Q. $$\int_0^2 \int_0^x \int_0^{2x+2y} e^{x+y+z}\, dz\, dy\, dx$$

$$\int_0^2 \int_0^x \left(e^{x+y+z}\right)_0^{2x+2y} dx\, dy$$

$$\int_0^2 \int_0^x e^{x+y+2x+2y} - \int_0^2 \int_0^x e^{x+y}\, dx\, dy$$

$$\int_0^2 \left[\frac{e^{3x+3y}}{3}\right]_0^x dx - \int_0^2 \left[e^{x+y}\right]_0^x dx$$

$$\int_0^2 \frac{e^{6x}}{3} - \frac{e^{3x}}{3} - \int_0^2 e^{2x} - e^x$$

$$\left(\frac{e^{6x}}{18} - \frac{e^{3x}}{9} - \frac{e^{2x}}{2} - e^x\right)_0^2$$

$$\left(\frac{e^{12}}{18} - \frac{e^6}{9} - \frac{e^4}{2} - e^2\right) - \left(\frac{1}{18} - \frac{1}{9} - \frac{1}{2} - 1\right)$$

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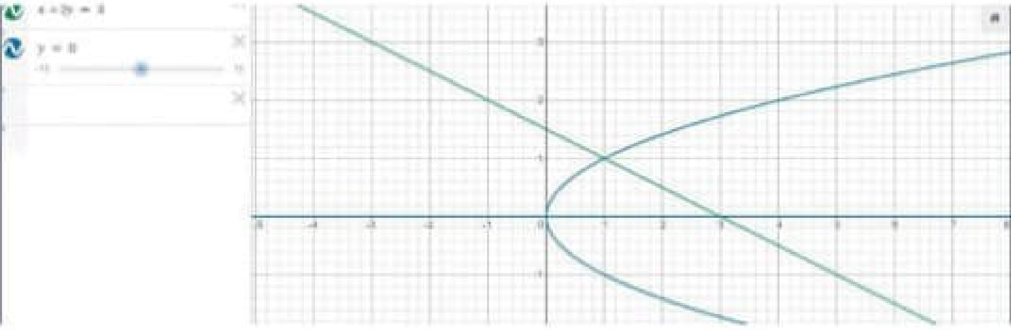
Math

5 points

Answer

14 5.0



Topic:

Integration

Solution:

We need to evaluate the following integral.

$$I = \iint_A (5 - 2x - y)\, dx\, dy$$

where A is given by $y = 0, x + 2y = 3$ and $x =$

- First of all plot the graphs for given equations and find the limits for the integral as the region A is given by provided equations.

Interpreting region A, we get the following limits:

$$I = \int_0^1 \int_{y^2}^{3-2y} (5 - 2x - y)\, dx\, dy$$

Solving the inner integral, w.r.t. x keeping y's as constant.

Q. find the area enclosed by the leminscate $r^2 = a^2 \cos 2\theta$

Sol^n

$$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 \, d\theta$$

$$A = 4\left[\frac{1}{2}\int_{0}^{\pi/4} a^2 \cos 2\theta \, d\theta\right]$$

$$A = 2a^2\left[\frac{1}{2}\sin 2\theta\right]_{0}^{\pi/4}$$

$$A = a^2\left[\sin\frac{1}{2}\pi - \sin 0\right]$$

$$A = a^2 \text{ unit}^2$$

Q.

$$-y^2\frac{y}{2|y|}\arcsin\left(\frac{2}{y}x\right)$$

$$\implies \int_1^y \sqrt{4x^2-y^2}\,dx = \frac{1}{2}\left(\frac{4x^2}{\sqrt{4x^2-y^2}} - y^2\frac{y}{2|y|}\arcsin\left(\frac{2}{y}x\right)\right)$$

.

[wrong] And it seems that after I evaluate the last one i get $\frac{y^2}{\sqrt{3y^2}} = \frac{y^2}{\sqrt{3}|y|}$

Then if I solve the integral of the expresion above for x I get $\frac{1}{2\sqrt{3}}$ if $y > 0$ and $\frac{-1}{2\sqrt{3}}$ if $y < 0$.[/wrong]

I'm almost sure i made a mistake somewhere. Can someone find any errors?

$$\iint_R r^3\,dr\,d\theta = \int_{-\pi/2}^{\pi/2}\int_{2\cos\theta}^{4\cos\theta} r^3\,dr\,d\theta = \int_{-\pi/2}^{\pi/2}\left(\frac{r^4}{4}\right)_{2\cos\theta}^{4\cos\theta}$$

$$= \int_{-\pi/2}^{\pi/2} \frac{1}{4}\left(256\cos^4\theta - 16\cos^4\theta\right)d\theta$$

$$= 60\int_{-\pi/2}^{\pi/2} \cos^4\theta\,d\theta$$

$$= 60\int_{-\pi/2}^{\pi/2} \cos^4\theta\,d\theta$$

$$= 120\int_{0}^{\pi/2} \cos^4\theta\,d\theta$$

$$= 120 \times \frac{3\times 1}{4\times 2} \times \frac{\pi}{2} = \frac{45\pi}{2} \text{ unit}^2$$

Q. find the area common to circles $x^2+y^2=a^2$ and $x^2+y^2=2ax$

Sol^n $C_1 \to x^2+y^2=a^2$

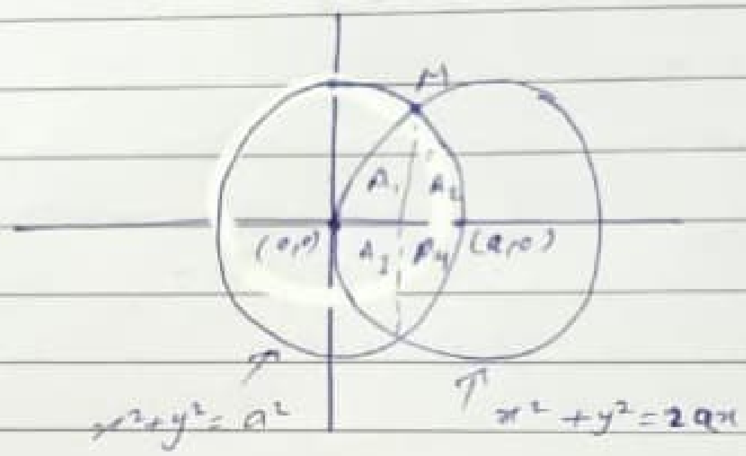
$C_2 \to x^2+y^2=2ax$

$x^2+y^2-2ax+a-a=0$

$(x-a)^2+y^2=a$

radius of both circles are a

$C_1(0,0)$ $C_2(a,0)$



$A_1=A_2=A_3=A_4$

Total Area = $4A_1$

$a^2=2ax$

$a(a-2x)=0$

$a=0$ $\quad a=2x$

$\boxed{x=a/2}$ $\quad \boxed{y=\frac{\sqrt{3}a}{2}}$

$M\left(\frac{a}{2}, \frac{\sqrt{3}a}{2}\right)$

$$A_1 = \int_0^{a/2}\int_0^{\sqrt{2ax-x^2}} dy\,dx$$

$$A_1 = \int_0^{a/2}\left[\sqrt{2ax-x^2}\right]dx$$

Q. Transform the integral $\int_0^1 \int_0^x f(x,y)\, dy\, dx$ to the integral in polar co-ordinates

Sol^n $x = r\cos\theta$

$y = r\sin\theta$ $\quad r^2 = x^2 + y^2$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

from jacobian determinant $dx\,dy = r\,dr\,d\theta$

$$\underbrace{\left\{ \int_0^1 \int_0^{r\cos\theta} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta \right\}}_{\text{Transform in Polar Coordinates}}$$

