

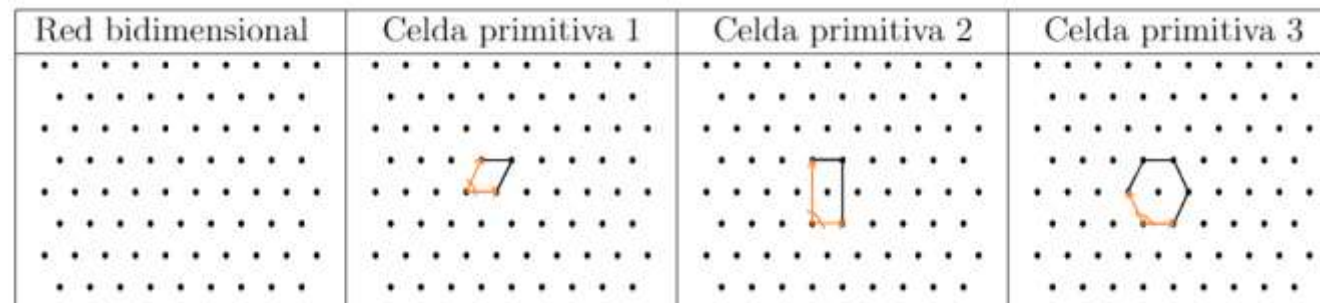
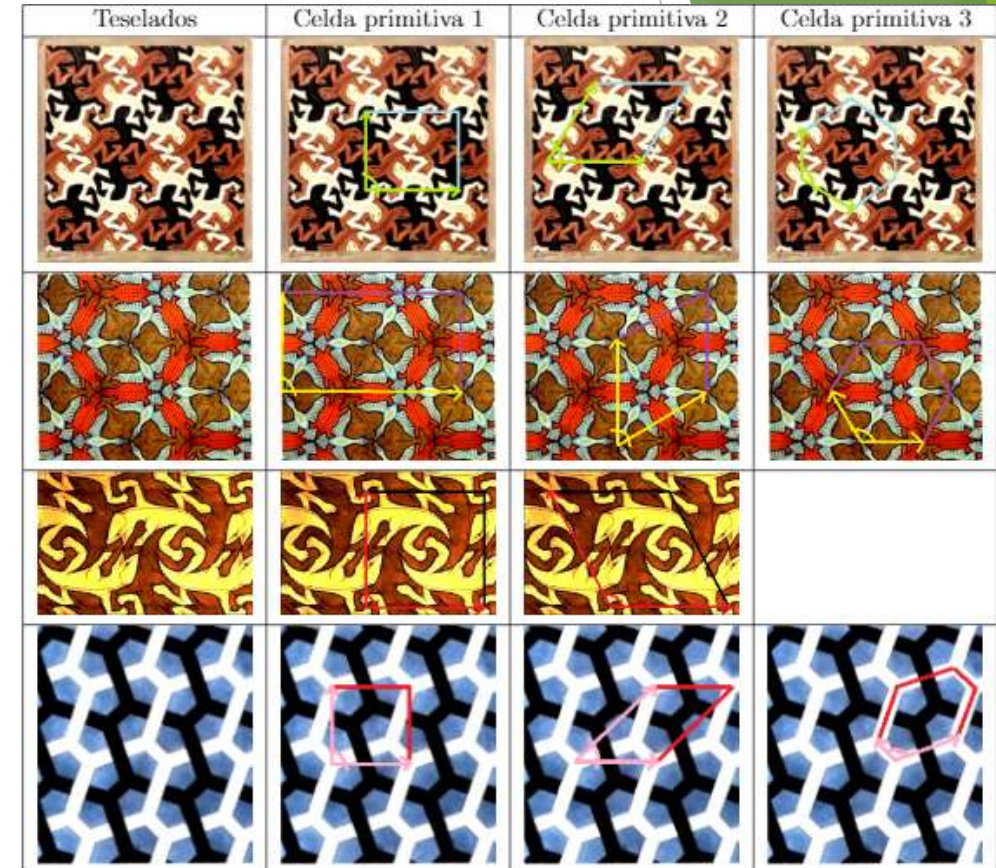
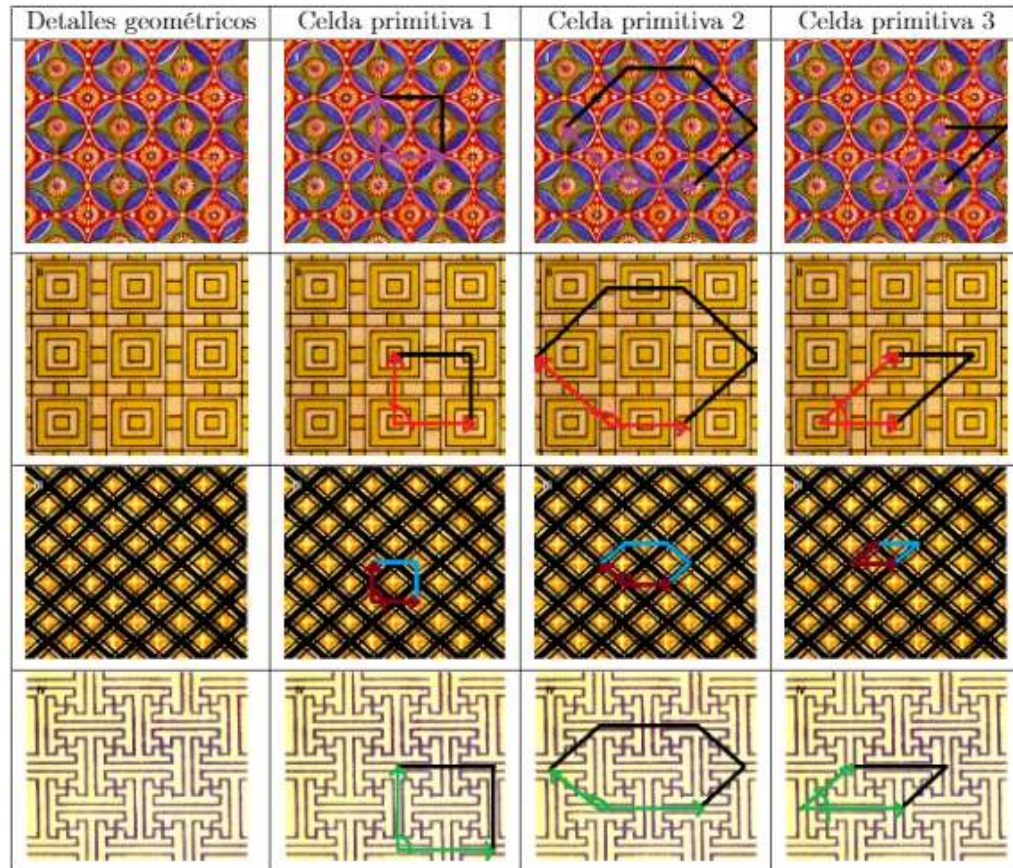


