Features x_1 to x_6 ... Rate them by usefulness for decision making.

"6 dimension data"

x_1	x_2	x_3	x_4	x_5	x_6
1	6	2	2	1	100
2	5	3	4	1	1
3	4	4	6	1	2
4	3	5	8	1	1
5	2	6	10	1	2
6	1	7	12	1	1

outlier

Negative correlation

Not useful

(linear shift of x_1)

Corre-lated

Not useful

derived from x_1

Not useful

"Same for all examples"

x_1 & x_2 = negatively correlated

x_3, $\cancel{x_4}$ = Can be calculated from x_1

x_5 = Same across all observation

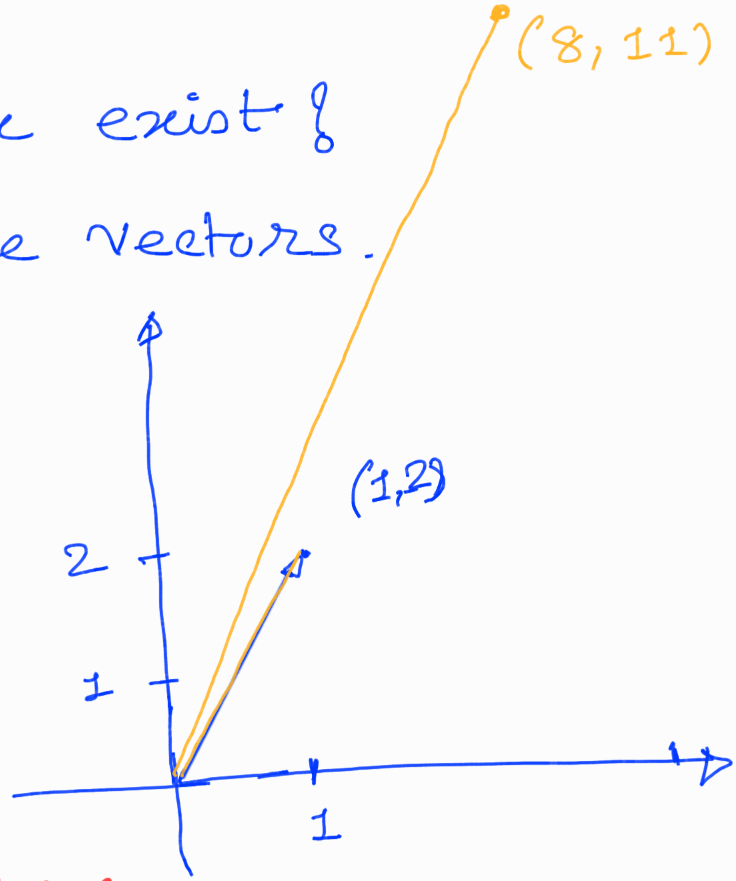
$\boxed{x_6 = \ ?}$ — has an outlier

"if we know x_1, $x_2, x_3, \cancel{x_4}$ are not required; because they are not providing any more info. than x_1."

Why does matrix exist?

→ To torture vectors.

$$\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$$



If matrix is Thanos far vectors; then there is super hero vector. (Iron Man)/ (Captain America)

- "Eigen Vector"

- When a matrix is multiplied with its Eigen vector, we get a scalar & Eigen vector

$$Av = \lambda v$$

Eigen vector

Eigen value

$$A = \begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix}$$ find out Eigen values & vectors.

$$Av - \lambda v = 0$$

$$(A - \lambda I)v = 0$$

$$\therefore \quad A - \lambda I = 0$$

use this to find Eigen values.

$$\begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\therefore \quad \begin{bmatrix} -6-\lambda & 3 \\ 4 & 5-\lambda \end{bmatrix} = 0$$

$$\therefore \quad (-6-\lambda)(5-\lambda) - 12 = 0$$

$$\therefore \quad \lambda^2 + \lambda - 42 = 0$$

$$\therefore \quad \lambda_1 = -7, \quad \lambda_2 = 6$$

$$A v = \lambda_1 v$$

$$\begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \lambda_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$-6v_1 + 3v_2 = -7v_1 \quad \therefore \quad v_1 = -3v_2$$

$$4v_1 + 5v_2 = -7v_2 \qquad v_1 = -3v_2$$

$$\boxed{V_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix} \qquad \lambda_1 = -7}$$

