Dev++

ECC and Transactions



Connect to wifi Install python3, virtualenv, git

```
$ git clone http://github.com/bitcoinedge/devplusplus
$ cd devplusplus
$ virtualenv -p python3 .venv
$ . .venv/bin/activate
$ pip install -r requirements.txt
$ jupyter notebook
```

Your web browser should open up a jupyter notebook

Class Structure

- Present some material
- Ask questions
- You have time to play with/study the code

What We'll Cover

- Foundational Math
- Elliptic Curve Cryptography
- Transactions



Finite Fields

What Is a Finite Field?



- Set of numbers
- Finite
- Closed under +, -, *, /, except division by 0
- Used with Elliptic Curves for Cryptography
- Prime fields are the most interesting

Example



Prime Field of 19 (Denoted F₁₉)

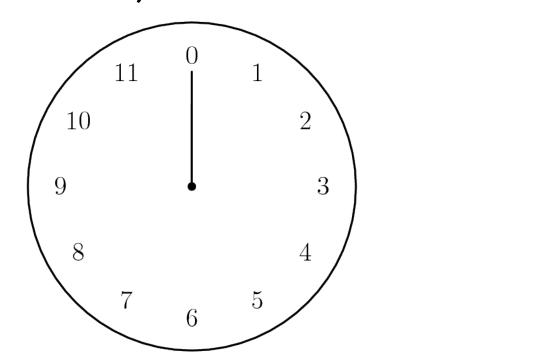
$$F_{19} = \{0, 1, 2, ... 18\}$$

 $F_{97} = \{0, 1, 2, ... 96\}$
 $F_{48947} = \{0, 1, 2, ... 48946\}$

Modular Arithmetic



Remainder math



Addition and Subtraction



Same as modulo P arithmetic (F₁₉)

$$11 + 6 = 17 \% 19 = 17$$

$$8 + 14 = 22 \% 19 = 3$$

$$4 - 12 = -8 \% 19 = 11$$

Multiplication and Exponentiation Programming Blockchain



Same as modulo P arithmetic (F₁₉)

$$2 * 4 = 8 % 19 = 8$$

$$7 * 3 = 21 % 19 = 2$$

$$11^3 = 1331 \% 19 = 1$$

Python: pow(11, 3, 19) == 1

Division



Inverse of Multiplication (F₁₉)

$$2 * 4 = 8 => 8 / 4 = 2$$

$$7 * 3 = 2 => 2 / 3 = 7$$

$$15 * 4 = 3 => 3 / 4 = 15$$

Fermat's Little Theorem



$$n^{p-1}=1 \mod p$$

Works for all n if p is prime. This means that

$$1/n=n^{-1}=n^{-1}*1=n^{-1}*n^{p-1}=n^{p-2} \mod p$$

This is how we do division.

Division



So how do we calculate it? (F_{19})

$$n^{p-1} = 1 => 1/n = n^{p-2}$$
 $2 / 3 = 2 * 1/3 = 2 * 3^{17} = 7$
 $3 / 15 = 3 * 1/15 = 3 * 15^{17} = 4$

Python: 1/n = pow(n, p-2, p)

Examples

from ecc import FieldElement

Programming Blockchain

```
a = FieldElement(2, 19)
b = FieldElement(15, 19)
# Add
print(a+b) # 17
# Subtract
print(a-b) # 6
# Multiply
print(a*b) # 11
# Exponentiate
print(b**5) # 2
# Divide
print(a/b) # 9
```



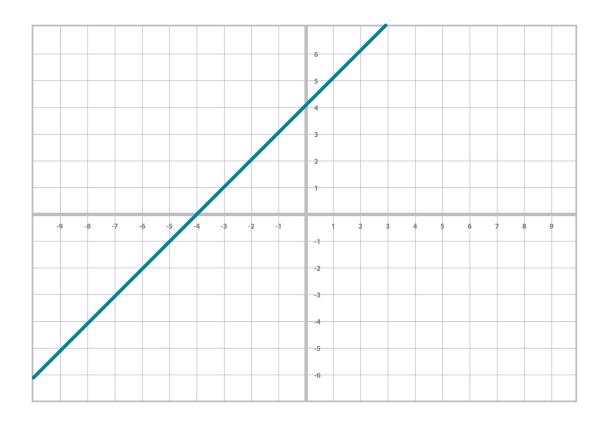


ecc.py:FieldElement
ecc.py:FieldElementTest

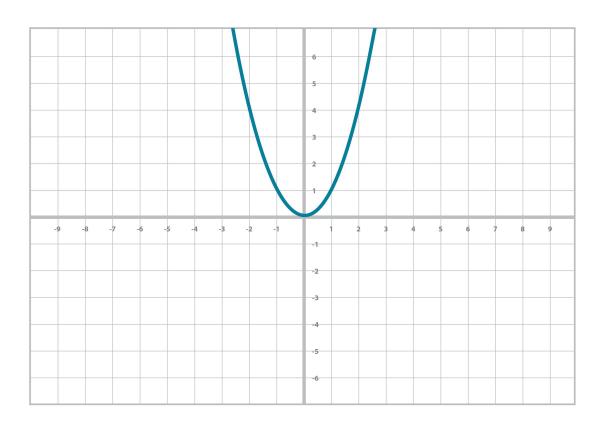
Questions?