Redes de Bravais

Métodos matemáticos para físicos I
Profesor: Luis Nuñez

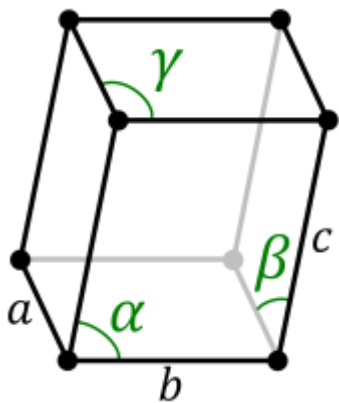
Gabriela Sánchez Ariza - 2200816
Nicolás Toledo - 2200017

Inciso a

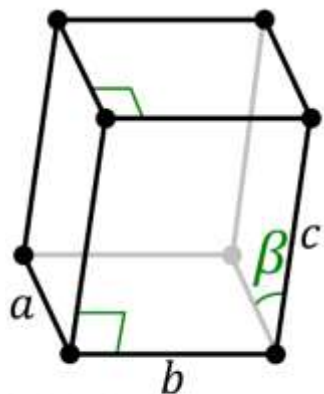


Inciso b

Triclínica



Monoclínica



$$|a| \neq |b| \neq |c| \text{ y } \alpha = \gamma = 90^\circ \\ \beta < 90^\circ$$

$$\mathbf{a} = (a_1, a_2, a_3), \quad \mathbf{b} = (b_1, b_2, b_3), \quad \mathbf{c} = (c_1, c_2, c_3)$$

$$v = \mathbf{c} * (\mathbf{a} \times \mathbf{b}) = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

