Linear Time-Invariant Systems

1. Discrete-Time LTI Systems: the Convolution Sum

sifting property

$$x[n]=\sum_{k=-\infty}^{+\infty}x[k]\delta[n-k]$$

unit impluse(sample) response $h[n]$, it is the output of the LTI system when $\delta[n]$ is the input.
then for an LTI system, we have

$$y[n]=\sum_{k=-\infty}^{+\infty}x[k]h[n-k]=x[n]*h[n]$$

This result is referred to as the **convolution sum** or **superposition sum**.
An LTI system is completely characterized by its response to its response to the unit impulse.

2.Continuous-Time LTI Systems: the Convolution Integral

sifting property

$$x(t)=\int_{-\infty}^{+\infty}x(\tau)\delta(t-\tau)\,\mathrm{d}\tau$$

unit impluse(sample) response $h(t)$, it is the output of the LTI system when $\delta(t)$ is the input.
similarly, we have

$$y(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)\,\mathrm{d}\tau=x(t)*h(t)$$

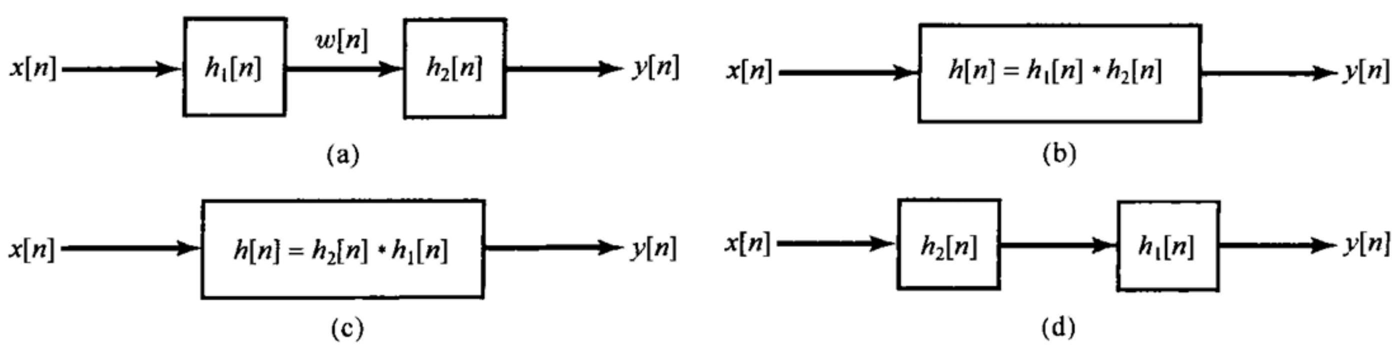
This result is referred to as the **convolution integral** or **superposition integral**.
As in discrete time, we see that a continuous-time LTI system is completely characterized by its impulse response.

3. Properties of LTI Systems

the commutative property $x[n] * h[n] = h[n] * x[n]$ and $x(t) * h(t) = h(t) * x(t)$
The output of an LTI system with input $x[n]$ and unit impulse response $h[n]$ is identical to the output of an LTI system with input $h[n]$ and unit impulse response $x[n]$.
the distributive property $x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$
A parallel combination of LTI systems can be replaced by a single LTI system whose unit impulse response is the sum of the individual unit impulse responses in the parallel combination.

as a consequence of the properties, we have $[x_1(t) + x_2(t)] * h(t) = x_1(t) * h(t) + x_2(t) * h(t)$
The response of an LTI system to the sum of two inputs must equal the sum of the responses to these signals individually.

the associative property $x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$
The unit impulse response of a cascade of two LTI systems does not depend on the order in which they are cascaded.



Being able to interchange the order of systems in a cascade is a characteristic particular to LTI systems.

The only way that it can be a system **without memory** for a discrete-time LTI system is if $h[n] = K\delta[n]$, where $K = h[0]$ is a constant.
invertibility of LTI systems a system with impulse response $h(t)$, the inverse system with impulse response $h_1(t)$,then$h(t) * h_1(t) = \delta(t)$
causality for LTI systems the impulse response of a causal discrete-time LTI system satisfy the condition $h[n] = 0 \quad \text{for } n < 0$
It is common terminology to refer to a signal as being causal if it is zero for $n < 0$ or $t < 0$.
stability for LTI systems if the impulse response is absolutely summable $\sum_{k=-\infty}^{+\infty} |h[k]| < \infty$ or absolutely integrable $\int_{-\infty}^{+\infty} |h(\tau)| \, \mathrm{d}\tau < \infty$, then the system is stable
unit step response $s[n] = h[n] * u[n] = \sum_{k=-\infty}^{n} h[k]$
or $s(t) = \int_{-\infty}^{t} h(\tau) \, \mathrm{d}\tau$

4. Singularity Functions

a basic property of the unit impulse $\delta(t) = \delta(t) * \delta(t)$
defining the unit impulse through convolution $x(t) = x(t) * \delta(t)$
unit doublet $u_1(t)$: $\frac{\mathrm{d}x(t)}{\mathrm{d}t} = x(t) * u_1(t)$
then we have $\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} = x(t) * u_2(t)$ and $u_2(t) = u_1(t) * u_1(t)$

similarly, based on $u(t) = \int_{-\infty}^{t} \delta(t)\,\mathrm{d}t$
then **unit ramp function** $u_{-2}(t) = u(t) * u(t) = tu(t)$