Elliptic Curves

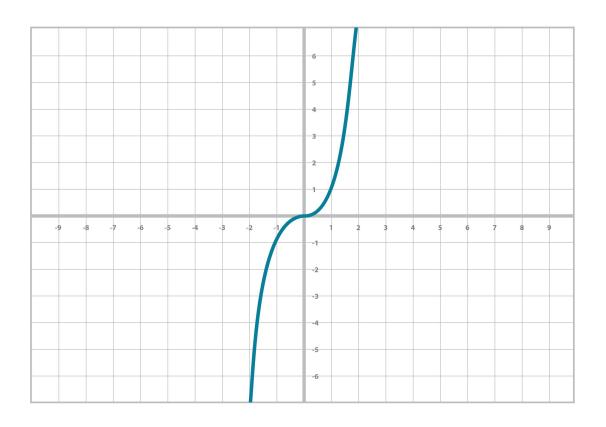
Linear (y = ax + b)



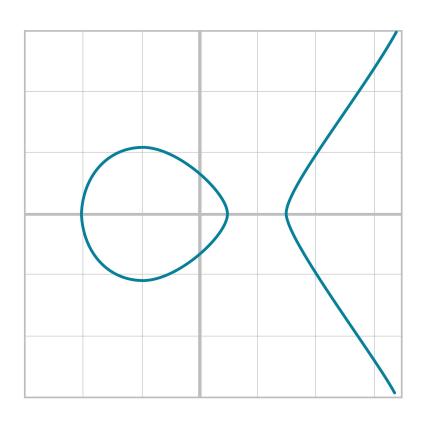
Quadratic: $(y = ax^2 + bx + c)$

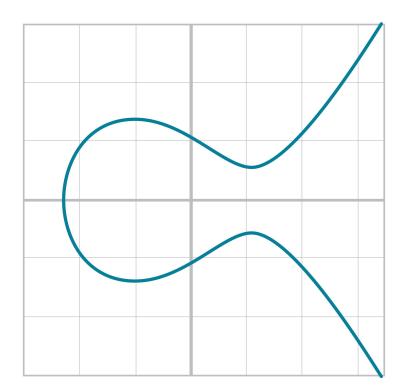


Cubic: $(y = ax^3 + bx^2 + cx + d)$

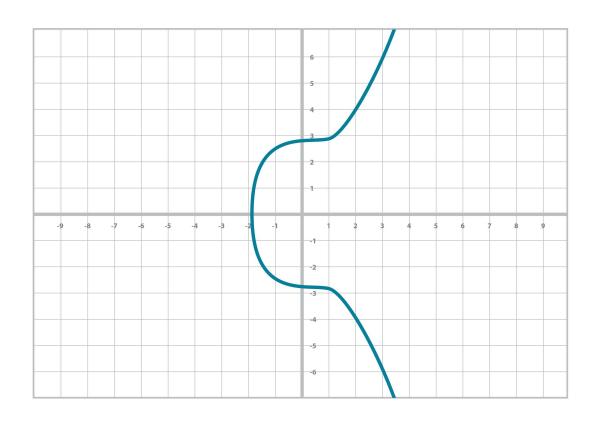


Elliptic: $(y^2 = x^3 + bx + c)$

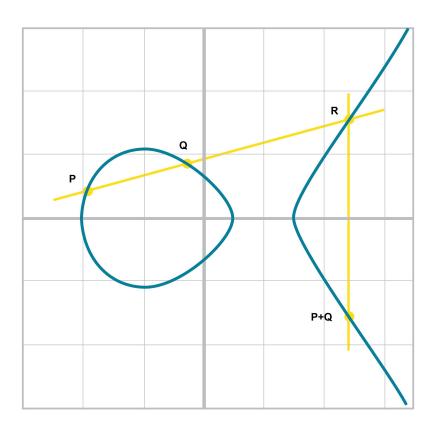




$secp256k1: y^2 = x^3 + 7$



Point Addition



Group Law for the point at ∞

Curve:
$$y^2=x^3+ax+b$$

 $(x_1, y_1)=(x_1, y_1)+(\infty, \infty)$
 $(x_1, y_1)+(x_1, -y_1)=(\infty, \infty)$

