5

Fundamentals of Cellular Communications

With the limitation in spectral width, the maximum number of users (capacity) that can be supported in a wireless system is an important performance measure. If the system is supported by a single base station, a high power transmitter is needed to support a large number of users. The system capacity (number of users) can be enlarged by arranging small cells, each requiring only a low power transmitter, in a cellular array. In this chapter, the rationale behind cellular operation as a means of increasing system capacity is explained. The increase in system capacity comes from the use of smaller cells, reuse of frequencies, and antenna sectoring. The ramifications of frequency reuse and the gain obtainable from cell splitting and antenna sectoring are described and discussed in detail.

5.1 INTRODUCTION

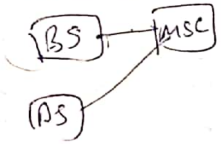
As discussed in Chapter 2, the wireless propagation channel exhibits impairments far more severe than those inherent in a guided wire. Severe impairments mean that the wireless channel yields a poorer signal-to-interference ratio (SIR) and, hence, higher transmission bit error rate at the output of the receiver compared with those in a wired channel. For example, the BER in an optical fiber is in the neighborhood of 10^{-10} while that in a wireless channel with multipath fading may be in the range of 10^{-2} to 10^{-3}. This high error rate has the effect of limiting the amount of throughput and rendering signal reception unreliable. Impairments in the propagation channel will result in a reduction of usable spectral width. Mitigation of interference due to channel impairments has

the effect of enlarging the spectral width. A wider spectrum, in turn, means more bandwidth to support more users. Certain sophisticated modulation and coding techniques have built-in properties that the receiver can exploit to compensate for channel impairments. Some of the modulation and detection methods that are relevant to wireless communications are discussed in Chapters 3 and 4.

A wider spectrum will allow for greater sharing of the spectral width to support more users and increase the throughput. System capacity can be interpreted as the largest number of users that can be supported during any one use of the channel. A larger population will mean a larger geographical coverage. If a single transmitter were used to cover a large geographical area, a very high power transmitter and very high antenna would be required. With a single high power transmitter, all users will share the same set of frequencies, or radio channels. The system capacity, in terms of the maximum number of users that can be supported, offered by a single high power transmitter will hit a limit. The reason for this limitation is that the radio resources (i.e., frequencies) are not efficiently utilized. If the same set of radio resources were assigned to serve a smaller geographical area and then reused to serve another small geographical area, it would be possible to expand the system capacity. However, care must be taken to ensure that use of the same set of frequencies to serve more than one geographical area does not introduce reception interference between users in the two areas. Or, if there is some interference, it must be at an acceptable level. This means that the geographical regions that use the same set of radio frequencies must be physically separated from each other so that the power level of the signal that spills out from one region to a neighboring region does not produce unacceptable interference. This way of replicating identically structured and operated geographical regions gives rise to the concept of *cellular communications*.

Reuse

A geographical region can be a single small area called a *cell* that is served by a single base station, or a cluster of cells. In cellular systems, each of the base stations in the cluster is connected to a mobile switching center (MSC) by wirelines. An MSC has more computing power and can perform many more functions than an individual base station. Therefore, most of the communications operations are handled by the MSC.



5.2 FREQUENCY REUSE AND MOBILITY MANAGEMENT

5.2.1 Cellular Communications and Frequency Reuse

If a given set of frequencies, or radio channels, can be reused without increasing the interference, then the large geographical area covered by a single high power transmitter can be divided into a number of small areas, each allocated a subset of frequencies. With a smaller geographical coverage, lower power transmitters with lower antennas can be used. Provided that the physical separation of two cells is sufficiently wide, the same subset of frequencies can be used in both cells. This is the concept of *frequency reuse* [92]. The ability to reuse the frequencies offers a means to expand the total system capacity without the need to employ high power transmitters. This plan of dividing the total large geographical coverage area into many small contiguous areas and using a low power transmitter with low antenna in each small area is referred to as *cellular communications*.

def

BS = AP   Cell = footprint   M→B uplink/Rev
B→M down/Fwd

Each small area is served by a transmit/receive unit called a *base station* (BS). A base station is the common resource shared by multiple users. User-to-user communication passes through the base station. Users transmit their signals to the base station, and the base station then relays the information-bearing signals to their destinations. We say that the users access the base station so that the base station is also called an *access point* (AP). The propagation channel that handles the transmissions from the base station to the users is called the *forward channel* or *downlink* while the channel used for transmissions from the users to the base station is called the *reverse channel* or *uplink*. Techniques for multiaccessing the base station will be discussed in Chapter 6. BS→Users U→B up

The radio coverage by one base station is referred to as a *cell*, which is also called a *footprint*. Cells arranged in a two-dimensional array form a cellular structure. From a conceptual point of view, it is highly desirable to construct the cellular system such that the cells do not overlap, and are tightly packed without dead spots. This form of layout requires the use of regular topologies (e.g., a square, triangular or hexagonal topology). Because of the differences in terrain and population densities, real footprints are amorphous in nature. On the other hand, cellular layouts using irregular structures are inefficient and limit growth. For this reason, cellular layouts and performance studies are based on regular topologies, even though they may just be conceptual. Also, regular topologies allow for systematic growth. In this chapter, all our discussions will be based on a hexagonal topology. The design and performance obtained using regular topologies may not correspond to real environments, but these topologies should provide valuable information and guidelines for structuring practical layouts.

While the cellular approach allows the use of low power transmitters and frequency reuse to expand system capacity, these advantages do not come without a price. Since a salient feature of wireless communications is the flexibility to support user roaming, smaller geographical areas will mean that mobile users can move out of one cell and into another cell quite frequently. To maintain continuous operation of an ongoing session, when the mobile migrates out of its current serving base station into the footprint of another, the connection must be handed off from the serving base station to the new base station.

An effective and efficient handoff mechanism must be in place to maintain service continuity and preserve end-to-end QoS (quality of service) requirements. The procedure to perform and to manage handoff is referred to as *handoff management*.

The principle of cellular communications is that a mobile host (MH)[1] is assigned a *home network*. The MH is identified by an *address*, called its home address. An agent in the home network, referred to as the *home agent*, keeps track of the MH's current location to facilitate delivery of messages destined for the MH. As the MH migrates away from its home network, the association between the MH and its home agent must be maintained so that the home agent can keep track of the MH's current location for message delivery purposes. In cellular communications, the procedure to keep track of the user's current location so as to maintain the association between the MH and its home agent is referred to as *location management*. Both handoff management and location management are necessitated by user mobility. These management functions are considered as two components of *mobility management*.

[1] For mobility and resource management at the networking layer, the term mobile host (MH) is often used instead of mobile station (MS).

MH = Cell/m vdc

5.2.2 Mobility Management

Mobility management consists of handoff management and location management. While mobility management issues are dealt with in Chapters 7 and 8, the salient aspects of handoff and location operations are described in the following paragraphs.

Handoff Management. When a mobile moves into a different cell while a session is in progress, the session has to be transferred to a new channel belonging to the new cell. This operation is referred to as handoff. The handoff operation involves the identification of a new base station and the allocation of channels to support both data and control signals in the new base station. As alluded to above, the MSC has the computing power to perform many different functions. Therefore, the handoff operation is normally handled by the MSC. The MSC keeps track of the resource usage of all cells under its jurisdiction. When a mobile moves into a different cell while a conversation is in progress, the MSC determines the availability of the unused channels in the new cell and makes transfer decisions. If the new base station has channels that can be allocated to handle both information carrying and control signals to accommodate the handoff connection, the handoff takes place. Otherwise handoff cannot occur.

Location Management. As previously mentioned, an MH is associated with a home network and its home address resides with its home agent. When the MH moves away from its home network, it enters a foreign territory. The network where the MH resides when it is away from home is called the foreign network. The MH has to register with its home agent, through the foreign agent, to let the home agent know its current location to facilitate message delivery. When an MH powers on, it registers with its home agent. When it moves to a foreign network, it has to register with its home agent, via the foreign agent. This means that there is an association between the home agent and the foreign agent. When the home agent has messages destined for the MH, it forwards them to the MH via the foreign agent. During the registration process, the home agent needs to ensure, from the identification conveyed through the foreign agent, that the mobile submitting a registration is the correct MH. The process of verifying that the identity provided during the registration process does belong to the correct MH is referred to as an *authentication* process.

5.3 CELL CLUSTER CONCEPT

In a wireless communications system, the channels used in the forward and reverse directions are separated in time or in frequency to permit duplexing. The total number of channels available in a cellular system is finite. The capacity of the cellular system is defined by the total number of channels available. The system capacity as a function of the total number of available channels depends on how the available channels are deployed. Specifically, if the available channels are reused for transmission of additional traffic, it is possible to expand the system capacity (i.e., support more users). Two or more different cells can use the same set of frequencies or radio channels if the nearest cells are separated such that the interference between cells at any given frequency is at an acceptable level. Cells which use the same set of frequencies are referred to as *cochannel cells*, and the interference between cochannel cells is referred to as *cochannel interference*. Frequencies or channels represent radio resources. In this text, we will use the terms *frequencies* and *channels* interchangeably.

