Computer Vision and Image Processing

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Image Formation

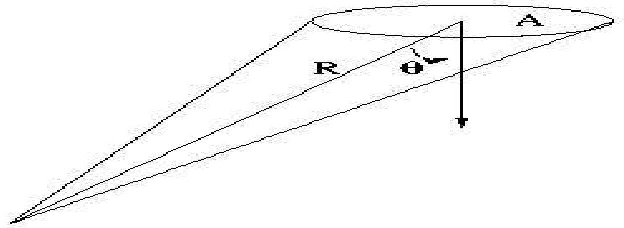
Image Brightness

- image of 3D object
  - shape
  - reflectance properties
  - distribution of light sources
  - position of object relative to the imaging system
- position and distribution of the source of illumination determines brightness pattern
- need to know how image is formed
  - brightness at a particular point in the image
- required to know Radiometry

Radiometry: Irradiance and Radiance

- irradiance: amount of light falling on a surface
  - power per unit area incident on the surface (watts per square meter)
- radiance: amount of light radiated from a surface
  - power per unit area per unit solid angle emitted from the surface (watts per square meter per steradian)
  - surface can radiate into a whole hemisphere of possible directions and can radiate different amount of energy in different directions
  - solid angle of a cone of directions is defined as the area cut out by the cone on the unit sphere
  - a hemisphere of directions has a solid angle 2π

Radiometry: Solid Angle



- A small planar patch of area A at distance R from the origin subtends a solid angle

$$\Omega = \frac{A \cos\theta}{R^2}$$

θ is the angle between a surface normal and a line connecting the patch to the origin

- foreshortened area - the surface area multiplied by the cosine of the angle between a perpendicular to the surface and the specified direction

Image Formation

- Brightness is determined by the amount of energy an imaging system receives per unit apparent area
- relation between the radiance at a point on an object (scene radiance) and the irradiance at the corresponding point in the image (image irradiance)

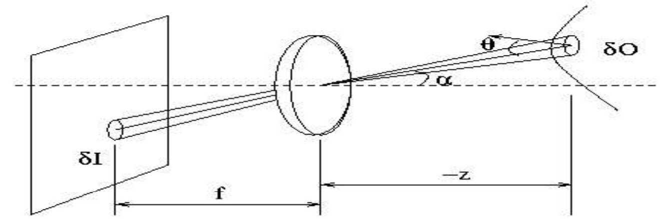


Image Formation

- the solid angle of the cone of rays leading to the patch on the object is equal to the solid angle of the cone of rays leading to image patch
- apparent area of the image patch as seen from the center of the lens is $\delta I \cos\alpha$
- the distance from center of the lens is $f/\cos\alpha$
- solid angle subtended by the image patch

$$\frac{\delta I \cos\alpha}{(f/\cos\alpha)^2}$$

- the solid angle of the object patch as seen from the center of the lens is

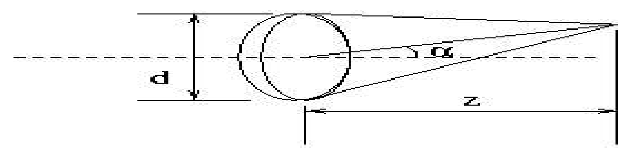
$$\frac{\delta O \cos\theta}{(z/\cos\alpha)^2}$$

- with two solid angles are to be equal

$$\frac{\delta O}{\delta I} = \frac{\cos\alpha}{\cos\theta}\left(\frac{z}{f}\right)^2$$

Image Formation

- how much of the light emitted by the surface makes its way through the lens



- solid angle subtended by the lens as seen from the object patch

$$\Omega = \frac{\pi}{4}\frac{d^2\cos\alpha}{(z/\cos\alpha)^2} = \frac{\pi}{4}\left(\frac{d}{z}\right)^2 \cos^3\alpha$$

- the power of the light originating on the patch and passing through the lens is

$$\delta P = L\delta O \Omega \cos\theta = L\delta O \frac{\pi}{4}\left(\frac{d}{z}\right)^2 \cos^3\alpha \cos\theta$$

L is the radiance of the surface in the direction toward the lens. This power is concentrated in the image.

