P21CO006

Question 5(a)

now we have set rules from moving cell

i) we can move i cells in any chain of hexagonal cells
ii) we can move 60° Anti counter clockwise direction

represented in below diagram.

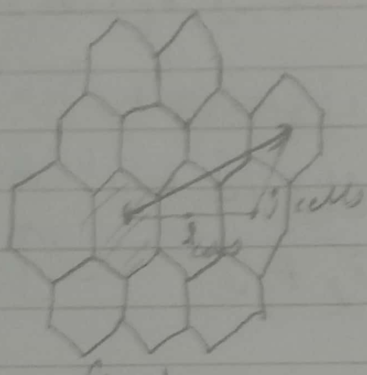

Fig 1

- now distance b/w two adjacent cells are

$\sqrt{3}R$

- now consider following figure 2 from Fig 1.

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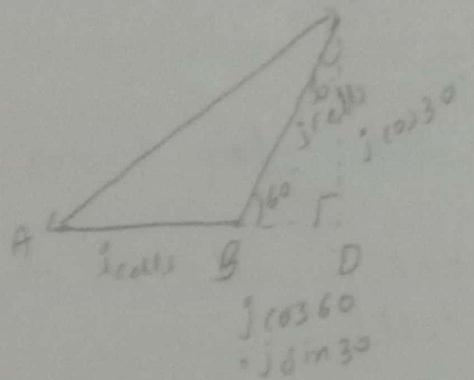

Fig 2

From fig 2 we can say that

$$AC^2 = \cancel{AB^2}\ AD^2 + \cancel{B}DC^2$$
$$= (i + j\sin 30)^2 + (j\cos 30)^2$$
$$= i^2 + 2ij\sin 30 + j^2\sin^2 30 + j^2\cos^2 30$$
$$= i^2 + 2(\tfrac{1}{2})\, ij + j^2(\sin^2 30 + \cos^2 30)$$
$$= i^2 + ij + j^2 \quad -(1)$$

now, if radius is R then for hexagon we can give area $A = \beta R^2$ where $\beta = \frac{3\sqrt{3}}{2} = 2.598$

So, Area of cell is $A_{cell} = \beta R^2$

& Area of first tier hexagon is $A_{large} = \beta D^2$

~~So, total no of cell in first tier hexagon is~~

$$\cancel{\frac{A_{large}}{A_{cell}} = i^2 + ij + j^2}$$

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from (i)

$$D_{norm} = \sqrt{i^2+ij+j^2}$$

so $D = \sqrt{3} R D_{norm}$
$= \sqrt{3} R \sqrt{i^2+ij+j^2}$

so, total no of cell in hexagonal first tier is

$$\frac{A_{larg}}{A_{cell}} = \frac{\cancel{k}\left(\sqrt{3}\sqrt{i^2+ij+j^2}\right)^2 R^2}{\cancel{k}\, R^2}$$

$$= 3(i^2+ij+j^2) \quad \text{— (ii)}$$

total no of cell form in first tier hexagon is

$$N + \frac{1}{3}(6N) = 3N \quad \text{— (iii)}$$

from (i) & (iii)

So, $3N = 3(i^2+ij+j^2)$

$\therefore N = i^2+ij+j^2$