The space between adjacent cochannel cells can be filled with other cells that use different frequencies to provide frequency isolation. A group of cells that use a different set of frequencies in each cell is called a *cell cluster*. Let N be the cluster size in terms of the number of cells within it and K be the total number of available channels without frequency reuse. The N cells in the cluster would then utilize all K available channels. In this way, each cell in the cluster contains one-Nth of the total number of available channels. In this sense, N is also referred to as the *frequency reuse factor* of the cellular system.

J: channels per cell
K: Total channels
N: cluster size (freq. reuse factor)

5.3.1 Capacity Expansion by Frequency Reuse

Suppose each cell is allocated J channels ($J \le K$). If the K channels are divided among the N cells into unique and disjoint channel groups, each with J channels, then

$J = K/N$

$$\boxed{K = JN.} \tag{5.3.1}$$

Collectively the N cells in a cluster use the complete set of available frequencies. Since K is the total number of available channels, from Eq. (5.3.1), it can be seen that a decrease in the cluster size N is accompanied by an increase in the number of channels J allocated per cell. Thus, by decreasing the cluster size, it is possible to increase the capacity per cell.

The cluster can be replicated many times to form the entire cellular communications system. Let M be the number of times the cluster is replicated and C be the total number of channels used in the entire cellular system with frequency reuse. C is then the system capacity and is given by

$$\boxed{C = MJN.} \tag{5.3.2}$$

If N is decreased and J is proportionally increased so that Eq. (5.3.1) is satisfied, it is necessary to replicate the smaller cluster more times in order to cover the same geographical area. This means that the value of M has to be increased. Since $JN(= K)$ remains constant and M is increased, Eq. (5.3.2) shows that the system capacity C is increased. That is, when N is minimized, C is maximized. We will see shortly that minimizing N will increase cochannel interference.

Example 5.1 Cellular System Capacity without freq reuse ie cluster

Consider a cellular system in which there are a total of 1001 radio channels available for handling traffic. Suppose the area of a cell is 6 km^2 and the area of the entire system is 2100 km^2.

a. Calculate the system capacity if the cluster size is 7.
b. How many times would the cluster of size 4 have to be replicated in order to approximately cover the entire cellular area?
c. Calculate the system capacity if the cluster size is 4.
d. Does decreasing the cluster size increase the system capacity? Explain.

Solution Given:

The total number of available channels $K = 1001$
Cluster size $N = 7$

Area of cell $A_{cell} = 6$ km^2
Area of cellular system $A_{sys} = 2100$ km^2

a. Since the number of channels per cell is $J = K/N$, then $J = 1001/7 = 143$ channels/cell. The coverage area of a cluster is

$$A_{cluster} = N \times A_{cell} = 7 \times 6 = 42 \text{ km}^2.$$

The number of times that the cluster has to be replicated to cover the entire cellular system is $M = A_{sys}/A_{cluster} = 2100/42 = 50$. Therefore,

$$C = MJN = 50 \times 143 \times 7 = 50{,}050 \text{ channels}.$$

b. For $N = 4$, $A_{cluster} = 4 \times 6 = 24$ km^2. Therefore,

$$M = A_{sys}/A_{cluster} = 2100/24 = 87.5 \simeq 87.$$

c. With $N = 4$, $J = 1001/4 \approx 250$ channels/cell. The system capacity is then

$$C = 87 \times 250 \times 4 = 87{,}000 \text{ channels}.$$

d. From (*a*) and (*c*), it is seen that a decrease in N from 7 to 4 is accompanied by an increase in M from 50 to 87, and the system capacity is increased from 50,050 channels to 87,000 channels. Therefore, decreasing the cluster size does increase the system capacity.

5.3.2 Cellular Layout for Frequency Reuse

As mentioned earlier, the discussion of cellular communications in this text is based on a two-dimensional chaining of hexagonal cells. The rule to find the nearest cochannel neighbor of a particular cell is as follows.

Rule for Determining the Nearest Cochannel Neighbors. The following two-step rule can be used to determine the location of the nearest cochannel cell:

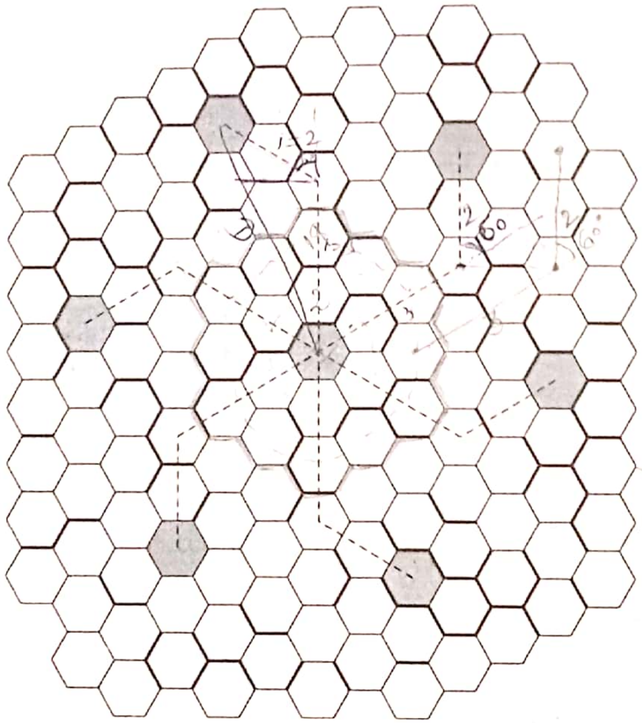
Step 1: Move i cells along any chain of hexagons;
Step 2: Turn 60 degrees counterclockwise and move j cells.

The method of locating cochannel cells in a cellular system using the preceding rule is shown in Figure 5.1 for $i = 3$ and $j = 2$, where the cochannel cells are the shaded cells.

The cluster concept and frequency reuse in a cellular network are illustrated in Figure 5.2, where cells with the same number use the same set of frequencies. These are cochannel cells that must be separated by a distance such that the cochannel interference is below a prescribed QoS threshold. The parameters i and j measure the number of nearest neighbors between cochannel cells; the cluster size, N, is related to i and j by the equation

$$N = i^2 + ij + j^2. \quad [9+6+4=19] \tag{5.3.3}$$

For example, in Figure 5.2(b), we have $i = 1$ and $j = 2$, so that $N = 7$. With a cluster size $N = 7$, the frequency reuse factor is seven since each cell contains one-seventh of the total number of available channels.



i=3
j=2

5

Figure 5.1 Locating cochannel cells in a cellular system.

Advantages of Cellular Systems. The advantages of operating in a cellular arrangement include:

a. the use of low power transmitter and,
b. an allowance for frequency reuse.

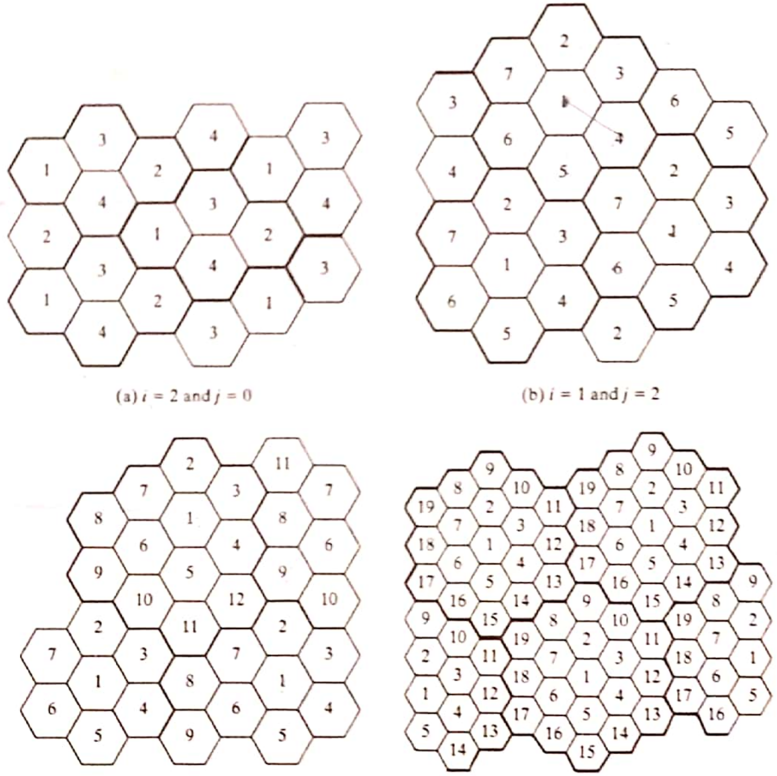
Frequency reuse needs to be structured so that cochannel interference is kept at an acceptable level. As the distance between cochannel cells increases, cochannel interference will decrease. If the cell size is fixed, the average signal-to-cochannel interference ratio will be independent of the transmitted power of each cell (discussed in Section 5.4). The distance between any two cochannel cells can be examined by making use of the geometry of hexagonal cells.