Think zero

Group Law for $x_1 \neq x_2$ Curve: $y^2=x^3+ax+b$ $(x_3, y_3) = (x_1, y_1) + (x_2, y_2)$ $s=(y_2-y_1)/(x_2-x_1)$ $x_{3} = s^{2} - x_{1} - x_{2}$ $y_3 = s(x_1 - x_3) - y_1$

Example

Curve:
$$y^2 = x^3 + 5x + 7$$

What is $(2,5) + (3,7)$?
 $s = (y_2 - y_1) / (x_2 - x_1) = (7-5) / (3-2) = 2$
 $x_3 = s^2 - x_1 - x_2 = 2^2 - 2 - 3 = -1$
 $y_3 = s(x_1 - x_3) - y_1 = 2(2 - (-1)) - 5 = 1$
 $(2,5) + (3,7) = (-1,1)$

Group Law for $x_1 = x_2$, $y_1 = y_2$ Curve: $y^2=x^3+ax+b$ $(x_3, y_3) = (x_1, y_1) + (x_1, y_1)$ $s=(3x_1^2+a)/(2y_1)$ $x_3 = s^2 - 2x_1$ $y_3 = s(x_1 - x_3) - y_1$

Examples



```
from ecc import Point
p0 = Point(x=None, y=None, a=5, b=7)
p1 = Point(x=-1, y=1, a=5, b=7)
p2 = Point(x=3, y=7, a=5, b=7)
# Add identity
print(p0+p1) # (-1,1)
# Add Different Points
print(p1+p2) # (0.25, -2.875)
# Add Same Points
print(p1+p1) # (18,-77)
```

Study

Programming Blockchain

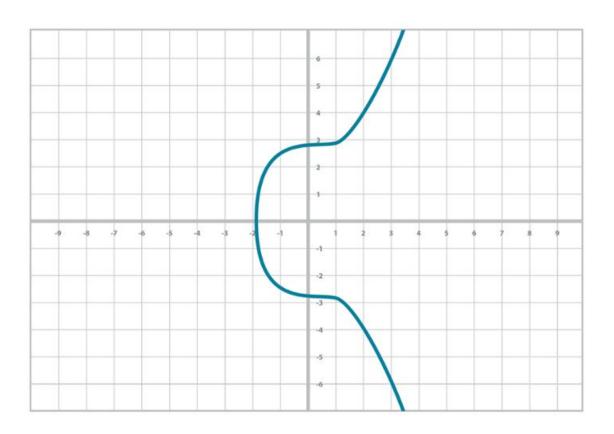
ecc.py:Point

ecc.py:PointTest

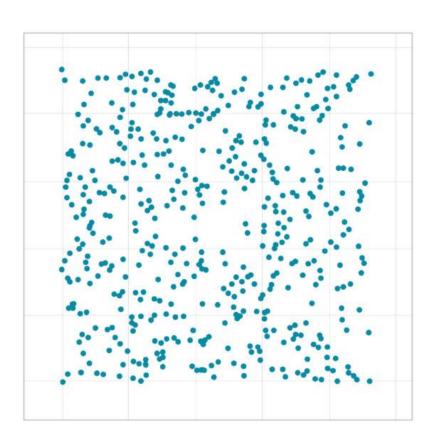
Questions?

Elliptic Curves over Finite Fields

Elliptic Curve over Reals



Elliptic Curve over Finite Field



Examples



from ecc import FieldElement, Point a = FieldElement(0, 137) b = FieldElement(7, 137)p0 = Point(x=None, y=None, a=a, b=b)p1 = Point(x=FieldElement(73, 137), y=FieldElement(128, 137), a=a, b=b) p2 = Point(x=FieldElement(46, 137), y=FieldElement(22, 137),

a=a, b=b





ecc.py:ECCTest:test_on_curve

ecc.py:ECCTest:test_add1

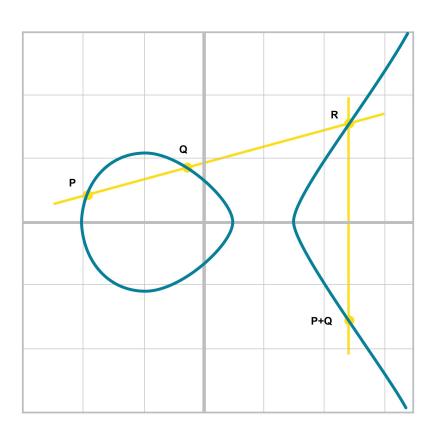
Questions?

