

Apply to Machine Learning

Week 4
Machine Learning

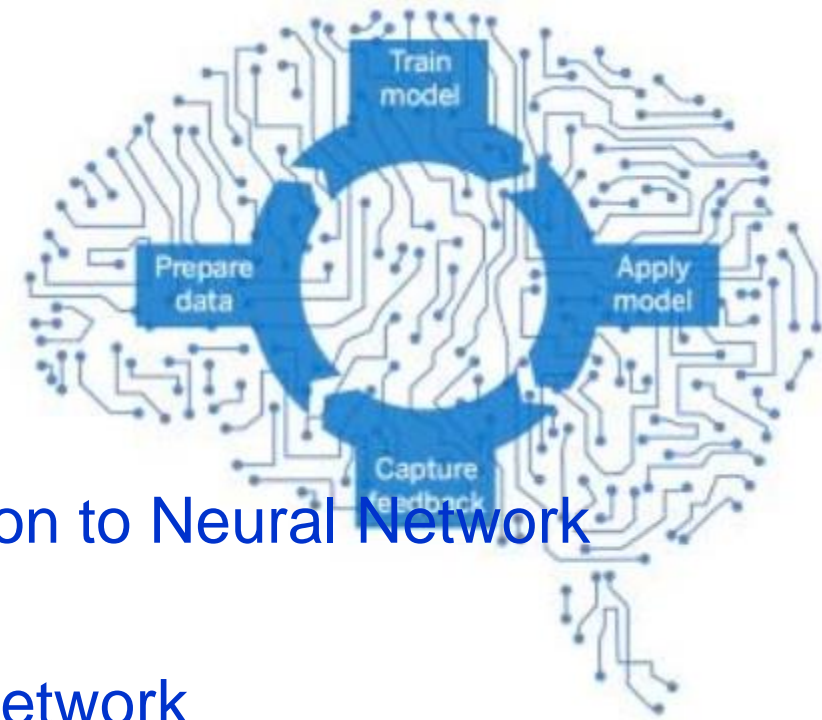
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인하대학교



Contents

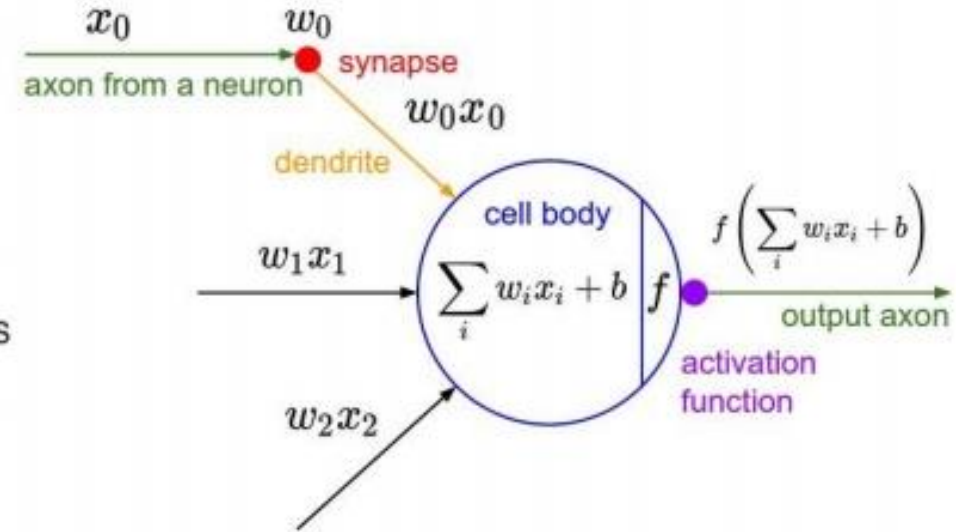
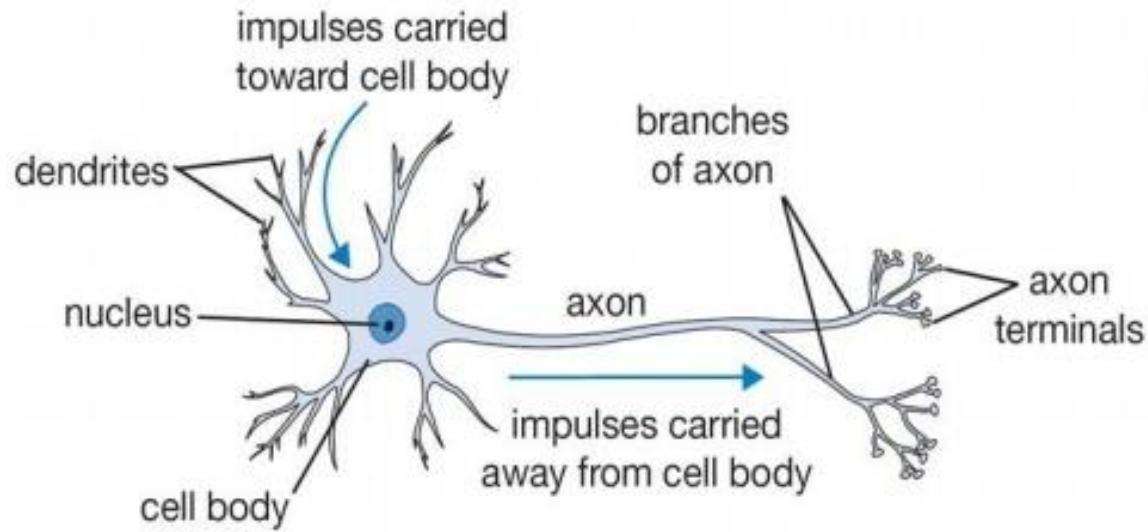


- 1 Perceptron to Neural Network
- 2 Nueral Network
- 3 Forward & Backward Propagation

1

Propagation to Neural Network

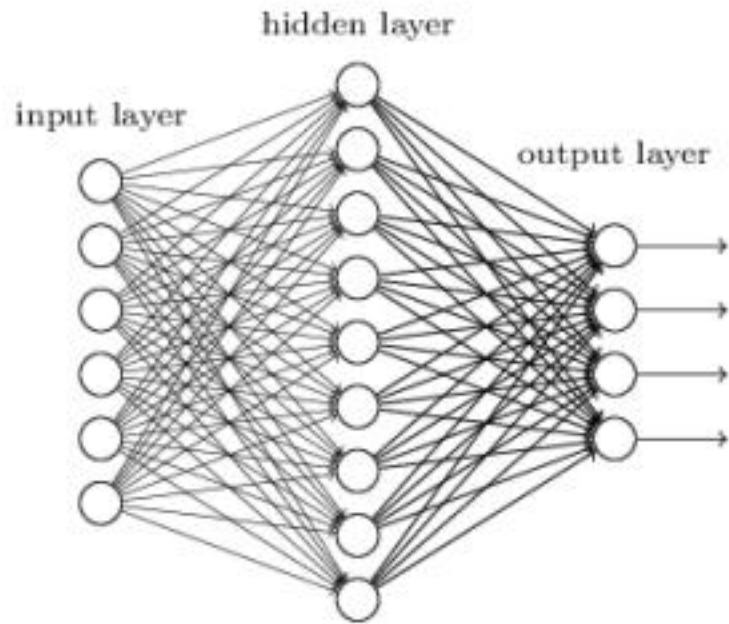
Neural Network



여러 자극이 들어오고 일정 기준을 넘으면 이를 다른 뉴런에 전달하는 구조

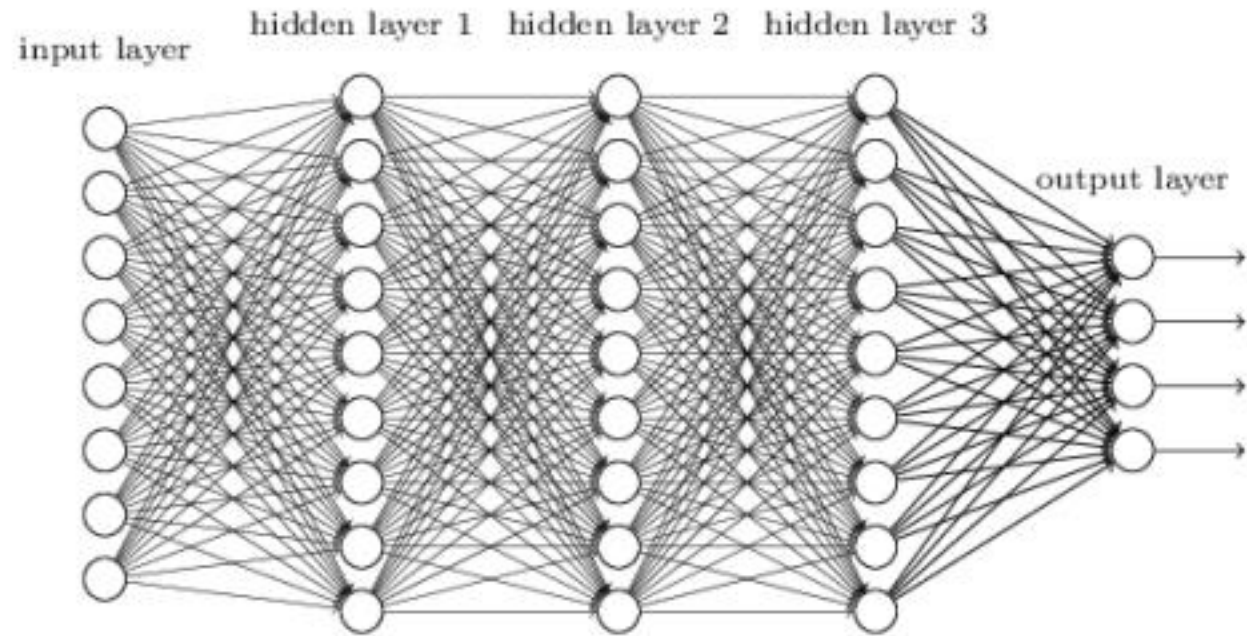
Neural Network

"Non-deep" feedforward neural network



$$y = w_2(\text{act}(w_1 * \text{input} + b_1)) + b_2$$

Deep neural network



$$y = w_4(\text{act}(w_3(\text{act}(w_2(\text{act}(w_1 * \text{input} + b_1)) + b_2)) + b_3)) + b_4$$

Neural Network

$$Y = W \cdot x + b$$

$$= \begin{bmatrix} x_{00} & x_{01} & \dots & \dots & \dots \\ x_{10} & x_{11} & \dots & \dots & \dots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ x_{mn} & \dots & \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} w_{00} & w_{01} & w_{02} & \dots & \dots \\ w_{10} & w_{11} & \dots & \dots & \dots \\ w_{20} & \dots & \dots & \dots & \dots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ w_{nl} & \dots & \dots & \dots & \dots \end{bmatrix} + \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ \vdots \\ \vdots \\ b_m \end{bmatrix}$$

$m \times n$ $n \times l$ $m \times 1$

Neural Network

$$Y = W \cdot x + b$$

Hand-drawn matrix representation of the equation $Y = W \cdot x + b$:

- The input matrix x is $m \times n$. It is shown as a large square bracket containing elements $x_{00}, x_{01}, \dots, x_{10}, x_{11}, \dots, x_{mn}$. The first row is highlighted with an orange box.
- The weight matrix W is $n \times l$. It is shown as a large square bracket containing elements $w_{00}, w_{01}, w_{02}, \dots, w_{10}, w_{11}, \dots, w_{20}, \dots, w_{nl}$. The first column is highlighted with an orange box.
- The bias vector b is $m \times 1$. It is shown as a large square bracket containing elements b_0, b_1, \dots, b_m . The first element b_0 is highlighted with an orange box.

The equation is represented as:

$$= \begin{bmatrix} x_{00} & x_{01} & \dots & \dots \\ x_{10} & x_{11} & \dots & \dots \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ x_{mn} \end{bmatrix} \begin{bmatrix} w_{00} & w_{01} & w_{02} & \dots \\ w_{10} & w_{11} & \dots & \dots \\ w_{20} & \dots & \dots & \dots \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ w_{nl} \end{bmatrix} + \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ b_m \end{bmatrix}$$

Dimensions are indicated below the matrices:

- $m \times n$ for the input matrix x .
- $n \times l$ for the weight matrix W .
- $m \times 1$ for the bias vector b .