Example 5.2 Number of Frequency Channels

Consider a cellular system with a total bandwidth of 30 MHz which uses two 25 kHz simplex channels to provide full duplex voice and control channels. Assuming that the system uses a nine-cell reuse pattern and 1 MHz of the total bandwidth is allocated for control channels,

N = 9

from 30 MHz



(a) $i = 2$ and $j = 0$

(b) $i = 1$ and $j = 2$

(c) $i = 2$ and $j = 2$

(d) $i = 2$ and $j = 3$

Figure 5.2 Cell clusters.

a. calculate the total available channels, (control + voice)
b. determine the number of control channels,
c. determine the number of voice channels per cell, and
d. determine an equitable distribution of control channels and voice channels in each cell.

Solution Given:

Total bandwidth = 30 MHz
Channel bandwidth = 25 kHz × 2 = 50 kHz/duplex channel

$K = \frac{30000}{50} = 600$

a. The total number of available channels = $\frac{30000}{50} = 600$.
b. The number of control channels = $\frac{1000}{50} = 20$.

$\frac{580}{9}$ = 600 - 20

c. The number of voice channels per cell $= \frac{600-20}{9} \simeq 64$.

d. Since only a maximum of 20 channels can be used as control channels, for $N = 9$, one way to allocate is 7 cells with 2 control channels and 64 voice channels each, and 2 cells with 3 control channels and 66 voice channels each.

Note that the channel allocation performed in part (*d*) is not unique.

5.3.3 Geometry of Hexagonal Cells

The geometry of an array of hexagonal cells is depicted in Figure 5.3, where R is the radius of the hexagonal cell (from the center to a vertex). A hexagon has exactly six equidistant neighbors. As can be seen from Figure 5.3, in a cellular array the lines joining the centers of any cell and each of its neighbors are separated by multiples of 60 degrees. Note that in Figure 5.3 the 60° angle is bounded by the vertical line and the 30° line, both of which join centers of hexagonal cells.

The distance between the nearest cochannel cells in a hexagonal area can be calculated from the geometry shown in Figure 5.3. For notational convenience, the cell under consideration will be referred to as the candidate cell. The distance between the centers of two adjacent hexagonal cells is $\sqrt{3}R$. Let D_{norm} be the distance from the center of the candidate cell to the center of a

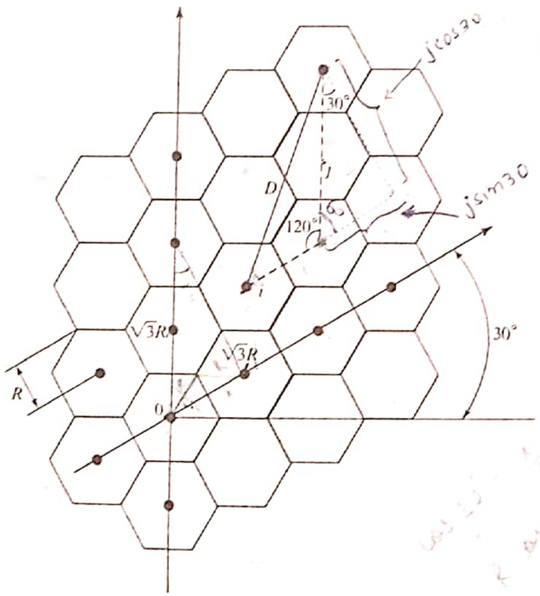


Figure 5.3 Distance between nearest cochannel cells.

No. of co-channel cells in 1st tier = 6
" " " kth " = 6K
Radius of 1st tier = D
" " kth tier = kD

nearest cochannel cell, normalized with respect to the distance between the centers of two adjacent cells, $\sqrt{3}R$. Note that the normalized distance between two adjacent cells (either with $i = 1$ and $j = 0$ or with $i = 0$ and $j = 1$) is unity. Let D be the actual distance between the centers of two adjacent cochannel cells. D is a function of D_{norm} and R.

From the geometry shown in Figure 5.3 we readily have

$$D^2_{norm} = j^2\cos^2(30^\circ) + (i + j\sin(30^\circ))^2$$
$$= i^2 + j^2 + ij. = N \quad (5.3.4)$$

This is for $i = 1, j = 2$; try for other values

From Eqs. (5.3.4) and (5.3.3),

$$D_{norm} = \sqrt{N}.$$

With the actual distance $\sqrt{3}R$ between the centers of two adjacent hexagonal cells, the actual distance between the center of the candidate cell and the center of a nearest cochannel cell is

$$D = D_{norm} \times \sqrt{3}R = \sqrt{3N}R. \quad (5.3.5)$$

For hexagonal cells, there are six nearest cochannel neighbors to each cell. Cochannel cells are located in tiers. In general, a candidate cell is surrounded by $6k$ cells in tier k. For cells with the same size, the cochannel cells in each tier lie on the boundary of the hexagon that chains all the cochannel cells in that tier. As D is the radius between two nearest cochannel cells, the radius of the hexagon chaining the cochannel cells in the kth tier is given by kD. For the frequency reuse pattern with $i = 2$ and $j = 1$ so that $N = 7$, the first two tiers of cochannel cells are illustrated in Figure 5.4. It can be readily observed from Figure 5.4 that the radius of the first tier is D and the radius of the second tier is $2D$.

Example 5.3 Number of Cells in a Cluster

Verify that the cell cluster size is $N = i^2 + ij + j^2$, where i and j are the integer parameters determining the cochannel cells as illustrated in Figure 5.1.

Solution A candidate cell has 6 nearest cochannel cells. By joining the centers of the 6 nearest neighboring cochannel cells, we form a large hexagon, as shown in Figure 5.5. This large hexagon has radius equal to D, which is also the cochannel cell separation. With the cell radius R, from Eq. (5.3.4), we have

$$D = \sqrt{3}R D_{norm} = \sqrt{3(i^2 + ij + j^2)}R.$$

In general, the area of a hexagon is proportional to the square of its radius. Let $\beta(\approx 2.598)$ be the proportional constant. Then the area of the large hexagon with radius D is

$$A_{large} = \beta D^2 = \beta[3(i^2 + ij + j^2)R^2] \qquad \beta = \frac{3\sqrt{3}}{2}$$

and the area of a cell (the small hexagon) with radius R is

$$A_{small} = \beta R^2.$$

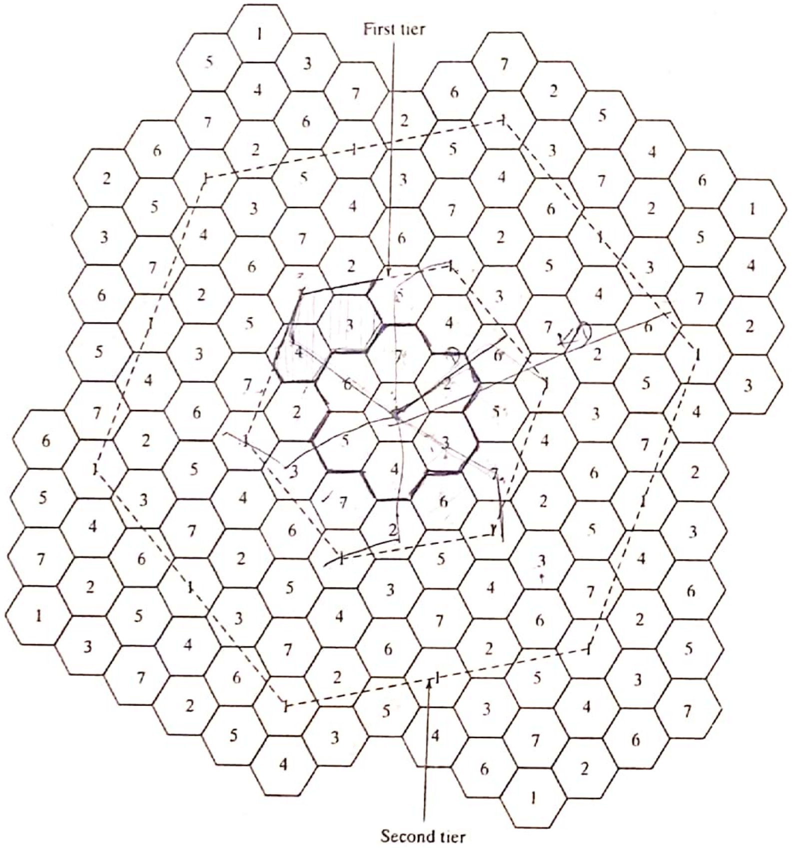


Figure 5.4 Two tiered interfering cells with $N = 7$.