Image Formation

- no light from other areas reaches this image patch

$$E = \frac{\delta P}{\delta I} = L \frac{\delta O}{\delta I} \frac{\pi}{4} \left(\frac{d}{z} \right)^2 \cos^3 \alpha \cos \theta$$

E is the irradiance of the image at the patch, substituting $\delta O / \delta I$

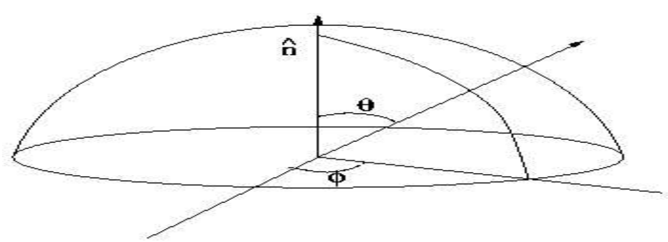
$$E = L \frac{\pi}{4} \left(\frac{d}{f} \right)^2 \cos^4 \alpha$$

- image irradiance is proportional to scene radiance
- this relationship exploited to recover information about object from its image
- inverse of the square of the effective f-number (f/d)
- brightness - image irradiance E is proportional to scene radiance L

Bidirectional Reflectance Distribution Function (BRDF)

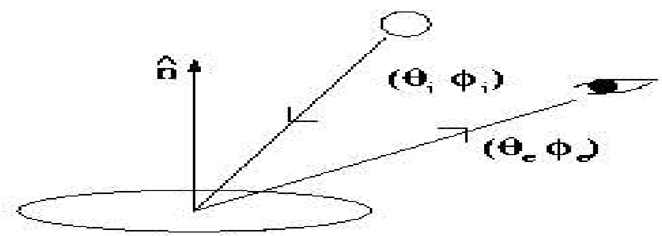
- scene radiance depends on
  - the amount of light that falls on a surface and the fraction of the incident light that is reflected
  - geometry of light reflection
- In short, radiance of a surface depends on the direction from which it is viewed and the direction from which it is illuminated
- surface normal $\hat{n}$, θ between a ray and the normal and ϕ between a perpendicular projection of the ray onto the surface and the reference line on the surface
- θ - polar angle
- ϕ - azimuth angle

BRDF



- the direction (θ_i, ϕ_i) from which light is falling on the surface
- the direction (θ_e, ϕ_e) into which it is emitted toward the viewer

BRDF



- BRDF $f(\theta_i, \phi_i, \theta_e, \phi_e)$ tells us how bright a surface appears when viewed from one direction while light falls on it from another
- the amount of light falling on the surface from the direction (θ_i, ϕ_i) the irradiance be $\delta E(\theta_i, \phi_i)$
- the brightness of the surface as seen from the direction (θ_e, ϕ_e) the radiance be $\delta L(\theta_e, \phi_e)$
- BRDF is the ratio of radiance to irradiance

$$f(\theta_i, \phi_i, \theta_e, \phi_e) = \frac{\delta L(\theta_e, \phi_e)}{\delta E(\theta_i, \phi_i)}$$

BRDF: Surfaces

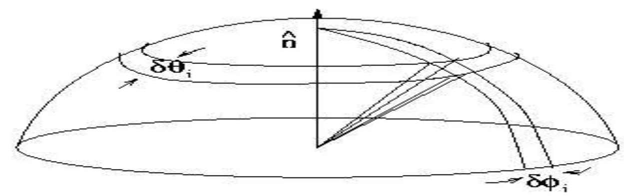
- for many surfaces the radiance is not altered if the surface is rotated about the surface normal
- BRDF depends only on the difference $\phi_e - \phi_i$
- this is true for matte surfaces and specularly reflecting surfaces but not for surfaces with oriented microstructure (tiger's eye and iridescent feathers of some birds)
- constraint on BRDF: if two surfaces are in thermal equilibrium, radiation reaching one from the other must be balanced by radiation flowing in the opposite direction
- otherwise surface receiving more radiation heat up and other would cool down which disturbs the equilibrium (second law of thermodynamics)
- BRDF is constrained by Helmholtz reciprocity condition

$$f(\theta_i, \phi_i; \theta_e, \phi_e) = f(\theta_e, \phi_e; \theta_i, \phi_i)$$

Extended Light Sources

- previously assumed that all light comes from one direction
- several light sources or extended source (sky)
- nonzero solid angle of directions required to obtain a nonzero radiance
- infinitesimal patch of the sky, of size $\delta\theta_i$ in polar angle and $\delta\phi_i$ in azimuth and this patch subtends a solid angle

$$\delta\omega = \sin\theta_i \delta\theta_i \delta\phi_i$$



BRDF: Extended Light Sources

- $E(\theta_i, \phi_i)$ be the radiance per unit solid angle coming from the direction (θ_i, ϕ_i), then the radiance from the patch is

$$E(\theta_i, \phi_i) \sin\theta_i \delta\theta_i \delta\phi_i$$

- the total irradiance of the surface is

$$E_0 = \int_{-\pi}^{\pi} \int_0^{\pi/2} E(\theta_i, \phi_i) \sin\theta_i \cos\theta_i d\theta_i d\phi_i$$

