① a) $\dfrac{1}{z^4+5z^2+6} = \dfrac{1}{(z^2+2)(z^2+3)}$

Raíces $\downarrow$ polos: $z^2+2=0$, $z^2=-2$, $z=\pm i\sqrt{2}$

$z^2+3=0$, $z^2=-3$, $z=\pm i\sqrt{3}$

Residuos

- $z=\sqrt{2}\,i$

$$\left.\frac{1}{4z^3+10z}\right|_{z=\sqrt{2}i} = \frac{1}{4(\sqrt{2}i)^3+10(\sqrt{2}i)} = \frac{1}{4(2)^{3/2}(-i)+10\sqrt{2}i}$$

$$= \frac{1}{i(2)^{3/2}(-4+5)} = \frac{1}{i(2)^{3/2}} = \boxed{-\frac{\sqrt{2}i}{4}}$$

- $z=-\sqrt{2}\,i$

$$\left.\frac{1}{4z^3+10z}\right|_{z=-\sqrt{2}i} = \frac{1}{4(-\sqrt{2}i)^3+10(-\sqrt{2}i)} = \frac{1}{4(2)^{3/2}(i)-10\sqrt{2}i}$$

$$= \frac{1}{i(2)^{3/2}(4-5)} = \frac{1}{-i(2)^{3/2}} = \boxed{\frac{\sqrt{2}i}{4}}$$

- $z=\sqrt{3}\,i$

$$\left.\frac{1}{4z^3+10z}\right|_{z=\sqrt{3}i} = \frac{1}{4(\sqrt{3}i)^3+10(\sqrt{3}i)} = \frac{1}{4(3)^{3/2}(-i)+10(3)^{1/2}i}$$

$$= \frac{1}{i(3)^{1/2}(-4(3)+10)} = \frac{1}{i(3)^{1/2}(-2)} = \boxed{\frac{\sqrt{3}}{6}i}$$

- $z=-\sqrt{3}\,i$

$$\left.\frac{1}{4z^3+10z}\right|_{z=-\sqrt{3}i} = \frac{1}{4(-\sqrt{3}i)^3+10(-\sqrt{3}i)} = \frac{1}{4(3)^{3/2}(i)-10(3)^{1/2}i}$$

$$= \frac{1}{i(3)^{1/2}(4(3)-10)} = \frac{1}{i(3)^{1/2}(2)} = \boxed{-\frac{\sqrt{3}}{6}i}$$

$\dfrac{1}{(z^2-1)^2}$

Raíces $\downarrow$ polos: $z^2-1=0$, $z^2=1$, $z=1$ y $z=-1$

Residuos

$$\operatorname{Res} f(z) = \lim_{z\to 1}\left(\frac{1}{(2-1)!}\frac{d^{2-1}}{dz^{2-1}}\left[(z-1)^2\frac{1}{\cancel{(z-1)}(z+1)\cancel{(z-1)}(z+1)}\right]\right)$$

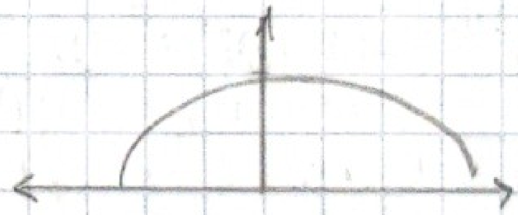
$$\operatorname{Res} f(z) = \lim_{z\to 1}\left(\frac{d}{dz}\left[\frac{1}{(z+1)^2}\right]\right) = \lim_{z\to 1}\left(-2\,\frac{1}{(z+1)^3}\right) = -2\left(\frac{1}{2^3}\right)$$

$$= \boxed{-\frac{1}{4}}$$

$$\operatorname{Res} f(z) = \lim_{z\to -1}\left(\frac{d}{dz}\left[(z\cancel{+1})^2\,\frac{1}{(z-1)(z\cancel{+1})(z-1)(z\cancel{+1})}\right]\right) = \lim_{z\to -1}\left(\frac{d}{dz}\left[\frac{1}{(z-1)^2}\right]\right)$$

$$= \lim_{z\to -1}\left(-2\,\frac{1}{(z-1)^3}\right) = -2\left(\frac{1}{(-2)^3}\right) = \boxed{\frac{1}{4}}$$

b) $$\int_0^\infty \frac{x^2}{x^4+5x^2+6}\,dx = \int_0^\infty \frac{z^2}{z^4+5z^2+6}\,dz = i\pi \sum \operatorname{Res} f(z)$$