Neural Network

$$y = \text{act}(wx + b)$$

$$= \text{activation} \left(\begin{bmatrix} wx + b \end{bmatrix} \right)$$

$m \times 1$

Neural Network

만약 activation function 이 없다면 아래의 식은 결국 linear function.

$$y = w_4(\text{act}(w_3(\text{act}(w_2(\text{act}(w_1 * \text{input} + b_1) + b_2)) + b_3)) + b_4$$

Neural Network

만약 activation function 이 없다면 아래의 식은 결국 linear function.

$$y = w_4(\text{act}(w_3(\text{act}(w_2(\text{act}(w_1 * \text{input} + b_1) + b_2)) + b_3)) + b_4$$

activation function 으로 non-linearity 를 추가해야 함

Neural Network

만약 activation function 이 없다면 아래의 식은 결국 linear function.

$$y = w4(act(w3(act(w2(act(w1 * input + b1) + b2)) + b3)) + b4$$

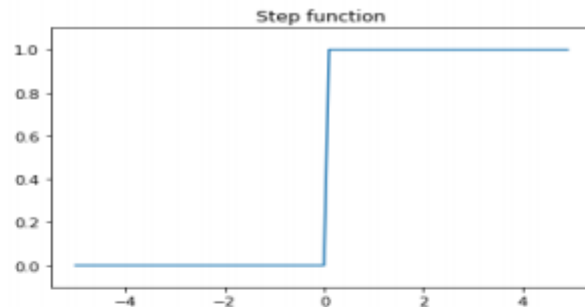
activation function 으로 non-linearity 를 추가해야 함

그렇다면 어떤 activation function 을 써야 할까 ?

Neural Network

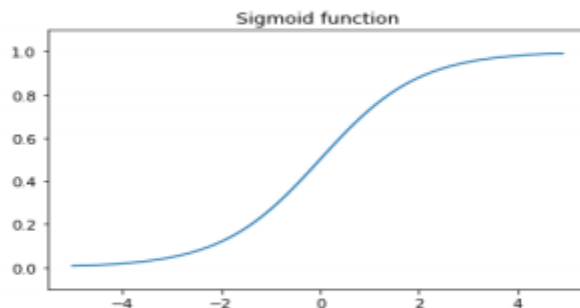
- Step function

$$h(x) = \begin{cases} 0 & (x \leq 0) \\ 1 & (x > 0) \end{cases}$$



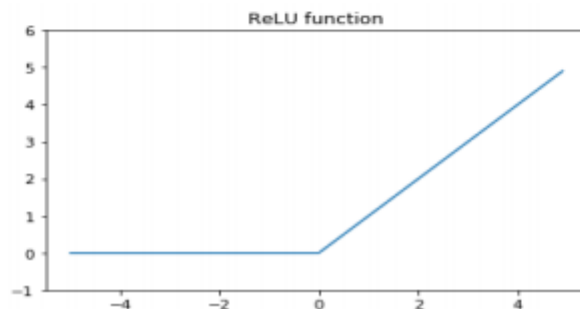
- Sigmoid

$$h(x) = \frac{1}{1 + \exp(-x)}$$



- ReLU (Rectified Linear Unit)

$$h(x) = \begin{cases} 0 & (x < 0) \\ x & (x \geq 0) \end{cases}$$



활성화 함수로는 반드시
비선형 함수를 사용한다
→ 선형 함수는 층을 깊게
하더라도 의미가 없기 때문

Neural Network

- Step function

```
def step_function(x):  
    return np.array(x > 0, dtype=np.int)
```

$x > 0$ 의 True/False를 (*numpy*)*int*로
변환하여 0 또는 1의 값으로 return

- Sigmoid

```
def sigmoid(x):  
    return 1 / (1 + np.exp(-x))
```

$\text{sigmoid}(x) = \frac{1}{1+\exp(-x)}$ 식의 값을 return

- ReLU (Rectified Linear Unit)

```
def relu(x):  
    return np.maximum(0, x)
```

0과 x 를 비교하여 큰 값을 출력한다.
 $x \geq 0$ 일 때는 x 를, $x < 0$ 일 때는 0을 출력

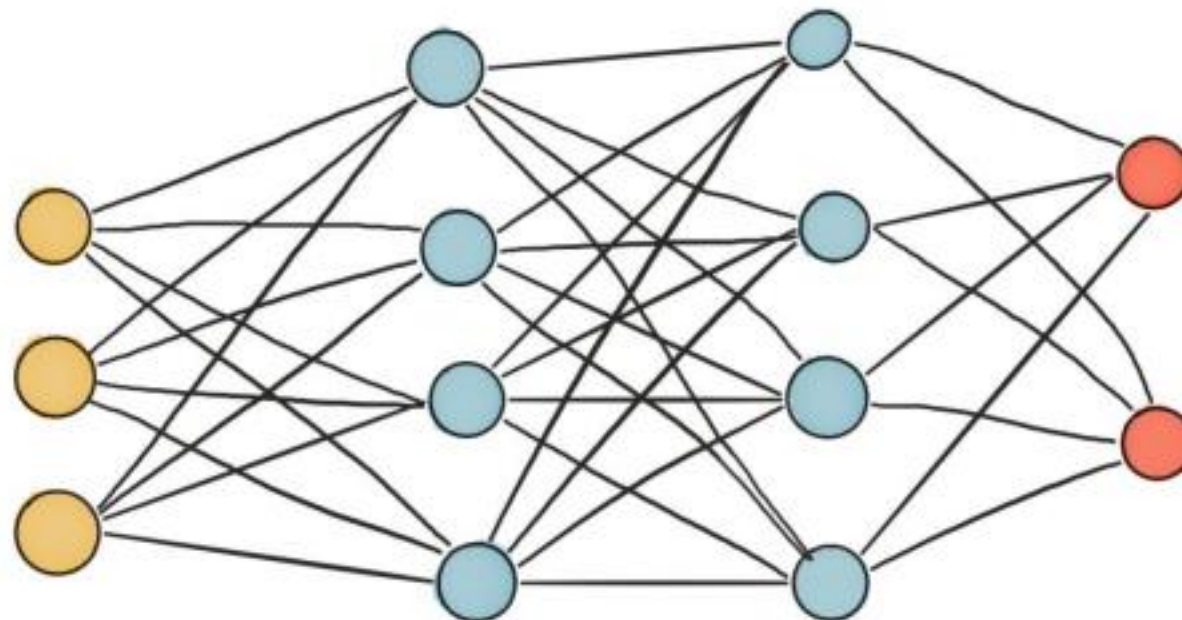
Neural Network

- Softmax 함수

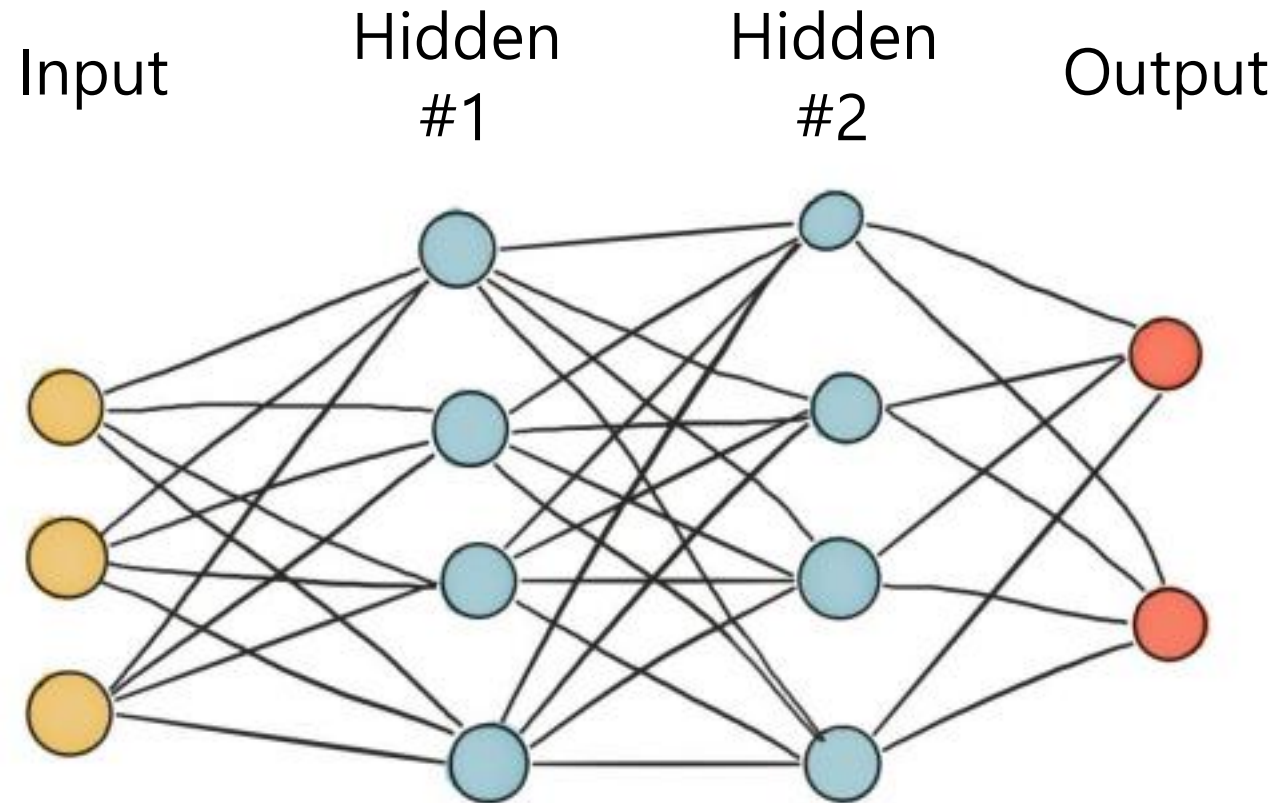
```
def softmax(x):  
    c = np.max(x)  
    exp_x = np.exp(x - c) # overflow prevention  
    sum_exp_x = np.sum(exp_x)  
    return exp_x / sum_exp_x
```

- 지수 함수(exp)를 사용하기 때문에 오버플로 (inf 값 발생)가 날 수 있다.
=> x 중에서의 $np.max$ 값 c 를 빼서 값이 무수히 커지는 것을 막아준다.
- $softmax(x) = \frac{\exp(x_k - C)}{\sum_{i=1}^n \exp(x_i - C)}$ 의 값을 return

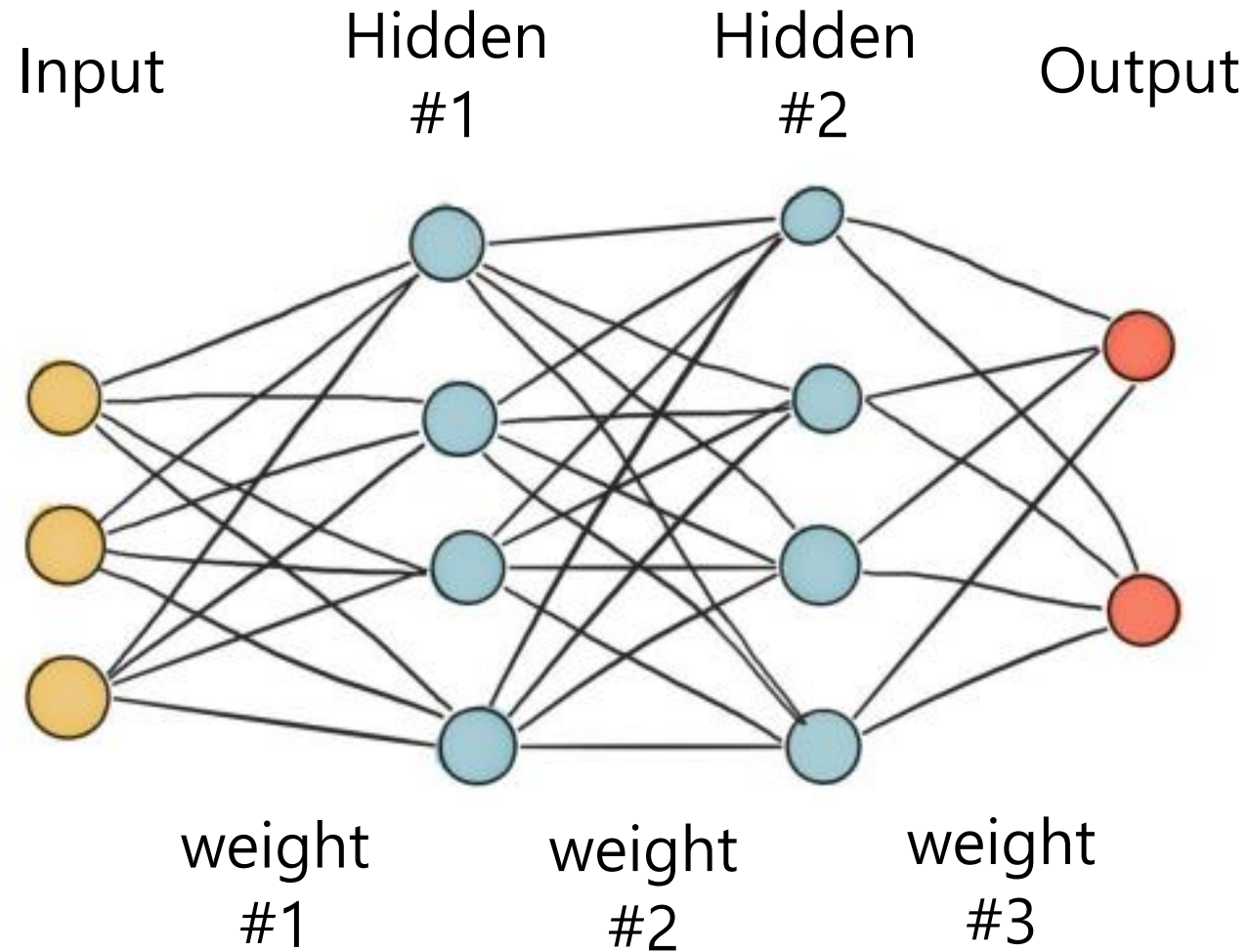
Forward & Back Prop.



