Nicolás Toledo - 2200017

3 (3) (b) ① $\int_0^{\infty} \frac{x^2}{x^4+5x^2+6}\,dx$, es par, entonces es equivalente a

$$\int_{-\infty}^{\infty} \frac{x^2}{x^4+5x^2+6}\,dx$$

Extensión analítica:

$$\int_{-R}^{R} \frac{z^2}{(z^2+2)(z^2+3)}\,dz$$ Polos simples

$$\oint_C f(z)\,dz = \cancel{\int_\Gamma f(z)\,dz} + \int_{-r}^{r} f(x)\,dx = 2i\pi \sum \operatorname{Res} f(z)$$

0

Polos: $\pm\sqrt{2}\,i$, $\pm\sqrt{3}\,i$

$$\operatorname*{Res}_{(\sqrt{2}\,i)} f(z) = \lim_{z\to\sqrt{2}\,i} \left( \frac{1}{0!} \left[ (z-\sqrt{2}\,i)\cdot f(z) \right. \right)$$

Escaneado con CamScanner

Polos: $\pm\sqrt{2}\,i$ , $\pm\sqrt{3}\,i$

$$\underset{(\sqrt{2}\,i)}{\operatorname{Res} f(z)} = \lim_{z\to\sqrt{2}\,i} \left( \frac{1}{0!} \left[ (z-\sqrt{2}\,i)\cdot f(z) \right] \right)$$

$$\parallel$$

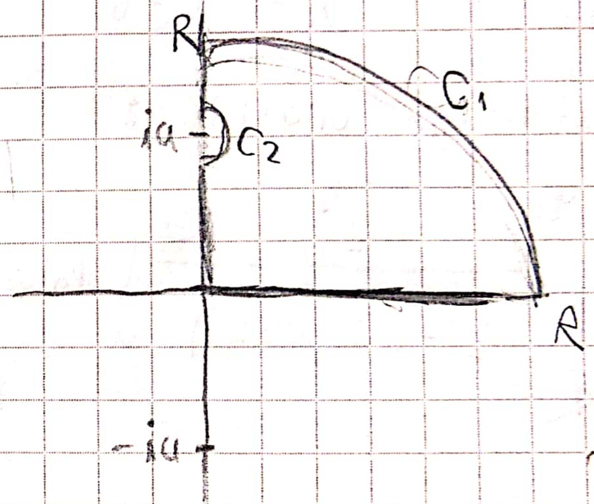
$$\frac{p(z_0)}{q'(z_0)} = \left.\frac{z^2}{4z^3+10z}\right|_{z=\sqrt{2}\,i} = i\frac{\sqrt{2}}{2} = \operatorname{Res} f(z)\Big|_{z=\sqrt{2}\,i}$$

$$\xrightarrow{z=\sqrt{3}\,i} -\frac{\sqrt{3}}{2}\,i = \operatorname{Res} f(z)\Big|_{z=\sqrt{3}\,i}$$

$$= 2\,i\,\pi\left( i\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\,i \right) = -\pi\left(\sqrt{2}-\sqrt{3}\right)$$

Scribe

(2) $\displaystyle\int_0^{\infty} \frac{x \operatorname{Sen} x}{x^2+a^2}\,dx \;,\; a \in \mathbb{R}$ Integral impropia




Ext. analítica

Polo → $Z_0 = \pm ia$

$$\rightarrow \oint_c \frac{z e^{iz}}{z^2+a^2}\,dz$$

No hay polos de 0 a $\infty$

$$\oint_c = \int_0^R \frac{x e^{ix}}{x^2+a^2} + \int_{C_1} \frac{z e^{iz}}{z^2+a^2} + \int_{Ri}^{(a+\varepsilon)i} + \int_{C_2} + \int_{(a-\varepsilon)i}^{0}$$

---

(1) $\dfrac{1}{\ldots} = \dfrac{1}{(\ldots)(\ldots)}$ Polos $\left\{ \pm\sqrt{2}\,i \right.$