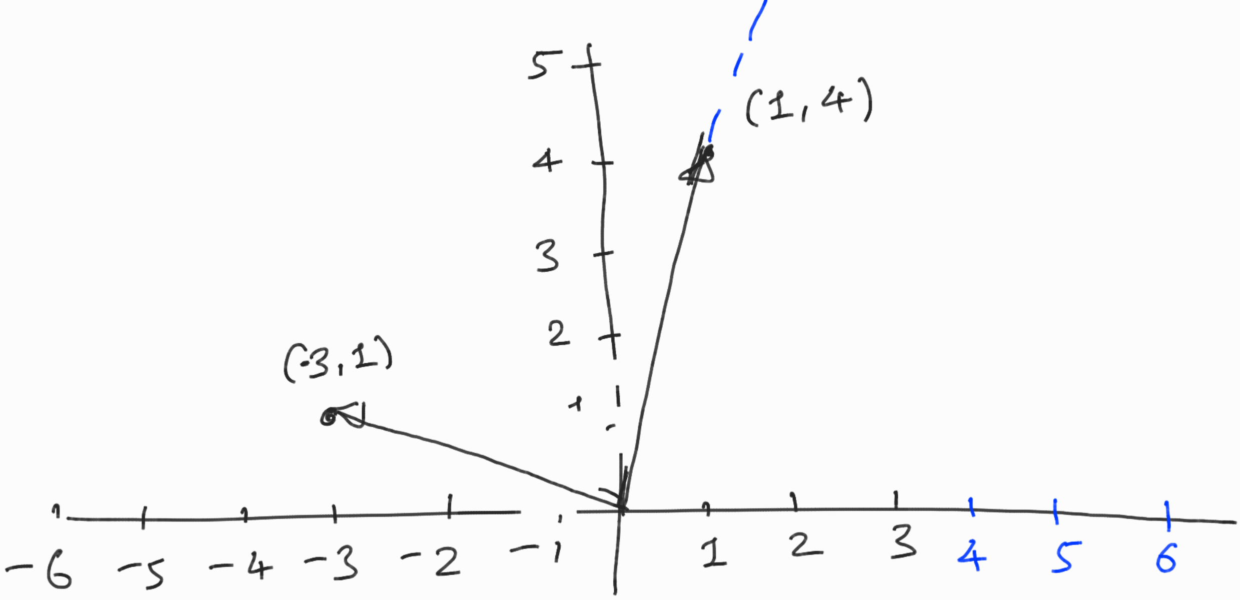
$$\begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 6 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$-6v_1 + 3v_2 = 6v_1$$

$$3v_2 = 12v_1$$

$$\boxed{v_2 = 4v_1}$$

$$V_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \qquad \lambda_2 = 6$$



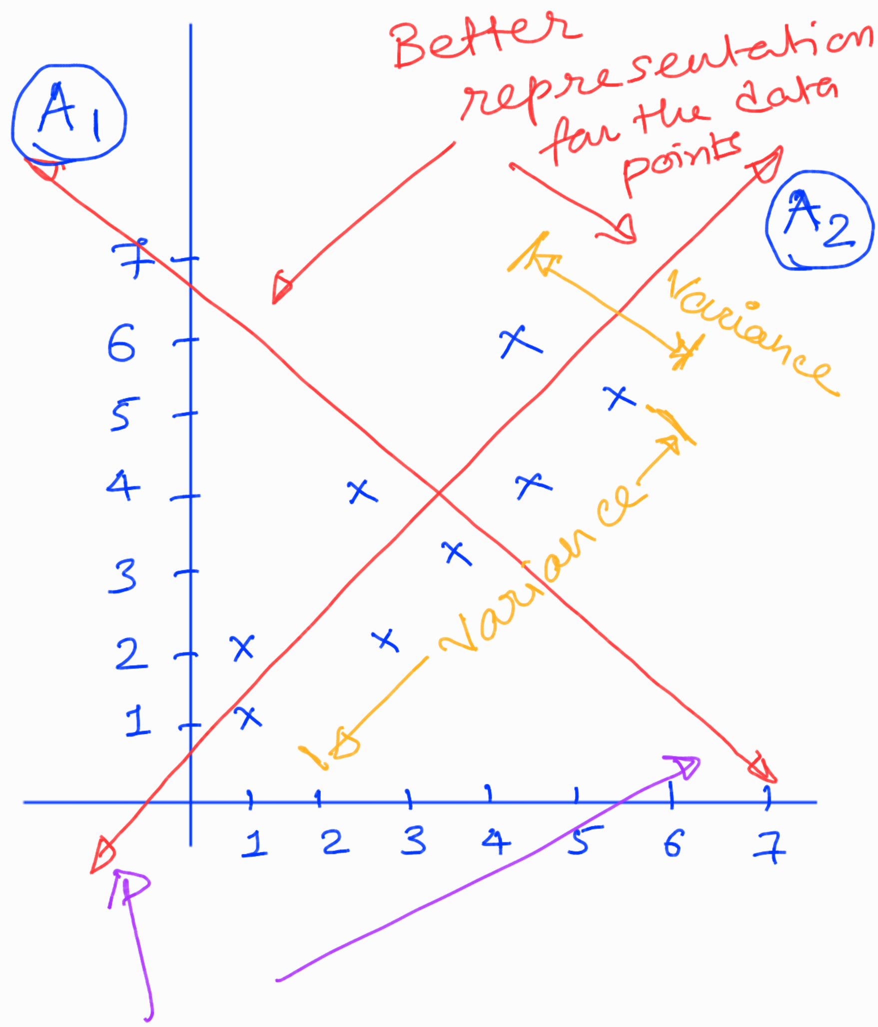
"Eigen Vectors are orthogonal to each other."

