EN.550.661: Nonlinear Optimization I

Instructor: Daniel P. Robinson

Homework Assignment #5

Starred exercises require the use of Matlab.

Exercise 5.1: Consider the problem of finding a zero of a continuously differentiable function $F : \mathbb{R}^n \to \mathbb{R}^n$ by finding a first-order solution to the optimization problem

$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \;\; f(x) = \tfrac{1}{2}\|F(x)\|_2^2 \tag{1}$$

(a) State the precise condition(s) that ensure(s) that a first-order solution to (1) is also a zero of F. Prove that this is true.

(b) Let x_k be the current estimate of a zero of F and suppose that $J(x_k)^T F(x_k) \neq 0$, $\lambda > 0$, and

$$\left[J(x_k)^T J(x_k) + \lambda I\right] p_k = -J(x_k)^T F(x_k).$$

Prove that p_k is a descent direction for f at x_k.

Exercise 5.2: Let x_k and x_{k+1} be consecutive iterates for finding a zero of a function $F : \mathbb{R}^n \to \mathbb{R}^n$ and define

$$s_k = x_{k+1} - x_k \quad \text{and} \quad y_k = F(x_{k+1}) - F(x_k).$$

(a) If $B_k \in \mathbb{R}^{n \times n}$ is a matrix associated with the kth iteration, prove that

$$B_{k+1} = B_k + \frac{(y_k - B_k s_k) s_k^T}{s_k^T s_k} \tag{2}$$

satisfies the *secant equation* $B_{k+1} s_k = y_k$.

(b) Prove that B_{k+1} as defined in (2) solves the problem

$$\underset{B \in \mathbb{R}^{n \times n}}{\text{minimize}} \;\; \|B - B_k\|_2 \;\; \text{subject to} \;\; B s_k = y_k.$$