The number of cells in the large hexagon is then

$$\frac{A_{\text{large}}}{A_{\text{small}}} = 3(i^2 + ij + j^2). \tag{5.3.6}$$

On the other hand, from the geometry (e.g., Figure 5.4) it can be seen that, in general, the large hexagon encloses the center cluster of N cells plus 1/3 the number of the cells associated with six other peripheral large hexagons. Hence, the total number of cells enclosed by the large hexagon is

$$N + 6\left(\frac{1}{3}N\right) = 3N. \tag{5.3.7}$$

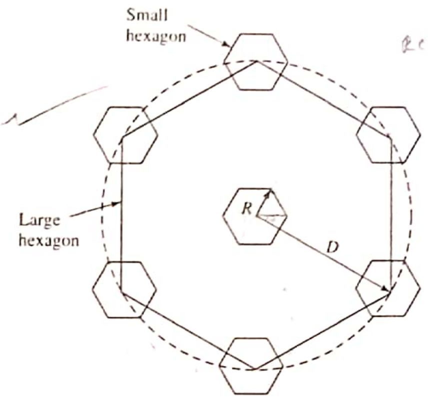


Figure 5.5 First tier cochannel interfering cells.

From Eqs. (5.3.6) and (5.3.7), we have

$$N = i^2 + ij + j^2.$$

5.3.4 Frequency Reuse Ratio

The frequency reuse ratio, q, is defined as

$$q \triangleq \frac{D}{R}. \quad = \frac{\sqrt{3N}\,R}{R} = \sqrt{3N} \tag{5.3.8}$$

Because frequency reuse leads to cochannel cell operation, q is also referred to as the cochannel reuse ratio.

Table 5.1 Frequency Reuse Ratio and Cluster Size

Frequency Reuse Pattern (i, j)	Cluster Size N	Frequency Reuse Ratio q
(1, 1)	3	3.00
(2, 0)	4	3.46
(2, 1)	7	4.58
(3, 0)	9	5.20
(2, 2)	12	6.00
(3, 1)	13	6.24
(3, 2)	19	7.55
(4, 1)	21	7.94
(3, 3)	27	9.00
(4, 2)	28	9.17
(4, 3)	37	10.54

Substituting Eq. (5.3.5) into Eq. (5.3.8), the frequency reuse ratio q is related to the cluster size (or frequency reuse factor) N by

$$q = \sqrt{3N}. \tag{5.3.9}$$

Since q increases with N and since a smaller value of N has the effect of increasing the capacity of the cellular system (see Subsection 5.3.1) and, at the same time, increasing cochannel interference, the choice of q or N has to be made such that the signal-to-cochannel interference ratio is at an acceptable level. Several frequency reuse patterns, together with the corresponding cluster sizes and frequency reuse ratios, are tabulated in Table 5.1 for easy reference.

5.4 COCHANNEL AND ADJACENT CHANNEL INTERFERENCE

Consider the performance of the candidate cell in a cellular array. A given base station provides the capacity to handle the services of many mobile users. The signal received from a target user at the cell-site (base station) receiver is subject to interference from transmissions of other mobiles in the same cell, background noise, and interference from transmissions by mobiles in neighboring cells. Under the assumption of a proper separation between uplink transmissions and downlink transmissions, either in the time domain (i.e., time division duplexing) or in the frequency domain (i.e., frequency division duplexing), interference from transmissions in the other link can be neglected. Interference from other mobiles at the cell-site receiver in the same cell is *intracell interference*. Interference from other cells is *intercell interference*. Intercell interference in the downlink that affects the reception at the individual mobile hosts may be more of a problem than uplink interference at the cell-site receiver. The reason for this can be attributed to the fact that the cell-site receiver may be more sophisticated than the receivers of the individual mobile users.

If the different cells in the entire cellular system were to use different sets of frequencies, intercell interference would be kept at a minimum. However, the system capacity would be limited; deployment of frequency reuse (see Subsection 5.3.1) is necessary to enlarge the system capacity. On the other hand, frequency reuse will introduce cochannel interference from cells using the same set of frequencies. Therefore, frequency reuse needs to be carefully planned so that cochannel interference is kept at an acceptable level.

5.4.1 Cochannel Interference

As mentioned in Chapter 1, wireless channels are interference limited. Except for the cochannel cells, other neighboring cells will operate at frequencies different from those of the candidate cell so that interference from noncochannel cells will be minimal. Intercell interference is thus dominated by cochannel interference. It is thus of interest to assess system performance, taking into consideration interference from cochannel cells. For simplicity in the following analysis, we will consider only the average channel quality as a function of the distance dependent path loss, without going into details of channel statistics due to propagation shadowing and multipath fading.

We will use the symbols S and I to denote respectively the power of the desired signal and the power of the cochannel interference at the output of the receiver demodulator. Let N_I be the number of cochannel interfering cells and I_i be the interference power caused by transmissions

from the ith interfering cochannel cell base station. The signal-to-cochannel interference ratio (S/I) at the desired mobile receiver is given by

$$\frac{S}{I} = \frac{S}{\sum_{i=1}^{N_I} I_i}.$$

The average received signal strength at any point decays as a power law of the distance between the transmitter and the receiver, as discussed in Section 2.4.

Let D_i be the distance between the ith interferer and the mobile. The received interference, I_i, at a given mobile due to the ith interfering cell is proportional to $(D_i)^{-\kappa}$, where κ is the path loss exponent. The path loss exponent κ is normally determined by measurement. In many cases it is in the range $2 \le \kappa \le 5$.

In addition to cochannel interference, there is always the inherent background noise. However, in an interference dominated environment, we may neglect the background noise. It is noted that the desired received signal power S is proportional to $r^{-\kappa}$, where r is the distance between the mobile and the serving base station. When the transmit powers from all base stations are equal, and the path loss exponent is the same throughout the geographical coverage area, the cochannel interference from the ith cochannel cell, I_i, for all i, depends on D_i and κ only. The S/I at a typical mobile receiver can be approximated by

$$\frac{S}{I} = \frac{r^{-\kappa}}{\sum_{i=1}^{N_I} D_i^{-\kappa}}. \tag{5.4.1}$$

The degree of cochannel interference is a function of the location of the mobile within the cell of the serving base station. When the mobile is located at the cell boundary (i.e., $r = R$), the worst case cochannel interference occurs as the power of the desired signal is minimum. With hexagon shaped cellular systems, there are always six cochannel interfering cells in the first tier. If we neglect cochannel interference from the second and other higher tiers, this means that $N_I = 6$. In the case that $r = R$ and using $D_i \approx D$ for $i = 1, 2, \ldots, N_I$,

worst case condition — at same assumed

$$\frac{S}{I} = \frac{(D/R)^{\kappa}}{N_I} = \frac{q^{\kappa}}{N_I} = \frac{(\sqrt{3N})^{\kappa}}{N_I}. \tag{5.4.2}$$

Thus, the frequency reuse ratio can be expressed as

$$q = \left(N_I \times \frac{S}{I}\right)^{1/\kappa} = \left(6 \times \frac{S}{I}\right)^{1/\kappa} \tag{5.4.3}$$

For the U.S. AMPS analog FM system, a value of $S/I = 18$ dB or greater is acceptable. With a path loss exponent of $\kappa = 4$, the frequency reuse ratio q is determined as

$$q = (6 \times 10^{1.8})^{1/4} = (6 \times 63.1)^{0.25} \simeq 4.41.$$

Therefore, the cluster size N should be

$$N = q^2/3 = 6.49 \simeq 7.$$

Example 5.4 S/I Ratio versus Cluster Size

Suppose the acceptable signal-to-cochannel interference ratio in a certain cellular communications situation is $S/I = 20$ dB or 100. Also, from measurements, it is determined that $\kappa = 4$. What is the minimum cluster size?

Solution The frequency reuse ratio can be calculated, using Eq. (5.4.3), as

$$q = (6 \times 100)^{1/4} = 4.9492.$$

Then, from Eq. (5.3.9), the cluster size is given by

$$N = q^2/3 = 8.165 \simeq 9.$$

In this case, a 9-reuse pattern is needed for an S/I ratio of at least 20 dB. Since

$$q = D/R \quad \text{or} \quad D = qR,$$

D can be determined, given the cell radius R, and vice versa. Note that if N is less than 9, the S/I value would be below the acceptable level of 20 dB.

Consider that the mobile is at the cell boundary, where it experiences worst case cochannel interference on the forward channel. If we use a better approximation of the distance between the mobile and the first tier interfering base stations as illustrated in Figure 5.6, then from Eq. (5.4.1) the S/I ratio can be expressed as (see Problem 5-3)

$$\frac{S}{I} = \frac{R^{-\kappa}}{2(D-R)^{-\kappa} + 2D^{-\kappa} + 2(D+R)^{-\kappa}}. \tag{5.4.4}$$

Denom

Recall that $D/R = q$. With a path loss exponent of $\kappa = 4$, Eq. (5.4.4) can be written as

$$\frac{S}{I} = \frac{1}{2(q-1)^{-4} + 2q^{-4} + 2(q+1)^{-4}}. \tag{5.4.5}$$

N: f. reuse factor
q: f. ratio

Example 5.5 Worst Case Cochannel Interference

Consider a cellular system that requires an S/I ratio of 18 dB.

a. For a frequency reuse factor of 7, calculate the worst-case signal-to-cochannel interference ratio.

b. Is a frequency reuse factor of 7 acceptable in terms of cochannel interference? If not, what would be a better choice of frequency reuse factor?