Mathematical Group

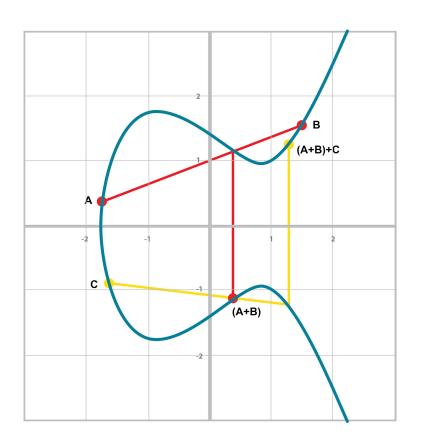
Mathematical Group

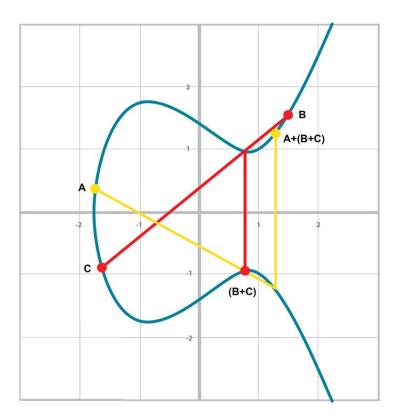
- Single operation
- Closed (if A, B in G, A+B in G)
- Associative ((A+B)+C = A+(B+C))
- Commutative (A+B=B+A)
- Invertible (if A in G, there's a -A in G)
- Identity (0 exists)
- Point addition gets us a group!

Closed, Commutative, Invertible



Associative





Scalar Multiplication

Start with an Elliptic Curve over a Finite Field. Pick a point G (generator point). G+G=2G, G+G+G=3G ... nG where nG=0 (point at ∞) $\{0, G, 2G, \dots (n-1)G\}$ is a finite group

Examples



from ecc import FieldElement, Point

```
a = FieldElement(0, 137)
b = FieldElement(7, 137)
p0 = Point(x=None, y=None, a=a, b=b)
p1 = Point(x=FieldElement(73, 137), y=FieldElement(128,
137), a=a, b=b)
current = p1
n = 1
while current != p0:
    current += p1
    n += 1
print(n, p1, n*p1) # order of p1 is 69
```





ecc.py:ECCTest:test_rmul

Questions?

Scalar Multiplication

- Imagine a really large group n~2²⁵⁶
- P=sG where s is really, really large
- Finding P when we know s is easy
- Finding s when we know P is not
- Sometimes referred to as "Secret Exponent"
- P=G^s => Log_GP=s (Discrete Log Problem)
- Convention: lower-case letters for scalar, upper-case letters for points

Defining an Elliptic Curve

- Elliptic Curve Equation (a and b of y²=x³+ax+b)
- Finite Field Prime Number (p)
- Generator Point (G)
- Number of points in the group (n)

secp256k1

- Equation $y^2 = x^3 + 7$ (a=0, b=7)
- Prime Field (p) = 2^{256} 2^{32} 977
- Generator Point (G) = (79BE667EF9DCBBAC55A06295CE870B07029BFCDB2DCE28D959F2815B16F81798, 483ADA7726A3C4655DA4FBFC0E1108A8FD17B448A68554199C47D08FFB10D4B8)
- SEC = Standards for Efficient Cryptography
- 256 = number of bits in the prime field

2²⁵⁶ is really big

- \bullet 2²⁵⁶ \sim 10⁷⁷
- Number of atoms in and on earth ~ 10⁵⁰
- Number of atoms in the solar system ~ 10⁵⁷
- Number of atoms in the galaxy ~ 10⁶⁸
- Number of atoms in the universe ~ 10⁸⁰
- Trillion computers doing a trillion operations every picosecond (10⁻¹² seconds) for a trillion years < 10⁵⁶ operations.

Public Key Cryptography

- Private key is the scalar (Denoted w/lower case letter "s")
- Public key is the resulting point sG (Denoted w/upper case letter "P")
- Public key is a point (x, y) and thus has 2 numbers

Getting a public key from private

from ecc import G

```
secret = 999
point = secret*G
print(point)
Point(9680241112d370b56da22eb535745d9e314380e568229e09f72410
66003bc471,
ddac2d377f03c201ffa0419d6596d10327d6c70313bb492ff495f946285d
8f38)
```

Study

Programming Blockchain

ecc.py:S256Field

ecc.py:S256Point

Questions?