polos $\rightarrow$ $z = \pm\sqrt{2}\,i \;\wedge\; z = \pm\sqrt{3}\,i$

solo tomo $z = \sqrt{2}\,i \;\wedge\; z = \sqrt{3}\,i$

residuos

- $z = \sqrt{2}\,i$

$$\left.\frac{z^2}{4z^3+10z}\right|_{z=\sqrt{2}i} = \frac{(\sqrt{2}\,i)^2}{4(\sqrt{2}\,i)^3+10(\sqrt{2}\,i)} = -2\,\frac{(-\sqrt{2}\,i)}{4} = \boxed{\frac{\sqrt{2}\,i}{2}}$$

- $z = \sqrt{3}\,i$

$$\left.\frac{z^2}{4z^3+10z}\right|_{z=\sqrt{3}i} = \frac{(\sqrt{3}\,i)^2}{4(\sqrt{3}\,i)^3+10(\sqrt{3}\,i)} = -3\,\frac{\sqrt{3}}{6}\,i = \boxed{-\frac{\sqrt{3}\,i}{2}}$$

$$i\pi \sum \operatorname{Res} f(z) = i\pi \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \right) i = -\pi(-0{,}15892) = 0{,}4993$$

$$\hookrightarrow \int_0^{\infty} \frac{z^2}{z^4+5z^2+6}\, dz = \boxed{0{,}4993}$$

$$\int_0^{\infty} \frac{x \sin x}{x^2+a^2}\, dx = \pi \sum \operatorname{Re} \operatorname{Res} \left| f(z) e^{iz} \right|$$

polos $\to$ $z^2 = -a^2$

$z = \pm ai$

solo tomo $z = ai$

$$f(z) = \frac{z}{z^2+a^2}$$

Residuos

- $$\left. \frac{z e^{iz}}{2z} \right|_{z=ai} = \frac{ai\, e^{i(ai)}}{2(ai)} = \frac{e^{-a}}{2} = \frac{1}{2e^a}$$

$$\int_0^{\infty} \frac{x \sin x}{x^2+a^2}\, dx = \boxed{\frac{\pi}{2e^a}}$$

② $x = \cosh(\omega)\cos(\upsilon)$, $y = \operatorname{senh}(\omega)\operatorname{sen}(\upsilon)$, $z = z$

a) El vector en coordenadas elípticas se puede escribir como

$$|r\rangle = \cosh(\omega)\cos(\upsilon)\,|e_x\rangle + \operatorname{senh}(\omega)\operatorname{sen}(\upsilon)\,|e_y\rangle + z\,|e_z\rangle$$

base asociada para el vector: $|e_\omega\rangle = \dfrac{1}{h_\omega}\dfrac{\partial|r\rangle}{\partial\omega}$ $\quad |e_z\rangle = 1$

$$\frac{\partial|r\rangle}{\partial\omega} = \cos(\upsilon)\operatorname{senh}(\omega)\,|e_x\rangle + \operatorname{sen}(\upsilon)\cosh(\omega)\,|e_y\rangle$$

$$h_\omega = \left\|\frac{\partial|r\rangle}{\partial\omega}\right\| = \|\cos(\upsilon)\operatorname{senh}(\omega)\,|e_x\rangle + \operatorname{sen}(\upsilon)\cosh(\omega)\,|e_y\rangle\|$$

$$= \sqrt{\cos^2(\upsilon)\operatorname{senh}^2(\omega) + \operatorname{sen}^2(\upsilon)\cos^2h(\omega)}$$
$$= \sqrt{\cos^2\operatorname{senh}^2(\omega) + \operatorname{sen}^2(\upsilon)\,(1+\operatorname{sen}^2h(\omega))}$$
$$= \sqrt{\operatorname{senh}^2(\omega)\,(\cos^2(\upsilon)+\operatorname{sen}^2(\upsilon)) + \operatorname{sen}^2(\upsilon)} = \sqrt{\operatorname{senh}^2(\omega)+\operatorname{sen}^2(\upsilon)}$$

$$|e_\omega\rangle = \frac{\cos(\upsilon)\operatorname{senh}(\omega)\,|e_x\rangle + \operatorname{sen}(\upsilon)\cosh(\omega)\,|e_y\rangle}{(\operatorname{senh}^2(\omega)+\operatorname{sen}^2(\upsilon))^{1/2}}$$

