EN.550.661: Nonlinear Optimization I

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Homework Assignment #2

Starred exercises require the use of MATLAB.

Exercise 2.1* Write a MATLAB m-function that performs Newton's Method for finding a zero of a function $F : \mathbb{R}^n \to \mathbb{R}^n$. The function call should have the form

```
[x,F,J,iter,status] = newton( Fun,x0,maxit,printlevel,tol )
```

where `Fun` is of type *string* that holds the name of a Matlab m-function, `x0` is an initial guess at a zero, `maxit` is the maximum number of iterations allowed, `printlevel` determines the amount of printout required, and `tol` is the final stopping tolerance. The Matlab m-function `Fun` should have the form

```
[F,J] = Fun( x )
```

where `F` and `J` should contain the value and Jacobian of a desired function at the point `x`. In the code, if the parameter `printlevel` has the value zero, then no printing should occur; otherwise, a single line of output is printed (in column format) per iteration. On output, the parameters `x`, `F`, and `J` should contain the final iterate, function value, and Jacobian matrix computed by the algorithm. The paramter `iter` should contain the total number of iterations performed. Finally, `status` should have the value 0 if the final stopping tolerance was obtained and the value 1 otherwise.

Exercise 2.2* Let A be a given real symmetric matrix.

(a) Define an iteration of Newton's Method for solving the $n+1$ nonlinear equations

$$(A - \lambda I)x = 0 \quad \text{and} \quad x^T x = 1$$

in the $n+1$ unknowns (x, λ). Note that a zero (x, λ) is an eigenpair of the matrix A.

(b) Use the code you wrote for Exercise 2.1 to find an eigenpair (x, λ) for the matrix

$$A = \begin{pmatrix} 4 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad \text{with starting point} \quad x_0 = \begin{pmatrix} 1/5 \\ -1/5 \\ 4/5 \end{pmatrix} \quad \text{and} \quad \lambda_0 = 1.$$

Exercise 2.3: Consider the function $f : \mathbb{R}^3 \to \mathbb{R}$ defined by

$$f(x) = e^{x_3} x_1^2 + x_2^2 + x_3^2 \cos(x_1)$$

(a) Derive the gradient $\nabla f(x)$ and Hessian $\nabla^2 f(x)$.

(b) Compute $f(x)$, $\nabla f(x)$, and $\nabla^2 f(x)$ at $\bar{x} = (0,0,0)$ and $\widehat{x} = (0,0,-1)$. In both cases, compute the spectral decomposition of the Hessian matrix and indicate whether the necessary and sufficient conditions for being a local unconstrained minimizer are satisfied.

(c) For each of the points $\bar{x}$ and $\widehat{x}$, compute the Newton direction p^N and determine if it is a descent direction for f.

(d) If the Hessian is indefinite at $\bar{x}$ or $\widehat{x}$, compute a *modified* Newton direction p^M. Find the value of the directional derivative along p^M.