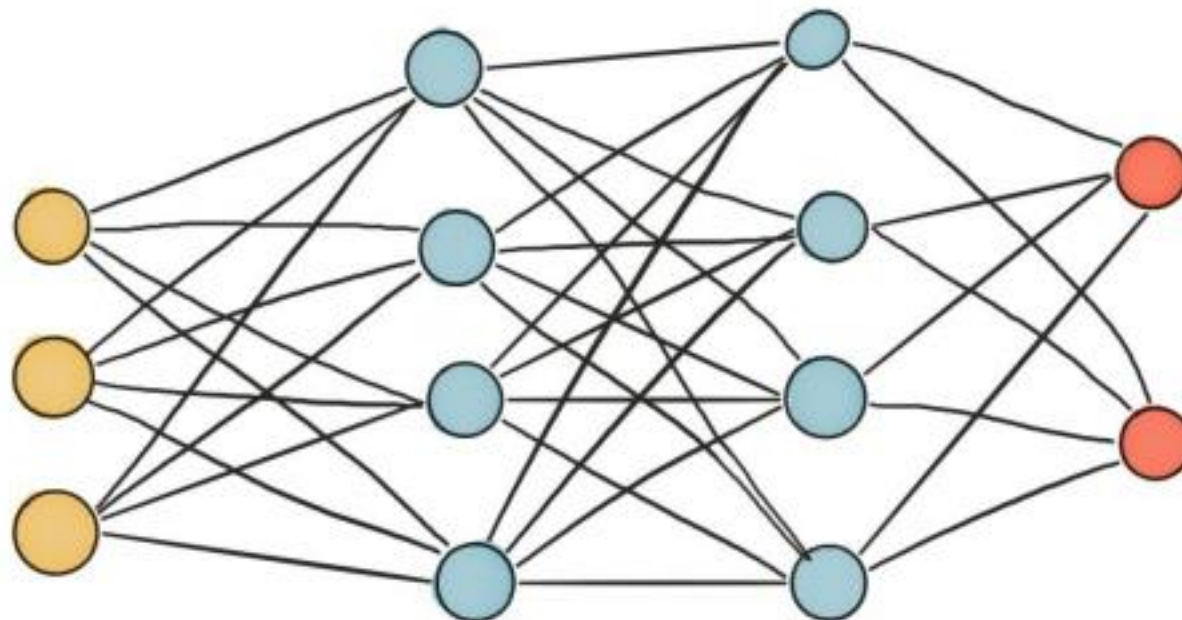
Forward & Back Prop.



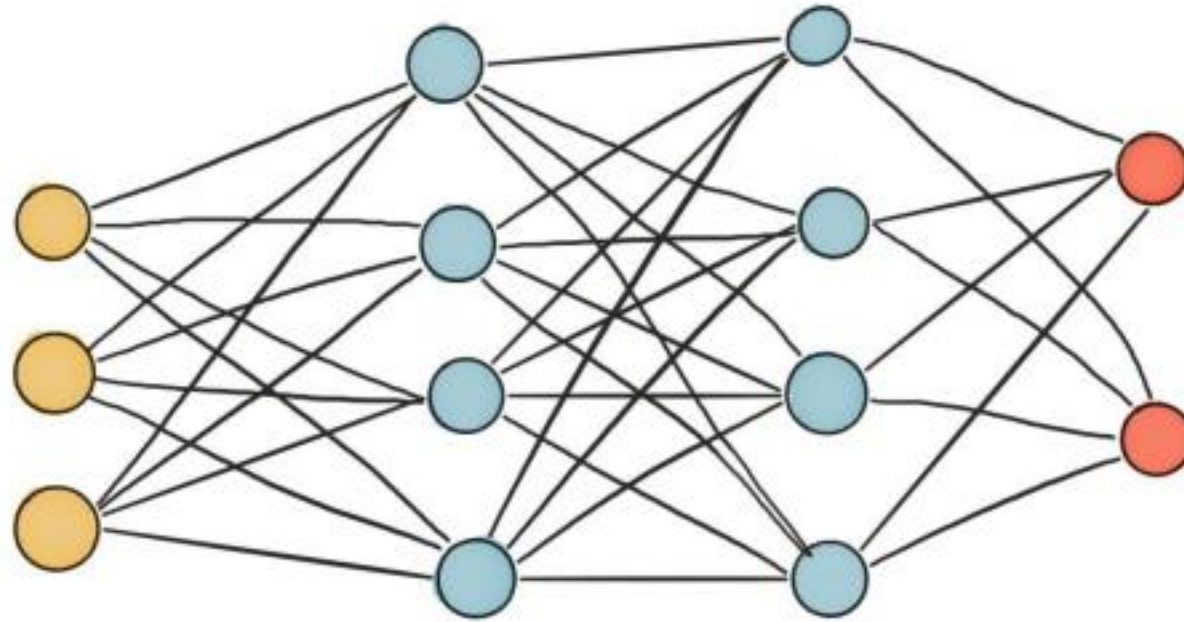
Forward & Back Prop.



Forward & Back Prop.

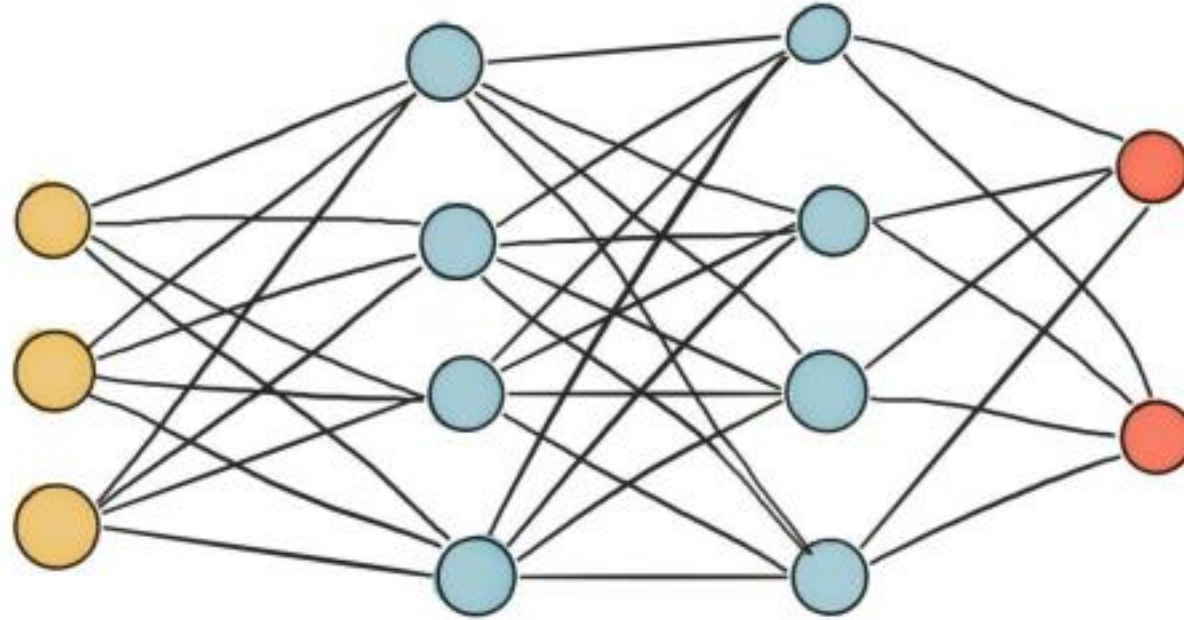


Forward & Back Prop.



$$\begin{bmatrix} w_{00} & w_{01} & w_{02} & w_{03} \\ w_{10} & w_{11} & w_{12} & w_{13} \\ w_{20} & w_{21} & w_{22} & w_{23} \end{bmatrix} \times \begin{bmatrix} w_{00} & w_{01} & w_{02} & w_{03} \\ w_{10} & w_{11} & w_{12} & w_{13} \\ w_{20} & w_{21} & w_{22} & w_{23} \\ w_{30} & w_{31} & w_{32} & w_{33} \end{bmatrix} \times \begin{bmatrix} w_{00} & w_{01} \\ w_{10} & w_{11} \\ w_{20} & w_{21} \\ w_{30} & w_{31} \end{bmatrix}$$

Forward & Back Prop.



$$\begin{bmatrix} w_{00} & w_{01} & w_{02} & w_{03} \\ w_{10} & w_{11} & w_{12} & w_{13} \\ w_{20} & w_{21} & w_{22} & w_{23} \end{bmatrix}$$

3x4

x

$$\begin{bmatrix} w_{00} & w_{01} & w_{02} & w_{03} \\ w_{10} & w_{11} & w_{12} & w_{13} \\ w_{20} & w_{21} & w_{22} & w_{23} \\ w_{30} & w_{31} & w_{32} & w_{33} \end{bmatrix}$$

4x4

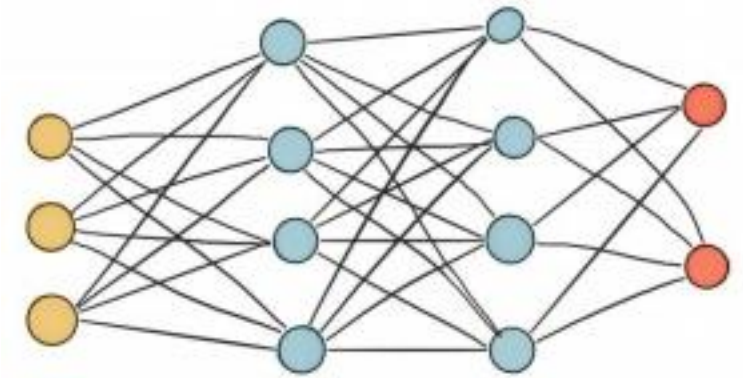
x

$$\begin{bmatrix} w_{00} & w_{01} \\ w_{10} & w_{11} \\ w_{20} & w_{21} \\ w_{30} & w_{31} \end{bmatrix}$$

4x2

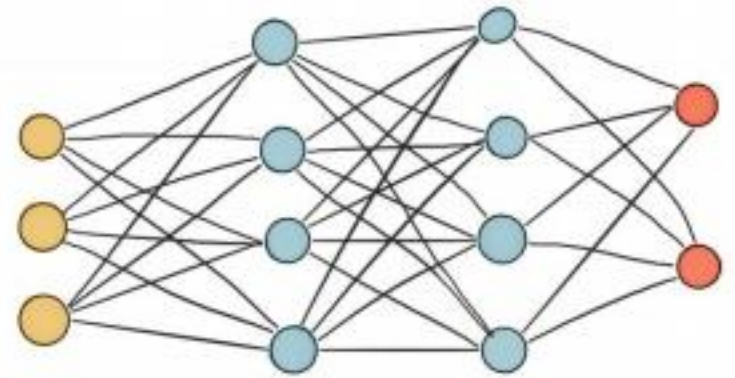
Forward & Back Prop.

$$y^* = w3 * sig(w2 * sig(w1 * x + b1) + b2) + b3$$



쉽게 이해되도록
loss = 예측값 - 실제로 설정

Forward & Back Prop.