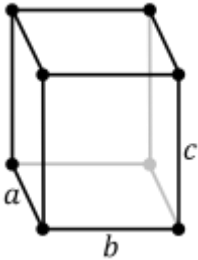
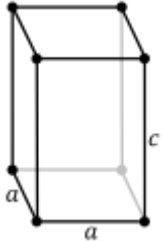
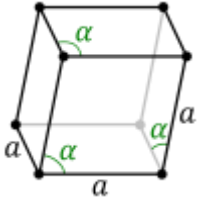
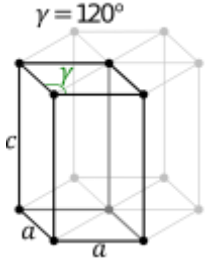
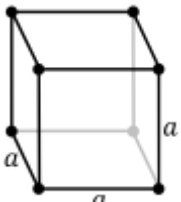
$$v^2 = \text{Det}(\mathbf{D}\mathbf{D}^T) = \begin{vmatrix} \mathbf{c} \cdot \mathbf{c} & \mathbf{c} \cdot \mathbf{a} & \mathbf{c} \cdot \mathbf{b} \\ \mathbf{c} \cdot \mathbf{a} & \mathbf{a} \cdot \mathbf{a} & \mathbf{a} \cdot \mathbf{b} \\ \mathbf{c} \cdot \mathbf{b} & \mathbf{a} \cdot \mathbf{b} & \mathbf{b} \cdot \mathbf{b} \end{vmatrix}$$

$$v^2 = \text{Det}(\mathbf{D}\mathbf{D}^T) = (\mathbf{c} \cdot \mathbf{c})((\mathbf{a} \cdot \mathbf{a})(\mathbf{b} \cdot \mathbf{b}) - (\mathbf{a} \cdot \mathbf{b})^2) - (\mathbf{c} \cdot \mathbf{a})((\mathbf{c} \cdot \mathbf{a})(\mathbf{b} \cdot \mathbf{b}) - (\mathbf{a} \cdot \mathbf{b})(\mathbf{c} \cdot \mathbf{b})) + (\mathbf{c} \cdot \mathbf{b})((\mathbf{c} \cdot \mathbf{a})(\mathbf{a} \cdot \mathbf{b}) - (\mathbf{a} \cdot \mathbf{a})(\mathbf{c} \cdot \mathbf{b}))$$

$$v = |\mathbf{a}||\mathbf{b}||\mathbf{c}|\sqrt{1 - \cos^2 \gamma - \cos^2 \beta - \cos^2 \alpha + 2\cos\gamma\cos\beta\cos\alpha}$$

$$v = |\mathbf{a}||\mathbf{b}||\mathbf{c}|\sqrt{1 - \cos^2 \beta}$$

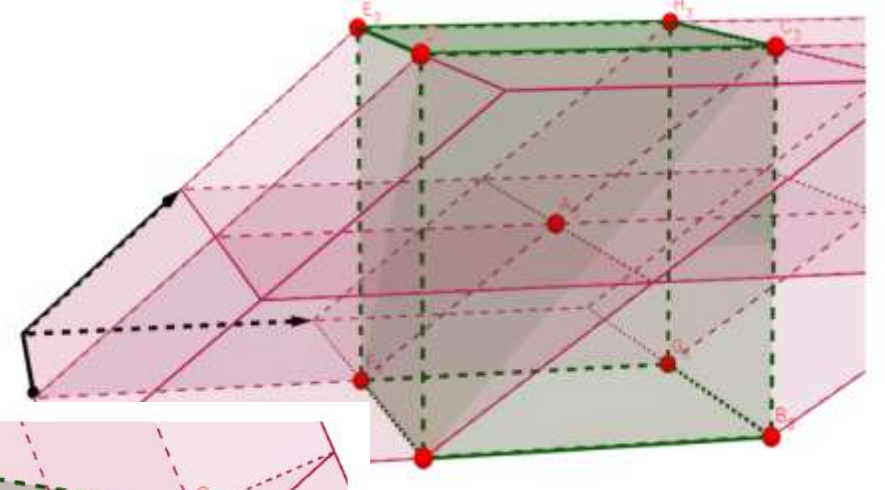
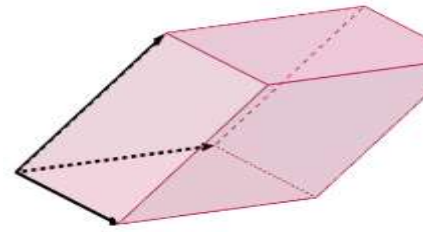
$$v = |\mathbf{a}||\mathbf{b}||\mathbf{c}|\sin\beta$$