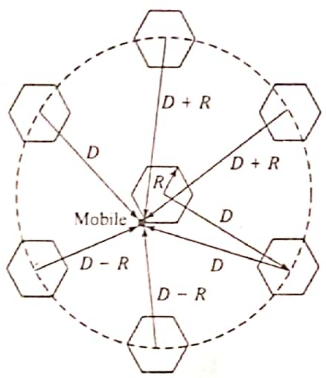


Figure 5.6 Worst case cochannel interference scenario for $N = 7$.

Solution

a. For $N = 7$, the frequency reuse ratio is $q = \sqrt{3N} = 4.6$. Assuming a path loss exponent of $\kappa = 4$, then, from Eq. (5.4.5), the worst-case signal-to-cochannel interference ratio is

$$\frac{S}{I} = 54.3 \quad \text{or} \quad 17.3 \text{ dB}.$$

$= (\log_{10} 54.3 \times 10)$

b. The value of S/I for a 7 frequency reuse factor is below the acceptable level of 18 dB. To increase S/I, we need to decrease I. This can be achieved by increasing the frequency reuse factor, N. For $N = 9$, we have $q = \sqrt{3N} = 5.20$, so that $S/I = 95.66$ or 19.8 dB. This value of S/I is above the acceptable level of 18 dB. Therefore, a frequency reuse factor of 9 is a suitable choice.

Although an increase of frequency reuse factor from 7 to 9 in Example 5.5 yields an acceptable S/I level, this increase in N is accompanied by a decrease in system capacity, since a 9-cell reuse offers a spectrum utilization of 1/9 within each cell, whereas a 7-cell reuse offers a spectrum utilization of 1/7. The capacity reduction may not be tolerable. From an operational point of view, catering to the worst-case scenario, which rarely occurs, may not be desirable. The designer may want to seek the best tradeoff by accepting the fact that, with a small but nonzero probability, the worst-case scenario may occur and, hence, degrade the performance to a level below specification during some interval of the holding time of a call.

From the preceding analysis, it is clear that cochannel interference determines link performance which, in turn, dictates the frequency reuse plan and the overall capacity of cellular systems.

5.4.2 Adjacent Channel Interference

Adjacent channel interference (ACI) results from signals which are adjacent in frequency to the desired signal. ACI is mainly due to imperfect receiver filters which allow nearby frequencies to leak into the passband. Consider the uplink transmissions from two mobile users using adjacent channels, one very close to the base station and the other very close to the cell boundary. Without proper transmission power control, the received power from the mobile close to the base station is much larger than that from the other mobile far away. This near-far effect can significantly increase the ACI from the strong received signal to the weak received signal. To reduce ACI, we should: (*a*) use modulation schemes which have low out-of-band radiation (e.g., MSK is better than QPSK and GMSK is better than MSK); (*b*) carefully design the bandpass filter at the receiver front end; (*c*) use proper channel interleaving by assigning adjacent channels to different cells; (*d*) avoid using adjacent channels in adjacent cells to further reduce ACI if the cell cluster size is large enough; and (*e*) separate the uplink and downlink properly by TDD or FDD.

5.5 CALL BLOCKING AND DELAY AT THE CELL-SITE

Signal-to-interference ratio, which determines the transmission bit error rate, is a QoS factor at the output of the cell-site receiver. From the user's perspective, quality of service is more than an acceptable transmission accuracy. In fact, there are two crucial questions: (*a*) How successfully can a new user get a connection established? (*b*) After connection establishment, how successfully will the connection be maintained as the user moves from one cell to another? The first question refers to the admission of new calls, while the second question refers to the admission of handoff calls. The performance measure is the probability that a call (new or handoff) is blocked.

To formulate the probability of call blocking, consider a radio cell which has been allocated J channels. Assume a large population size of mobile users in the cell. Suppose during the connection time of a call, each user occupies one channel. If the number of active users during any epoch equals J, all available channels will be occupied. Then, with probability 1, a call request will be denied (i.e., blocked). If the number of ongoing calls is fewer than J, a call will be blocked with probability smaller than 1. This is equivalent to the condition that the trunk traffic load in Erlangs is less than J, the number of available channels. One Erlang represents the amount of traffic load carried by a channel that is completely occupied, such as one call-hour per hour. If a channel is busy for 30 minutes during a one hour period, then the channel is said to carry 0.5 Erlangs of traffic. Offered traffic refers to the amount of traffic sent by the users, while carried traffic refers to the amount of traffic served.

To obtain an expression for call blocking probability, we model the cell-site as a bufferless system. With no buffer, blocked calls are lost. Assume that (*a*) there are $L(\gg 1)$ users in the system; (*b*) the aggregate arrival traffic is Poisson distributed with rate λ; (*c*) the duration of a call is exponentially distributed with parameter μ_1; and (*d*) the residence time of each user in a cell is exponentially distributed with parameter μ_2. As an exponential random variable is memoryless, the channel holding time is the minimum of the call duration and the cell residence time, which is also exponentially distributed with parameter $\mu = \mu_1 + \mu_2$. That is, the mean channel holding time of the call is μ^{-1}, corresponding to a mean service rate of μ for the call. Since the channel holding time is exponentially distributed, the service time of each of the servers is also exponentially distributed. With Poisson arrivals and exponential service times, the underlying queueing process is Markovian.

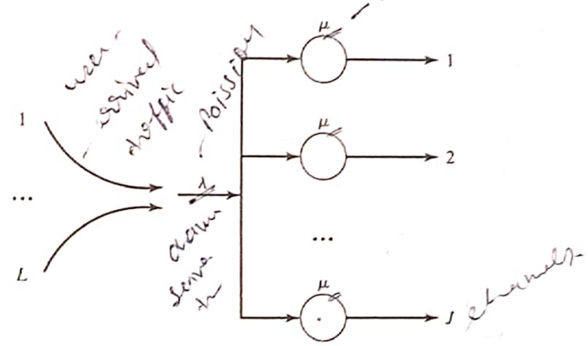


Figure 5.7 *J*-server bufferless model.

The cell-site receiver can be modeled as a J-server system, in which each server serves traffic at a mean rate μ. The J-server system for a population of size L and aggregate arrival rate λ is illustrated in Figure 5.7. The state transition rate diagram of the multiserver system with Poisson arrival and exponential service time is shown in Figure 5.8. As shown in the diagram, when the system is in state j, for $j = 0, 1, \cdots, J$, there are j ongoing calls, and j servers, each with mean service rate μ, are being engaged. When the system is in state $j = J$, all J servers are engaged and new requests will be blocked. Note that, if $L \leq J$, the number of users is fewer than the number of servers, there will be no blocking. But this would not be the best way to utilize the available resources.

The traffic intensity is defined as $\rho \triangleq \lambda/\mu$. In terms of the traffic intensity ρ, the probability of blocking is given by the *Erlang loss formula*

Erlang B

$$P(\text{blocking}) = \frac{\rho^J/J!}{\sum_{j=0}^{J} \rho^j/j!}. \tag{5.5.1}$$

Equation (5.5.1) is also known as the Erlang-B loss formula. The call blocking probability is considered to be the grade of service (GoS) parameter for the Erlang-B system. The traffic intensity in Erlangs for a prescribed GoS is tabulated in Appendix F.

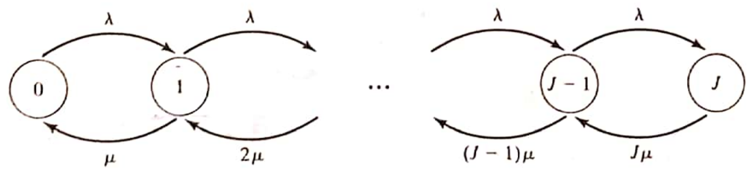


Figure 5.8 State transition rate diagram of a *J*-server system.

In the bufferless J-server system, the delay for any request is zero. The request is either granted or blocked. If some queueing delay can be tolerated, the J-server system in Figure 5.7 can also have a common buffer for queueing requests when all J servers are fully engaged. Upon arrival, a request (new or handoff), finding all J servers are busy, can join the queue. When a server becomes available, the head of the queue will then receive service. In this case, the queueing delay would be nonzero. The reason for queueing requests is to decrease the call blocking probability. The probability of queueing is given by [31, 158]

$$P(\text{queueing}) = \frac{\dfrac{J\rho^J}{J!(J-\rho)}}{\left[\dfrac{J\rho^J}{J!(J-\rho)}\right] + \displaystyle\sum_{j=0}^{J-1}\left(\frac{\rho^j}{j!}\right)}. \tag{5.5.2}$$

Queueing gives rise to delay. The probability of nonzero delay is given by

$$P(\text{delay} > 0) = \frac{\rho^J}{\rho^J + J!\left(1 - \frac{\rho}{J}\right)\displaystyle\sum_{j=0}^{J-1}\frac{\rho^j}{j!}}. \tag{5.5.3}$$

Erlang-C

Equations (5.5.2) and (5.5.3) are basically the same. Both equations are known as the Erlang-C formula. The probability of nonzero delay that can be tolerated is also considered to be the GoS parameter for the Erlang-C system. The offered loads in Erlangs corresponding to the number of channels available for different GoS, given by the Erlang-C formula, are tabulated in Appendix F.