  $\cos\theta_i$ term accounts for the foreshortening of the surface as seen from the direction (θ_i, ϕ_i)
- for radiance - integrate the product of the BRDF and the irradiance over the hemisphere of possible directions from which light can fall on a surface

$$L(\theta_e, \phi_e) = \int_{-\pi}^{\pi} \int_0^{\pi/2} f(\theta_i, \phi_i; \theta_e, \phi_e) E(\theta_i, \phi_i) \sin\theta_i \cos\theta_i d\theta_i d\phi_i$$

  function of two variable θ_e and ϕ_e; specify the direction of the ray emitted toward the viewer

BRDF: Extended Light Sources

- ideal Lambertian surface is one that appears equally bright from all viewing directions and reflects all incident light, absorbing none, i.e., BRDF must be a constant for such a surface
- integrate the radiance of the surface over all directions and equate the total radiance so obtained to the total irradiance

$$\int_{-\pi}^{\pi} \int_0^{\pi/2} fE \cos\theta_i \sin\theta_e \cos\theta_e d\theta_e d\phi_e = E \cos\theta_i$$

  or

$$2\pi f \int_0^{\pi/2} \sin\theta_e \cos\theta_e d\theta_e d\phi_e = 1$$

- $\pi f = 1$ for an ideal Lambertian surface, BRDF is constant $f(\theta_i, \phi_i; \theta_e, \phi_e) = \frac{1}{\pi}$
- the radiance $L = \frac{1}{\pi} E_0$

Reflectance Map

- makes explicit relationship between surface orientation and brightness
- encodes information about surface reflectance properties and light-source distributions
- representational tool for recovering surface shape from images
- a source of radiance E illuminating a Lambertian surface
- scened radiance $L = \frac{1}{\pi} E \cos\theta_i$ (θ_i is angle between the surface normal and the direction toward source)

$$\cos\theta_i = \frac{1 + p_s p + q_s q}{\sqrt{1 + p^2 + q^2}\sqrt{1 + p_s^2 + q_s^2}}$$

- called reflectance map $R(p, q)$; depends on the properties of surface material of object and the distribution of light sources

Reflectance Map

- Image irradiance is proportional to a number of constants; the inverse of the square of f-number and the fixed brightness of the source
- for this reason, the reflectance map is normalized in some way, having maximum value one
- for Lambertian surface illuminated by a single distant point source $R(p, q)$ is defined as above
- reflectance map gives the dependence of scene radiance on surface orientation
- plot surface $R(p, q)$ - function of gradient (p, q); pq plane called gradient space
- every point in it corresponds to a particular surface orientation
- the point at the origin represents the orientation of all planes that are perpendicular to the viewing direction

Reflectance Map

- contour map in gradient space used to depict a reflectance map
- Lambertian surface; contours of constant brightness are nested conic sections in pq-plane $R(p, q) = c$

$$(1 + p_s p + q_s q)^2 = c^2(1 + p^2 + q^2)(1 + p_s^2 + q_s^2)$$

- $R(p, q)$ is maximum at $(p, q) = (p_s, q_s)$
- Shading in Images
  - brightness pattern in image affected by shape of an object
  - surface patches with different orientations appear with different brightness
  - the variation of brightness is called shading
  - reflectance map captures the dependence of brightness on surface orientation

Reflectance Map

- image irradiance $E(x, y)$ a particular point in the image proportional to the radiance at the corresponding point on the surface imaged
- surface gradient at that point is (p, q); the radiance there is $R(p, q)$
- normalizing by setting the constant of proportionality to one

$$E(x, y) = R(x, y)$$

- sphere with a Lambertian surface illuminated by a point source at essentially the same place as the viewer
- $\theta_e = \theta_i$ and $(p_s, q_s) = (0, 0)$

$$R(p, q) = \frac{1}{\sqrt{1 + p^2 + q^2}}$$

Reflectance Map: Example

- with sphere on the optical axis; equation for its surface

$$z = z_0 + \sqrt{r^2 - (x^2 + y^2)}$$

r is radius, z_0 is the distance of its center from the lens and $x^2 + y^2 \leq r^2$