Ortorrómbica		$v = \mathbf{a} \mathbf{b} \mathbf{c} $
Tetragonal		$v = \mathbf{a} ^2 \mathbf{c} $
Romboédrico		$v = \mathbf{a} ^3 \sqrt{1 - 3 \cos^2 \alpha + 2 \cos^3 \alpha}$
Hexagonal		$v = \mathbf{a} ^2 \mathbf{c} \frac{\sqrt{3}}{2}$
Cúbico		$v = \mathbf{a} ^3$

Inciso c

Parte I: BCC $\mathbf{a} = a\hat{\mathbf{i}}, \mathbf{b} = a\hat{\mathbf{j}}, \mathbf{c} = a(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})/2$

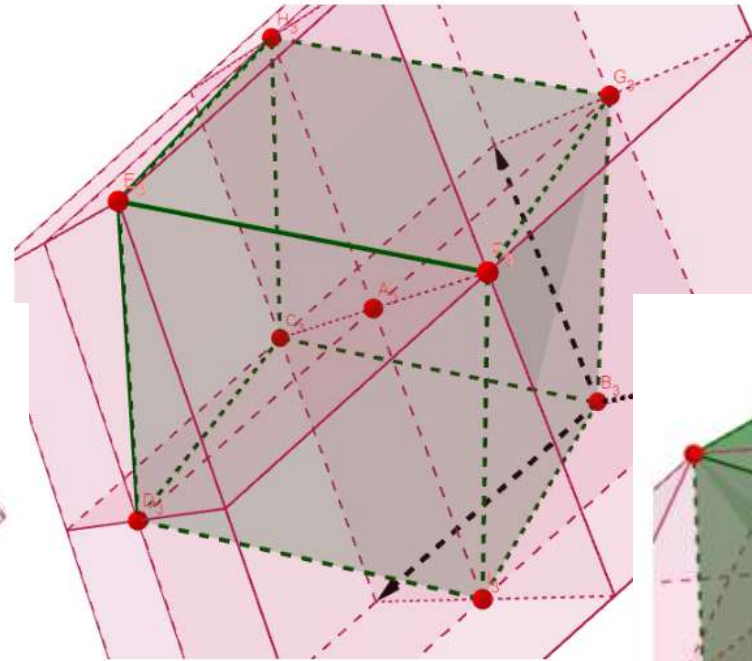
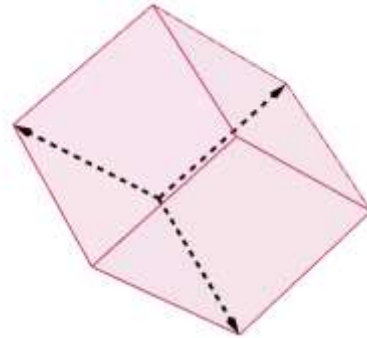
$$\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) = \begin{vmatrix} \frac{a}{2} & \frac{a}{2} & \frac{a}{2} \\ a & 0 & 0 \\ 0 & a & 0 \end{vmatrix} = \frac{a^3}{2}$$



Parte II: BCC

$\mathbf{a} = a(\hat{\mathbf{j}} + \hat{\mathbf{k}} - \hat{\mathbf{i}})/2, \mathbf{b} = a(\hat{\mathbf{i}} + \hat{\mathbf{k}} - \hat{\mathbf{j}})/2, \mathbf{c} = a(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})/2$