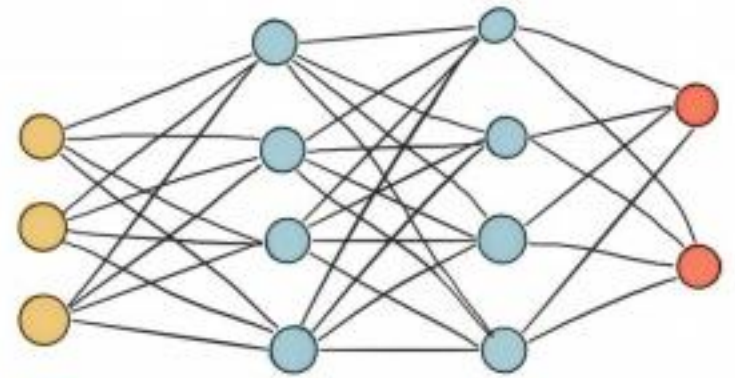


$$y^* = w3 * sig \ w2 * sig \ w1 * x + b1 \ + b2 \ + b3$$

$$\begin{aligned} loss &= y^* - y \\ &= w3 * sig \ w2 * sig \ w1 * x + b1 \ + b2 \ + b3 - y \end{aligned}$$

쉽게 이해되도록
loss = 예측값 - 실제로 설정

Forward & Back Prop.



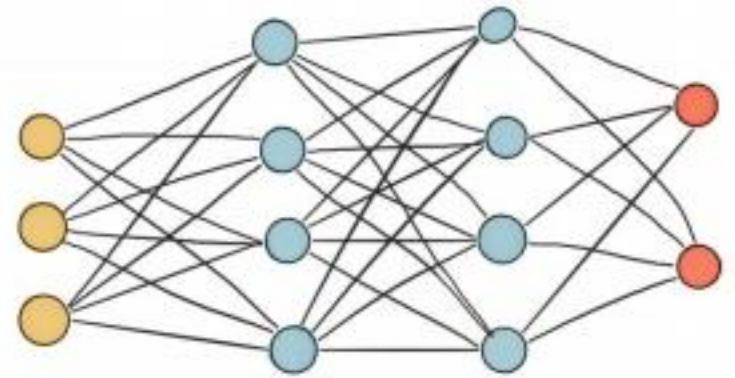
$$y^* = w3 * sig(w2 * sig(w1 * x + b1) + b2) + b3$$

쉽게 이해되도록
loss = 예측값 - 실제로 설정

$$\begin{aligned} loss &= y^* - y \\ &= w3 * sig(w2 * sig(w1 * x + b1) + b2) + b3 - y \end{aligned}$$

$$\frac{\partial loss}{\partial w3} = sig(w2 * sig(w1 * x + b1) + b2)$$

Forward & Back Prop.

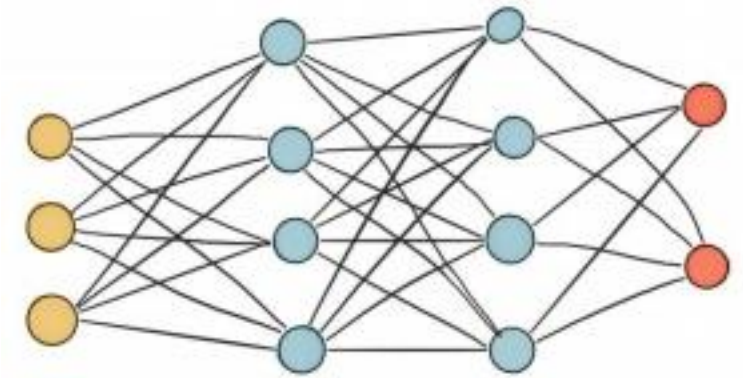


쉽게 이해되도록
loss = 예측값 - 실제로 설정

$$\frac{\partial loss}{\partial w3} = sig(w2 * sig(w1 * x + b1) + b2)$$

$$\frac{\partial loss}{\partial b3} = 1$$

Forward & Back Prop.



쉽게 이해되도록
loss = 예측값 - 실제로 설정

$$y^* = w3 * sig(w2 * sig(w1 * x + b1) + b2) + b3$$

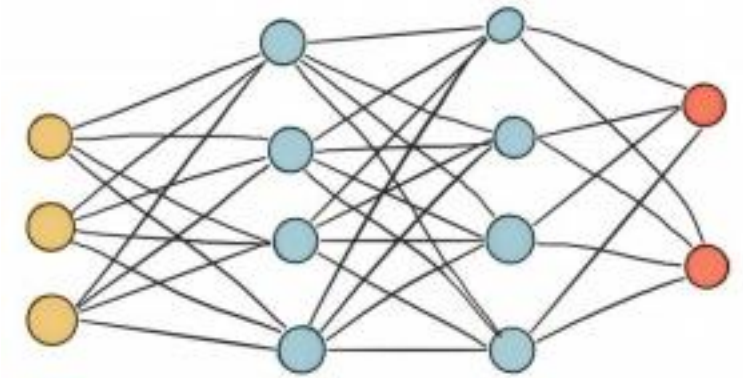
$$\begin{aligned} loss &= y^* - y \\ &= w3 * sig(w2 * sig(w1 * x + b1) + b2) + b3 - y \end{aligned}$$

$$\frac{\partial loss}{\partial w3} = sig(w2 * sig(w1 * x + b1) + b2)$$

$$\frac{\partial loss}{\partial b3} = 1$$

$$\frac{\partial loss}{\partial w2} = ??$$

Forward & Back Prop.



쉽게 이해되도록
loss = 예측값 - 실제로 설정

$$y_* = w3 * sig(w2 * sig(w1 * x + b1) + b2) + b3$$

$$\begin{aligned} loss &= y_* - y \\ &= w3 * sig(w2 * sig(w1 * x + b1) + b2) + b3 - y \end{aligned}$$

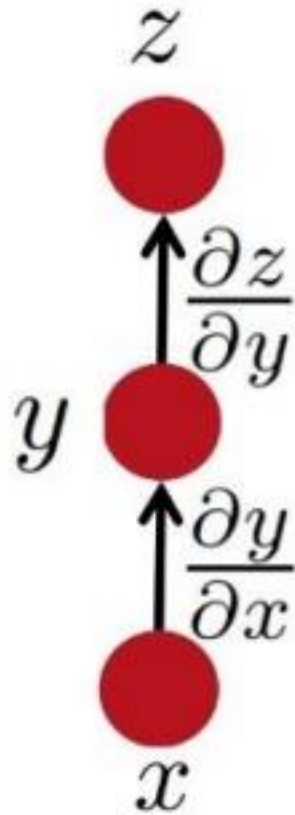
$$\frac{\partial loss}{\partial w3} = sig(w2 * sig(w1 * x + b1) + b2)$$

$$\frac{\partial loss}{\partial b3} = 1$$

$$\frac{\partial loss}{\partial w2} = chain\ rule \quad \text{!!}$$

Forward & Back Prop.

Simple Chain Rule



$$\Delta z = \frac{\partial z}{\partial y} \Delta y$$

$$\Delta y = \frac{\partial y}{\partial x} \Delta x$$

$$\Delta z = \frac{\partial z}{\partial y} \frac{\partial y}{\partial x} \Delta x$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} \frac{\partial y}{\partial x}$$

Forward & Back Prop.

Backpropagation: a simple example

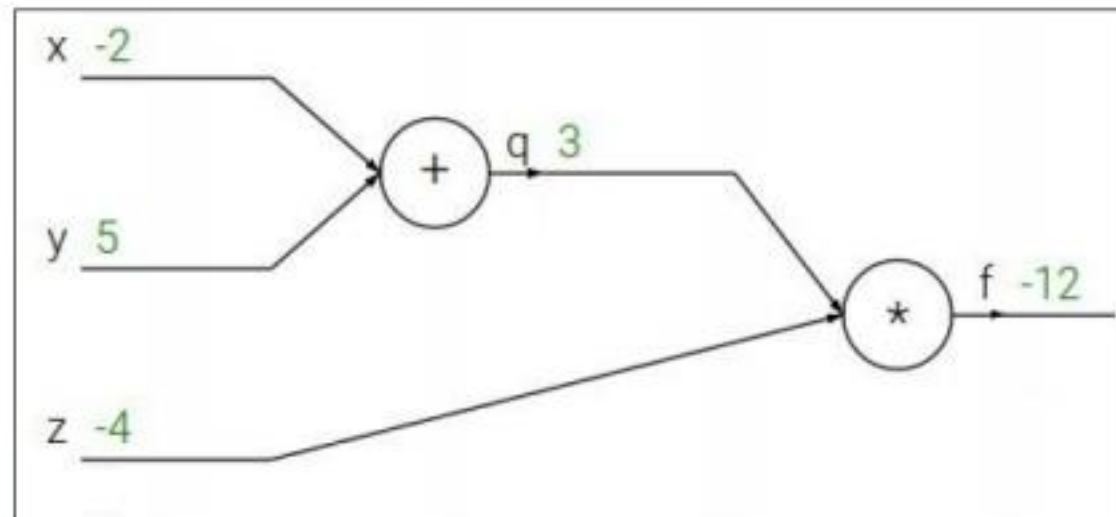
$$f(x, y, z) = (x + y)z$$

e.g. $x = -2, y = 5, z = -4$

$$q = x + y \quad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \quad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



Forward & Back Prop.

Backpropagation: a simple example

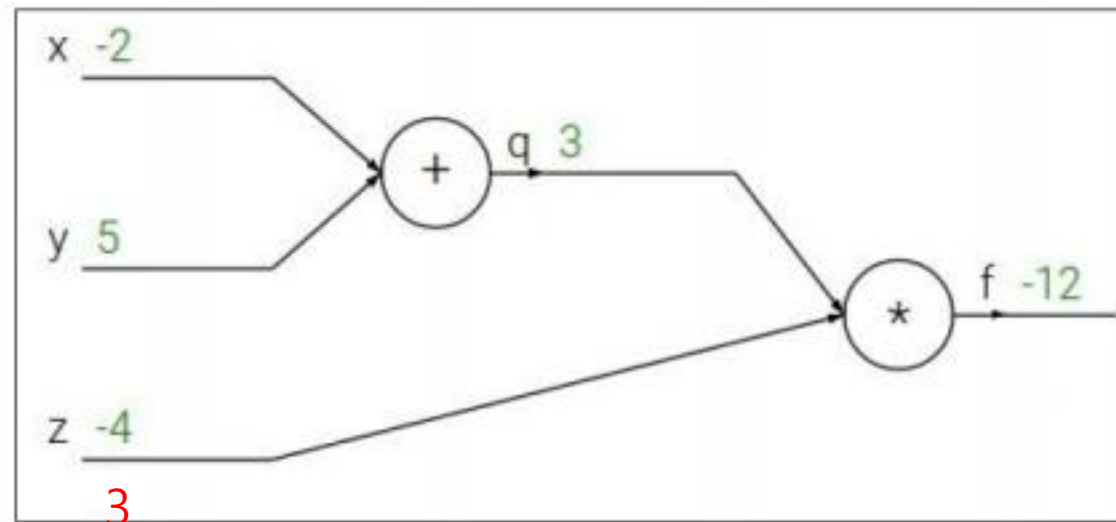
$$f(x, y, z) = (x + y)z$$

e.g. $x = -2, y = 5, z = -4$

$$q = x + y \quad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \quad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



$$\frac{\partial f}{\partial z} = q = x + y = -2 + 5 = 3$$

Forward & Back Prop.