A given request for network access can only tolerate a finite amount of delay. It is of interest to know the probability that the delay exceeds a given delay threshold. Let t be the delay threshold. Given that the mean channel holding time of a call is μ^{-1}, the probability that the delay exceeds t is given by

$$\begin{aligned} P(\text{delay} > t) &= P(\text{delay} > 0) \times P(\text{delay} > t \mid \text{delay} > 0) \\ &= P(\text{delay} > 0)\exp[-(J-\rho)\mu t]. \end{aligned} \tag{5.5.4}$$

The average delay, $\bar{D}$, for all calls in the queueing system is given by

$$\bar{D} = P(\text{delay} > 0) \times \frac{1}{\mu(J-\rho)}. \tag{5.5.5}$$

In the Erlang-B or Erlang-C system, a channel is allocated to a user on a call by call basis, thus allowing the system to serve a number of subscribers much greater than the number of available channels. The system performance can be described by *trunking efficiency*. Trunking efficiency is defined as the carried traffic intensity in Erlangs per channel, which is a value between zero and one, and is a function of the number of channels per cell and the specified GoS parameters, such as call blocking rate in the Erlang-B system and average queueing delay in the Erlang-C system.

For a comparison of the Erlang-B and Erlang-C systems, given (*a*) the same number of channels per cell and (*b*) the call blocking rate in the Erlang-B system being the same as the probability of nonzero queueing delay in the Erlang-C system, the offered traffic intensity of the Erlang-B system is larger than that of the Erlang-C system. For example, with $J = 20$, $P(\text{blocking}) = P(\text{queueing}) = 0.1$, from Appendix F, the offered traffic intensity is 17.613 Erlangs in the Erlang-B system and is 14.116 Erlangs in the Erlang-C system. This is because, in the Erlang-B system, the blocked calls are cleared and are not served by the system; while in the Erlang-C system, the new calls are buffered (not dropped) when all the channels are busy and are then served when a channel becomes available. As a result, some offered traffic (the blocked calls) in the Erlang-B system is not carried by the system, but all the offered traffic in the Erlang-C system is carried by the system, under the assumption that the incoming calls can tolerate any queueing delay.

Example 5.6 Traffic Load and Call Blocking Probability

Consider a cellular system with 416 radio channels available for handling traffic. Suppose 21 of these channels are designated as control channels. Let the average channel holding time of a call be 3 minutes, the blocking probability during busy hours be 2%, and the frequency reuse factor be 9.

a. Determine the number of calls per cell per hour.
b. Determine the signal-to-cochannel interference ratio, S/I, in dB.

P = 2%
N = 9

Solution

a. In a cellular system with frequency reuse, all the available radio channels are allocated to handle services in a single cluster of cells. Therefore, the number of voice channels in a cluster is $416 - 21 = 395$. With a frequency reuse factor of 9, the effective number of voice channels per cell $= \frac{395}{9} \simeq 44$. With 44 available channels and a 2% blocking probability, the traffic load is (from the Erlang-B table in Appendix F) 34.683 Erlangs.

Page 320

$$\text{The number of calls per cell per unit time} = \frac{\text{number of calls per cell}}{\text{average holding time of a call}}.$$

Therefore,

$$\begin{aligned} \text{the number of calls per cell per unit time} &= \frac{34.683}{3} \text{ calls per cell per minute} \\ &= \frac{34.683}{3} \times 60 \approx 693 \text{ calls per cell per hour.} \end{aligned}$$

b. The frequency reuse ratio $q = \sqrt{3N} = \sqrt{3 \times 9} = 5.1962$. For $\kappa = 4$, the signal-to-cochannel interference ratio is

$$S/I = \frac{1}{6} \times q^{\kappa} = \frac{1}{6} \times (5.1962)^4 = 121.5 \quad \text{or} \quad 20.845 \text{ dB}.$$

$\log 121.5$

$q = (6 \times \frac{S}{I})^{\frac{1}{4}}$ $\quad \frac{1}{6} \times q^K = \frac{S}{I}$

5.6 OTHER MECHANISMS FOR CAPACITY INCREASE

As discussed in Subsection 5.3.1, the capacity of a cellular system can be enlarged through frequency reuse. The capacity can also be improved based on cellular layout and antenna design using

a. cell splitting, and
b. antenna sectoring.

5.6.1 Cell Splitting

One way to perform cell splitting, as illustrated in Figure 5.9, is to subdivide a congested cell into smaller cells, each with its own base station and a corresponding reduction in antenna height and transmit power. With more cells, there will be more clusters in the same coverage area. This is equivalent to replicating a cell cluster more times. In the context of Subsection 5.3.1, the replication factor M is increased. Hence, cell splitting increases the capacity of a cellular system since it increases the number of times that channels are reused. In Figure 5.9, the central area is assumed to be saturated with traffic (i.e., the call blocking probability in the area exceeds an acceptance level). The original large cell with radius R in the center is split into the medium cells with radius $R/2$ and the medium cell in the center is further split into the small cells with radius $R/4$. The cell splitting reduces the call blocking probability in the area, and increases the frequency with which mobiles hand off from cell to cell.

Let d be the distance between the transmitter and the receiver, and d_0 be the distance from the transmitter to a close-in reference point. Let P_0 be the power received at the close-in reference point. From Section 2.4, the average received power, P_r, is proportional to P_0, and is given by

$$P_r = P_0 \left(\frac{d}{d_0}\right)^{-\kappa} \tag{5.6.1}$$

where $d \geq d_0$ and κ, as defined earlier, is the path loss exponent. Taking the logarithm, Eq. (5.6.1) can be expressed as

$$P_{r(\text{dBW})} = P_{0(\text{dBW})} - 10\kappa \log_{10}\left(\frac{d}{d_0}\right), \quad d \geq d_0. \tag{5.6.2}$$

Let P_{t1} and P_{t2} be the transmit power of the large cell base station and the medium cell base station, respectively. The received power, P_r at the large (old) cell boundary is proportional to $P_{t1}R^{-\kappa}$, and P_r at the medium (new) cell boundary is proportional to $P_{t2}(R/2)^{-\kappa}$. On the basis of equal received power, we have

$$P_{t1}R^{-\kappa} = P_{t2}(R/2)^{-\kappa} \quad \text{or} \quad P_{t1}/P_{t2} = 2^{\kappa}.$$

Taking the logarithm, the above can be expressed as

$$10\log_{10}\left(\frac{P_{t1}}{P_{t2}}\right) = 10\kappa \log_{10} 2 \simeq 3\kappa \text{ dB}.$$

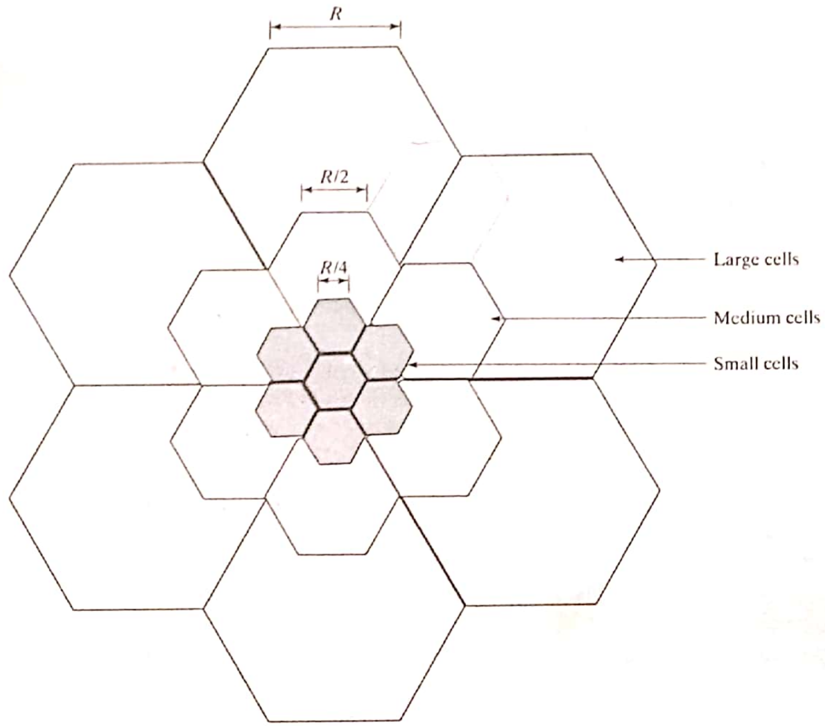


Figure 5.9 Illustration of cell splitting from radius R to $R/2$ and to $R/4$.

For $\kappa = 4$, $P_{t1}/P_{t2} = 12$ dB. Thus, with cell splitting, where the radius of the new cell is one-half of that of the old cell, we can achieve a 12 dB reduction in the transmit power.

