

Assignment 3

Solving Question 1

Loading packages

```
library(lpSolve)
library(lpSolveAPI)
```

creating lp with 9 variables and 0 constraints

```
lp<- make.lp(0,9)
```

##Specify the objective function

```
set.objfn(lp, c(420,420,420,360,360,360,300,300,300))
lp.control(lp, sense = 'max')
```

```

## $anti.degen
## [1] "fixedvars" "stalling"
##
## $basis.crash
## [1] "none"
##
## $bb.depthlimit
## [1] -50
##
## $bb.floorfirst
## [1] "automatic"
##
## $bb.rule
## [1] "pseudononint" "greedy"          "dynamic"          "rcostfixing"
##
## $break.at.first
## [1] FALSE
##
## $break.at.value
## [1] 1e+30
##
## $epsilon
##      epsb      epsd      epsel      epsint  epsperturb  epspivot
##      1e-10      1e-09      1e-12      1e-07      1e-05      2e-07
##
## $improve
## [1] "dualfeas" "thetagap"
##
## $infinite
## [1] 1e+30
##
## $maxpivot
## [1] 250
##
## $mip.gap
## absolute relative
##      1e-11      1e-11
##
## $negrange
## [1] -1e+06
##
## $obj.in.basis
## [1] TRUE
##
## $pivoting
## [1] "devex"      "adaptive"
##
## $presolve
## [1] "none"
##
## $scalelimit
## [1] 5
##

```

```
## $scaling
## [1] "geometric" "equilibrate" "integers"
##
## $sense
## [1] "maximize"
##
## $simplextype
## [1] "dual" "primal"
##
## $timeout
## [1] 0
##
## $verbose
## [1] "neutral"
```

Adding Constraints

```
add.constraint(lp, c(1,1,1,0,0,0,0,0,0), "<=", 750 )
add.constraint(lp, c(0,0,0,1,1,1,0,0,0), "<=", 900)
add.constraint(lp, c(0,0,0,0,0,0,1,1,1), "<=", 450)
add.constraint(lp, c(20,15,12,0,0,0,0,0,0), "<=", 13000)
add.constraint(lp, c(0,0,0,20,15,12,0,0,0), "<=", 12000)
add.constraint(lp, c(0,0,0,0,0,0,20,15,12), "<=", 5000)
add.constraint(lp, c(1,1,1,0,0,0,0,0,0), "<=", 900)
add.constraint(lp, c(0,0,0,1,1,1,0,0,0), "<=", 1200)
add.constraint(lp, c(0,0,0,0,0,0,1,1,1), "<=", 750)
add.constraint(lp ,c(900,-750,0,900,-750,0,900,-750,0), "=", 0)
add.constraint(lp ,c(0,450,-900,0,450,-900,0,450,-900), "=", 0)
add.constraint(lp ,c(450,0,-750,450,0,-750,450,0,-750), "=", 0)
```

Assignining rownames and column names

```
RowNames <-c("1-ProductionCapacity","2-ProductionCapacity","3-ProductionCapacity",
             "1-StorageSpace","2-StorageSpace","3-StorageSpace",
             "ForecastLarge","ForecastMedium","ForecastSmall",
             "PercentCapP1andP2","PercentCapP2andP3","PercentCapP1andP3")

ColNames <- c("1-PlantLarge","2-PlantLarge","3-PlantLarge",
             "1-PlantMedium","2-PlantMedium","3-PlantMedium",
             "1-PlantSmall","2-PlantSmall","3-PlantSmall")
```

Getting Constraints

```
solve(lp)
```

```
## [1] 0
```

```
get.objective(lp)
```

```
## [1] 699026.5
```

```
get.constraints(lp)
```

```
## [1] 750.0000 858.4071 250.0000 13000.0000 12000.0000 5000.0000
## [7] 750.0000 858.4071 250.0000 0.0000 0.0000 0.0000
```

```
dimnames(lp) <- list(RowNames, ColNames)
lp
```

```
## Model name:
## a linear program with 9 decision variables and 12 constraints
```

Solving Question 2

Identifying the shadow prices, dual solution, and reduced costs

```
# Reduced Costs
get.sensitivity.obj(lp)
```

```
## $objfrom
## [1] 4.200e+02 4.125e+02 4.200e+02 -1.000e+30 3.600e+02 3.480e+02 2.800e+02
## [8] -1.000e+30 -1.000e+30
##
## $objtill
## [1] 4.40e+02 4.20e+02 4.32e+02 3.60e+02 3.75e+02 3.60e+02 1.00e+30 3.15e+02
## [9] 3.24e+02
```

```
#Shadow Prices
get.sensitivity.rhs(lp)
```

```
## $duals
## [1] 60.0000000 0.0000000 0.0000000 22.3008850 22.3008850 19.3008850
## [7] 0.0000000 0.0000000 0.0000000 -0.0339823 0.0000000 -0.1231858
## [13] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
## [19] 0.0000000 -15.0000000 -24.0000000
##
## $dualsfrom
## [1] 7.287611e+02 -1.000000e+30 -1.000000e+30 1.216129e+04 9.120000e+03
## [6] 1.904762e+03 -1.000000e+30 -1.000000e+30 -1.000000e+30 0.000000e+00
## [11] -1.000000e+30 0.000000e+00 -1.000000e+30 -1.000000e+30 -1.000000e+30
## [16] -8.628319e+01 -1.000000e+30 -1.000000e+30 -1.000000e+30 -8.495575e+01
## [21] -1.039823e+02
##
## $dualstill
## [1] 8.075221e+02 1.000000e+30 1.000000e+30 1.335821e+04 1.267143e+04
## [6] 7.285714e+03 1.000000e+30 1.000000e+30 1.000000e+30 0.000000e+00
## [11] 1.000000e+30 0.000000e+00 1.000000e+30 1.000000e+30 1.000000e+30
## [16] 6.371681e+01 1.000000e+30 1.000000e+30 1.000000e+30 1.150442e+02
## [21] 1.438053e+02
```

```
#Dual solution
get.dual.solution(lp)
```

```
## [1] 1.0000000 60.0000000 0.0000000 0.0000000 22.3008850 22.3008850
## [7] 19.3008850 0.0000000 0.0000000 0.0000000 -0.0339823 0.0000000
## [13] -0.1231858 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
## [19] 0.0000000 0.0000000 -15.0000000 -24.0000000
```

