

$$a = \frac{\sum y_i x_i - b \sum x_i}{\sum x_i^2} \quad 1^\circ$$

subst. 1° to $\frac{\partial \text{MSE}}{\partial b}$

solve b and subst. back to a and solve a

$$\frac{\partial}{\partial b} \sum (y_i - \hat{y}_i)^2 = \frac{\partial}{\partial b} \sum (y_i - ax_i - b)^2 = 0$$

$$\frac{1}{n} \cdot 2 \sum (y_i - ax_i - b) \cdot (-1) = 0$$

$$\Rightarrow \sum (y_i - ax_i - b) = 0$$

$$\Rightarrow \sum y_i - a \sum x_i - bn = 0$$

$$\Rightarrow bn = \sum y_i - a \sum x_i \quad || : n$$

$$b = \frac{\sum y_i - a \sum x_i}{n}$$

$$a = \frac{\sum y_i x_i - b \sum x_i}{\sum x_i^2}$$

$$b = \frac{\sum y_i - a \sum x_i}{N}$$

$$\Rightarrow \frac{\sum y_i x_i - \left(\frac{\sum y_i - a \sum x_i}{N} \right) \sum x_i}{\sum x_i^2}$$

$$\Rightarrow \frac{\sum y_i x_i - \frac{\sum y_i \sum x_i - a \sum x_i \sum x_i}{N}}{\sum x_i^2} \quad || \cdot N$$

$$\Rightarrow \frac{\sum y_i x_i - (\sum y_i \sum x_i - a \sum x_i^2)}{N \sum x_i^2}$$

$$\Rightarrow -a (\sum x_i)^2 = \frac{\sum y_i x_i - \sum y_i \sum x_i}{N \sum x_i^2} \quad || : -(\sum x_i)^2$$

$$a = \frac{\sum y_i x_i - \sum y_i \sum x_i}{N \sum x_i^2 - (\sum x_i)^2}$$

$$a = \frac{\sum_{i=1}^n y_i x_i - \sum_{i=1}^n y_i \sum_{i=1}^n x_i}{N \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2}$$