How cell splitting increases system capacity is best illustrated by an example.

Example 5.7 Cell Splitting and Capacity Increase

Consider the cellular system shown in Figure 5.10, where the original cells have radius R. These cells are split into smaller cells, each with radius $R/2$. Suppose each base station is allocated 60 channels regardless of the cell size. There are obviously more small cells than original cells in the same coverage area. Since the number of channels allocated in a small cell is the same as that in a large cell, it is obvious that cell splitting increases the number of channels within the same coverage area. Find the number of channels contained in a 3×3 km^2 area centered around (small) cell "A" for the following cases:.

a. without cell splitting (i.e., just the original large cells), and
b. with cell splitting (i.e., using the small cells (microcells)).

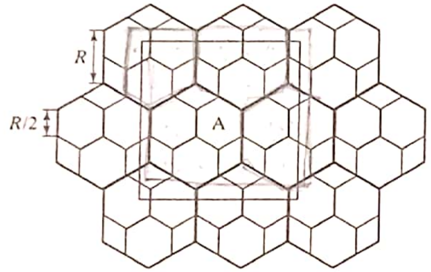


Figure 5.10 A cell splitting example with $R = 1$ km.

Solution To cover an area of 3×3 km^2 centered around cell A, we need to cover 1.5 km to the right, left, top and bottom of base station A, as shown in Figure 5.10. From Figure 5.10, it is observed that the 3×3 km^2 square centered around cell "A" contains more small cells than large cells. However, because of edge effect, the number of either type of cells contained within the square can only be an estimate. A reasonable estimate is that there are approximately 4 large cells (from visual observation). With a 1/2 radius split, the number of small cells within the square would be at most

$$\left(\frac{R}{R/2}\right)^2 \times \text{ the number of large cells } = 4 \times 4 = 16 \text{ small cells.}$$

The preceding calculation would be correct if the enclosed area is infinitely large. With a finite area, it is necessary to take edge effect into consideration, so that the number of small cells contained within the 3×3 km^2 area would be less than 16. A reasonable estimate would be 15 small cells.

a. With an estimate of 4 base stations contained within the square, the number of channels equals $4 \times 60 = 240$.
b. With an estimate of 15 small cells, the number of channels contained in the square, with cell splitting, is $15 \times 60 = 900$ channels, which is 3.75 times the channel number in the nonsplitting case.

Note that the upper bound is a 4-fold increase.

5.6.2 Directional Antennas (Sectoring)

In the basic form, antennas are omnidirectional. Directional antennas can increase the system capacity relative to that of omnidirectional antennas. From Eq. (5.4.1) the worst-case S/I is given by

$$\frac{S}{I} = \frac{R^{-\kappa}}{\sum_{i=1}^{N_I} (D_i)^{-\kappa}},$$

where the value of N_I depends on the form of antenna used. In the omnidirectional case, $N_I = 6$ for the first tier of cochannel cells. Assuming $D_i \approx D$, $i = 1, 2, \ldots, N_I$,

$$\left(\frac{S}{I}\right)_{omni} = \frac{1}{6} \times q^{\kappa},$$

where $q = D/R$. In terms of capacity increase through sectorization, the omnidirectional case can be used as the benchmark.

With hexagonal cells, as illustrated in Figure 5.11, sectorization can be done in multiples of 60°. Assuming a 7-cell reuse, for the 3-sector case (with 120° in each sector) the number of interferers in the first tier is reduced from 6 to 2.

With $D_i \approx D$,

$$\left(\frac{S}{I}\right)_{omni} = \frac{1}{6} \times q^{\kappa} \quad \text{and} \quad \left(\frac{S}{I}\right)_{120^\circ} = \frac{1}{2} \times q^{\kappa}.$$

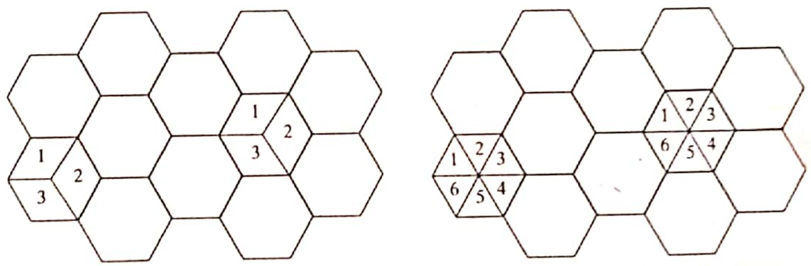
The increase in the signal-to-interference ratio is then

$$\frac{(S/I)_{120^\circ}}{(S/I)_{omni}} = 3.$$

That is, theoretically, the capacity increase from the omnidirectional antenna case is given by the number of sectors in each cell resulting from the use of directional antennas. Note that, within each cell, mobiles may need to hand off from sector to sector. However, the handoff process can be easily managed by the base station. If the total number of channels available to each cell needs to be partitioned for the sectors, the trunking efficiency for each cell is reduced from that without sectoring.

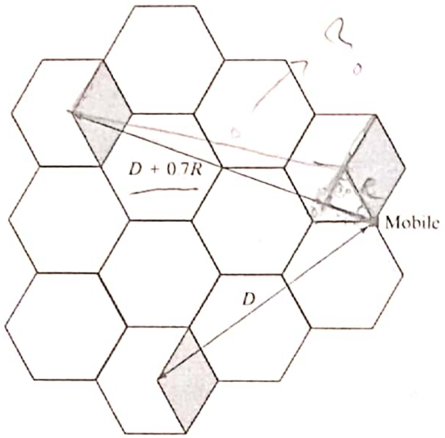
A worst-case scenario in a 120° sectorization is shown in Figure 5.12, where the mobile is located at the corner of the cell, R is the cell radius, and D is the distance between the adjacent cochannel cells. In the 3-sector case, the mobile experiences interference from one sector of each of the two interfering cells. With the distance approximation shown and a path loss exponent of $\kappa = 4$, we have

$$\left(\frac{S}{I}\right)_{120^\circ} = \frac{R^{-4}}{D^{-4} + (D + 0.7R)^{-4}} = \frac{1}{q^{-4} + (q + 0.7)^{-4}}. \tag{5.6.3}$$



(a) 3 sectors of 120° each (b) 6 sectors of 60° each

Figure 5.11 Antenna sectorization.



0.5
15
Sin 30 R
0.56 R

Figure 5.12 Worst-case scenario in 120° sectoring.

Example 5.8 Cochannel Interference with Sectoring

In Example 5.6, it is shown that, with a frequency reuse factor of 7, base stations using omnidirectional antennas cannot satisfy the 18 dB signal-to-cochannel interference ratio requirement. Determine whether the use of 120° sectoring and 7-cell frequency reuse would satisfy the 18 dB requirement.

Solution For a 7-cell reuse, we have $q = \sqrt{3 \times 7} = 4.6$. Substituting for q in Eq. (5.6.3), we get

$$\left(\frac{S}{I}\right)_{120^\circ} = 285 \quad \text{or} \quad 24.5 \text{ dB}.$$

Since this is greater than 18 dB, the 3-sector worst case for a 7-cell reuse is acceptable.

5.7 CHANNEL ASSIGNMENT STRATEGIES

There are essentially two channel assignment approaches: (*a*) fixed channel assignment, and (*b*) dynamic channel assignment.

Fixed Channel Assignment (FCA). In FCA, each cell is allocated a predetermined set of voice channels. Any call attempt within the cell can only be served by the unused channels in that particular cell. To improve utilization, a borrowing option may be considered. With the borrowing option, a cell is allowed to borrow channels from a neighboring cell if all of its own channels are already occupied and the neighboring cell has spare channels. Borrowing is normally supervised by the MSC.

As mentioned in Subsection 5.2.2, since handoff is to be performed by the MSC, the MSC has full knowledge of the capacity usage of the cluster of cells within its jurisdiction. Therefore, the MSC is the natural subsystem to oversee functions such as channel borrowing.

Dynamic Channel Assignment (DCA). In DCA, voice channels are not allocated to different cells on a permanent basis [33]. Each time a call request is made, the serving base station requests a channel from the MSC. The MSC determines (dynamically) the availability of a channel and executes its allocation procedure accordingly. The MSC only allocates a given frequency (radio channel) if that frequency (radio channel) is not presently in use in the cell, or any other cell which falls within the minimum restricted distance of frequency reuse to avoid cochannel interference.

Dynamic channel assignment reduces the likelihood of call blocking, which increases the trunking capacity of the system, since all available channels under the control of the MSC are accessible to all of the cells. Dynamic channel assignment strategies require the MSC to collect real-time data on channel occupancy, traffic distribution, and radio signal quality of all channels on a continuous basis. In any case, the MSC needs to do this data collection in order to manage handoff.

