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Seccion 1.5.7

Ejercicio 2

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} = x^i \hat{i}$$

$$\vec{a} = \vec{a}(r) = \vec{a}(x,y,z)$$
$$\vec{b} = \vec{b}(r) = \vec{b}(x,y,z)$$

$$\phi = \phi(\vec{r}) = \phi(x,y,z) \qquad \psi = \psi(\vec{r}) = \psi(x,y,z)$$

a) $\vec{\nabla}(\phi\psi) = \phi\vec{\nabla}\psi + \psi\vec{\nabla}\phi$

$$(\vec{\nabla}(\phi\psi))^i = \partial^i(\phi\psi) = (\partial^i\phi)\psi + (\partial^i\psi)\phi$$
$$= (\nabla\phi)\psi + (\nabla\psi)\phi$$

2. d) $\nabla \cdot (\nabla \times \vec{a})$

d) $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{a}) = \partial^i (\vec{\nabla} \times \vec{a})_i = \partial^i (\epsilon^{ijk} \partial_j (a_k))$

$= \epsilon^{ijk} \partial^i (\partial_j a_k) = 0$

1,2,3 2,3,1 3,1,2
1,3,2 3,2,1 2,1,3

$\partial^1(\partial_2 a_3) + \partial^2(\partial_3 a_1) + \partial^3(\partial_1 a_2) - \partial^1(\partial_3 a_2) - \partial^3(\partial_2 a_1)$
$- \partial^2(\partial_1 a_3) = 0$

$\partial^1 \partial_2 = \partial^2 \partial_1$

$\vec{\nabla} \times (\vec{\nabla} \cdot \vec{a}) = \epsilon^{ijk} \partial_j (\vec{\nabla} \cdot \vec{a})_k = \epsilon^{ijk} \partial_j \partial_k a^i$

El producto cruz entre un vector y un escalar no está definido

f) $\vec{\nabla} \times (\vec{\nabla} \times \vec{a}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{a}) - \vec{\nabla}^2 \vec{a}$

$(\vec{\nabla} \times (\vec{\nabla} \times \vec{a}))^i = \epsilon^{ijk} \partial_j (\vec{\nabla} \times \vec{a})_k$

$= \epsilon^{ijk} \partial_j \epsilon_{kmn} \partial^m a^n$

$= \epsilon^{ijk} \epsilon_{mnk} \partial_j \partial^m a^n$

$= (\delta^i_m \delta^j_n - \delta^i_n \delta^j_m) \partial_j \partial^m a^n$

$= \delta^i_m \delta^j_n \partial_j \partial^m a^n - \delta^i_n \delta^j_m \partial_j \partial^m a^n$

$= \partial_j \partial^i a^j - \partial_j \partial^j a^i$

$= \partial^i (\partial_j a^j) - a^i (\partial_j \partial^j)$

$= \partial^i (\vec{\nabla} \cdot \vec{a}) - (\vec{\nabla} \cdot \vec{\nabla}) a^i$

$\vec{\nabla} \times (\vec{\nabla} \times \vec{a}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{a}) - \vec{\nabla}^2 \vec{a}$