Backpropagation: a simple example

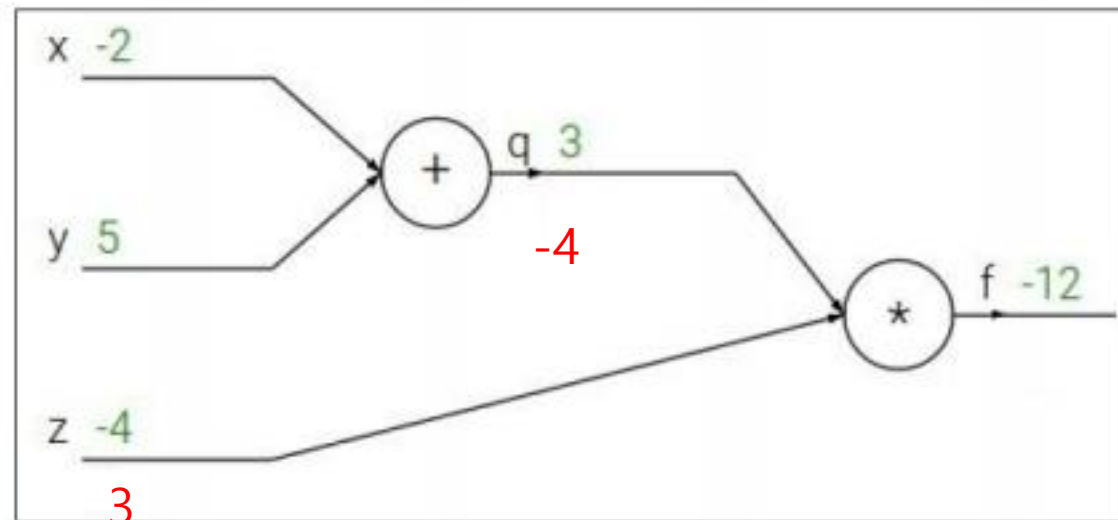
$$f(x, y, z) = (x + y)z$$

e.g. $x = -2, y = 5, z = -4$

$$q = x + y \quad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \quad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



$$\frac{\partial f}{\partial z} = q = x + y = -2 + 5 = 3$$

$$\frac{\partial f}{\partial q} = z = -4$$

Forward & Back Prop.

Backpropagation: a simple example

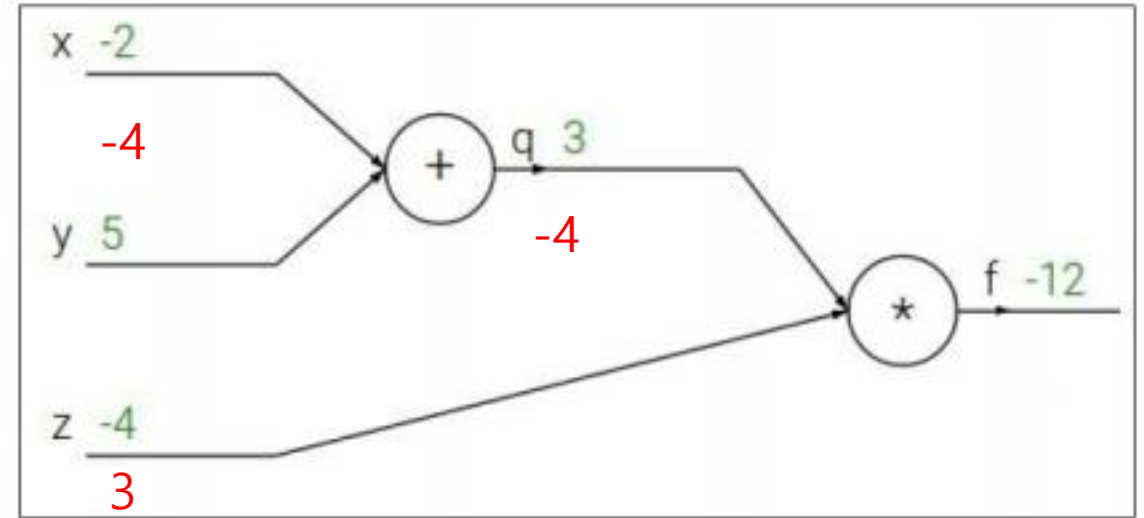
$$f(x, y, z) = (x + y)z$$

e.g. $x = -2, y = 5, z = -4$

$$q = x + y \quad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \quad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



$$\frac{\partial f}{\partial z} = q = x + y = -2 + 5 = 3$$

$$\frac{\partial f}{\partial q} = z = -4 = \frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x} = -4 * 1 = -4$$

Forward & Back Prop.

Backpropagation: a simple example

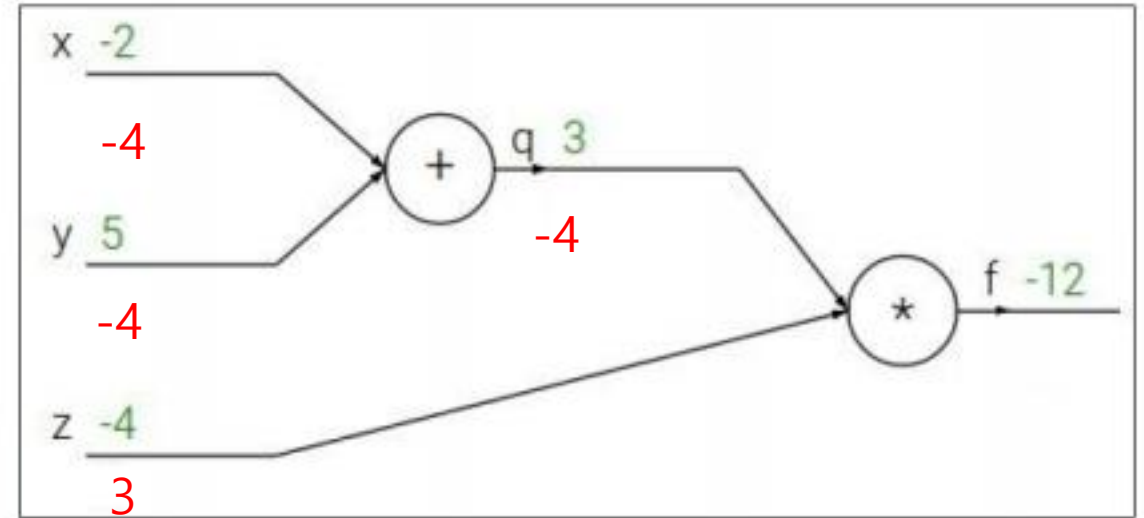
$$f(x, y, z) = (x + y)z$$

e.g. $x = -2, y = 5, z = -4$

$$q = x + y \quad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \quad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

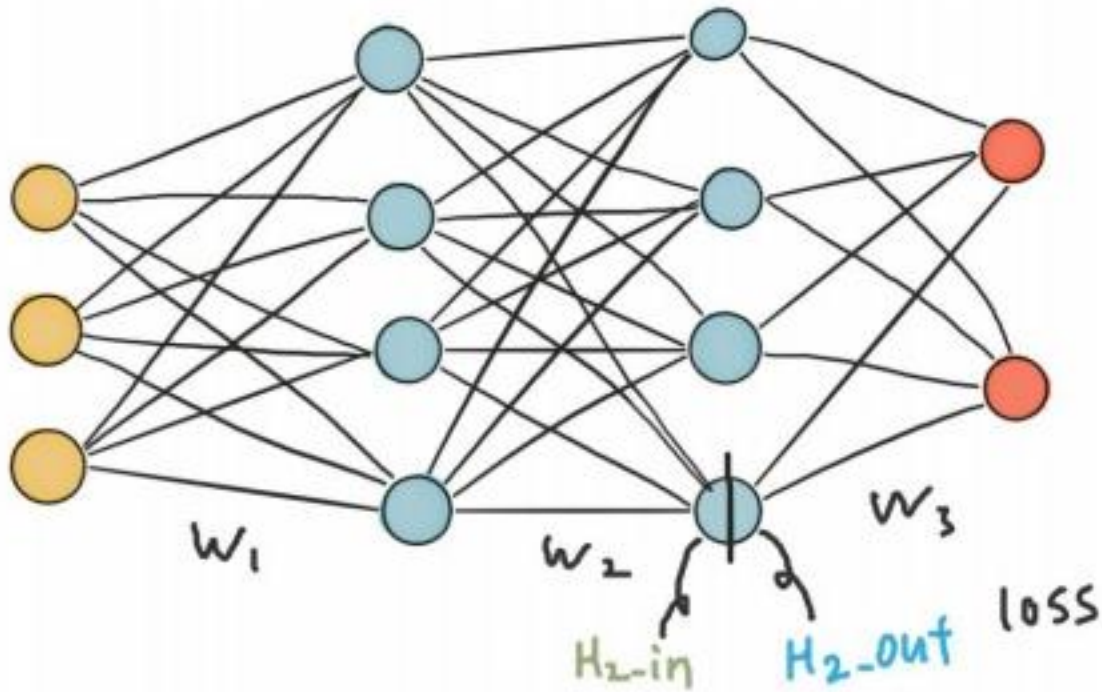


$$\frac{\partial f}{\partial z} = q = x + y = -2 + 5 = 3$$

$$\frac{\partial f}{\partial q} = z = -4 = \frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x} = -4 * 1 = -4$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y} = -4 * 1 = -4$$

Forward & Back Prop.



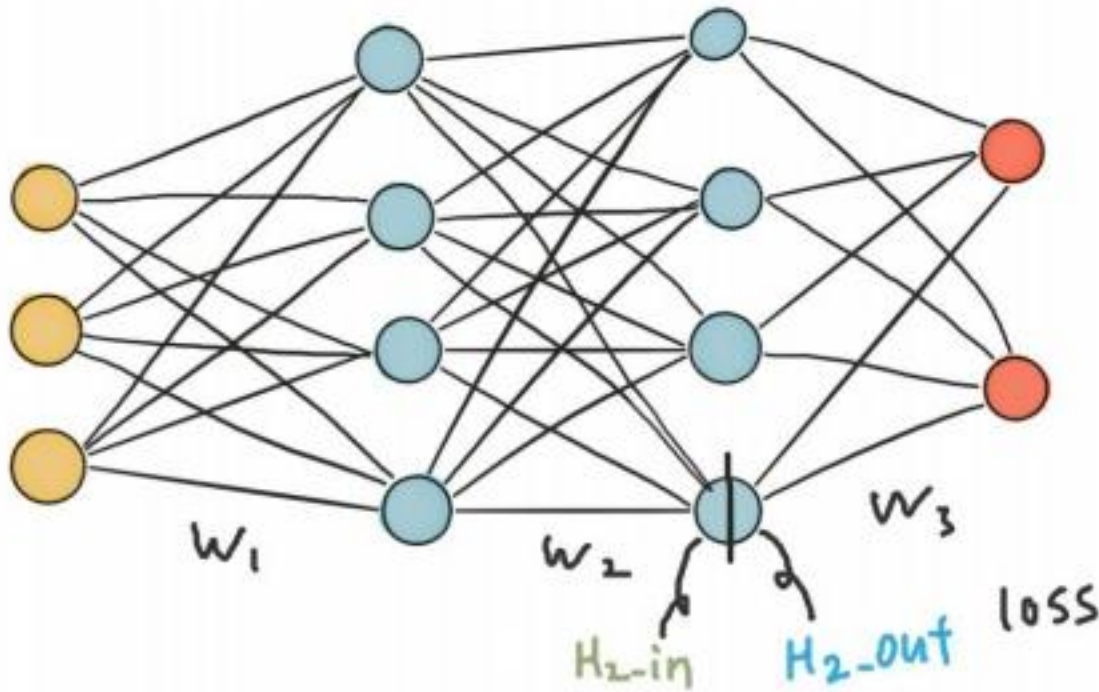
$$loss = w_3 \times \text{sig}(\underbrace{w_2 \times \text{sig}(w_1 x + b_1) + b_2}_{H_2\text{-in}}) + b_3 - y$$

$H_2\text{-out}$

$$\frac{\partial loss}{\partial w_2} = \frac{\partial loss}{\partial H_2\text{-out}} \times \frac{\partial H_2\text{-out}}{\partial H_2\text{-in}} \times \frac{\partial H_2\text{-in}}{\partial w_2}$$

$\frac{\partial loss}{\partial w_2}$ $\frac{\partial H_2\text{-out}}{\partial H_2\text{-in}}$ $\frac{\partial H_2\text{-in}}{\partial w_2}$
 $H_2\text{-out}$ 의 비례 $H_2\text{-in}$ 의 비례 w_2 의 비례

Forward & Back Prop.



$$loss = w_3 \times \text{sig}(\underbrace{w_2 \times \text{sig}(w_1 x + b)}_{H_2\text{-in}} + b_2) + b_3 - y$$

$H_2\text{-out}$

$$\frac{\partial loss}{\partial w_2} = \frac{\partial loss}{\partial H_2\text{-out}} \times \frac{\partial H_2\text{-out}}{\partial H_2\text{-in}} \times \frac{\partial H_2\text{-in}}{\partial w_2}$$

$\frac{\partial loss}{\partial w_2}$ $\frac{\partial H_2\text{-out}}{\partial H_2\text{-in}}$ $\frac{\partial H_2\text{-in}}{\partial w_2}$
 $H_2\text{-out}$ 의 비례 $H_2\text{-out}$ $H_2\text{-in}$ 의 비례 w_2 의 비례

$$\frac{\partial loss}{\partial w_2} = w_3 * \text{sigmoid}(h2_in)' * \text{sigmoid}(w_1 * x + b)$$

Forward & Back Prop.