Q. $\iint r^3 dr\, d\theta$

Sol^n $\tan\theta = 1$

Area =

$$A_1 = \int_0^{a/2} \left[\sqrt{a^2-(x-a)^2}\right] dx$$

$$A_1 = \left[\frac{x-a}{2}\sqrt{ax-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right)\right]_0^{a/2}$$

$$A = \left[\frac{\left(\frac{a}{2}-a\right)}{2}\sqrt{\frac{a^2}{2}-\frac{a^2}{4}} + \frac{a^2}{2}\sin^{-1}\left(\frac{a/2-a}{a}\right)\right] - \left[\frac{-a}{2}\times 0 + \frac{a^2}{2}\sin^{-1}(-1)\right]_0$$

$$A = \left[\frac{-a}{4}\times\frac{a}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{-1}{2}\right) - \frac{a^2}{2}\sin^{-1}(-1)\right]$$

$$A = \left[\frac{-a^2}{8} + \frac{a^2}{2}\times\left(\frac{-\pi}{6}\right) - \frac{a^2}{2}\left(\frac{-\pi}{2}\right)\right]$$

$$A_1 = -\frac{a^2}{8} - \frac{a^2\pi}{12} + \frac{a^2\pi}{4}$$

$$A_1 = \frac{-3a^2 - 2a^2\pi + 6a^2\pi}{24} = \frac{-3a^2 + 4a^2\pi}{24}$$

Total Area $(A) = 4A_1$

$$= 4 \times \frac{4a^2\pi - 3a^2}{24}$$

$$\boxed{A = \frac{4a^2\pi - 3a^2}{6}}$$

Surprise Test - 2

ABHISHEK KUMAWAT

$$\int_0^1 \int_0^{\sqrt{\frac{1}{2}(1-y^2)}} \frac{dx\,dy}{\sqrt{1-x^2-y^2}}$$

$$= \int_0^1 \int_0^{\sqrt{\frac{1-y^2}{2}}} \frac{dx\,dy}{\sqrt{(1-y^2)-x^2}}$$

$$= \int_0^1 \left[\sin^{-1}\left[\frac{x}{\sqrt{1-y^2}}\right]\right]_0^{\sqrt{\frac{1-y^2}{2}}}$$

$$= \int_0^1 \int_0^1 \left[\sin^{-1}\frac{1}{\sqrt{2}} - \sin^{-1}0\right] dy$$

$$= \int_0^1 \frac{\pi}{4}\, dy = \frac{\pi}{4}\left[y\right]_0^1 = \frac{\pi}{4}$$

Q:- $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{(a^2-x^2)-y^2}\, dy\, dx$

$$\int_0^a \left[\frac{y}{2}\sqrt{a^2-x^2-y^2} + \frac{(\sqrt{a^2-x^2})^2}{2} \sin^{-1}\frac{y}{\sqrt{a^2-x^2}} \right]_0^{\sqrt{a^2-x^2}} dx$$

$$= \int_0^a \left[\frac{\sqrt{a^2-x^2}}{2} \times 0 + \frac{a^2-x^2}{2} \sin^{-1} 1 - 0 \right] dx$$

$$= \int_0^a \left[\frac{a^2}{2} \cdot \pi/2 - \frac{x^2}{2}\pi \right] dx$$

$$= \frac{\pi a^2}{4} [x]_0^a - \frac{\pi}{\cancel{8}} \left[\frac{x^3}{\cancel{3}} \right]_0^a$$

$$= \frac{\pi a^3}{4} - \frac{\pi a^3}{6} = \frac{\cancel{2}\pi a^3}{\cancel{24}12} = \frac{\pi a^3}{12}.$$

Q:- determine the area of region bounded by the curves $xy=2$, $4y=x^2$, $y=4$

$\Rightarrow$ Intersection pt.

$y=2/x$ and $y=4 \longrightarrow x=1/2$

$y=4$ and $y=x^2/4 \longrightarrow x=4$

$y=2/x$ and $y=x^2/4 \longrightarrow x=2$.

$A1 =$ integral $1/2$ to $2\,(2/x) = \ln 8$

$A2 =$ integral 2 to $4\,(x^2/4) = 14/3$

$A3 =$ integral $1/2$ to $2\,(4) = 6$

$A4 =$ integral 2 to $4\,(4) = 8$.

$A5 = A3 - A1 = 6 - \ln 8$

$= 6 - 2.079 = 3.921$

$A6 = A4 - A2 = 8 - 4/3 = 10/3 = 3.33$

$A5 + A6 = 3.921 + 3.33 = \underline{7.25}$.

Q:- $\iint r\sin\theta\, dr\, d\theta$ over the area of the cardiod $r = a(1+\cos\theta)$ above the initial line.

$$\Rightarrow \int_0^{\pi/2}\int_0^{a(1+\cos\theta)} r\sin\theta\, dr\, d\theta$$

$$\int_0^{\pi/2}\left[\frac{r^2\sin\theta}{2}\right]_0^{a(1+\cos\theta)} d\theta$$

$$\int_0^{\pi/2} a^2\frac{(1+\cos\theta)^2\sin\theta}{2}\, d\theta = \frac{a^2}{2}\int_0^{\pi/2}(\sin\theta + \cos^2\theta\sin\theta + 2\sin\theta\cos\theta)\, d\theta$$

$$= \frac{a^2}{2}\left[(-\cos\theta)_0^{\pi/2} + \left(-\tfrac{1}{3}\cos^3\theta\right)_0^{\pi/2} + \left(-\cos 2\theta/2\right)_0^{\pi/2}\right.$$

$$= \frac{a^2}{2}\left[1 + \tfrac{1}{3} + 1\right] = \frac{7a^2}{2}.$$

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Q2. $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2 \sqrt{x^2+y^2}\, dx\, dy$ by changing into polar Co-ordinates.

$$\Longrightarrow \int_0^a \left(\frac{y}{2}\sqrt{a^2-x^2-y^2} + \left(\frac{\sqrt{a^2-x^2}}{2}\right)^2 \sin^{-1} \frac{y}{\sqrt{a^2-x^2}} \right]_0^{\sqrt{a^2-x^2}} dx$$

$$= \int_0^a \left[\frac{a^2-x^2}{2} \times 0 + \frac{a^2-x^2}{2} \sin^{-1} 1 - 0 \right] dx$$

$$\int_0^a \left[\frac{a^2}{2} \cdot \frac{\pi}{2} - \frac{x^2}{2} \pi \right] dx$$

$$\frac{\pi a^2}{4} \left[x\right]_0^a - \frac{\pi}{6} \left[\frac{x^3}{3}\right]_0^a$$

$$\frac{\pi a^3}{4} - \frac{\pi a^3}{6}$$

$$= \frac{\cancel{2}\pi a^3}{\cancel{24}\,12}$$

$$= \boxed{\frac{\pi a^3}{12}}$$

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Q.2

$$\int_0^2 \left[\int_0^x \left(\int_0^{2x+2y} e^{\cancel{x+y+z}}\, dz\right) dy\right]^? dx$$

$$= \int_0^2 \left(\int_0^x \left(e^{x+y+z}\right)_0^{2x+2y} dy\right) dx$$

$$= \int_0^2 \left(\int_0^x \left(e^{3x+3y} - e^{x+y}\right) dy\right) dx$$

$$= \int_0^2 \left(\frac{e^{3x+3y}}{3} - e^{x+y}\right)_0^x dx$$

$$= \int_0^2 \frac{e^{6x}}{3} - e^{2x} - e^{3x} - e^{x}\, dx$$

$$= \left(\frac{e^{6x}}{18} - \frac{e^{2x}}{2} - \frac{e^{3x}}{3} - e^{x}\right)_0^2$$

$$= \frac{e^{12}}{18} - \frac{e^4}{2} - \frac{e^6}{3} - e^2 - \left(\frac{1}{18} - \frac{1}{2} - \frac{1}{3} - 1\right)$$

$$= \frac{e^{12}}{18} - \frac{e^6}{3} - \frac{e^4}{2} - e^2 + \frac{16}{9}$$ Ans

Date / /

Q4 $\int_0^1 \int_0^{\sqrt{\frac{1}{2}(1-y^2)}} \frac{dx\,dy}{\sqrt{1-x^2-y^2}}$

$\left[\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)\right]$

$$I = \int_0^1 \int_0^{\sqrt{\frac{1}{2}(1-y^2)}} \frac{dx\,dy}{\sqrt{1-x^2-y^2}}$$

$$= \int_0^1 \int_0^{\sqrt{\frac{1}{2}(1-y^2)}} \frac{dx\,dy}{\sqrt{(\sqrt{1-y^2})^2-x^2}}$$

$$= \int_0^1 \left[\sin^{-1}\frac{x}{\sqrt{1-y^2}}\right]_0^{\sqrt{\frac{1}{2}(1-y^2)}} dy$$

$$= \int_0^1 \sin^{-1}\frac{\sqrt{\frac{1}{2}(1-y^2)}}{\sqrt{1-y^2}} - \sin^{-1}(0)\; dy$$

$$= \int_0^1 \sin^{-1}\sqrt{\frac{1}{2}}\,\frac{\cancel{\sqrt{(1-y^2)}}}{\cancel{\sqrt{1-y^2}}}\, dy$$

