Department of Computer Engineering, SVNIT, Surat

M Tech I - 1^{st} Semester, End-Semester Examinations, December 2021

CO 603: Algorithms and Computational Complexity

09:30 hrs to 12:30 hrs, 20^{th} December 2021

Instructions:

1. *Write your Admission number clearly in the answer books along with the other details.*
2. *Assume any necessary data but giving proper justifications.*
3. *Be brief, precise, clear and to the point in answering the questions. Unnecessary elaboration WILL NOT fetch more marks.*
4. *Each MAIN question carries 10 marks, maximum.*
5. *All sub-questions carry equal marks unless stated otherwise.*

1. (a) Show that if n is a power of 2, say $n = 2^k$, then $\sum_{i=0}^{k} lg(n/2^i) = \theta(lg^2 n)$

   (b) Thinking a minor variation on the algorithm used to solve the Activity Selection Problem that was taught in the class, – suppose that instead of selecting the first activity to finish, we instead select the last activity to start that is compatible to all the previous activities. Will this approach still be a greedy algorithm and would it yield an optimal solution ? With logical arguments prove the answer.

2. (a) Movers and Packers own and operate their Air Transport Service, which ships cargo by plane to most large cities in the India. The remaining capacity for one of the flights from New Delhi to Chennai is 10 tons. There are four different items that M&P can ship between New Delhi to Chennai. Each item has a weight in tons, a net profit in thousands of rupees, and a total number of that item that is available for shipping. This information is presented below in the figure. Identify which items should they load to maximize the profit. Assuming each item

ITEM	WEIGHT	PROFIT/UNIT	NUMBER AVAILABLE
1	1	3	1
2	4	9	1
3	3	8	1
4	2	5	1

Figure 1: Figure for problem 2(a)

   is available in a single number, design a recurrence to solve this problem. Draw an instance of the table showing all the intermediate values of the optimal value V[i]. Compute the maximum value of the profit. Write an algorithm that implements the recurrence designed and solves the problem. [7]

   (b) What is a dynamic programming version of a recursive algorithm ? Briefly, by drawing an appropriate graph, explain how this version improves the efficiency of the solution. [3]

3. (a) Discuss briefly the two types of randomized algorithms. Specifying its type, give the pseudocode of the Randomized-Quicksort and show how does it improve the efficiency of the Quicksort algorithm.

   (b) The popular site Amazon.com has been using item-based Collaborative Filtering (e.g. users who bought x also bought y) to prompt and guide its users in selecting and purchasing a book. How does a Collaborative Filtering algorithm work ? Design an algorithm that enables one to implement the Collaborative Filtering and analyze the time complexity of the same.

OR

3. (a) Suppose you own two stores, A and B. On each day you can be either at A or B. If you are currently at store A (or B) then moving to store B the next day (or A) will cost C amount of money. For each day i, $i = 1, ..., n$ we are also given the profits $PA(i)$ and $PB(i)$ that you will make if you are store A or B on day i respectively. Design an algorithm to get an a schedule which tells where you should be on each day so that the overall money earned (profit minus the cost of moving between the stores) is maximized. Your design of the recurrence could start with defining two arrays $TA[]$ and $TB[]$, such that $TA[i]$ gives the most profitable schedule for days $i, ..., n$ given that we start at store A on day i. Define $TB[i]$ similarly. [7]

   (b) Prove that a prefix code can always be represented as a full binary tree. Prove also that a full binary tree representation always gives optimal code OR a binary tree that is not full cannot correspond to an optimal prefix code. [3]

4. (a) Prove that the problem maximum independent set of a given graph G=(V,E), is polynomially equivalent to the CNF problem 3-SAT. You can use the 3-SAT expression viz. $(\bar{x}\ V\ y\ V\ \bar{z})\ (x\ V\ \bar{y}\ V\ z)(x\ V\ y\ V\ z)(\bar{x}\ V\ \bar{y}\ V\ \bar{z})$ for designing the gadget.

   (b) Using the reduction shown in (a) above as part of the answer, prove that the problem maximum independent set is an NPC problem.

5. (a) Design the algorithm Approximate_Vertex_Cover to solve the minimum vertex cover problem. Prove that the algorithm so designed works and compute its time complexity.

   (b) Given n jobs, to execute the jobs we have m identical machines, M_1, M_2 . . . ,M_m , available. Executing job j on any of the machines takes time t_j, where $t_j > 0$. If we denote the collection of jobs assigned to machine M_i by $A(i)$, then what is the load T_i of machine M_i? What is the total time for which M_i is busy? What is the expression for the *Makespan* of the assignment. If the goal is to find an assignment of jobs to machines that minimizes the *makespan*, where each job is assigned to a single machine – scheduling is non-preemptive, design the algorithm to do so. Computes its approximation ratio value ρ.

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