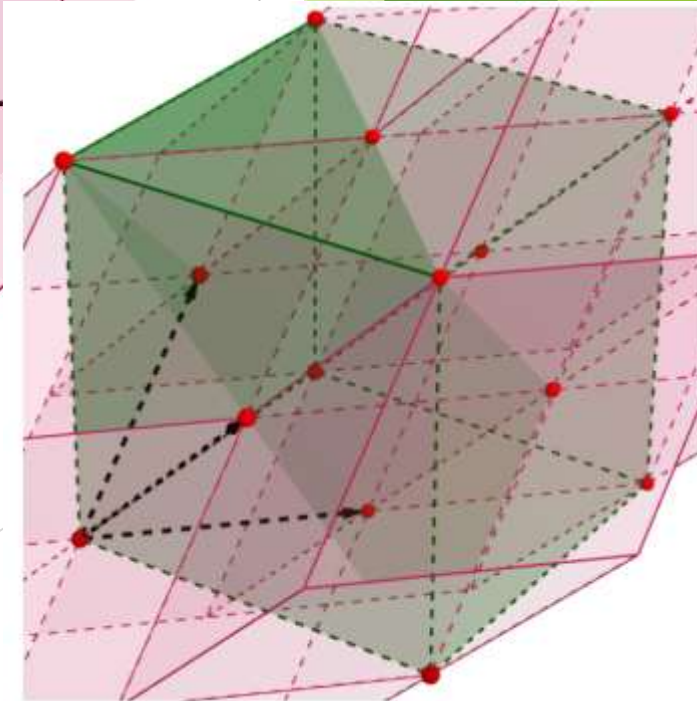
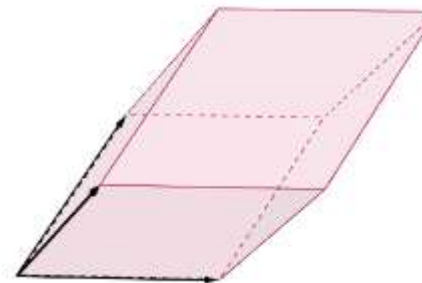
$$\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) = \begin{vmatrix} \frac{a}{2} & \frac{a}{2} & -\frac{a}{2} \\ -\frac{a}{2} & \frac{a}{2} & \frac{a}{2} \\ \frac{a}{2} & -\frac{a}{2} & \frac{a}{2} \end{vmatrix} = \frac{a^3}{2}$$



Parte III: FCC

$\mathbf{a} = a(\hat{\mathbf{j}} + \hat{\mathbf{k}})/2, \mathbf{b} = a(\hat{\mathbf{i}} + \hat{\mathbf{k}})/2, \mathbf{c} = a(\hat{\mathbf{i}} + \hat{\mathbf{j}})/2$

$$\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) = \begin{vmatrix} \frac{a}{2} & \frac{a}{2} & 0 \\ 0 & \frac{a}{2} & \frac{a}{2} \\ \frac{a}{2} & 0 & \frac{a}{2} \end{vmatrix} = \frac{a^3}{4}$$



Inciso d

	Vectores primitivos:	Vectores recíprocos	Volumen celda recíproca
Cúbico simple	$\mathbf{a} = a\hat{i}, \mathbf{b} = a\hat{j}, \mathbf{c} = a\hat{k}$	$\mathbf{a}' = (\frac{1}{a}, 0, 0) \quad \mathbf{b}' = (0, \frac{1}{a}, 0)$ $\mathbf{c}' = (0, 0, \frac{1}{a})$	$\mathbf{c}' \cdot (\mathbf{a}' \times \mathbf{b}') = V = \frac{1}{a^3}$
BCC (C-1)	$\mathbf{a} = a\hat{i}, \mathbf{b} = a\hat{j},$ $\mathbf{c} = a(\hat{i} + \hat{j} + \hat{k})/2$	$\mathbf{a}' = (\frac{1}{a}, 0, -\frac{1}{a}) \quad \mathbf{b}' = (0, \frac{1}{a}, -\frac{1}{a})$ $\mathbf{c}' = (0, 0, \frac{2}{a})$	$\mathbf{c}' \cdot (\mathbf{a}' \times \mathbf{b}') = V = \frac{2}{a^3}$
BCC (C-2)	$\mathbf{a} = \frac{a(\hat{j} + \hat{k} - \hat{i})}{2},$ $\mathbf{b} = \frac{a(\hat{i} + \hat{k} - \hat{j})}{2},$ $\mathbf{c} = a(\hat{i} + \hat{j} - \hat{k})/2$	$\mathbf{a}' = (0, \frac{1}{a}, \frac{1}{a}) \quad \mathbf{b}' = (\frac{1}{a}, 0, \frac{1}{a})$ $\mathbf{c}' = (\frac{1}{a}, \frac{1}{a}, 0)$	$\mathbf{c}' \cdot (\mathbf{a}' \times \mathbf{b}') = V = \frac{2}{a^3}$
FCC(C-3)	$\mathbf{a} = \frac{a(\hat{j} + \hat{k})}{2}, \mathbf{b} = \frac{a(\hat{i} + \hat{k})}{2},$ $\mathbf{c} = a(\hat{i} + \hat{j})/2$	$\mathbf{a}' = (-\frac{1}{a}, \frac{1}{a}, \frac{1}{a}) \quad \mathbf{b}' = (\frac{1}{a}, -\frac{1}{a}, \frac{1}{a})$ $\mathbf{c}' = (\frac{1}{a}, \frac{1}{a}, -\frac{1}{a})$	$\mathbf{c}' \cdot (\mathbf{a}' \times \mathbf{b}') = V = \frac{4}{a^3}$