Bitcoin Addresses

SEC Format

- Public Key (point on curve) serialized
- Uncompressed

<mark>04</mark>7211a824f55b505228e4c3d5194c1fcfaa15a456abdf37f9b9d97a4040afc073<mark>dee6c8906498</mark> 4f03385237d92167c13e236446b417ab79a0fcae412ae3316b77

- o 04 Marker
- x coordinate 32 bytes
- y coordinate 32 bytes
- Compressed

0349fc4e631e3624a545de3f89f5d8684c7b8138bd94bdd531d2e213bf016b278a

- 02 if y is even, 03 if odd Marker
- x coordinate 32 bytes

Addresses

- Take either compressed or uncompressed SEC format
- SHA-256 the result and then RIPEMD160 the result (aka HASH160)
- Prepend with network prefix (00 for mainnet,
 6F for testnet)
- Add a 32-bit double-SHA256 checksum at the end
- Encode in Base58

Example

from ecc import G

```
secret = 999
point = secret*G
print(point.address(compressed=True, testnet=False))
print(point.address(compressed=False, testnet=False))
print(point.address(compressed=True, testnet=True))
print(point.address(compressed=False, testnet=True))
```

Study

Programming Blockchain

ecc.py:S256Test

Questions?

ECDSA

Elliptical Curve Digital Signature Algorithm

Intuition

- sG=P
- uG+vP where u, v≠0
- Say you can choose u and v
- Can only manipulate the sum if you know how G and P are related (that is, you know the private key)
- uG+vP=uG+vsG=(u+sv)G
- If you know s, you can manipulate u+sv, if you don't you can't.

Signature Algorithm

- Start with the hash of what you're signing (z)
- Next assume your secret is e and the public point P=eG
- Get a new random number k
- Compute kG. The x coordinate = r
- Compute s=(z+re)/k (Division is the same as field division: 1/x = pow(x, n-2, n))
- Signer can compute s since he has e, nobody else can compute s.
- Signature is simply the pair, (r, s)
- Note s has to be less than n/2. If s>n/2, use n-s instead.

Verification Algorithm

- Start with the hash of what you're signing (z)
- Next assume you have the public point eG = P
- Signature is (r,s) where s=(z+re)/k
- Compute u = z / s
- Compute v = r / s
- Compute uG + vP
- uG+vP=(z/s)G+(r/s)P=(z/s)G+(re/s)G= ((z+re)/s)G=((z+re)k/(z+re))G=kG=(r,y)
- If the x coordinate matches r, you have a valid sig.

Example

from ecc import PrivateKey

```
z = 432089432098342098234089098423098324089
secret = 999
priv_key = PrivateKey(secret)
pub_key = priv_key.point
sig = priv_key.sign(z)
print(sig)
print(pub_key.verify(z, sig)) # True
```





- ecc.py:Signature
- ecc.py:SignatureTest
- ecc.py:PrivateKey
- ecc.py:PrivateKeyTest

Questions?

DER Signature Format

Encoding r and s

DER Format

Signature (r,s) serialized

30<mark>4502</mark>21<mark>00ed81ff192e75a3fd2304004dcadb746fa5e24c5031ccfcf213</mark>

20b0277457c98f02207a986d955c6e0cb35d446a89d3f56100f4d7f67801

c31967743a9c8e10615bed

- 30 Marker
- 45 Length of sig
- 02 Marker for r value
- 21 r value length
- 00ed...8f r value
- 02 Marker for s value
- 20 s value length
- 7a98...ed s value

Example

```
from binascii import hexlify
from random import randint
from ecc import PrivateKey
z = randint(0, 2**256)
secret = 999
priv_key = PrivateKey(secret)
sig = priv_key.sign(z)
print(hexlify(sig.der()))
```





ecc.py:Signature:serialize
ecc.py:Signature:test_der

Questions?

Transaction Structure

What is a Transaction?

- Assignment of bitcoin from one script to another.
- Note addresses are really compressed scripts.

Bitcoin Transaction Elements

- Version (4 bytes)
- Inputs
- Outputs
- Locktime

Inputs

- Two types of inputs
 - Coinbase
 - Previous transaction output (a.k.a. tx out, utxo, spendable, outpoint)

Input

- Elements
 - Previous tx hash
 - Vout (output index in that transaction)
 - Sequence Used for RBF
 - Scriptsig involves SCRIPT language
- Note: no amount here! You have to look it up.
- All nodes have to validate these inputs as legitimate.

Output

- Elements
 - Amount
 - ScriptPubKey involves SCRIPT language
- What we think of as assigning to an address is actually a script in ScriptPubKey
- Note: Amount can be zero in certain instances (OP_RETURN)

Locktime

 Designed to tell nodes not to let a tx into a block until a certain time or a certain block height

