

Average Runtime Complexity of the Non-Random Pivot version of Quick Sort

Assume a bad split $\Rightarrow T\left(\frac{9}{10}n\right) + T\left(\frac{1}{10}n\right)$.

Solve with substitution method : can assume best & worst.

$$n \lg n \leq \text{avg} \leq n^2$$

Guess : $T(n) \leq cn \lg n$

$$T(n) = T\left(\frac{9n}{10}\right) + T\left(\frac{n}{10}\right) + n$$

Assume : $T(n) \leq cn \lg n$

$$T(n) = T\left(\frac{9n}{10}\right) + T\left(\frac{n}{10}\right) + n \leq cn \lg n$$

$$T\left(\frac{9n}{10}\right) \leq c \frac{9n}{10} \log \frac{9n}{10}$$

$$T\left(\frac{n}{10}\right) \leq c \frac{n}{10} \log \frac{n}{10}$$

$$\therefore T(n) = T\left(\frac{9n}{10}\right) + T\left(\frac{n}{10}\right) + n \leq c \frac{9n}{10} \log \left(\frac{9}{10}n\right) + \frac{c}{10} \log \left(\frac{n}{10}\right) + n$$

$$= \frac{c}{10} \left[9n \log \left(\frac{n}{10/9}\right) + \log(n) - \log(10) \right]$$

$$= \frac{c}{10} \left[9n (\log(n) + \log(9/10)) + \log(n) - 1 \right]$$

$$= \frac{c}{10} \left[9n \log n + n(9 \log(9) - 9) + \log(n) - 1 \right]$$

$$\leq \frac{c}{10} \left[9n \log n + \log(n) \right]$$

$$\leq \frac{c}{10} \left[9n \log n + n \log n \right]$$

$$\leq \frac{c}{10} \left[10 n \log n \right] \Rightarrow c n \log n$$

$$\therefore T(n) = O(n \log n)$$