"For an $n \times n$ matrix, there are n Eigen vectors."

Image

$$\begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 24 \end{bmatrix} = 6 \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

X_1	X_2
1	1
1	2
3	2
3	4
4	3
5	4
5	6
6	5



- Eigen Vectors

- Eigen values is proportion -al to variance

$$\boxed{\text{Eigen value corresponding to } A_2 > \text{Eigen value corresponding to } A_1}$$

- Computations of any ML algo. increases with number of dimensions.
- By reducing dimension we can avoid unnecessary computations.
- We want to keep important attributes and remove redundant or unnecessary attributes.

$\boxed{\text{Race Religion gender color appearance}}$

"Don't use this features in decision making."

	x_1	x_2	$x_1-\mu_{x_1}$	$x_2-\mu_{x_2}$	$(x_1-\mu_{x_1})^2$	$(x-\mu_{x_2})^2$	$(x_1-\mu_{x_1})\cdot(x_2-\mu_{x_2})$
samples	1	2	-1	-1	1	1	1
	2	3	0	0	0	0	0
	3	4	1	1	1	1	1
	$\mu_{x_1}=2$	$\mu_{x_2}=3$			$\frac{2}{2}=1$	$\frac{2}{2}=1$	$\frac{2}{2}=1$

1 - Remove mean

$$\sum x_i / n$$

2 - Calculate Variance

$$\frac{\sum (x_i-\mu_{x_i})^2}{(n-1)}$$

3 - Calculate Covariance

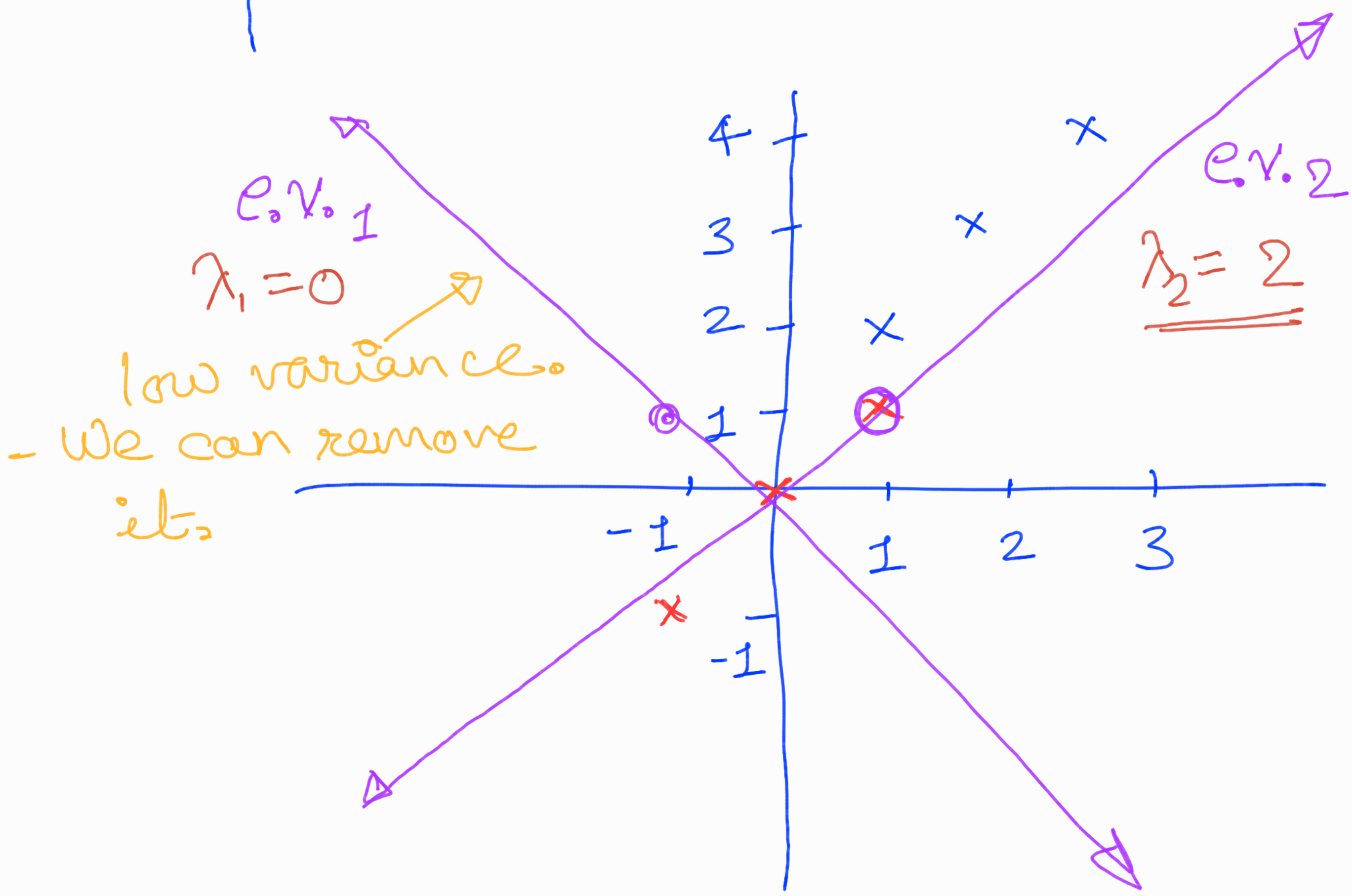
$$\forall_{i,j} \quad \frac{\sum (x_i-\mu_{x_i})(x_j-\mu_{x_j})}{(n-1)}$$

$Var(x_1) = 1$, $Var(x_2) = 1$,

$Covar(x_1, x_2) = 1$

Covariance Matrix x_1, x_2

	x_1	x_2
x_1	1	1
x_2	1	1



④ Find Eigen values & vector of co-variance matrix

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\begin{bmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix} = 0$$

