$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \frac{1}{m} \begin{bmatrix} f_{B,x}^{\alpha} \\ f_{B,y}^{\alpha} \end{bmatrix}$$

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Problem 3.

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} f_{B,x} \\ f_{B,y} \\ f_{B,z} \end{bmatrix} \cdot \frac{1}{m} - \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$\dot{V} = \frac{u\dot{u} + v\dot{v} + w\dot{w}}{V}$$

$u = V \cos\alpha \cos\beta$

$v = V \sin\beta$

$w = V \sin\alpha \cos\beta.$

$$\dot{\alpha} = \frac{\dot{w}u - \dot{u}w}{u^2 + w^2}$$

$$\dot{\beta} = \frac{-vu\dot{u} + (u^2 + w^2)\dot{v} - vw\dot{w}}{V^2 \sqrt{u^2 + w^2}}$$

$$\begin{bmatrix} f_{B,x} \\ f_{B,y} \\ f_{B,z} \end{bmatrix} = m \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} + m \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$f_{B,x} = m\dot{u} + m(qw - rv)$

$f_{B,y} = m\dot{v} + m(ur - pw)$

$f_{B,z} = m\dot{w} + m(vp - qu)$

$$\dot{V} = \cos\alpha \cos\beta \, \dot{u} + \sin\beta \, \dot{v} + \sin\alpha \cos\beta \, \dot{w} = X$$

we need to prove this.

$$\dot{V} = \frac{1}{m} \cos\alpha \cos\beta \, f_{B,x} + \frac{1}{m} \sin\beta \, f_{B,y} + \frac{1}{m} \sin\alpha \cos\beta \, f_{B,z}.$$

$$\dot{V} = \overbrace{\cos\alpha\cos\beta \, \dot{u} + \sin\beta \, \dot{v} + \sin\alpha\cos\beta \, \dot{w}}^{X} + (qw - rv)\cos\alpha\cos\beta +$$

$$+ \sin\beta \, (ur - pw) + \sin\alpha\cos\beta \, (vp - qu) =$$

$$= X + (qw - rv)\frac{u}{V} + \frac{v}{V}(ur - pw) + \frac{w}{V}(vp - qu) = X$$

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$\dot{V}$ term is correct

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Problem 3

$$\cancel{\dot{\alpha} = \frac{V^2}{(\tan\beta)^2}\left(V\cos\alpha\cos\beta\,\dot{w} - \dot{u}\,V\sin\alpha\cos\beta\right)}$$

$$\cancel{\frac{pV^2}{u^2+w^2} - (\tan\beta)^2}$$

$$\dot{\alpha} = \frac{1}{u^2+w^2}(\dot{w}u - \dot{u}w) = \frac{(\tan\beta)^2}{V^2}\left(\dot{w}V\cos\alpha\cos\beta - \dot{u}\,V\sin\alpha\cos\beta\right) =$$

$$(\tan\beta)^2 = \frac{V^2}{u^2+w^2} \qquad = \frac{\sin\beta^2}{V^2\sin\beta^2}\left(\dot{w}\,V\frac{\cos\alpha}{\cos\beta} - \dot{u}\,V\frac{\sin\alpha}{\cos\beta}\right) =$$

$$= \frac{1}{V}\left(\dot{w}\frac{\cos\alpha}{\cos\beta} - \dot{u}\frac{\sin\alpha}{\cos\beta}\right) = \dot{\alpha} \quad \frac{1}{V}$$

let's check.

$$\dot{\alpha} = q - (p\cos\alpha + r\sin\alpha)\tan\beta + \frac{1}{V}\left(-\frac{\sin\alpha}{\cos\beta}\dot{u} + \frac{\cos\alpha}{\cos\beta}\dot{w} + \frac{\cos\alpha}{\cos\beta}(vp - qu) - \frac{\sin\alpha}{\cos\beta}(qw - rv)\right) =$$

$$= \cancel{\dot{\alpha}}$$

Because.

$$\dot{\alpha} = \frac{1}{V}\left(\dot{w}\left(\frac{\cos\alpha}{\cos\beta}\right) - \dot{u}\left(\frac{\sin\alpha}{\cos\beta}\right)\right) = \frac{1}{V}\left(\dot{w}\frac{\cos\alpha}{\cos\beta} - \dot{u}\frac{\sin\alpha}{\cos\beta}\right) + q - (p\cos\alpha + r\sin\alpha)\tan\beta + \frac{1}{V}\left(\frac{\cos\alpha}{\cos\beta}(vp - qu)\right) - \frac{1}{V}\left(\frac{\sin\alpha}{\cos\beta}(qw - rv)\right)$$

let's prove.

$$q - (p\cos\alpha + r\sin\alpha)\tan\beta = \frac{1}{V}\left(\frac{\sin\alpha}{\cos\beta}(qw - rv)\right) - \frac{1}{V}\left(\frac{\cos\alpha}{\cos\beta}(vp - qu)\right)$$

let's insert w, v, and q.

$$q - (p\cos\alpha + r\sin\alpha)\sin\beta = \left(\sin\alpha\left(q\sin\alpha\cos\beta - r\sin\beta\right) - \left(\cos\alpha\left(p\sin\beta - q\cos\alpha\cos\beta\right)\right)\right.$$

$$q - (p\cos\alpha + r\sin\alpha)\sin\beta = q - r\sin\beta\sin\alpha - p\sin\beta\cos\alpha.$$

true!

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$$\dot{\beta} = \frac{\tan\beta\left(-u\dot{u} + \frac{u^2+w^2}{V}\dot{v} - w\dot{w}\right)}{V^2} =$$

$$= \frac{\sin\beta\left(-V\cos\alpha\,\dot{u} + \frac{V\cos\beta}{\sin\beta}\dot{v} - V\sin\alpha\,\dot{w}\right)}{V^2} =$$

$$= \frac{1}{V}\left(-\sin\beta\cos\alpha\,\dot{u} + \cos\beta\,\dot{v} - \sin\alpha\sin\beta\,\dot{w}\right).$$

prove:

$$\dot{\beta} = p\sin\alpha - r\cos\alpha + \frac{1}{V}\left(-\cos\alpha\sin\beta\,\dot{u} + \cos\beta\,\dot{v} - \sin\alpha\sin\beta\,\dot{w}\right) -$$

$$- \frac{1}{V}\left(\cos\alpha\sin\beta\,(qw - rv)\right) + \frac{1}{V}\cos\beta\,(ur - pw) -$$

terms with q ~~are dia~~ become zero.

Because.

$$\frac{1}{V}\sin\alpha\sin\beta\,(vp - qu)$$

$$\dot{\beta} = \frac{1}{V}\left(-\sin\beta\cos\alpha\,\dot{u} + \cos\beta\,\dot{v} - \sin\alpha\sin\beta\,\dot{w}\right)$$

So, true!

$$-\cos\alpha\sin\beta\,qw + \sin\alpha\sin\beta\,qu = 0.$$

$$Vp\sin\alpha = \sin\alpha\sin\beta\cdot vp - \cos\beta\,pw.$$