Q.
$$\int_0^{\frac{a}{\sqrt{2}}}\int_0^{x} x\,dx\,dy + \int_{\frac{a}{\sqrt{2}}}^{a}\int_0^{\sqrt{a^2-x^2}} x\,dx\,dy$$

$$\int_0^{a/\sqrt{2}} (xy)_0^{x}\,dx + \int_{\frac{a}{\sqrt{2}}}^{a} (xy)_0^{\sqrt{a^2-x^2}}\,dx$$

$$\int_0^{a/\sqrt{2}} x^2\,dx + \int_{\frac{a}{\sqrt{2}}}^{a} x\sqrt{a^2-x^2}\,dx$$

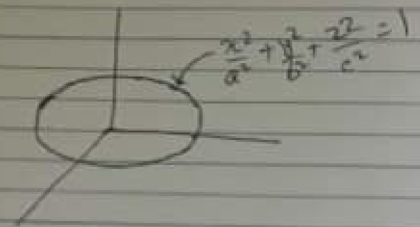
$$a^2-x^2=t^2$$
$$-2x\,dx = 2t\,dt$$
$$x\,dx = -t\,dt$$

$$\left(\frac{x^3}{3}\right)_0^{a/\sqrt{2}} + \int_{\frac{a}{\sqrt{2}}}^{0} -t^2\,dt$$

$$\frac{a^3}{6\sqrt{2}} - \left(\frac{t^3}{3}\right)_{\frac{a}{\sqrt{2}}}^{0}$$

$$\frac{a^3}{6\sqrt{2}} + \frac{a^3}{6\sqrt{2}} = \frac{a^3}{3\sqrt{2}} \text{ unit}^2$$

Ellipsoid : $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}$



Cartesian Co-ordinates → Spherical co-ordinate sy^m
$(x,y,z)\to(r,\theta,\phi)$

Put $x = ar\sin\theta\cos\phi$
$y = br\sin\theta\sin\phi$
$z = cr\cos\theta$

$dx\,dy\,dz = abc\,r^2\sin\theta\,dr\,d\theta\,d\phi$

$\therefore \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=r^2$

$f(x,y,z) = \sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}} = \sqrt{1-r^2} = f(r,\theta,\phi)$

Limits $0\le r\le 1$
$0\le\theta\le\pi/2$
$0\le\phi\le\pi/2$

$\therefore I = 8\iiint\sqrt{1-r^2}\;abc\,r^2\sin\theta\,d\theta\,dr\,d\phi$

$= 8\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{1}\sqrt{1-r^2}\;abc\,r^2\sin\theta\,dr\,d\theta\,d\phi$

$= 8abc\int_0^{\pi/2}\sin\theta\,d\theta\int_0^{\pi/2}d\phi\int_0^{1}\sqrt{1-r^2}\,r^2\,dr$

$= 8abc[-\cos\theta]_0^{\pi/2}[\phi]_0^{\pi/2}\int_0^{\pi/2}\cos t\sin^2 t\cos t\,dt$
[Put $r=\sin$

$8abc\left(\frac{\pi}{2}\right)\left(\frac{\pi}{8}\right)$ - - - $\{$using β $f^n\}$

$\boxed{\therefore I = \frac{\pi^2}{2}abc}$

(1) $\int_0^1\left(\int_0^y (x^2+y^2)\,dx\right)dy + \int_0^2\left(\int_0^{2-y} (x^2+y^2)\,dx\right)dy.$

$= \int_0^1 \left(\frac{x^3}{3}+y^2x\right)_0^y dy + \int_0^2 \left(\frac{x^3}{3}+y^2x\right)_0^{2-y} dy.$

$= \int_0^1 \left(\frac{y^3}{3}+y^3\right) dy + \int_0^2 \left(\frac{(2-y)^3}{3}+y^2(2-y)\right) dy$

$= \int_0^1 \frac{4y^3}{3} dy + \int_0^2 \frac{8-3\times4\times y+3\times2\times y^2-y^3}{3} + 2y^2-y^3$

$= \left(\frac{4}{3}\frac{y^4}{4}\right)_0^1 + \int_0^2 \frac{8-12y+6y^2-y^2+6y^2-3y^3}{3} dy$

$= \left(\frac{y^4}{3}\right)_0^1 + \frac{1}{3}\int_0^2 \left(-4y^3+11y^2-12y+8\right) dy$

$= \frac{1}{3} + \frac{1}{3}\left(-y^4+\frac{11}{3}y^3-6y^2+8y\right)_0^2$

$= \frac{1}{3}+\frac{1}{3}\left(-\cancel{16}+\frac{88}{3}-24+\cancel{16}\right)$

$= \frac{1}{3}+\frac{1}{3}\left(\frac{88-72}{3}\right)$

$= \frac{1}{3}+\frac{16}{9}$

$= \frac{19}{9}$ Ans

(1) $\iiint \sqrt{\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}}\, dx\,dy\,dz$ over the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Soln → In ellipsoid →

$$x = a r \sin\theta \cos\phi$$
$$y = b r \sin\theta \sin\phi$$
$$z = c r \cos\theta$$

$$dx\,dy\,dz = abc\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$
$$0 \le r \le (1)$$
$$0 \le \theta \le \pi/2$$
$$0 \le \phi \le \pi/2$$

$$f(x,y,z) = \sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}} = \sqrt{1-r^2} = f(r,\theta,\phi)$$

$$I = 8\iiint \sqrt{1-r^2}\, abc\, r^2 \sin\, dr\, d\theta\, d\phi$$

$$I = 8\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{r} \sqrt{1-r^2}\, abc\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= 8abc \int_0^{\pi/2} \sin\theta\, d\theta \int_0^{\pi/2} d\phi \int_0^{r} \sqrt{1-r^2}\, dr$$

Put $r = \sin t$

$$= 8abc\left[-\cos\theta\right]_0^{\pi/2}\left[\phi\right]_0^{\pi/2}\int_0^{\pi/2}\cos t\, \sin^2 t\, \cos t\, dt$$

$$= 8abc\left(\pi/2\right)\left(\frac{\pi}{8}\right) = \left\{\frac{\pi^2}{2}abc\right\}$$ ANS

$\int_2 \int_3$ ... $\int_3 \int_1$

Soln (1)

$$\int_0^1 \left[\int_0^{1-x} e^{2x+3y}\, dy \right] dx.$$

$$= \int_0^1 \left[\frac{1}{3} \cdot e^{2x+3y} \right]_0^{1-x} dx$$

$$= \frac{1}{3} \int_0^1 \left[e^{2x+3(1-x)} - e^{2x+3\times 0} \right] dx$$

$$= \frac{1}{3} \int_0^1 \left(e^{3-x} - e^{2x} \right) dx$$

$$= \frac{1}{3} \left[-e^{3-x} - \frac{e^{2x}}{2} \right]_0^1$$

$$= -\frac{1}{3} \left[e^{3-1} + \frac{e^2}{2} - e^{3-0} - \frac{e^0}{2} \right].$$

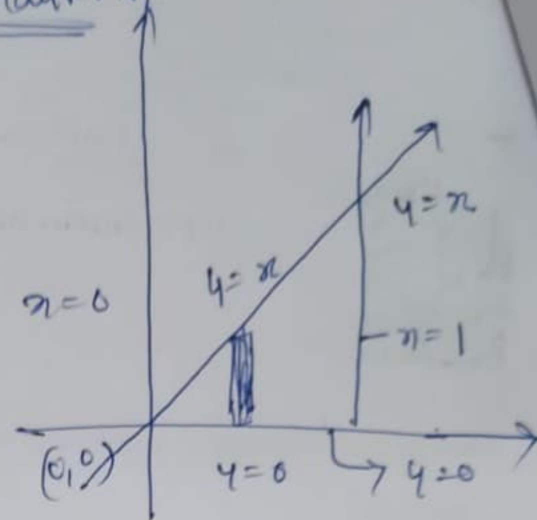
$$= \frac{-1}{3} \left[e^2 + \frac{e^2}{2} - e^3 - \frac{1}{2} \right] = \frac{-1}{3} \left[\frac{2e^2 + e^2 - 2e^3 - 1}{2} \right]$$

$$= \frac{-1}{3} \left[\frac{1}{6} \quad e^2 . \right.$$

$$\frac{1}{6} (e-1)^2 (2e+1). \text{ Ans.}$$

Que (8)

21BCS7206
Ashwin Gautam



$$= \iint_A \sqrt{4x^2-y^2}\,dx\,dy.$$

$$= \iint_A \sqrt{(2x)^2-y^2}\,dx\,dy.$$

putting limits.