$$\therefore \quad (1-\lambda)^2 - 1 = 0$$

$$\therefore \quad 1 - 2\lambda + \lambda^2 - 1 = 0$$

$$\therefore \quad \lambda(\lambda - 2) = 0$$

Eigen values: $\lambda_1 = 0, \quad \lambda_2 = 2$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$v_1 + v_2 = 0$$

$$\boxed{v_1 = -v_2}$$

V_1

$$\begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$\left(-\tfrac{1}{\sqrt{2}}\right)^2 + \left(\tfrac{1}{\sqrt{2}}\right)^2 = 1$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 2 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$v_1 + v_2 = 2v_1$$

$$v_2 = v_1$$

V_2

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \approx \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$\boxed{\begin{array}{ll} \lambda_1 = 0 & \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\ \lambda_2 = 2 & \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{array}}$$

Principal Component Analysis (PCA)

1. Remove mean from the data
2. Calculate Covariance matrix
3. Find Eigen values & vectors

from the covariance matrix

4. Eigen vectors are the new principal components.

5. Arrange Eigen vectors in descending order of Eigen values.

6. Remove Eigen vector with low Eigen values to reduce dimension.

- Principal Components are the new features constructed from the original features.

1. Eigen values & vectors are difficult to compute.

2. Co-variance matrix is square matrix $(n \times n)$