$$|e_v\rangle = \frac{1}{h_v}\frac{\partial|r\rangle}{\partial v}$$

$$\frac{\partial|r\rangle}{\partial v} = -\cosh(w)\,\mathrm{sen}(v)\,|e_x\rangle + \mathrm{senh}(w)\cos(v)\,|e_y\rangle$$

$$h_v = \left\| \frac{\partial|r\rangle}{\partial v} \right\| = \| -\cosh(w)\,\mathrm{sen}(v)\,|e_x\rangle + \mathrm{senh}(w)\cos(v)\,|e_y\rangle \|$$

$$= \sqrt{\cos^2h(w)\,\mathrm{sen}^2(v) + \mathrm{senh}^2(w)\cos^2(v)}$$

$$= \sqrt{(1+\mathrm{sen}^2h(w))\,\mathrm{sen}^2(v) + \mathrm{senh}^2(w)\cos^2(v)}$$

$$= \sqrt{\mathrm{sen}^2h(w)\,(\mathrm{sen}^2(v)+\cos^2(v)) + \mathrm{sen}^2(v)} = \sqrt{\mathrm{senh}^2(w)+\mathrm{sen}^2(v)}$$

$$|e_v\rangle = \frac{-\cosh(w)\,\mathrm{sen}(v)\,|e_x\rangle + \mathrm{senh}(w)\cos(v)\,|e_y\rangle}{(\mathrm{senh}^2(w)+\mathrm{sen}^2(v))^{1/2}}$$

b) $|a\rangle = 5\hat{i} + 0\hat{j} + 2\hat{k} = a_1\,|e_w\rangle + a_2\,|e_v\rangle + a_3\,|e_z\rangle$

$\cosh(w)\cos(v) = 5$

$\mathrm{senh}(w)\,\mathrm{sen}(v) = 0$

$z = 2.$

$\boxed{v = 2\pi}$

$\cosh(w) = 5 \quad w = \cosh^{-1}(5) = 2{,}29$

$$5 = \frac{a_1\,(23{,}8812)}{(23{,}8812)}$$

$|a\rangle = 5\,|e_w\rangle + 2\,|e_z\rangle$

c) $g_{ij} = \begin{pmatrix} h_1^2 & 0 & 0 \\ 0 & h_2^2 & 0 \\ 0 & 0 & h_3^2 \end{pmatrix} = \begin{pmatrix} \operatorname{senh}^2(\omega) + \operatorname{sen}^2(v) & 0 & 0 \\ 0 & \operatorname{senh}^2(\omega) + \operatorname{sen}^2(v) & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\langle a| = g_{ij} |a\rangle = (\operatorname{senh}^2(\omega) + \operatorname{sen}^2(v)) 5 |e_\omega\rangle + (\operatorname{senh}^2(\omega) + \operatorname{sen}^2(v)) 0 |e_v\rangle$

$+ 1(2) |e_z\rangle = 5(\operatorname{senh}^2(\omega) + \operatorname{sen}^2(v)) |e_\omega\rangle + 2 |e_z\rangle$

$= 120 \langle e_\omega| + 2 \langle e_z|$ $\qquad g_{ij} = \begin{pmatrix} 24 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

d) $T_{ij} = \begin{pmatrix} 2 & 1 & 3 \\ 2 & 3 & 4 \\ 1 & 2 & 2 \end{pmatrix}$ $\qquad T^{ij} = g^{im} g^{jn} T_{ij} - g^{im} T_{ij} g^{jn}$

$$T^{ij} = \begin{pmatrix} \frac{1}{\operatorname{senh}^2(\omega) + \operatorname{sen}^2(v)} & 0 & 0 \\ 0 & \frac{1}{\operatorname{senh}^2(\omega) + \operatorname{sen}^2(v)} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 3 \\ 2 & 3 & 4 \\ 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} \frac{1}{\operatorname{senh}^2(\omega) + \operatorname{sen}^2(v)} & 0 & 0 \\ 0 & \frac{1}{\operatorname{senh}^2(\omega) + \operatorname{sen}^2(v)} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T^{ij} = \frac{1}{(\sinh^2(\omega) + \operatorname{sen}^2(v))^2} \begin{pmatrix} 2 & 1 & 3 \\ 2 & 3 & 4 \\ 1 & 2 & 2(\sinh^2(\omega) + \operatorname{sen}^2(v))^2 \end{pmatrix}$$