$$= \int_0^1 \int_0^x \sqrt{(2x)^2-y^2}\,dx\,dy$$

$$= \int_0^1 \left\{ \frac{y}{2}\sqrt{4x^2-y^2} + \frac{4x^2}{2}\sin^{-1}\left(\frac{y}{2x}\right) \right\}_0^x dx$$

$$= \int_0^1 \frac{x}{2}\sqrt{4x^2-x^2} - 0 + 2x^2 \sin^{-1}\frac{1}{2} - 0)\,dx.$$

$$= \int_0^1 \left(\frac{\sqrt{3}}{2}x^2 + 2x^2\,\pi/4 \right) dx.$$

$$= \int_0^1 \left(\frac{\sqrt{3}}{2} + \pi/2 \right) x^2\,dx.$$

$$= \int_0^1 \left(\frac{\sqrt{3}+\pi}{2} \right) x^2\,dx. = \left(\frac{\sqrt{3}+\pi}{2} \right) \int_0^1 x^2\,dx.$$

$$= \left(\frac{\sqrt{3}+\pi}{2} \right) \left[\frac{x^3}{3} \right]_0^1 = \frac{\sqrt{3}+\pi}{6} \text{ Ans}$$

Q. 2. $$\int_0^1\int_0^y (x^2+y^2)\,dx\,dy + \int_1^2\int_0^{2-y} (x^2+y^2)\,dx\,dy$$

$$\int_0^1\left[\left(\frac{x^3}{3}+xy^2\right)\right]_0^y dy + \int_1^2\left(\frac{x^3}{3}+xy^2\right)_0^{2-y} dy$$

$$\int_0^1\left[\frac{y^3}{3}+y^3\right] dy + \int_1^2\left(\frac{(2-y)^3}{3}+(2-y)y^2\right) dy$$

$$\int_0^1\left(\frac{4y^3}{3}\right) dy + \left[\frac{(2-y)^4}{-12}+\frac{2y^3}{3}-\frac{y^4}{4}\right]_1^2$$

$$\left(\frac{4y^4}{4\times 3}\right)_0^1 + \left[0+\frac{2\times 8}{3}-\frac{16}{4}-\left(\frac{-1}{12}+\frac{2}{3}-\frac{1}{4}\right)\right]$$

$$= \frac{1}{3} + \frac{16}{3}-\frac{16}{4}+\frac{1}{12}-\frac{2}{3}+\frac{1}{4}$$

$$= \frac{4+64-48+1-8+3}{12}$$

$$= \frac{16}{12} = \cancel{0}$$

$$= \frac{4}{3}\ \text{unit}^2$$

Solution

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2}\, dydx$$

Solution

Show Steps

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2}\, dydx = \frac{\pi a^3}{6}$$

Steps

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2}\, dydx$$

$$\int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2}\, dy = \left(a^2-x^2\right)\frac{\pi}{4}$$

Show Steps

$$= \int_0^a \left(a^2-x^2\right)\frac{\pi}{4} dx$$

$$\int_0^a \left(a^2-x^2\right)\frac{\pi}{4} dx = \frac{\pi a^3}{6}$$

Show Steps

$$= \frac{\pi a^3}{6}$$

Got a different answer? Check if it's correct

Enter your answer

$$3\sqrt{2}\int_0^{\pi/2}\left(4-4\sin^2\theta\right)^{3/2} 2\cos\theta \, d\theta$$

$$= 24\cdot 2\sqrt{2}\int_0^{\pi/2}\cos^3\theta\,\cos\theta\, d\theta$$

$$= 48\sqrt{2}\int_0^{\pi/2}\cos^4\theta\, d\theta$$

$$= 48\sqrt{2}\quad \frac{3\times 1}{4\times 2}\times\frac{\pi}{2}$$

$$= 9\sqrt{2}\,\pi$$

(4) $\int_0^{a/\sqrt{2}} \int_0^{x} x\,dx\,dy + \int_{a/\sqrt{2}}^{a} \int_0^{\sqrt{a^2-x^2}} x\,dx\,dy$

$$\int_0^{a/\sqrt{2}} \left[xy \Big|_0^{x} dx \right] + \int_{a/\sqrt{2}}^{a} \left[xy \Big|_0^{\sqrt{a^2-x^2}} dx \right]$$

$$\int_0^{a/\sqrt{2}} x^2 dx + \int_{a/\sqrt{2}}^{a} \underline{x\sqrt{a^2-x^2}\,dx}$$

$$\frac{x^3}{3}\Big|_0^{a/\sqrt{2}} +$$

$u \Rightarrow a^2 - x^2$

$du = -2x\,dx$

$\frac{du}{-2} = x\,dx$

$$\therefore \frac{a^3}{6\sqrt{2}} + \int_0^{a^2/2} \frac{du}{-2}\sqrt{u}$$

$a^2 - \frac{a^2}{2}$

$\frac{a^2}{2}$

$$\frac{a^3}{6\sqrt{2}} + -\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right|_0^{a^2/2}$$

$$\frac{a^3}{6\sqrt{2}} - \frac{1}{3}\left[\left(\frac{a^2}{2}\right)^{3/2}\right]$$

$$\frac{a^3}{6\sqrt{2}} - \frac{1}{3}\frac{a^3}{(2\sqrt{2}} = 0 \quad \underline{\underline{\text{Ans}}}$$

Que (2) $\displaystyle\int_1^2\int_3^4 (xy+e^y)\,dy\,dx = \int_3^4\int_1^2 (xy+e^y)\,dx\,dy.$

Soln $\displaystyle I = \int_2^1 \left[\int_3^4 (xy+e^y)\,dy\right] dx$

$\displaystyle = \int_2^1 \left[\frac{xy^2}{2} + e^y\right]_3^4 dx$

$\displaystyle = \int_2^1 \left[8x + e^4 - \frac{9x}{2} - e^3\right] dx$

$\displaystyle = \left[\frac{8x^2}{2} + (e^4-e^3)x - \frac{9x^2}{4}\right]_2^1$

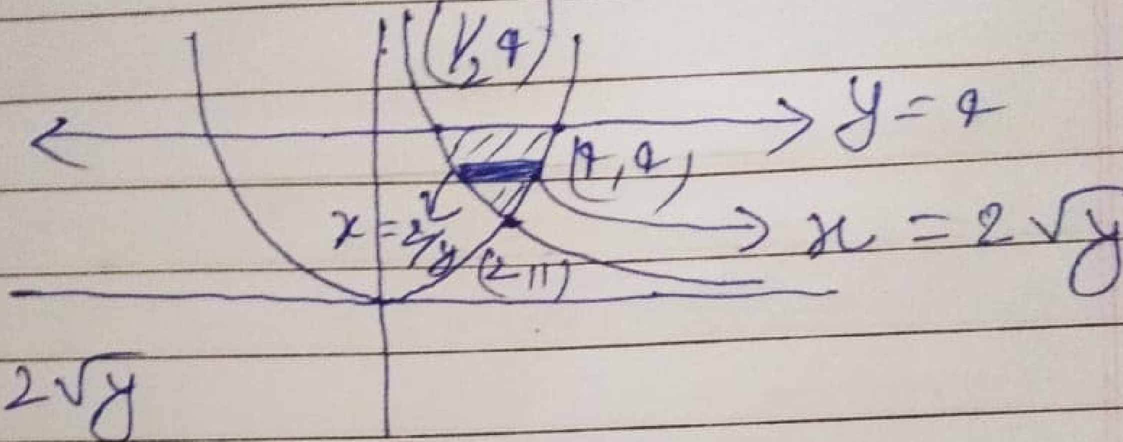
$\displaystyle = \left[4 + (e^4-e^3) - \frac{9}{4}\right.$

$\displaystyle 16 \pm 2(e^4-e^3) - \frac{\cancel{36}\,9}{\cancel{4}}$

$\displaystyle = \left(e^4 - e^3 + \frac{21}{4}\right)$

$\displaystyle II = \int_3^4 \left[\int_1^2 xy + e^y)\,dx\right] dy.$

$\displaystyle = \int_3^4 \left[\frac{yx^2}{2} + e^y\right]_1^2 dy$



$$\int_1^4 \int_{2/y}^{2\sqrt{y}} dx\, dy$$

$$= \int_1^4 \left(2\sqrt{y} - 2/y\right) dy$$

$$= \frac{2\, y^{3/2}}{3/2}\Bigg|_1^4 - 2\ln y\Bigg|_1^4$$

$$= \frac{4}{3}\cdot 8 - \frac{4}{3}\, 1 - 2\ln 4 + 0$$

$$= \frac{32}{3} - \frac{4}{3} - 4\ln 2$$

$$= \left(\frac{28}{3} - 4\ln 2\right)$$

Que (4)

