Nombres: Nicolas Toledo Parra 2200017
Gabriela Sánchez Ariza 2200816

Sección 1.6.6

Ejercicio 2

a) $\cos(3\alpha) = \cos^3(\alpha) - 3\cos(\alpha)\,\text{sen}^2(\alpha)$

$$\cos(3\alpha) + i\,\text{sen}(3\alpha) = e^{i3\alpha} = (e^{i\alpha})^3$$

$$= (\cos(\alpha) + i\,\text{sen}(\alpha))^3$$
$$= \cos^3(\alpha) + 3\cos^2(\alpha)(i\,\text{sen}(\alpha)) + 3(i\,\text{sen}(\alpha))^2\cos(\alpha) + (i\,\text{sen}(\alpha))^3$$
$$= \cos^3(\alpha) + 3\cos^2(\alpha)(i\,\text{sen}(\alpha)) - 3\,\text{sen}^2(\alpha)\cos(\alpha) - i\,\text{sen}^3(\alpha)$$
$$= \cos^3(\alpha) - 3\cos(\alpha)\,\text{sen}^2(\alpha) + i(3\cos^2(\alpha)\,\text{sen}(\alpha) - \text{sen}^3(\alpha))$$

Como la parte real de la expresión "$\cos(3\alpha) + i\,\text{sen}(3\alpha)$" debe ser igual a la parte real de la expresión obtenida, se tiene que:

$$\cos(3\alpha) = \cos^3(\alpha) - 3\cos(\alpha)\,\text{sen}^2(\alpha)$$

b) $\text{sen}(3\alpha) = 3\cos^2(\alpha)\,\text{sen}(\alpha) - \text{sen}^3(\alpha)$

De la misma forma que la demostración pasada se obtiene que la parte imaginaria de la expresión "$\cos(3\alpha) + i\,\text{sen}(3\alpha)$" debe ser igual a la parte imaginaria de la expresión obtenida, se tiene que:

$$\text{sen}(3\alpha) = 3\cos^2(\alpha)\,\text{sen}(\alpha) - \text{sen}^3(\alpha).$$

Ejercicio 5

encuentre las raices de

a) $(2i)^{1/2} = (2e^{i(\frac{\pi}{2}+2\pi n)})^{\frac{1}{2}} = \sqrt{2}\, e^{i(\frac{\pi}{4}+\pi n)}$

$= \sqrt{2}\left[\cos(\frac{\pi}{4}+\pi n) + i\,sen(\frac{\pi}{4}+\pi n)\right]$

$= \sqrt{2}\left[\cos(\frac{\pi}{4})\cos(\pi n) - sen(\frac{\pi}{4})\underbrace{sen(\pi n)}_{0} + i(sen(\frac{\pi}{4})\cos(\pi n)\right.$

$\left. + \underbrace{sen(\pi n)}_{0}\cos(\frac{\pi}{4}))\right]$

$= \sqrt{2}\left[\frac{\sqrt{2}}{2}(\pm 1) + i\frac{\sqrt{2}}{2}(\pm 1)\right] = \pm 1 + i(\pm 1)$

soluciones $z_1 = 1 + i$
$z_2 = -1 - i$

b) $(1-\sqrt{3}i)^{1/2} = ((1+3)^{1/2} e^{i\tan^{-1}(-\sqrt{3})})^{1/2}$

$= \sqrt{2}(e^{i(-\frac{\pi}{3}+2\pi n)})^{1/2} = \sqrt{2}\, e^{i(-\frac{\pi}{6}+\pi n)}$

$= \sqrt{2}\left[\cos(-\frac{\pi}{6}+\pi n) + i\,sen(-\frac{\pi}{6}+\pi n)\right]$

$= \sqrt{2}\left[\cos(\frac{\pi}{6})\cos(\pi n) + sen(\frac{\pi}{6})\underbrace{sen(\pi n)}_{0} + i(-sen(\frac{\pi}{6})\cos(\pi n)\right.$

$\left. + \underbrace{sen(\pi n)}_{0}\cos(\frac{\pi}{6}))\right]$

$= \sqrt{2}\left[\frac{\sqrt{3}}{2}(\pm 1) + i\left(-\frac{1}{2}(\pm 1)\right)\right] = \frac{\sqrt{6}}{2}(\pm 1) + i\left(\mp\frac{\sqrt{2}}{2}\right)$

soluciones: $z_1 = \frac{\sqrt{6}}{2} - i\frac{\sqrt{2}}{2}$

$z_2 = -\frac{\sqrt{6}}{2} + i\frac{\sqrt{2}}{2}$

c) $(-1)^{1/3} = \left(e^{i(\pi+2\pi n)}\right)^{1/3} = e^{i\left(\frac{\pi}{3}+\frac{2\pi}{3}n\right)}$

