EN.550.661: Nonlinear Optimization I

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Homework Assignment # 6

Starred exercises require the use of MATLAB.

Exercise 6.1*. This problem asks you to explore the performance of various coordinate minimization approaches for solving the very simple unconstrained optimization problem

$$\underset{x\in\mathbb{R}^n}{\text{minimize}} \; f(x) := \tfrac{1}{2}x^T H x \tag{1}$$

for various choices of the symmetric matrix H.

 (i) Code the following coordinate minimization methods specifically written to solve problem (1).

   (a) *cyclic* coordinate minimization with *exact* linesearch minimization (closed form solution);

   (b) *cyclic* coordinate descent with "optimal" *fixed step size* given by $\alpha = 1/\|H\|$;

   (c) *random* coordinate descent with "optimal" *fixed step size* given by $\alpha = 1/\|H\|$; and

   (d) *Gauss-Southwell* coordinate descent with "optimal" *fixed step size* given by $\alpha = 1/\|H\|$.

 (ii) Use your codes to *evaluate* the performance of methods (a)–(d) on problems of the form (1), where the matrix H is randomly created in MATLAB using the built in command SPRANDSYM.

   (a) How do they perform for $n = 10$ and $\text{cond}(H) \in \{10^1, 10^2, 10^3, 10^4\}$?

   (b) How do they perform for $n = 100$ and $\text{cond}(H) \in \{10^1, 10^2, 10^3, 10^4\}$?

   (c) How do they perform for $n = 1000$ and $\text{cond}(H) \in \{10^1, 10^2, 10^3, 10^4\}$?

 (iii) Describe what you have learned from this exercise?