21BCS7206

Ashutosh Gautam

$$= \iint_R r^3\, dr\, d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \int_{2\cos\theta}^{4\cos\theta} r^3\, dr\, d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \left(\frac{r^4}{4}\right)_{2\cos\theta}^{4\cos\theta} d\theta.$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{4}\left(256\cos^4\theta - 16\cos\theta\right) d\theta.$$

$$= 60 \int_{-\pi/2}^{\pi/2} \cos^4\theta\, d\theta.$$

$$= 60 \int_{-\pi/2}^{\pi/2} \cos^4\theta\, d\theta = 120 \int_0^{\pi/2} \cos^4\theta\, d\theta.$$

$$= \frac{120 \times 3 \times 1 \times \pi/2}{4 \times 2} = \frac{45\pi}{2} \text{ unit}^2 \quad \underline{\text{Ans}}$$

$$\int_0^1\int_0^{1-x}\int_0^{1-x^2-y^2} dz\,dy\,dx.$$

$$\int_0^1\int_0^{1-x} 1-x^2-y^2\,dy\,dx.$$

$$\int_0^1 (1-x-0)-x^2(1-x-0)-\frac{(1-x)^3}{3}+0\,dx$$

$$=\int_0^1 1-x-x^2(1-x)-\frac{(1-x)^3}{3}\,dx$$

[redacted]

$$=\int_0^1 1-x-x^2-x^3-\left(\frac{1-x}{3}\right)^3\,dx$$

$$= x\Big|_0^1-\frac{x^2}{2}\Big|_0^1-\frac{x^3}{3}\Big|_0^1-\frac{x^4}{4}\Big|_0^1+\frac{(1-x)^4}{4}\Big|_0^1$$

$$= 1-\frac{1}{2}-\frac{1}{3}-\frac{1}{4}+0-\frac{1}{4}$$

$$= 1-\tfrac{1}{2}-\tfrac{1}{3}-\tfrac{1}{2}$$

$$= 1-1-\tfrac{1}{3}$$

$$= -\tfrac{1}{3}$$

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$$4abc\,\pi/3 \int_0^{\pi/2} (1-\sin^2\theta)^{3/2} \cos\theta \, d\theta$$

$$= 4abc\,\pi/3 \int_0^{\pi/2} \cos^3\theta \cos\theta \, d\theta$$

$$= 4abc\,\pi/3 \; \frac{3\times 1}{4\times 2\times 1} \times \pi/2$$

$$= \frac{\pi^2}{4} abc$$

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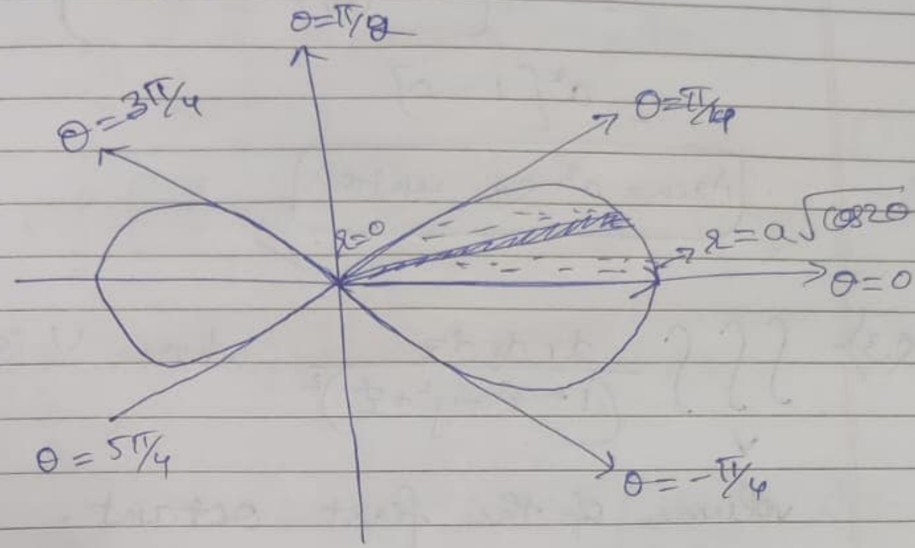
ASSIGNMENT - 2

Q1) Find the area enclosed by the leminscate $r^2 = a^2 \cos 2\theta$



$r^2 = a^2 \cos 2\theta$

$\Rightarrow r = a\sqrt{\cos 2\theta}$

In 1st Quadrant,

$\theta = 0$ to $\pi/4$

$\Rightarrow r = 0$ to $r = a\sqrt{\cos 2\theta}$

The given curve is symmetrical about both initial and extended axes.

$$\therefore \text{Area} = 4 \int_0^{\pi/4} \left(\int_0^{a\sqrt{\cos 2\theta}} r\,dr \right) d\theta$$

$$= 4 \int_0^{\pi/4} \left[\frac{r^2}{2} \right]_0^{a\sqrt{\cos 2\theta}} d\theta$$

$$= 2 \int_0^{\pi/4} a^2 \cos 2\theta \, d\theta$$

$$= \frac{\tan^{-1} a}{2} \int_0^{\pi/2} -[\cos\theta]_0^{\pi/2} \, d\phi$$

$$= \frac{\tan^{-1} a}{2} \int_0^{\pi/2} -(0-1) \, d\phi$$

$$= \frac{\tan^{-1} a}{2} \int_0^{\pi/2} 1 \, d\phi$$

$$= \frac{\tan^{-1} a}{2} [\phi]_0^{\pi/2}$$

$$\therefore \boxed{\text{Volume} = \frac{\pi}{4} \tan^{-1} a}$$

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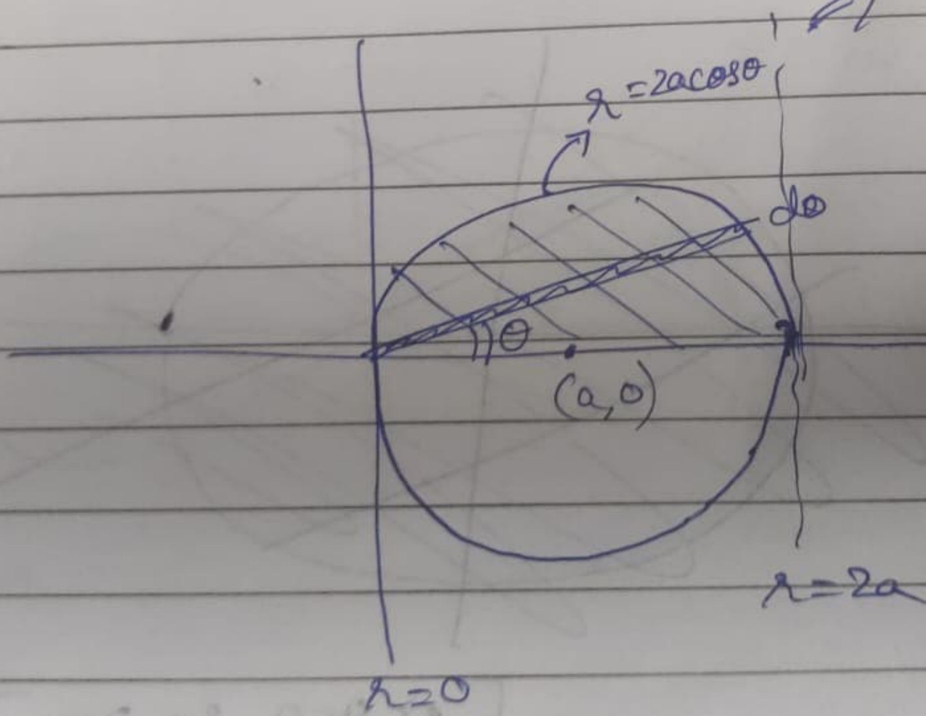
Q.2) $\iint_A r^2 \sin\theta \, d\theta \, dr$ where A is $r = 2a\cos\theta$ above initial line.