$$p = -\frac{x}{z - z_0} \quad \text{and} \quad q = -\frac{y}{z - z_0}$$

$$E(x, y) = R(p, q) = \frac{z - z_0}{r} = \sqrt{1 - \frac{x^2 + y^2}{r^2}}$$

- sphere with different reflectance properties, say,

$$R(p, q) = \frac{1 + p_s p + q_s q}{\sqrt{1 + p_s^2 + q_s^2}}$$

$(p_s, q_s) = (0, 0)$ obtains a uniformly bright disk in the image

Photometric Stereo

- reflectance map useful in CG: image is created from a description of the shape of an object
- Given an image recover the shape
- unique mapping from surface orientation specified by (p, q) to radiance, given by the reflectance map $R(p, q)$
- but inverse mapping is not unique; an infinite number of surface orientations give rise to the same brightness
- contour of constant $R(p, q)$ connects such a set of orientations in the reflectance map
- surface orientation can be determined uniquely for some special points; where brightness is maximum or minimum of $R(p, q)$ Lambertian surface $R(p, q) = 1$ only when $(p, q) = (p_s, q_s)$
- mapping from brightness to surface orientation cannot be unique, since brightness has one degree of freedom while orientation has two

Photometric Stereo

- to determine two unknowns p and q; need two equations
- two images with different lighting yield two equations for each image point

$$R_1(p, q) = E_1 \quad \text{and} \quad R_2(p, q) = E_2$$

- these equations are linear and independent then unique solution for p and q is possible

$$R_1(p, q) = \sqrt{\frac{1 + p_1 p + q_1 q}{r_1}} \quad \text{and} \quad R_2(p, q) = \sqrt{\frac{1 + p_2 p + q_2 q}{r_2}}$$

$$r_1 = \sqrt{1 + p_1^2 + q_1^2} \quad \text{and} \quad r_2 = \sqrt{1 + p_2^2 + q_2^2}$$

$$p = \frac{(E_1^2 r_1 - 1)q_2 - (E_2^2 r_2 - 1)q_1}{p_1 q_2 - q_1 p_2} \quad q = \frac{(E_2^2 r_2 - 1)p_1 - (E_1^2 r_1 - 1)p_2}{p_1 q_2 - q_1 p_2}$$

Photometric Stereo

- if equations are nonlinear; no solutions or several solutions

$$R_1(p, q) = \frac{1 + p_1 p + q_1 q}{\sqrt{1 + p^2 + q^2}\sqrt{1 + p_1^2 + q_1^2}}$$

$$R_2(p, q) = \frac{1 + p_2 p + q_2 q}{\sqrt{1 + p^2 + q^2}\sqrt{1 + p_2^2 + q_2^2}}$$

- can be two solutions, one solution or none depending on the values of R_1 and R_2

Recovering Albedo

- surface with non-uniform reflectance properties
- radiance is product of reflectance factor (albedo) and some function of orientation
- Albedo: a number between zero and one that indicates how much light the surface reflects relative to some ideal surface with the same geometric dependence in the BRDF
- matte surface like Lambertian surface but does not reflect all of the incident light
- its brightness is $\rho \cos \theta_i$
- recover ρ and (p, q) need three images

Recovering Albedo

- unit vectors in the directions of three source positions

$$\hat{\mathbf{s}}_i = \frac{(-p_i, -q_i, 1)^T}{\sqrt{1 + p_i^2 + q_i^2}} \qquad i = 1, 2, 3$$

$$E_i = \rho(\hat{\mathbf{s}}_i \cdot \hat{\mathbf{n}}) \quad \hat{\mathbf{n}} = \frac{(-p, -q, 1)^T}{\sqrt{1 + p^2 + q^2}}$$

$$E_1 = \rho(\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{n}}) \;\; E_2 = \rho(\hat{\mathbf{s}}_2 \cdot \hat{\mathbf{n}}) \;\; E_3 = \rho(\hat{\mathbf{s}}_3 \cdot \hat{\mathbf{n}})$$

$$\mathbf{E} = \rho \mathbf{S} \hat{\mathbf{n}} \quad \rho \hat{\mathbf{n}} = \mathbf{S}^{-1} \mathbf{E}$$

$$\rho \hat{\mathbf{n}} = \frac{1}{[\hat{\mathbf{s}}_1 \hat{\mathbf{s}}_2 \hat{\mathbf{s}}_3]} (E_1(\hat{\mathbf{s}}_2 \times \hat{\mathbf{s}}_3) + E_2(\hat{\mathbf{s}}_3 \times \hat{\mathbf{s}}_1) + E_3(\hat{\mathbf{s}}_1 \times \hat{\mathbf{s}}_2))$$

  where $[\hat{\mathbf{s}}_1 \hat{\mathbf{s}}_2 \hat{\mathbf{s}}_3]$ is the triple product $\hat{\mathbf{s}}_1 \cdot (\hat{\mathbf{s}}_2 \times \hat{\mathbf{s}}_3)$
- unique result is assured