SUMMARY

Cellular communication is designed to enhance system capacity while allowing for the use of low power transmitters and frequency reuse. The cellular concept is to divide a large geographical area into many small geographical coverage areas, and to reuse the frequencies used in a given small area elsewhere. Since the geographical coverage areas are now smaller, a mobile will move out of one base station coverage area and into a neighboring area more frequently. Therefore, the tradeoff in attaining capacity expansion is the need to handle handoff when the mobile roams from cell to cell, in order to maintain continuous service of a connection. This chapter has examined the basic characteristics of cellular systems, including system capacity and frequency reuse properties. Handoff management issues will be considered in Chapters 7 and 8.

ENDNOTES

1. For the cellular concept, see the special issue of the *Bell System Technical Journal* [19] which includes papers by MacDonald [92] and Young [163]. For system aspects of cellular radio, see the paper by Steele, Whitehead and Wong [148].
2. The analysis on the cochannel interference (CCI) given in Section 5.4 considers the average performance versus the propagation distance. In practice, the interference changes randomly, depending on channel fading (due to multipath propagation) and shadowing (depending on the propagation terrain), power control mechanism, receiver structure, traffic load and distribution, etc., in addition to the propagation distance. As a result, the interference should be modeled as a random process as the mobile user moves. The main probabilistic performance measure is the probability of CCI (also called the outage probability), defined as the probability that the instantaneous signal-to-cochannel interference ratio is below a given threshold. The threshold

value depends on the link quality requirement. The book by Stüber [147] provides an in-depth analysis on CCI modeling, the probability of CCI, and related references such as [63, 50, 88, 120, 146, 161].

3. For the fundamentals of queueing theory and detailed analysis of the Erlang systems, see the books by Cooper [31] and by Wolff [158].
4. For cellular system layout, performance measures, and traffic load analysis, see the paper by Everitt [48], Chapter 7 of the book edited by Jakes [42], and Chapter 2 of the book by Rappaport [128].
5. For effects of cell sectorization on the spectrum efficiency of cellular radio systems, see the paper by Chan [26].
6. For dynamic channel assignment, see the papers by Cox and Reudnik [33, 34, 35].

PROBLEMS

P5-1 Consider a cellular system with hexagonal cells. The cell radius is R. The service area is partitioned into cell clusters. Frequency is reused from cell cluster to cell cluster. The geometric relation between adjacent cochannel cells can be described by the two nonnegative integers i and j.

a. With pictorial illustration, explain what i and j represent.
b. Determine the distance D between the centers of two adjacent cochannel cells.
c. Derive an expression for the number of cells, N, in each cell cluster.
d. What should be considered in choosing a value for N?

P5-2 Show that the frequency reuse factor for a cellular system is given by K/J, where J is the average number of channels per cell and K is the total number of channels available to the cellular network provider without frequency reuse. The cell cluster can be replicated M times to provide a total capacity of C channels. Discuss the changes in the value of C when you increase or decrease the frequency reuse factor while keeping K constant.

Pg-174

P5-3 Verify the distance approximations shown in Figure 5.6 and Eq. (5.4.4).

P5-4 In the radio cell layout, in addition to the hexagonal topology, a square or an equilateral triangle topology can also be used.

a. Given the same distance between the cell center and its farthest perimeter points, compare the cell coverage areas among the three regular polygons (hexagon, square, and triangle);
b. Discuss the advantages of using the hexagonal cell shape over the square and triangle cell shapes.

P5-5 A cellular system has a total of 500 duplex voice channels (without frequency reuse). The service area is divided into 150 cells. The required signal-to-cochannel interference ratio is 18 dB. Consider the path loss exponent κ equal to 3, 4, and 5, respectively. Based on Eqs. (5.3.9) and (5.4.3), determine

a. the cell cluster size;
b. the number of cell clusters in the service area; and
c. the maximum number of users in service at any instant.

Discuss effects of the path loss exponent on the frequency reuse and on the transmit power (when the cell size is fixed).

P5-6 Consider a cellular system with a total bandwidth of 30 MHz. Each full duplex voice or control channel uses two 25 kHz simplex channels. It is assumed that (1) the system uses a 9-cell reuse pattern and 0.75 MHz of the total bandwidth is allocated for control channels; (2) the system service area consists of 50 cells; (3) the call blocking probability bound is 2%, as given by the Erlang-B formula. If the offered traffic per user is 0.025 Erlangs, calculate

a. the traffic load of each cell and the trunking efficiency,
b. the total number of users in each cell and in the system, respectively,
c. the number of mobile users per channel in each cell and in the system, respectively, and
d. the maximum number of users in service at any instant in the system.

P5-7 Consider the cellular system in Example 5.1 with the cell cluster of size 7. Given that the traffic load per user is 0.03 Erlangs and the average number of calls per hour per user is 1.5, for an Erlang-C system with a probability of delaying a call being 5%, determine

a. the traffic load per cell,
b. the number of users per km^2 that can be supported by this system,
c. the mean duration of a call,
d. the probability that a delayed call will have to wait for more than 10 s, and 0.062
e. the probability that a call will be delayed for more than 10 s. 0.0831

P5-8 Repeat the calculations in Problem 5-7 for the case where the cell cluster size is increased to nine. Discuss effects of the cluster size on the service quality.

P5-9 Consider a cellular system with a total of 395 traffic channels and a 7-cell frequency reuse. Suppose the probability of call blocking is to be no more than 1%. Assume that every subscriber makes 1 call per hour and each call lasts 3 minutes, on average.

a. For omnidirectional antennas, determine the traffic load in Erlangs per cell and the number of calls per cell per hour.
b. Repeat part (a) for a 120° sectoring.
c. Repeat part (a) for a 60° sectoring.
d. Discuss the effect of sectoring, using the information from parts (b) and (c), on the S/I ratio and on the trunk efficiency.

P5-10 Consider the system in Problem 5-9.

a. Determine the minimum frequency reuse factors for no sectoring, 120° sectoring and 60° sectoring, respectively, taking into consideration that a S/I value of 18 dB or better is satisfactory.
b. Determine the traffic loads per cell and trunk efficiencies with no sectoring, 120° sectoring, and 60° sectoring.

P5-11 A cellular network provider uses a digital TDMA scheme that can tolerate a S/I ratio of 15 dB in the worst case, in a propagation environment with path loss exponent $\kappa = 4$.

a. Find the best value of N for (1) omnidirectional antennas, (2) 120° sectoring, and (3) 60° sectoring.

b. Should sectoring be used?
c. If sectoring is used, should you use 120° or 60° sectoring? Explain.

P5-12 Consider a cellular system that employs omnidirectional antennas.

a. If the path loss exponent is $\kappa = 4$, show that a cell can be split into 4 smaller cells, each with a radius that is one-half of the radius of the original cell, and 1/16 of the transmitter power of the original base station.
b. If it is determined, through careful experimentation, that $\kappa = 3$ is the correct value, how should the transmitter power be changed in order to split a cell into 4 smaller cells?
c. Provide drawings that show how the new cells would fit within the original macrocells, so that all the base stations before splitting are also used after splitting.

P5-13 One method to increase the capacity of a cellular system is cell splitting, in which a large cell is divided into several smaller cells. This normally takes place in regions where there are heavy concentrations of users. Cell splitting needs to preserve the frequency reuse plan of the original large cells.

a. While cell splitting increases system capacity, this gain comes with the introduction of other networking problems. Describe two main problems arising from cell splitting and explain how you propose to address these problems.
b. The base station can be placed anywhere within the cell. Assuming that base stations are located at the centers of the large cells and using appropriate labeling, construct a cell splitting scenario starting from the center of a large cell such that the original $N = 7$-cell frequency reuse pattern is preserved.

P5-14 Here we want to investigate the cochannel interference in a more practical environment based on computer simulation. Consider a cellular system with hexagonal cells and with a frequency reuse factor of 7. Consider the forward link transmission with the cochannel interference resulting only from the 6 cochannel cells (base stations) in the first tier. The propagation environment is characterized by the lognormal shadowing as described by Eqs. (2.4.15)–(2.4.16), with path loss exponent κ and the standard deviation σ_ϵ. All the base stations have the same transmit signal power and the same values for d_0 and $\bar{L}_p(d_0)$, respectively. The required instantaneous signal-to-cochannel interference ratio (S/I) is 18 dB.

a. Assume that the mobile user location is uniformly distributed in the cell with $d > d_0$, find the probability that the instantaneous (S/I) value is below 18 dB, (1) for $\sigma_\epsilon = 8$ dB and $\kappa = 2, 2.5, 3, 3.5$, and 4, respectively, and (2) for $\kappa = 4$ and $\sigma_\epsilon = 7, 8$, and 9 dB, respectively.
b. Consider the worst-case scenario where the mobile user is at the cell boundary. Find the probability that the instantaneous (S/I) value is below 18 dB, (1) for $\sigma_\epsilon = 8$ dB and $\kappa = 2, 2.5, 3, 3.5$, and 4, respectively, and (2) for $\kappa = 4$ and $\sigma_\epsilon = 7, 8$, and 9 dB, respectively.
c. From the simulation results, comment on the effects of the user location and the channel parameters κ and σ_ϵ on the (S/I) ratio.