$r = 2a\cos\theta$

$\Rightarrow r^2 = 2ar\cos\theta$

$x^2 + y^2 = 2ax$

$(x^2 - 2ax + a^2) + y^2 = a^2$

$\boxed{(x-a)^2 + (y-0)^2 = a^2}$

Circle with centre $(a,0)$ and radius $= a$

when $\theta = 0 \Rightarrow r = 2a$

$\theta = \frac{\pi}{2} \Rightarrow r = 0$ (above initial line)

$$r = 2a\cos\theta$$
$$\frac{r}{2a} = \cos\theta$$
$$\theta = \cos^{-1}\left(\frac{r}{2a}\right)$$

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$$\text{Area} = \int_{r=0}^{r=2a} \int_{\theta=0}^{\theta=\cos^{-1}\left(\frac{r}{2a}\right)} r^2 \sin\theta \, d\theta \, dr$$

$$= \int_0^{2a} r^2 \left[\int_0^{\cos^{-1}\left(\frac{r}{2a}\right)} \sin\theta \, d\theta \right] dr$$

$$= -\int_0^{2a} r^2 \left[\cos\theta\right]_0^{\cos^{-1}\left(\frac{r}{2a}\right)} dr$$

$$= -\int_0^{2a} r^2 \left[\cos\left(\cos^{-1}\left(\frac{r}{2a}\right)\right) - \cos(0)\right] dr$$

$$= -\int_0^{2a} r^2 \left[\frac{r}{2a} - 1\right] dr$$

$$= \int_0^{2a} r^2 \left(1 - \frac{r}{2a}\right) dr$$

$$= \int_0^{2a} \left(r^2 - \frac{r^3}{2a}\right) dr$$

$$= \frac{1}{3}\left[r^3\right]_0^{2a} - \frac{1}{2a}\left(\frac{1}{4}\right)\left[r^4\right]_0^{2a}$$

$$= \frac{8a^3}{3} - \frac{1}{8a}\left[16a^4\right]$$

$$= \frac{8a^3}{3} - 2a^3$$

$$\therefore \boxed{\text{Area} = \frac{2a^3}{3} \text{ sq. units}}$$

$$= \frac{1}{3}\int_0^1 z\left[\frac{(1-z)^5}{2} - \frac{(1-z)^5}{5} - (1-z)^5 + \frac{3}{4}(1-z)^5\right]dz$$

$$= \frac{1}{3}\int_0^1 z(1-z)^5\left[\frac{1}{2} - \frac{1}{5} - 1 + \frac{3}{4}\right]dz$$

$$= \frac{1}{3}\int_0^1 z(1-z)^5\left[\frac{20-8-40+30}{40}\right]dz$$

$$= \frac{1}{3}\int_0^1 z(1-z)^5\left(\frac{2}{40}\right)dz$$

$$= \frac{1}{60}\int_0^1 z\left[1-5z+10z^2-10z^3+5z^4-z^5\right]dz$$

$$= \frac{1}{60}\int_0^1\left[z-5z^2+10z^3-10z^4+5z^5-z^6\right]dz$$

$$= \frac{1}{60}\left(\left[\frac{z^2}{2}\right]_0^1 - \frac{5}{3}\left[z^3\right]_0^1 + \frac{10}{4}\left[z^4\right]_0^1 - \frac{10}{5}\left[z^5\right]_0^1 + \frac{5}{6}\left[z^6\right]_0^1 - \frac{1}{7}\left[z^7\right]_0^1\right)$$

$$= \frac{1}{60}\left[\frac{1}{2} - \frac{5}{3} + \frac{5}{2} - 2 + \frac{5}{6} - \frac{1}{7}\right]$$

$$= \frac{1}{60}\left[\frac{21-70+105-84+35-6}{42}\right]$$

$$= \frac{1}{60}\left[\frac{161-160}{42}\right] = \frac{1}{2520} \text{ cubic units}$$

$$= 4\left[2(\sin\theta)_0^{\pi/2} + \frac{1}{2}\cdot\frac{\pi}{2}\right]$$

$$= 4\left[2 + \frac{\pi}{4}\right]$$

$$= (8 + \pi) \text{ sq. units}$$

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4.) $\iiint x^2 yz \, dx\, dy\, dz$ throughout the volume bounded by $x=0, y=0, z=0,$ $x+y+z=1$.

$$\text{Volume} = \int_0^1 \int_0^{(1-z)} \int_0^{(1-y-z)} x^2 y z \, dx\, dy\, dz$$

$$= \int_0^1 z \left[\int_0^{(1-z)} y \left[\int_0^{(1-y-z)} x^2 \, dx \right] dy \right] dz$$

$$= \int_0^1 z \int_0^{(1-z)} \frac{y}{3} \left[x^3 \right]_0^{(1-y-z)} dy\, dz$$

$$= \frac{1}{3} \int_0^1 z \int_0^{(1-z)} y (1-y-z)^3 \, dy\, dz$$

$$= \frac{1}{3} \int_0^1 z \int_0^{(1-z)} y \left((1-z) - y \right)^3 dy\, dz$$

$$= \frac{1}{3} \int_0^1 z \int_0^{(1-z)} y \left[(1-z)^3 - y^3 - 3(1-z)^2 y + 3(1-z) y^2 \right] dy\, dz$$

$$= \frac{1}{3} \int_0^1 z \int_0^{(1-z)} \left[y(1-z)^3 - y^4 - 3(1-z)^2 y^2 + 3(1-z) y^3 \right] dy\, dz$$

$$= \frac{1}{3} \int_0^1 z \left[\frac{(1-z)^3}{2} \left[y^2 \right]_0^{(1-z)} - \frac{1}{5} \left[y^5 \right]_0^{(1-z)} - (1-z)^2 \left[y^3 \right]_0^{(1-z)} + \frac{3}{4} (1-z) \left[y^4 \right]_0^{(1-z)} \right] dz$$

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$$= \int_{\pi/4}^{\pi/2} \cos\theta \left[\int_0^a r^2\, dr \right] d\theta$$

$$= \int_{\pi/4}^{\pi/2} \frac{\cos\theta}{3} \left[r^3 \right]_0^a d\theta$$

$$= \int_{\pi/4}^{\pi/2} \frac{\cos\theta}{3} \left[a^3 \right] d\theta$$

$$= \frac{a^3}{3} \int_{\pi/4}^{\pi/2} \cos\theta\, d\theta$$

$$= \frac{a^3}{3} \left[\sin\theta \right]_{\pi/4}^{\pi/2}$$

$$= \frac{a^3}{3} \left[\sin\frac{\pi}{2} - \sin\frac{\pi}{4} \right]$$

$$= \frac{a^3}{3} \left[1 - \frac{1}{\sqrt{2}} \right]$$

$$= \frac{a^3}{3\sqrt{2}} (\sqrt{2} - 1) \text{ sq. units}$$

$$r = 2a\cos\theta$$
$$\frac{r}{2a} = \cos\theta$$
$$\theta = \cos^{-1}\left(\frac{r}{2a}\right)$$

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$$\text{Area} = \int_{r=0}^{r=2a} \int_{\theta=0}^{\theta=\cos^{-1}\left(\frac{r}{2a}\right)} r^2 \sin\theta \, d\theta \, dr$$

$$= \int_0^{2a} r^2 \left[\int_0^{\cos^{-1}\left(\frac{r}{2a}\right)} \sin\theta \, d\theta \right] dr$$

$$= -\int_0^{2a} r^2 \left[\cos\theta\right]_0^{\cos^{-1}\left(\frac{r}{2a}\right)} dr$$

$$= -\int_0^{2a} r^2 \left[\cos\left(\cos^{-1}\left(\frac{r}{2a}\right)\right) - \cos(0)\right] dr$$

$$= -\int_0^{2a} r^2 \left[\frac{r}{2a} - 1\right] dr$$

$$= \int_0^{2a} r^2 \left(1 - \frac{r}{2a}\right) dr$$

$$= \int_0^{2a} \left(r^2 - \frac{r^3}{2a}\right) dr$$

$$= \frac{1}{3}\left[r^3\right]_0^{2a} - \frac{1}{2a}\left(\frac{1}{4}\right)\left[r^4\right]_0^{2a}$$

$$= \frac{8a^3}{3} - \frac{1}{8a}\left[\overset{2}{\cancel{16}}a^4\right]$$

$$= \frac{8a^3}{3} - 2a^3$$

$$\therefore \boxed{\text{Area} = \frac{2a^3}{3} \text{ sq. units}}$$

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Q.2) $\iint_A r^2 \sin\theta \, d\theta \, dr$ where A is $r = 2a\cos\theta$ above initial line.