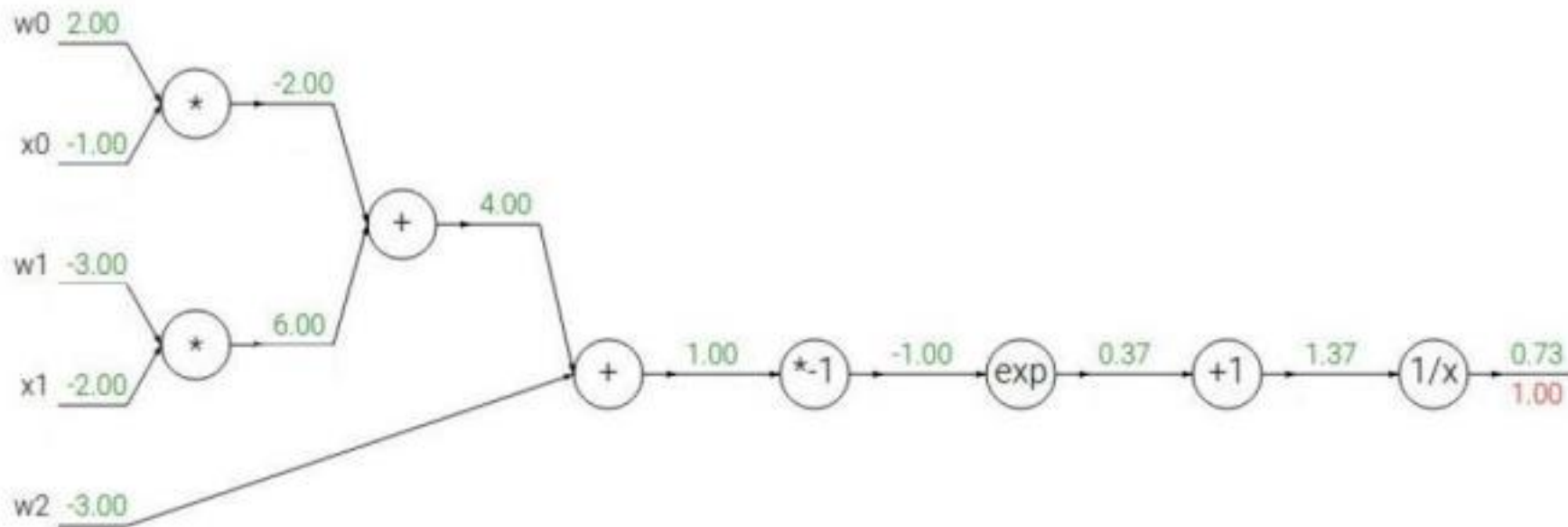
(참고) sigmoid 함수의 미분

$$\sigma(x)' = \frac{\delta\{1+e^{-x}\}^{-1}}{\delta x} = -(1+e^{-x})^{-2} \cdot e^{-x} = \frac{e^{-x}}{(1+e^{-x})^2}$$

$$\sigma(x)(1-\sigma(x)) = \frac{1}{1+e^{-x}} \left(1 - \frac{1}{1+e^{-x}}\right) = \frac{1}{1+e^{-x}} \left(\frac{e^{-x}}{1+e^{-x}}\right) = \frac{e^{-x}}{(1+e^{-x})^2}$$

Forward & Back Prop.

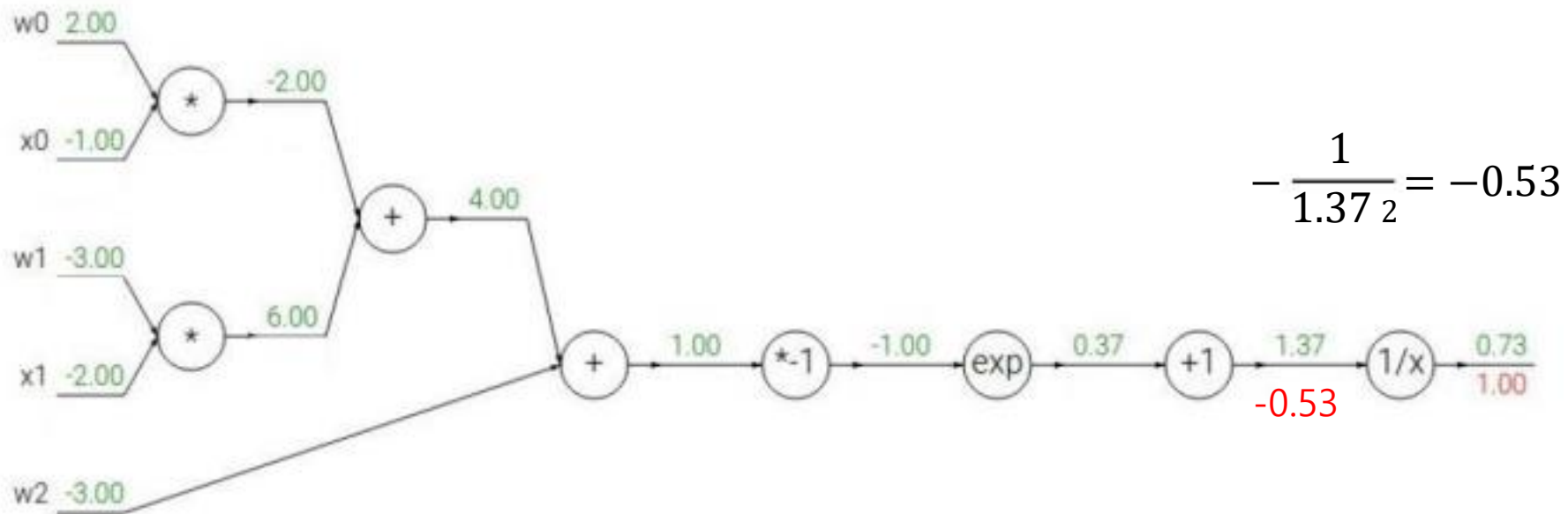
Another example: $f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$



$f(x) = e^x$	\rightarrow	$\frac{df}{dx} = e^x$		$f(x) = \frac{1}{x}$	\rightarrow	$\frac{df}{dx} = -1/x^2$
$f_a(x) = ax$	\rightarrow	$\frac{df}{dx} = a$		$f_c(x) = c + x$	\rightarrow	$\frac{df}{dx} = 1$

Forward & Back Prop.

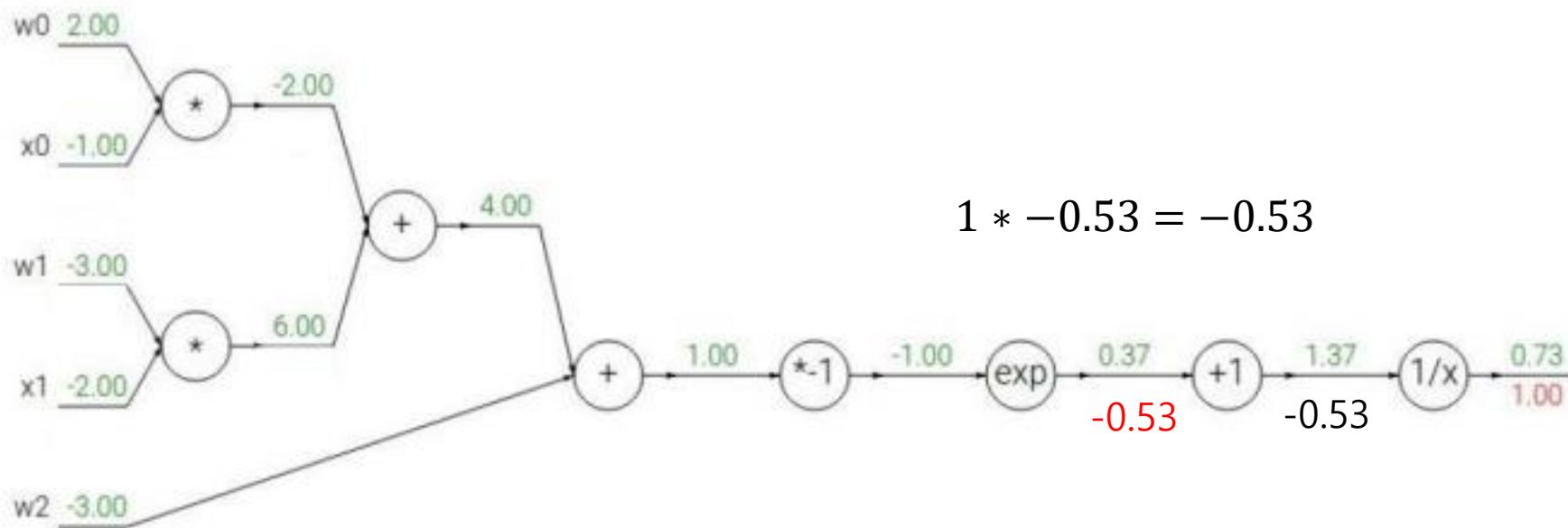
Another example: $f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$



$f(x) = e^x$	→	$\frac{df}{dx} = e^x$		$f(x) = \frac{1}{x}$	→	$\frac{df}{dx} = -1/x^2$
$f_a(x) = ax$	→	$\frac{df}{dx} = a$		$f_c(x) = c + x$	→	$\frac{df}{dx} = 1$

Forward & Back Prop.

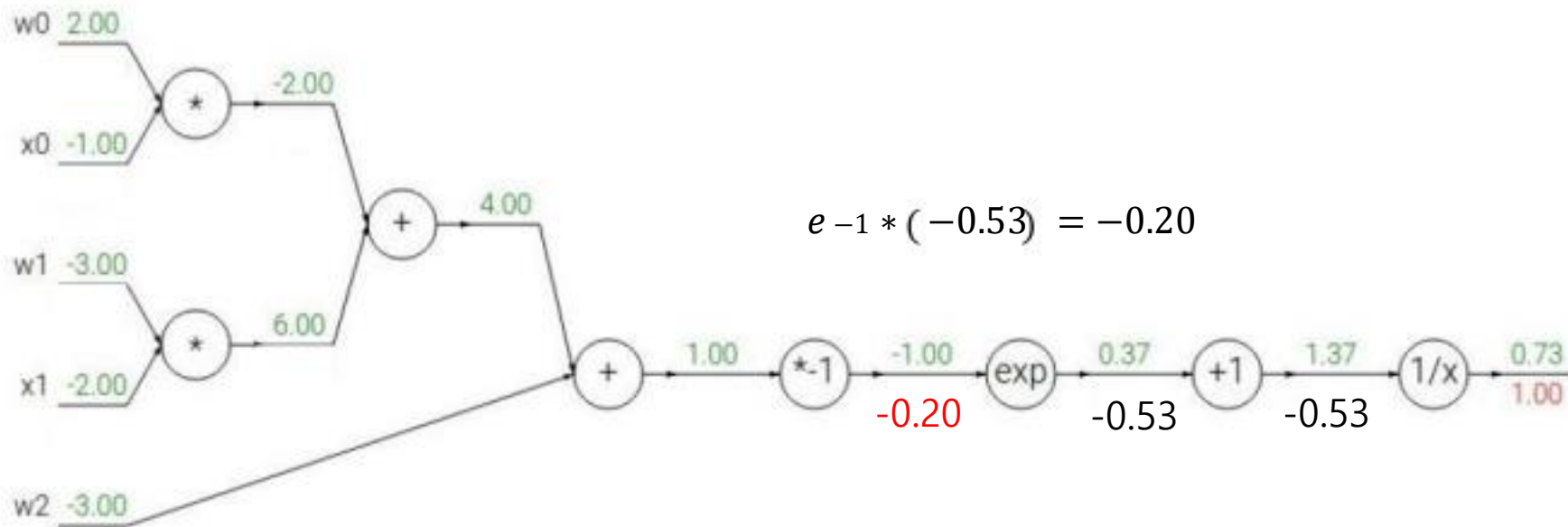
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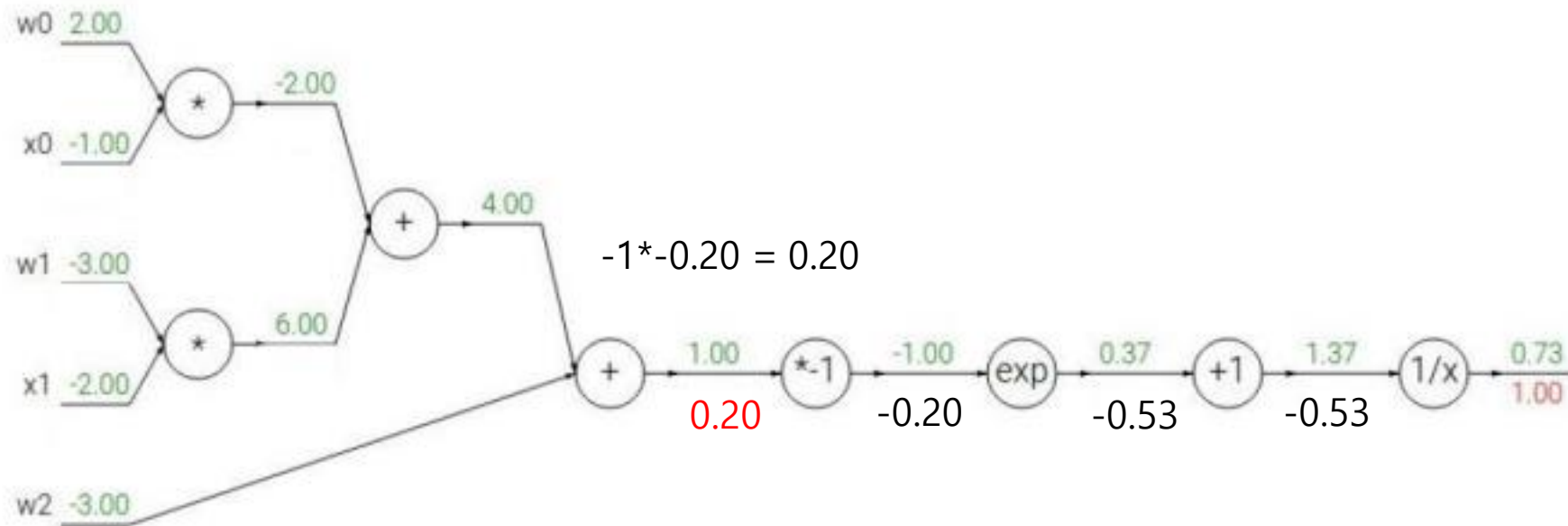
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Forward & Back Prop.

