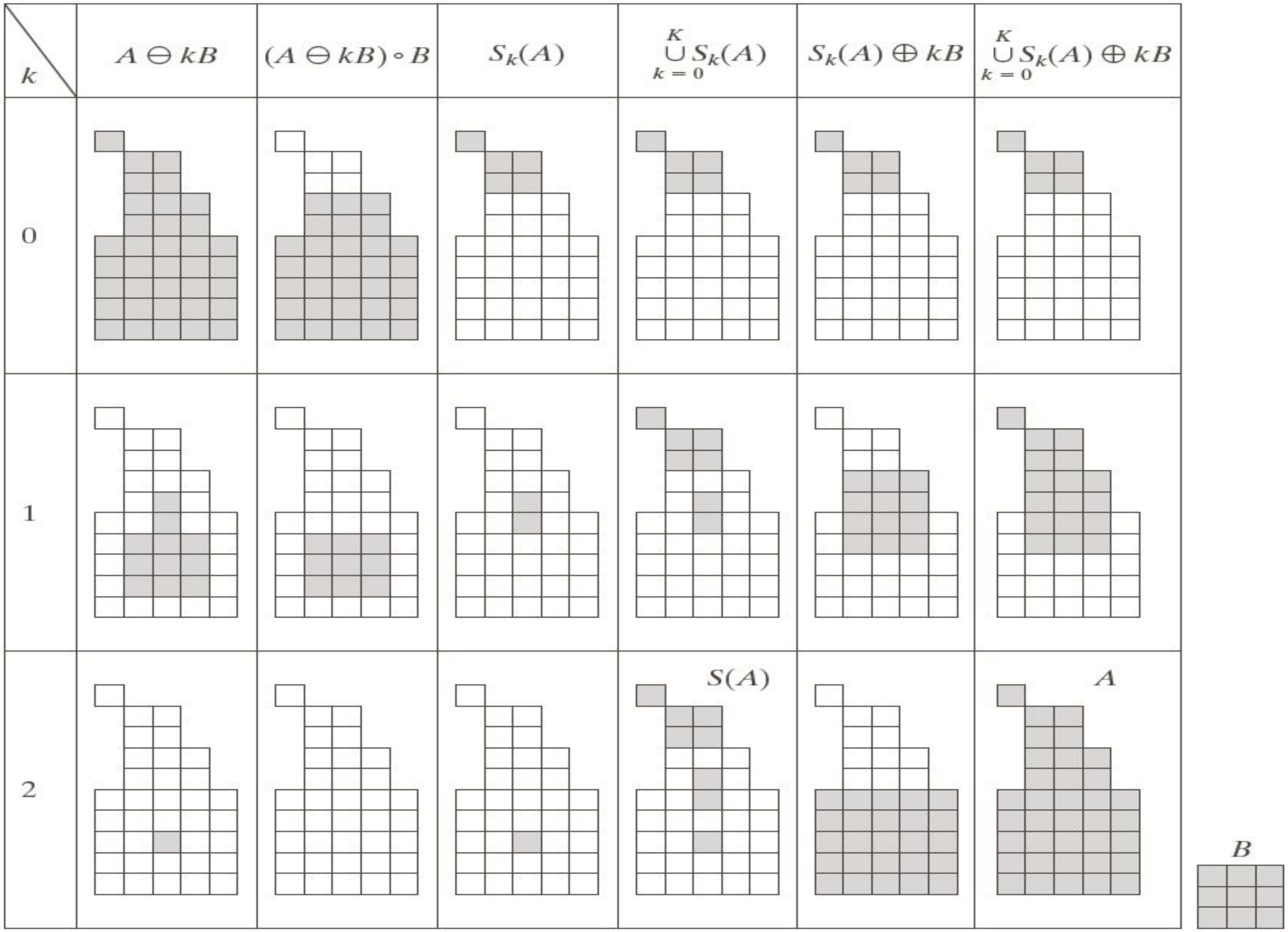
$$(A\Theta kB)=\Big(\big(\cdots((A\Theta B)\Theta B)\cdots\big)\Theta B\Big): \quad k \text{ times}$$

$$\text{K}=\max\left\{\text{k}\,\middle|\,(A\Theta kB)\neq\varnothing\right\}$$

Reconstruction:

$$\text{A}=\bigcup_{\text{k}=0}^{\text{K}}\left(S_k(A)\oplus kB\right)$$

$$(A\oplus kB)=\Big(\big(\cdots((A\oplus B)\oplus B)\cdots\big)\oplus B\Big): \quad k \text{ times}$$


k
$A \ominus kB$
$(A \ominus kB) \circ B$
$S_k(A)$
$\bigcup_{k=0}^{K} S_k(A)$
$S_k(A) \oplus kB$
$\bigcup_{k=0}^{K} S_k(A) \oplus kB$
0
1
2
$S(A)$
A
B

Thank you