$r = 2a\cos\theta$

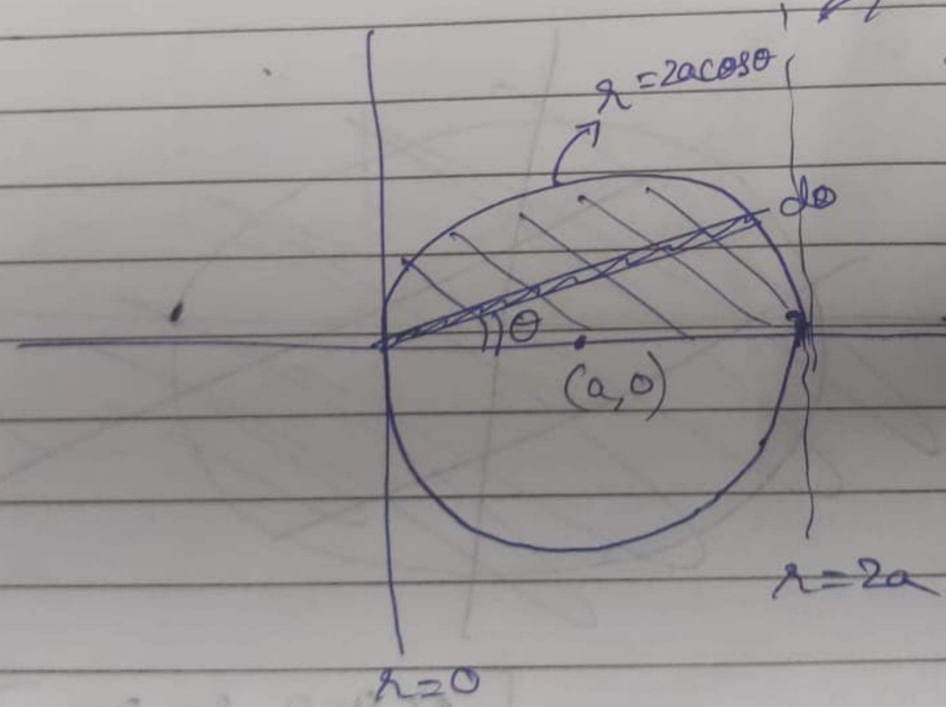
$\Rightarrow r^2 = 2ar\cos\theta$

$x^2 + y^2 = 2ax$

$(x^2 - 2ax + a^2) + y^2 = a^2$

$\boxed{(x-a)^2 + (y-0)^2 = a^2}$

Circle with centre $(a,0)$ and radius $= a$



when $\theta = 0 \Rightarrow r = 2a$

$\theta = \frac{\pi}{2} \Rightarrow r = 0$ (above initial line)

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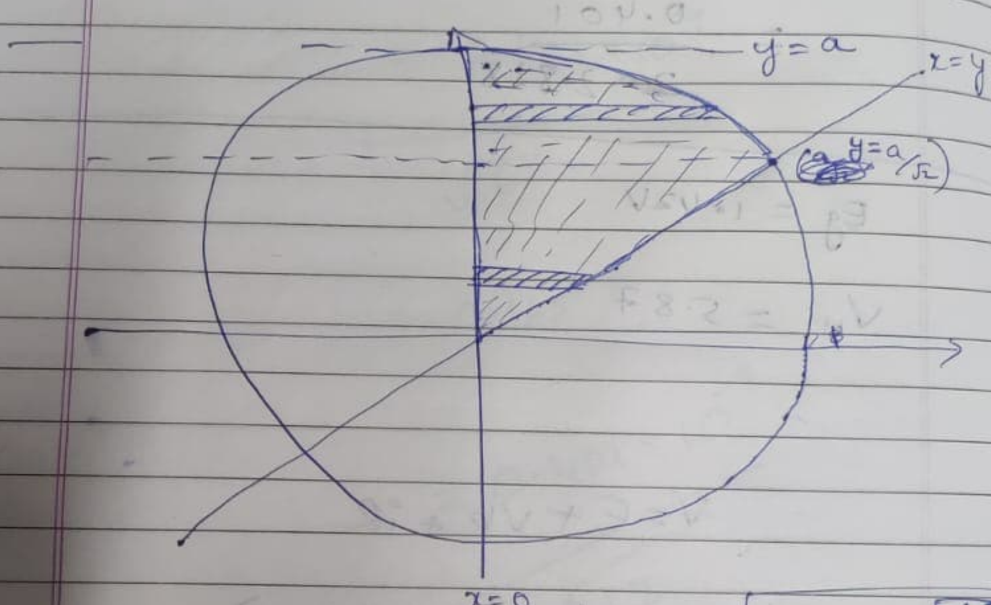
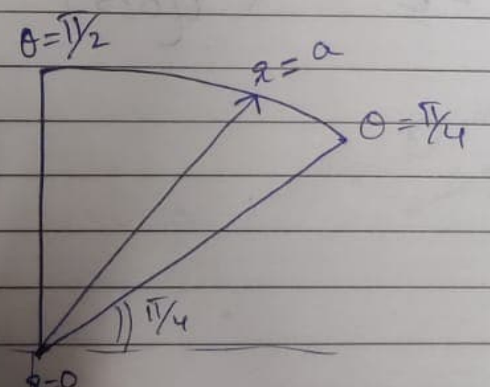
3) Express as single integral and evaluate:

$$\int_0^{a/\sqrt{2}} \int_0^{y} x\,dx\,dy + \int_{a/\sqrt{2}}^{a} \int_0^{\sqrt{a^2-y^2}} x\,dx\,dy$$





$x = \sqrt{a^2-y^2}$

$\Rightarrow x^2+y^2 = a^2$

$x = y$

$\Rightarrow 2y^2 = a^2$

$\Rightarrow \boxed{y = \frac{\pm a}{\sqrt{2}}}$

Changing from cartesian to polar form, we get,

$x = r\cos\theta$, $y = r\sin\theta$

$$\Rightarrow \int_0^{a/\sqrt{2}} \int_0^{y} x\,dx\,dy + \int_{a/\sqrt{2}}^{a} \int_0^{\sqrt{a^2-y^2}} x\,dx\,dy = \int_{\pi/4}^{\pi/2} \int_0^{a} r\cos\theta \cdot r\,dr\,d\theta$$

$$= \frac{1}{3}\int_0^1 z\left[\frac{(1-z)^5}{2} - \frac{(1-z)^5}{5} - (1-z)^5 + \frac{3}{4}(1-z)^5\right]dz$$

$$= \frac{1}{3}\int_0^1 z(1-z)^5\left[\frac{1}{2} - \frac{1}{5} - 1 + \frac{3}{4}\right]dz$$

$$= \frac{1}{3}\int_0^1 z(1-z)^5\left[\frac{20-8-40+30}{40}\right]dz$$

$$= \frac{1}{3}\int_0^1 z(1-z)^5\left(\frac{2}{40}\right)dz$$

$$= \frac{1}{60}\int_0^1 z\left[1-5z+10z^2-10z^3+5z^4-z^5\right]dz$$

$$= \frac{1}{60}\int_0^1 \left[z-5z^2+10z^3-10z^4+5z^5-z^6\right]dz$$

$$= \frac{1}{60}\left(\left[\frac{z^2}{2}\right]_0^1 - \frac{5}{3}\left[z^3\right]_0^1 + \frac{10}{4}\left[z^4\right]_0^1 - \frac{10}{5}\left[z^5\right]_0^1 + \frac{5}{6}\left[z^6\right]_0^1 - \frac{1}{7}\left[z^7\right]_0^1\right)$$

$$= \frac{1}{60}\left[\frac{1}{2} - \frac{5}{3} + \frac{5}{2} - 2 + \frac{5}{6} - \frac{1}{7}\right]$$

$$= \frac{1}{60}\left[\frac{21-70+105-84+35-6}{42}\right]$$

$$= \frac{1}{60}\left[\frac{161-160}{42}\right] = \frac{1}{2520} \text{ cubic units}$$

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$$= \cancel{2}a^2 \left[\frac{\sin 2\theta}{\cancel{2}}\right]_0^{\pi/4}$$

$$= a^2 \left[\sin 2\left(\frac{\pi}{4}\right) - \sin 2(0)\right]$$

$$= a^2[1-0]$$

$$\therefore \boxed{\text{Area} = a^2 \text{ sq. units}}$$

Q.3) $\iiint_V \frac{dx\,dy\,dz}{(1+x^2+y^2+z^2)^2}$ where V is the

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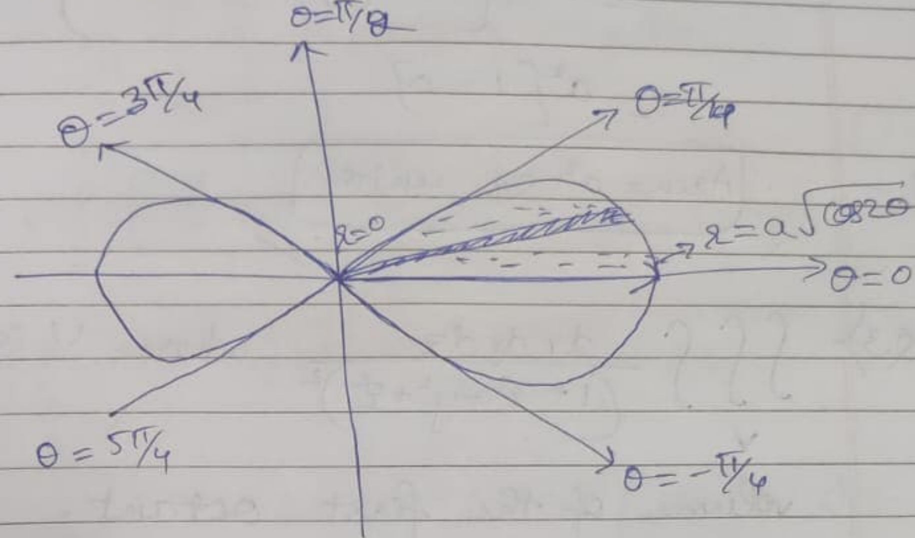
ASSIGNMENT - 2

Q1) Find the area enclosed by the leminscate $r^2 = a^2 \cos 2\theta$



$r^2 = a^2 \cos 2\theta$

$\Rightarrow r = a\sqrt{\cos 2\theta}$

In 1st Quadrant,

$\theta = 0$ to $\pi/4$

$\Rightarrow r = 0$ to $r = a\sqrt{\cos 2\theta}$

The given curve is symmetrical about both initial and extended axes.