Bitcoin Transaction

010000001813f79011acb80925dfe69b3def355fe914bd1d96a3f5f71bf8303c6a989c7d100000006b483045022100ed81ff192e75a3fd2304004dcadb746fa5e24c5031ccfcf21320b0277457c98f02207a986d955c6e0cb35d446a89d3f56100f4d7f67801c31967743a9c8e10615bed01210349fc4e631e3624a545de3f89f5d8684c7b8138bd94bdd531d2e213bf016b278afeffffff02a135ef01000000001976a914bc3b654dca7e56b04dca18f2566cdaf02e8d9ada88ac99c3980000000001976a9141c4bc762dd5423e332166702cb75f40df79fea1288ac19430600

```
01000000 - version
01 - # of inputs
02 - # of outputs
813f...d1 - previous tx hash
00000000 - previous tx index
6b00...8a - scriptSig
19430600 - locktime
feffffff - sequence
```

Example

```
from binascii import unhexlify
from io import BytesIO
from tx import Tx
raw tx =
BytesIO(unhexlify('0100000001813f79011acb80925dfe69b3def355fe914bd1d96a3f5f71b
f8303c6a989c7d1000000006b483045022100ed81ff192e75a3fd2304004dcadb746fa5e24c503
1ccfcf21320b0277457c98f02207a986d955c6e0cb35d446a89d3f56100f4d7f67801c31967743
a9c8e10615bed01210349fc4e631e3624a545de3f89f5d8684c7b8138bd94bdd531d2e213bf016
b278afeffffff02a135ef01000000001976a914bc3b654dca7e56b04dca18f2566cdaf02e8d9ad
a88ac99c39800000000001976a9141c4bc762dd5423e332166702cb75f40df79fea1288ac19430
600'))
tx_obj = Tx.parse(raw_tx)
print(tx_obj)
```

Study

Programming Blockchain

tx.py:Tx

tx.py:TxTest

Questions?

Script

What is SCRIPT?

- Limited Programming Language
- Not Turing Complete.
- Programmable way to assign bitcoins
- Addresses are compressed scripts

How does SCRIPT work?

- There are two types of items: elements and operations.
- Elements are just data (signatures, keys, hashes, etc)
- Operations do something to elements.
- Each element is added to a stack, operations do something to the stack.
- At the end of processing all the items, there must be a single element that's not zero left on the stack to evaluate to True.

Some common operations

- OP_DUP duplicates the top element in the stack and puts it on top
- OP_HASH160 Does a SHA256 and then a RIPEMD160 to the top element.
- OP_CHECKSIG Takes the top two elements, the first being the pubkey, the second being the signature and checks if the signature is valid for the current transaction.
- OP_RETURN Marks transaction as invalid, but also allows 80 bytes of data to be put in.

Parsing SCRIPT

- Each byte is interpreted as an integer.
- If byte is between 1 and 75 inclusive, the next n bytes are an element.
- Otherwise, byte is an operation based on a lookup table
- $0x00 0P_0$, Put a 0 on top of the stack
- 0x05 Next 5 bytes are an element
- 0x48 Next 72 bytes are an element
- 0x76 OP_DUP, Put a copy of the top element of the stack on top of the stack
- 0x93 OP_ADD, Take the top two elements, add them and put on top of the stack
- 0xa9 OP_HASH160, Perform a HASH160 to the top element of the stack
- ... Many more

Some common elements

- Public keys SEC Format (33 or 65 bytes)
- Signatures DER Format (71, 72 or 73 bytes)
- Hash160 20 bytes

Common Scripts

- Addresses are compressed scripts
- p2pk pay to pub key
- p2pkh pay to pub key hash
- p2sh pay to script hash
- p2wpk pay to witness pub key
- p2wsh pay to witness script hash

SCRIPT Validation

- The ScriptSig field of the input has SCRIPT items which are processed one at a time until there are no more items
- Every non-coinbase input must specify the previous transaction and index.
- This outpoint has a ScriptPubKey, these SCRIPT items are processed one at a time until there are no more items
- If the result of the processing leaves a non-zero element, the SCRIPT is considered valid, otherwise, the SCRIPT is invalid.

P2PK - First Standard SCRIPT

```
Pay-to-Pubkey scriptPubKey (receiving)
```

41<mark>0411db93e1dcdb8a016b49840f8c53bc1eb68a382e97b1482ecad7b148a6909a5cb2e0eaddfb</mark> 84ccf9744464f82e160bfa9b8b64f9d4c03f999b8643f656b412a3ac

```
41 - length of the pubkey 0411...a3 - <pubkey>
ac - OP_CHECKSIG
```

scriptSig (spending)

47<mark>304402204e45e16932b8af514961a1d3a1a25fdf3f4f7732e9d624c6c61548ab5fb8cd410220</mark> 181522ec8eca07de4860a4acdd12909d831cc56cbbac4622082221a8768d1d0901

```
47 - length of the signature 3044...01 - <signature>
```

P2PK - First Standard SCRIPT



Script

Stack

Processing

<signature>

<pub/>pubkey>

OP_CHECKSIG

OP_CHECKSIG

Script Stack **Processing** <pub/>pubkey> <signature>

Script Stack Processing OP_CHECKSIG -<pub/>pubkey> Checks if the OP CHECKSIG <signature> signature is valid for the current transaction. Puts 1 back if valid, 0 otherwise.