$= \cos\left(\frac{\pi}{3}+\frac{2\pi n}{3}\right) + i\,\text{sen}\left(\frac{\pi}{3}+\frac{2\pi}{3}n\right)$

$= \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{2\pi n}{3}\right) - \text{sen}\left(\frac{\pi}{3}\right)\text{sen}\left(\frac{2\pi n}{3}\right) + i\left(\text{sen}\left(\frac{\pi}{3}\right)\cos\left(\frac{2\pi n}{3}\right)\right.$

$\left. + \text{sen}\left(\frac{2\pi n}{3}\right)\cos\left(\frac{\pi}{3}\right)\right)$

$= \frac{1}{2}\cos\left(\frac{2\pi n}{3}\right) - \frac{\sqrt{3}}{2}\,\text{sen}\left(\frac{2\pi n}{3}\right) + i\left(\frac{\sqrt{3}}{2}\cos\left(\frac{2\pi n}{3}\right) + \frac{1}{2}\,\text{sen}\left(\frac{2\pi n}{3}\right)\right)$

$n=0$

$z^1 = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

$(z^1)^2 = \frac{1}{4} - \frac{3}{4} + \frac{\sqrt{3}}{2}i = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$(z^1)^3 = -\frac{3}{4} - \frac{1}{4} = -1$

$n=1$

$z^2 = -\frac{1}{4} - \frac{3}{4} + i\left(-\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}\right) = -1$

$n=2$

$z^3 = \frac{1}{2} - \frac{\sqrt{3}}{2}i$

$(z^3)^2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}$ $\qquad (z^3)^3 = (-1)\left(\frac{1}{4} + \frac{3}{4}\right) = -1$

d) $8^{1/6} = (8e^{i2\pi n})^{1/6} = 8^{1/6}\, e^{\frac{i\pi n}{3}}$

$z^0 = 8^{\frac{1}{6}} e^0 = 8^{1/6}$

$z^1 = 8^{\frac{1}{6}} e^{\frac{i\pi}{3}} = 8^{\frac{1}{6}}\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$

$z^2 = 8^{\frac{1}{6}} e^{\frac{2\pi}{3}} = 8^{\frac{1}{6}}\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$z^3 = 8^{\frac{1}{6}} e^{\pi} = 8^{\frac{1}{6}}(-1)$

$z^4 = 8^{\frac{1}{6}} e^{\frac{4\pi}{3}} = 8^{\frac{1}{6}}\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$

$z^5 = 8^{\frac{1}{6}} e^{\frac{5\pi}{3}} = 8^{\frac{1}{6}}\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$

e) $(-8 - 8\sqrt{3}i)^{1/4} = \left(16\, e^{i\left(\frac{\pi}{3} + 2\pi n\right)}\right)^{1/4} = 2\, e^{i\left(\frac{\pi}{12} + \frac{\pi n}{2}\right)}$

$= 2\left[\cos\left(\frac{\pi}{12} + \frac{\pi n}{2}\right) + i\,\text{sen}\left(\frac{\pi}{12} + \frac{\pi}{2}n\right)\right]$

$n = 0$

$z_0 = 2\,[0{,}966 + i\,0{,}258]$

$n = 1$

$z_1 = 2\,[-0{,}258 + i\,0{,}966]$

Ejercicio 6:

a) $\log(-ie) = 1 - \frac{\pi}{2} i$

$\log(-ie) = \log\left(e\, e^{i\left(\frac{3\pi}{2} + 2\pi n\right)}\right) = \ln(e) + i\left(-\frac{\pi}{2} + 2\pi n\right)$

$n = 0 \Rightarrow 1 - \frac{\pi}{2} i$

b) $\log(1-i) = \frac{1}{2}\ln(2) - \frac{\pi}{4} i$

$\log(1-i) = \log\left(\sqrt{2}\, e^{i\left(-\frac{\pi}{4} + 2\pi n\right)}\right) = \ln\sqrt{2} - \frac{i\pi}{4}$

$n = 0 \Rightarrow \frac{1}{2}\ln 2 - \frac{\pi}{4} i$

c) $\log(e) = 1 + 2n\pi i$

$\log\left(e \cdot e^{i(2\pi n)}\right)$

$n = 0 \Rightarrow 1 + 2\pi n i$

d) $\log(i) = \left(2n + \frac{1}{2}\right)\pi i$

$\log(i) = \log\left(1\, e^{i\left(\frac{\pi}{2} + 2\pi n\right)}\right) = \ln(1) + i\left(\frac{\pi}{2} + 2n\pi\right)$

$= i\left(\frac{\pi}{2} + 2\pi n\right) = i\pi\left(\frac{1}{2} + 2n\right)$