Solving Question 3

Identifying the sensitivity of the above prices and costs.

```
Sensitivity<-data.frame(get.sensitivity.rhs(lp)$duals[1:21],get.sensitivity.rhs(lp)$dualsfrom[1:21],get.sensitivity.rhs(lp)$dualstill[1:21])
names(Sensitivity)<-c("Price","low","High")
```

Specifying the range of shadow prices and reduced cost within which the optimal solution will not change.

Sensitivity

Price
<dbl>

low
<dbl>

High
<dbl>

Price <dbl>	low <dbl>	High <dbl>
60.0000000	7.287611e+02	8.075221e+02
0.0000000	-1.000000e+30	1.000000e+30
0.0000000	-1.000000e+30	1.000000e+30
22.3008850	1.216129e+04	1.335821e+04
22.3008850	9.120000e+03	1.267143e+04
19.3008850	1.904762e+03	7.285714e+03
0.0000000	-1.000000e+30	1.000000e+30
0.0000000	-1.000000e+30	1.000000e+30
0.0000000	-1.000000e+30	1.000000e+30
-0.0339823	0.000000e+00	0.000000e+00

1-10 of 21 rows

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solving Question 4

Formulating the dual of the above problem.

```
lpdual <- make.lp(0,12)
set.objfn(lpdual, c(750,900,450,13000,12000,5000,900,1200,750,0,0,0))
lp.control(lpdual,sense='min',simplextype="dual")
```

```

## $anti.degen
## [1] "fixedvars" "stalling"
##
## $basis.crash
## [1] "none"
##
## $bb.depthlimit
## [1] -50
##
## $bb.floorfirst
## [1] "automatic"
##
## $bb.rule
## [1] "pseudononint" "greedy"          "dynamic"          "rcostfixing"
##
## $break.at.first
## [1] FALSE
##
## $break.at.value
## [1] -1e+30
##
## $epsilon
##      epsb      epsd      epsel      epsint  epsperturb  epspivot
##      1e-10      1e-09      1e-12      1e-07      1e-05      2e-07
##
## $improve
## [1] "dualfeas" "thetagap"
##
## $infinite
## [1] 1e+30
##
## $maxpivot
## [1] 250
##
## $mip.gap
## absolute relative
##      1e-11      1e-11
##
## $negrange
## [1] -1e+06
##
## $obj.in.basis
## [1] TRUE
##
## $pivoting
## [1] "devex"      "adaptive"
##
## $presolve
## [1] "none"
##
## $scalelimit
## [1] 5
##

```

```
## $scaling
## [1] "geometric" "equilibrate" "integers"
##
## $sense
## [1] "minimize"
##
## $simplextype
## [1] "dual" "dual"
##
## $timeout
## [1] 0
##
## $verbose
## [1] "neutral"
```

```
add.constraint(lp dual ,c(1,0,0,20,0,0,1,0,0,900,0,450), ">=", 420)
add.constraint(lp dual ,c(0,1,0,0,20,0,1,0,0,-750,450,0), ">=", 420)
add.constraint(lp dual ,c(0,0,1,0,0,20,1,0,0,0,-900,-750), ">=", 420)
add.constraint(lp dual ,c(1,0,0,15,0,0,0,1,0,900,0,450), ">=", 360)
add.constraint(lp dual ,c(0,1,0,0,15,0,0,1,0,-750,450,0), ">=", 360)
add.constraint(lp dual ,c(0,0,1,0,0,15,0,1,0,0,-900,-750), ">=", 360)
add.constraint(lp dual ,c(1,0,0,12,0,0,0,0,1,900,0,450), ">=", 300)
add.constraint(lp dual ,c(0,1,0,0,12,0,0,0,1,-750,450,0), ">=", 300)
add.constraint(lp dual ,c(0,0,1,0,0,12,0,0,1,0,-900,-750), ">=", 300)
```

Getting constraints

```
solve(lp dual)
```

```
## [1] 0
```

```
get.objective(lp dual)
```

```
## [1] 696000
```

```
get.variables(lp dual)
```

```
## [1] 0.0000000 0.0000000 0.0000000 12.0000000 20.0000000 60.0000000
## [7] 0.0000000 0.0000000 0.0000000 0.2000000 0.4666667 0.0000000
```

```
get.constraints(lp dual)
```

```
## [1] 420 460 780 360 360 480 324 300 300
```


In a primal-dual pair of linear programming problems, if either the primal or the dual problem has an optimal solution,

then the other does also, and the two optimal objective values are equal. Both the primal and the dual LP yield the same objective value. This is a consequence of the Strong Duality Theorem.

$(\text{Primal optimal}) = (\text{dual optimal})$ - Strong dual theorem.