$\therefore$ Area $= 4 \int_0^{\pi/4} \left(\int_0^{a\sqrt{\cos 2\theta}} r \, dr \right) d\theta$

$= 4 \int_0^{\pi/4} \left[\frac{r^2}{2} \right]_0^{a\sqrt{\cos 2\theta}} d\theta$

$= 2 \int_0^{\pi/4} a^2 \cos 2\theta \, d\theta$

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Q.4) $\iiint_V \frac{dx\,dy\,dz}{(1+x^2+y^2+z^2)^2}$, where V is the volume in the first octant.

We know,

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

and $dx\,dy\,dz = r^2\sin\theta\,dr\,d\theta\,d\phi$

$$\Rightarrow \iiint_V \frac{dx\,dy\,dz}{(1+x^2+y^2+z^2)^2}$$

$$\Rightarrow = \int_0^{\pi/2}\int_0^{\pi/2}\int_0^a \frac{r^2\sin\theta\,dr\,d\theta\,d\phi}{(1+r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta)^2}$$

$$= \int_0^{\pi/2}\int_0^{\pi/2}\int_0^a \frac{r^2\sin\theta\,dr\,d\theta\,d\phi}{(1+r^2\sin^2\theta(\cos^2\phi+\sin^2\phi)+r^2\cos^2\theta)^2}$$

$$= \int_0^{\pi/2}\int_0^{\pi/2}\int_0^a \frac{r^2\sin\theta\,dr\,d\theta\,d\phi}{(1+r^2(\sin^2\theta+\cos^2\theta))^2}$$

$$= \int_0^{\pi/2}\int_0^{\pi/2}\int_0^a \frac{r^2\sin\theta\,dr\,d\theta\,d\phi}{(1+r^2)^2}$$

$$= \cancel{2}a^2 \left[\frac{\sin 2\theta}{\cancel{2}}\right]_0^{\pi/4}$$

$$= a^2 \left[\sin 2\left(\frac{\pi}{4}\right) - \sin 2(0)\right]$$

$$= a^2[1 - 0]$$

$$\therefore \boxed{\text{Area} = a^2 \text{ sq. units}}$$

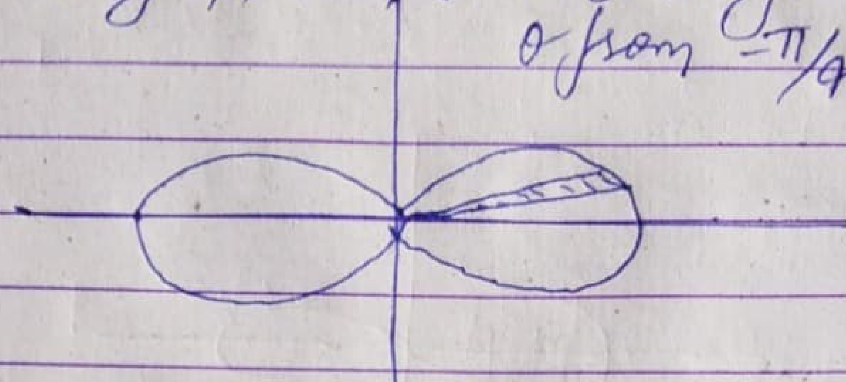
Q.3) $\displaystyle\iiint_V \frac{dx\,dy\,dz}{(1+x^2+y^2+z^2)^2}$ where V is the

$$= 4\left[2\left(\sin\theta\right)_0^{\pi/2} + \frac{1}{2}\cdot\frac{\pi}{2}\right]$$

$$= 4\left[2 + \frac{\pi}{4}\right]$$

$$= (8 + \pi) \text{ sq. units}$$

The curve is symmetrical with respect to the origin, and occurs only with values of θ from $-\pi/4$ to $\pi/4$



The area in the polar coordinates is

$$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 \, d\theta$$

$$A = 4\left[\frac{1}{2}\int_0^{\pi/4} a^2 \cos 2\theta \, d\theta\right]$$

$$A = 2a^2 \left[\frac{1}{2}\sin 2\theta\right]_0^{\pi/4}$$

$$A = a^2\left[\sin \pi/2 - \sin 0\right]$$

$$\boxed{A = a^2 \text{ unit}^2}$$ Ans

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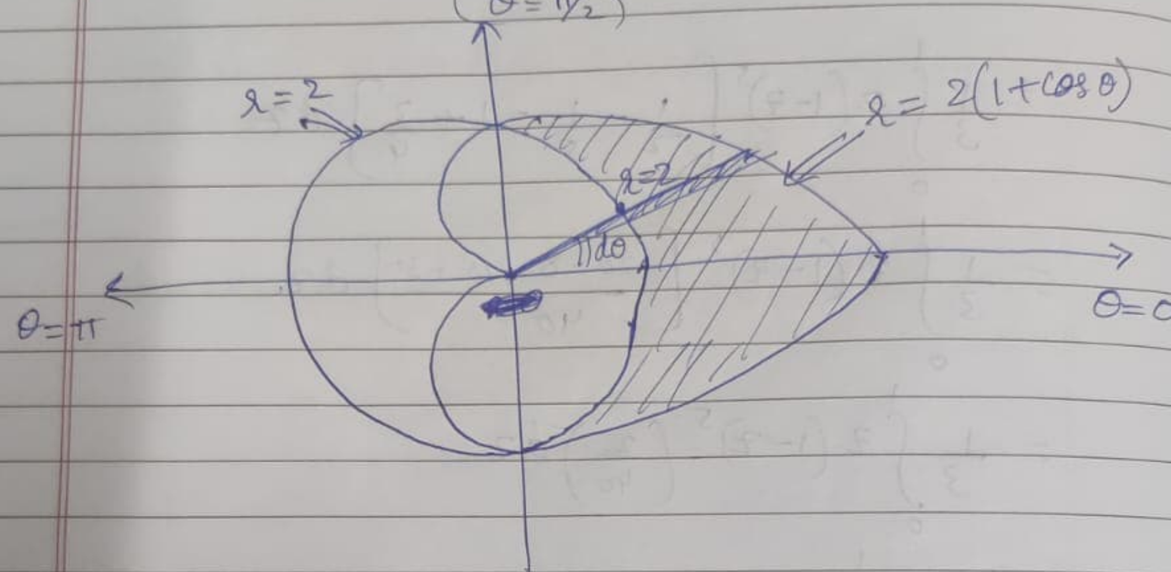
5.) Find the area outside the circle $r=2$ and inside the cardoid $r=2(1+\cos\theta)$.



The circle and cardiod are symmetrical about initial axis.

$$\therefore \text{Area} = 2\int_{\theta=0}^{\pi/2}\int_{r=2}^{r=2(1+\cos\theta)} r\, d\theta\, dr$$

$$= \cancel{2}\int_0^{\pi/2}\left(\frac{r^2}{\cancel{2}}\right)_2^{2(1+\cos\theta)} dr$$

$$= \int_0^{\pi/2}\left[4(1+\cos\theta)^2 - 4\right] dr$$

$$= 4\int_0^{\pi/2}\left(1+2\cos\theta+\cos^2\theta-1\right) dr$$

$$= 4\int_0^{\pi/2}\left(2\cos\theta+\cos^2\theta\right) d\theta$$

Express as single integral and evaluate :

$$\int_0^1 \int_0^y (x^2 + y^2)\, dx\, dy + \int_1^2 \int_0^{2-y} (x^2 + y^2)\, dx\, dy$$

format your answer