P2PK - First Standard SCRIPT

Script Stack Processing

OP_CHECKSIG Checks if the
signature is valid
for the current
transaction. Puts
1 back if valid, 0

otherwise.

P2PKH - Shorter & more secure

- These are the addresses that start with a "1"
- Shorter due to use of RIPEMD160
- More Secure due to requiring both ECC
 Discrete Log and Hash pre-images being needed.

P2PKH - Shorter & more secure

Pay-to-Pubkey-Hash scriptPubKey (receiving)

76a914bc3b654dca7e56b04dca18f2566cdaf02e8d9ada88ac

```
76 - OP_DUP
a9 - OP_HASH160
14 - Length of <hash>
bc3b...da - <hash>
88 - OP_EQUALVERIFY
ac - OP_CHECKSIG
```

P2PKH - Shorter & more secure

Pay-to-Pubkey-Hash scriptSig (spending)

48<mark>3045022100ed81ff192e75a3fd2304004dcadb746fa5e24c5031ccfcf2</mark> 1320b0277457c98f02207a986d955c6e0cb35d446a89d3f56100f4d7f678 01c31967743a9c8e10615bed01210349fc4e631e3624a545de3f89f5d868 4c7b8138bd94bdd531d2e213bf016b278a

```
48 - Length of <signature>
30...01 - <signature>
21 - Length of <pubkey>
0349...8a - <pubkey>
```

scriptPubKey

scriptSig

Script

OP_DUP

OP_HASH160

<hash>

OP_EQUALVERIFY

OP_CHECKSIG

<signature>

<pub/>pubkey>

<signature>

<pub/>pubkey>

OP_DUP

OP_HASH160

<hash>

OP_EQUALVERIFY

OP_CHECKSIG

Script <signature> <pub/>pubkey> OP_DUP OP_HASH160 <hash> OP_EQUALVERIFY

OP_CHECKSIG

Stack

Processing

Script Stack **Processing** <pub/>pubkey> OP_DUP OP_HASH160 <hash> OP_EQUALVERIFY <signature> OP_CHECKSIG

Script Stack **Processing** OP_DUP OP_HASH160 <hash> OP_EQUALVERIFY <pub/>pubkey> <signature> OP_CHECKSIG

Script Stack Processing OP_DUP duplicates the top element in OP_HASH160 the stack and <hash> puts it on top OP_EQUALVERIFY <pub/>pubkey> OP_DUP OP_CHECKSIG <signature>

Script Stack Processing OP_HASH160 <hash> <pub/>pubkey> OP_EQUALVERIFY <pub/>pubkey> OP_CHECKSIG <signature>

OP_DUP duplicates the
top element in
the stack and
puts it on top

Script Stack Processing OP_HASH160-Does a SHA256 and then a RIPEMD160 to <hash> <pub/>pubkey> the top element. OP_EQUALVERIFY <pub/>pubkey> OP_HASH160 OP_CHECKSIG <signature>

Script Stack Processing <hash> <hash> OP_EQUALVERIFY <pub/>pubkey> OP_CHECKSIG <signature>

OP_HASH160 Does a SHA256
and then a
RIPEMD160 to
the top element.

Script Stack Processing

<hash>

<pub/>pubkey>

<signature>

OP_EQUALVERIFY

OP_CHECKSIG

Script Stack **Processing** OP_EQUALVE RIFY - Checks that the top two <hash> elements are <hash> equal. If not, fails <pub/>pubkey> the whole script. OP_EQUALVERIFY OP_CHECKSIG <signature>

Script Stack **Processing** <pub/>pubkey> OP_CHECKSIG <signature>

OP_EQUALVE
RIFY - Checks
that the top two
elements are
equal. If not, fails
the whole script.

Script Stack Processing OP_CHECKSIG Checks if the signature is valid for the current transaction. Puts <pub/>pubkey> 1 back if valid, 0 <signature> OP_CHECKSIG otherwise.

Script Stack Processing OP_CHECKSIG -Checks if the signature is valid for the current transaction. Puts 1 back if valid, 0 otherwise.





script.py:Script
script.py:ScriptTest

Questions?