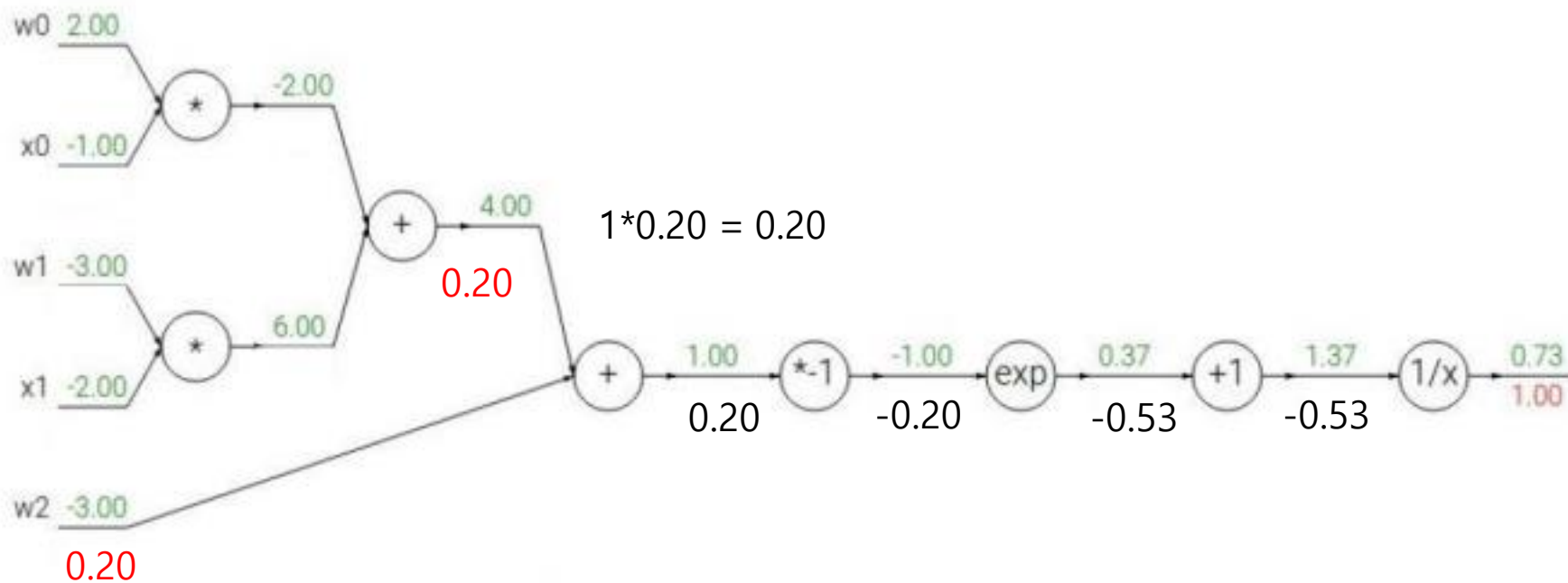
Another example: $f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$



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Forward & Back Prop.

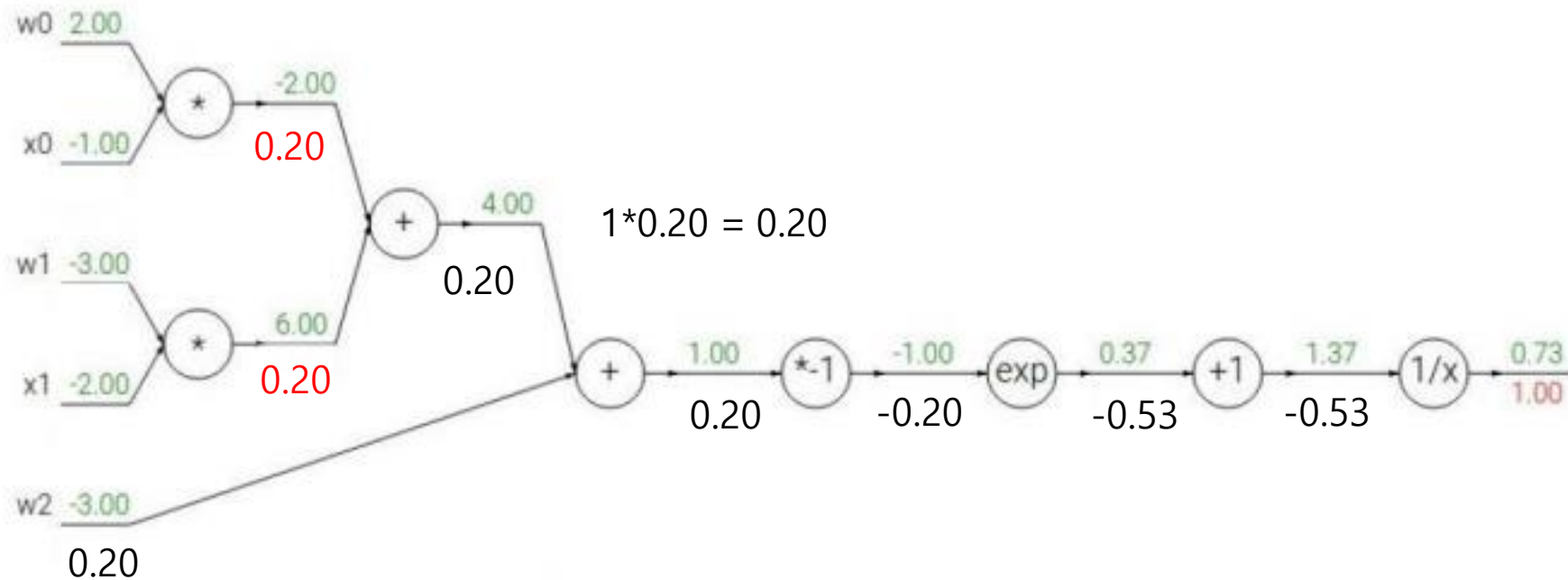
Another example: $f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$



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Forward & Back Prop.

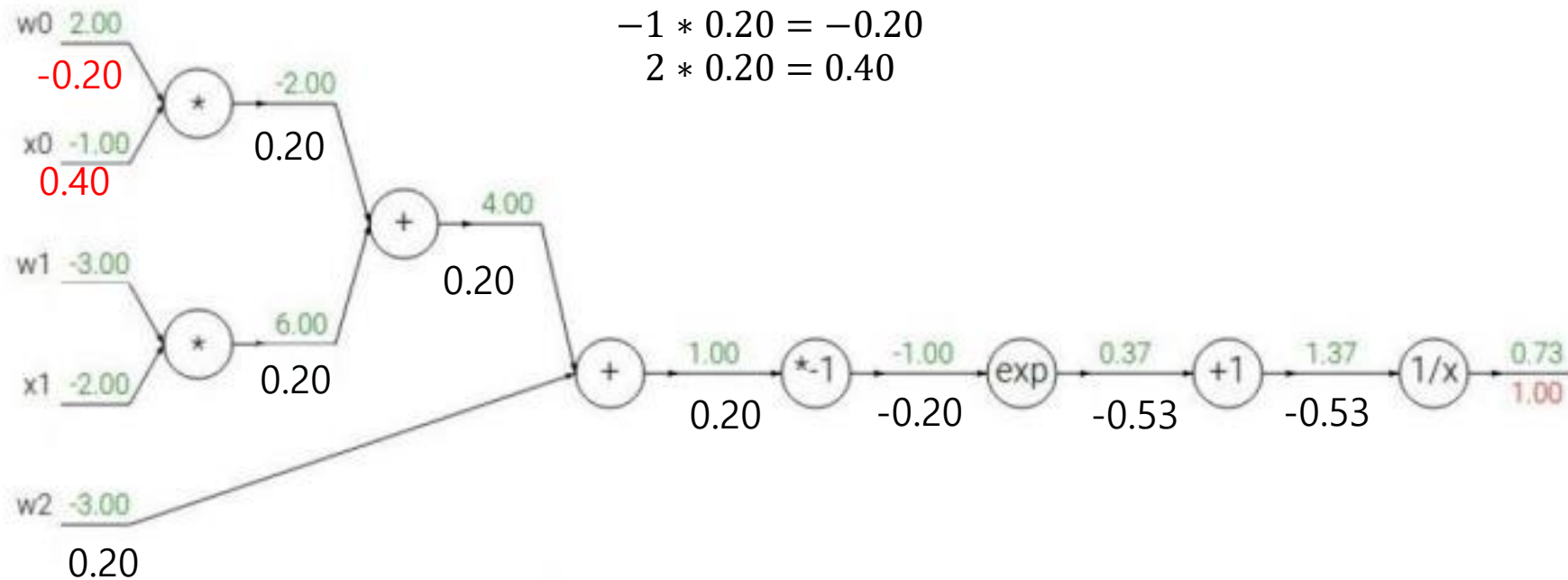
Another example: $f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$



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Forward & Back Prop.