Escaneado con CamScanner

$$\frac{1}{z^4+5z^2+6} = \frac{1}{(z^2+2)(z^2+3)} \text{, Polos } \begin{cases} \to \pm\sqrt{2}\,i \\ \to \pm\sqrt{3}\,i \end{cases}$$

$$\text{Residuos} \to \frac{P(z_0)}{q'(z_0)} \to \frac{1}{4z^3+10z} \begin{cases} \to \underset{z_0 \to \sqrt{2}i}{\operatorname{Res} f(z)} = -i\dfrac{\sqrt{2}}{4} \\ \to \underset{z_0 \to \sqrt{3}i}{\operatorname{Res} f(z)} = i\dfrac{\sqrt{3}}{6} \end{cases}$$

② $\dfrac{1}{(z^2-1)^2}$ , Polos orden $2 = \pm 1$

$$\operatorname{Res} f(z) = \lim_{z \to 1} \left( \frac{1}{(2-1)!} \frac{d}{dz} \left[ (z-1)^2 \; f(z) \right] \right)$$

$$= \lim_{z \to 1} -\frac{2}{(z+1)^3} = -\frac{1}{4} = \operatorname{Res} f(z)$$

Scribe

(2)

(a)

$x = \cosh(w)\cos(v)$

$y = \operatorname{senh}(w)\operatorname{sen}(v)$

$$\frac{\partial x^i}{\partial(v,w)} \left( \begin{matrix} \dfrac{\partial x}{\partial v} = -\cosh(w)\operatorname{sen}(v) & \dfrac{\partial x}{\partial w} = \cos(v)\operatorname{senh}(w) \\ \dfrac{\partial y}{\partial v} = \operatorname{senh}(w)\cos(v) & \dfrac{\partial y}{\partial w} = \operatorname{sen}(v)\cosh(w) \end{matrix} \right)$$

$$\begin{bmatrix} -\cosh(w)\operatorname{sen}(v) \\ \operatorname{senh}(w)\cos(v) \end{bmatrix} = \overbrace{\left(\begin{bmatrix} -\cosh(w)\operatorname{sen}(v) & \cos(v)\operatorname{senh}(w) \\ \operatorname{senh}(w)\cos(v) & \operatorname{sen}(v)\cosh(w) \end{bmatrix}\right)}^{T} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \cos(v)\operatorname{senh}(w) \\ \operatorname{sen}(v)\cosh(w) \end{bmatrix} = T \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(b)

$$= T \begin{bmatrix} 5 \\ 2 \end{bmatrix} \hat{i} \; ¿\hat{j}?$$

$$\frac{\partial f}{\partial v} = \operatorname{Senh}(w)\cos(v) \qquad \frac{\partial f}{\partial w} = -\operatorname{Sen}(v)\cosh(w)$$

$$\begin{bmatrix} -\cosh(w)\operatorname{Sen}(v) \\ \operatorname{Senh}(w)\cos(v) \end{bmatrix} = \overbrace{\begin{bmatrix} -\cosh(w)\operatorname{Sen}(v) & \cos(v)\operatorname{Senh}(w) \\ \operatorname{Senh}(w)\cos(v) & \operatorname{Sen}(v)\cosh(w) \end{bmatrix}}^{T} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \hat{\imath}$$

$$\begin{bmatrix} \cos(v)\operatorname{senh}(w) \\ \operatorname{Sen}(v)\cosh(w) \end{bmatrix} = T \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

b)

$$= T \begin{bmatrix} 5 \\ 2 \end{bmatrix} ¿\hat{\imath}?$$

$$\begin{bmatrix} 2\cos(v)\operatorname{senh}(w) - 5\operatorname{sen}(v)\cosh(w) \\ 5\cos(v)\operatorname{Senh}(w) + 2\operatorname{Sen}(v)\cosh(w) \end{bmatrix}$$

c)

Métrica $\begin{pmatrix} \langle \mathrm{Cu}_t \quad , \quad \rangle & \langle \quad , \quad \rangle \\ \langle \quad , \quad \rangle & \langle \quad , \quad \rangle \end{pmatrix}$

Escaneado con CamScanner