Another example: $f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$

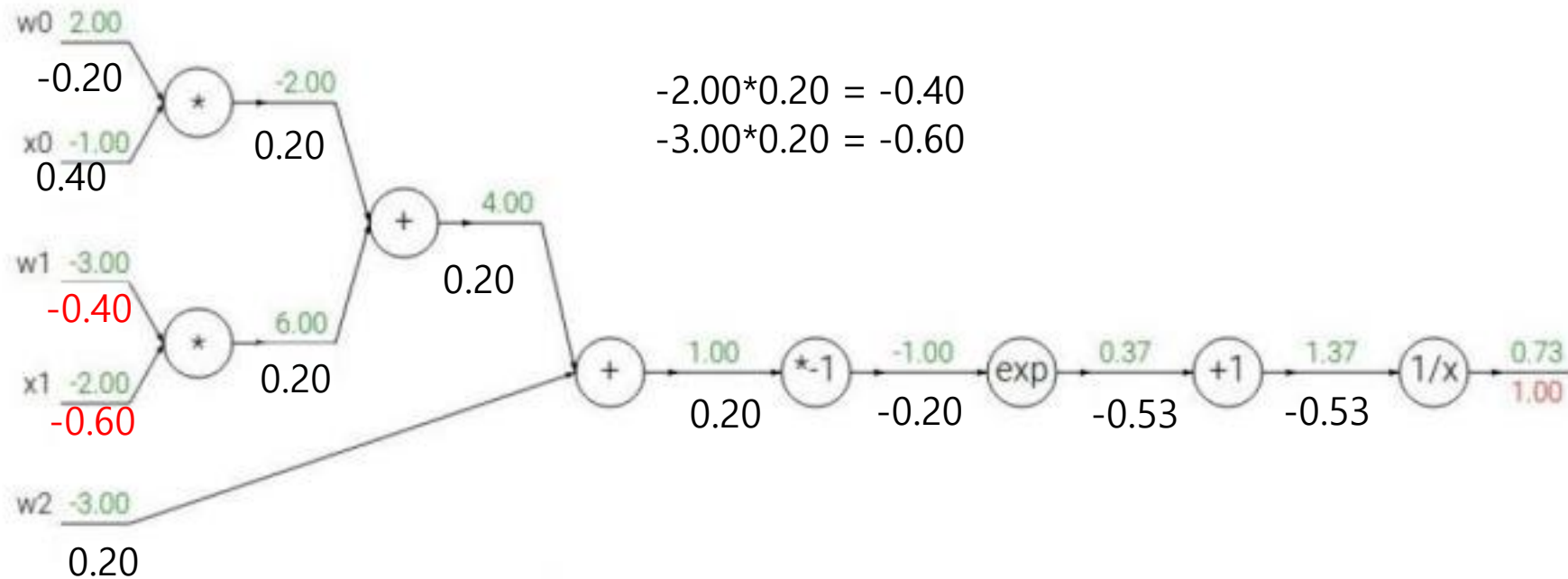


$$\begin{aligned} -1 * 0.20 &= -0.20 \\ 2 * 0.20 &= 0.40 \end{aligned}$$

$f(x) = e^x$	\rightarrow	$\frac{df}{dx} = e^x$		$f(x) = \frac{1}{x}$	\rightarrow	$\frac{df}{dx} = -1/x^2$
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Forward & Back Prop.

Another example: $f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2x_2)}}$



$f(x) = e^x$	\rightarrow	$\frac{df}{dx} = e^x$		$f(x) = \frac{1}{x}$	\rightarrow	$\frac{df}{dx} = -1/x^2$
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Neural Network

A mostly complete chart of Neural Networks

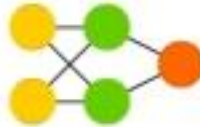
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-  Backfed Input Cell
-  Input Cell
-  Noisy Input Cell
-  Hidden Cell
-  Probabilistic Hidden Cell
-  Spiking Hidden Cell
-  Output Cell
-  Match Input Output Cell
-  Recurrent Cell
-  Memory Cell
-  Different Memory Cell
-  Kernel
-  Convolution or Pool

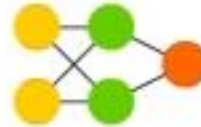
Perceptron (P)



Feed Forward (FF)



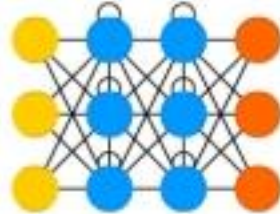
Radial Basis Network (RBF)



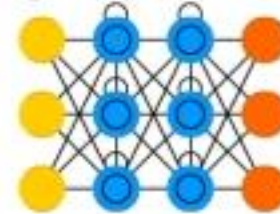
Deep Feed Forward (DFF)



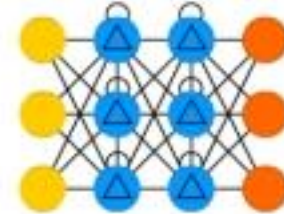
Recurrent Neural Network (RNN)



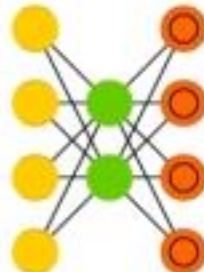
Long / Short Term Memory (LSTM)



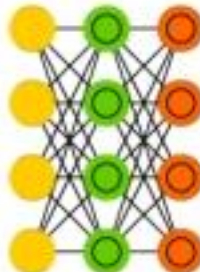
Gated Recurrent Unit (GRU)



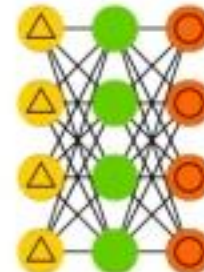
Auto Encoder (AE)



Variational AE (VAE)



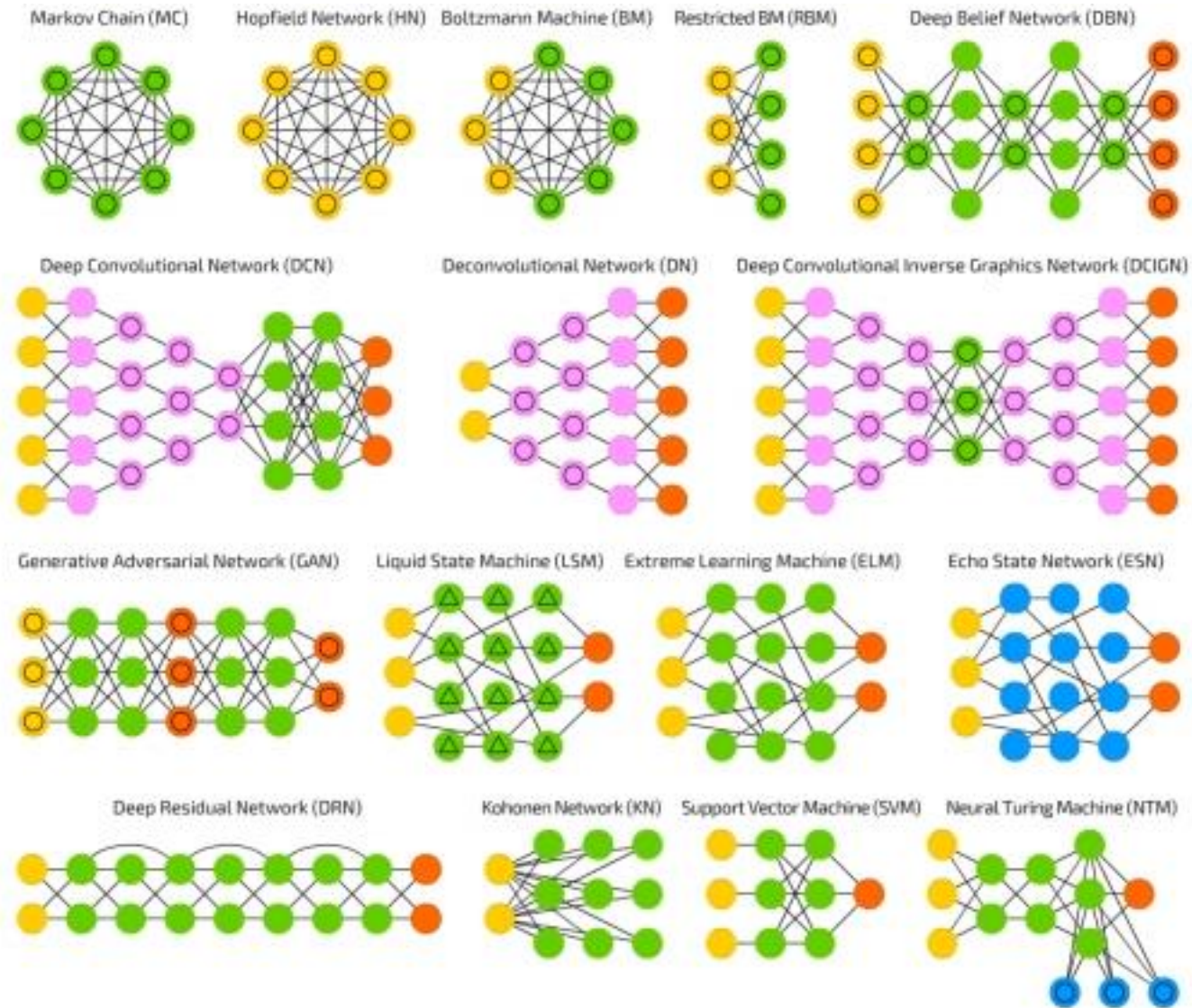
Denoising AE (DAE)



Sparse AE (SAE)



Neural Network



Forward & Back Prop.

$$y = \frac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}} = [1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}]^{-1}$$

$$\begin{aligned} \text{loss} &= \hat{y} - y \\ &= [1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}]^{-1} - y \end{aligned}$$

$\alpha = -1$
 $\beta = 0.37$
 $\gamma = 1.37$

$$\begin{aligned} w_0 &= 2 \\ w_1 &= -3 \\ w_2 &= -3 \\ x_0 &= -1 \\ x_1 &= -2 \end{aligned}$$

$$\frac{\partial \text{loss}}{\partial r} = (r^{-1})' = -\frac{1}{r^2} = -0.5327$$

$$\begin{aligned} \text{loss} &= r^{-1} - y \\ r &= 1 + \beta \\ \beta &= e^{\alpha} \end{aligned}$$

$$\frac{\partial r}{\partial \beta} = 1$$

$$\frac{\partial \beta}{\partial \alpha} = (e^{\alpha})' = e^{\alpha} = 0.3678$$

$$\frac{\partial \alpha}{\partial w_0} = -x_0 = 1$$

$$\begin{aligned} \frac{\partial \text{loss}}{\partial w_0} &= \frac{\partial \text{loss}}{\partial r} \times \frac{\partial r}{\partial \beta} \times \frac{\partial \beta}{\partial \alpha} \times \frac{\partial \alpha}{\partial w_0} \\ &= -0.5327 \times 1 \times 0.3678 \times 1 \\ &= -0.1959 \approx -0.2 \end{aligned}$$

Forward & Back Prop.

```
test.py x
1 import numpy as np
2 import torch
3 import torch.nn as nn
4 import torch.optim as optim
5 import torch.nn.init as init
6 from torch.autograd import Variable
7 from visdom import Visdom
8 viz = Visdom()
9
10 num_data = 1000
11 num_epoch = 5000
12
13 x = init.uniform(torch.Tensor(num_data,1), -15,15)
14 y = 8*(x**2) + 7*x + 3
15
16 noise = init.normal(torch.FloatTensor(num_data,1),std=1)
17 y_noise = y + noise
```

Forward & Back Prop.

필요한 라이브러리

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Forward & Back Prop.

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데이터 생성

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Forward & Back Prop.

```
21 model = nn.Sequential(  
22     nn.Linear(1,10),  
23     nn.ReLU(),  
24     nn.Linear(10,6),  
25     nn.ReLU(),  
26     nn.Linear(6,1),  
27 ).cuda()  
28  
29 loss_func = nn.L1Loss()  
30 optimizer = optim.SGD(model.parameters(), lr=0.001)  
31  
32 loss_arr = []  
33 label = Variable(y_noise.cuda())  
34 for i in range(num_epoch):  
35     output = model(Variable(x.cuda()))  
36     optimizer.zero_grad()  
37  
38     loss = loss_func(output, label)  
39     loss.backward()  
40     optimizer.step()  
41     if i % 100 == 0:  
42         print(loss)  
43     loss_arr.append(loss.cpu().data.numpy()[0])  
44  
45 param_list = list(model.parameters())  
46 print(param_list)
```

Forward & Back Prop.

Neural Network 모델 생성

loss function 및
gradient descent
optimizer 생성

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Forward & Back Prop.

Neural Network 모델 생성

loss function 및
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optimizer 생성

<training 단계 >

1. 모델로 결과값 추정
2. loss 및 gradient 계산
3. 모델 업데이트

```
21 model = nn.Sequential(  
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Forward & Back Prop.

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2. loss 및 gradient 계산
3. 모델 업데이트

training 이후 파라미터 값 확인

```
21 model = nn.Sequential(  
22     nn.Linear(1,10),  
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```

퍼셉트론의 학습과정

1. 임의의 w 와 b 를 설정
2. 주어진 훈련 데이터를 이용하여, 결과값(y)를 도출
3. 결과값(y)와 실제 결과값(\hat{y}) 사이의 오차 계산
4. 오차(Loss)를 줄이는 방향으로 w 와 b 를 재설정(학습)
5. 오차를 최소한으로 줄이도록 2~4의 과정을 계속반복진행

Book

주교재

밑바닥부터 시작하는 딥러닝 1, 사이토 고키 지음, 개앞맨시 옮김 .

(Deep Learning from Scratch의 번역서입니다 .)

부교재

Pytorch로 시작하는 딥러닝, 비슈누 수브라마니안 지음 김태완 옮김 .

(Deep Learning with PYTORCH 의 번역서입니다 .)