Transaction Validation

0100000001813f79011acb80925dfe69b3def355fe914bd1d96a3f5f71bf 8303c6a989c7d1000000006b483045022100ed81ff192e75a3fd2304004d cadb746fa5e24c5031ccfcf21320b0277457c98f02207a986d955c6e0cb3 5d446a89d3f56100f4d7f67801c31967743a9c8e10615bed01210349fc4e 631e3624a545de3f89f5d8684c7b8138bd94bdd531d2e213bf016b278afe ffffff02a135ef0100000000<mark>1976a914bc3b654dca7e56b04dca18f2566c</mark> daf02e8d9ada88ac99c3980000000000<mark>1976a9141c4bc762dd5423e33216</mark> 6702cb75f40df79fea1288ac19430600 Check the inputs that they're unspent d1c789a9c60383bf715f3f6ad9d14b91fe55f3deb369fe5d9280cb1a0179 3f81

Note you need the entire blockchain to check this.

0100000001813f79011acb80925dfe69b3def355fe914bd1d96a3f5f71bf
8303c6a989c7d1000000006b483045022100ed81ff192e75a3fd2304004d
cadb746fa5e24c5031ccfcf21320b0277457c98f02207a986d955c6e0cb3
5d446a89d3f56100f4d7f67801c31967743a9c8e10615bed01210349fc4e
631e3624a545de3f89f5d8684c7b8138bd94bdd531d2e213bf016b278afe
ffffff02a135ef01000000001976a914bc3b654dca7e56b04dca18f2566c
daf02e8d9ada88ac99c3980000000001976a9141c4bc762dd5423e33216
6702cb75f40df79fea1288ac19430600

Check the amounts and make sure that the inputs >= outputs.

Inputs = 42505594

Outputs = 32454049 + 10011545 = 42465594

42505594 >= 42465594 (note difference is the tx fee, which goes to the miner)

Tx Fee = 42505594 - 42465594 = 40000

010000001813f79011acb80925dfe69b3def355fe914bd1d96a3f5f71bf
8303c6a989c7d100000006b483045022100ed81ff192e75a3fd2304004d
cadb746fa5e24c5031ccfcf21320b0277457c98f02207a986d955c6e0cb3
5d446a89d3f56100f4d7f67801c31967743a9c8e10615bed01210349fc4e
631e3624a545de3f89f5d8684c7b8138bd94bdd531d2e213bf016b278afe
ffffff02a135ef01000000001976a914bc3b654dca7e56b04dca18f2566c
daf02e8d9ada88ac99c3980000000001976a9141c4bc762dd5423e33216
6702cb75f40df79fea1288ac19430600

Check the scriptSigs for inputs are valid (that is, combined script evals to TRUE). We'll cover that part (SCRIPT) later.

In practice, this means checking that the signature in the scriptSig is valid.

The best instructions for validating signatures are found here.

010000001813f79011acb80925dfe69b3def355fe914bd1d96a3f5f71bf 8303c6a989c7d1000000000000feffffff02a135ef010000000001976a914bc 3b654dca7e56b04dca18f2566cdaf02e8d9ada88ac99c39800000000019 76a9141c4bc762dd5423e332166702cb75f40df79fea1288ac19430600

To check the sig, we have to substitute the scriptSig with 00 or, an empty scriptSig, for every input.

0100000001813f79011acb80925dfe69b3def355fe914bd1d96a3f5f71bf 8303c6a989c7d1000000001976a914a802fc56c704ce87c42d7c92eb75e7 896bdc41ae88acfeffffff02a135ef01000000001976a914bc3b654dca7e 56b04dca18f2566cdaf02e8d9ada88ac99c39800000000001976a9141c4b c762dd5423e332166702cb75f40df79fea1288ac19430600

We substitute the scriptSig of the input we're signing with the scriptPubKey from the transaction output that we're spending.

Look up the tx output from the blockchain:

<u>d1c789a9c60383bf715f3f6ad9d14b91fe55f3deb369fe5d9280cb1a0179</u> 3f81

Reveals this scriptPubKey:

76a914a802fc56c704ce87c42d7c92eb75e7896bdc41ae88ac We prepend with the length (19) to complete it.

0100000001813f79011acb80925dfe69b3def355fe914bd1d96a3f5f71bf 8303c6a989c7d1000000001976a914a802fc56c704ce87c42d7c92eb75e7 896bdc41ae88acfeffffff02a135ef01000000001976a914bc3b654dca7e 56b04dca18f2566cdaf02e8d9ada88ac99c3980000000001976a9141c4b c762dd5423e332166702cb75f40df79fea1288ac1943060001000000

To check the sig, we also have to append the "hash type" or what the signature is good for. In this case, we're appending "SIGHASH_ALL" or "This sig is only good if the entire transaction goes through". There are others have different restrictions, but SIGHASH_ALL is the default and the most widely used.

Bitcoin Cash uses SIGHASH_ALL & SIGHASH_FORKID here for replay protection.

0100000001813f79011acb80925dfe69b3def355fe914bd1d96a3f5f71bf 8303c6a989c7d1000000001976a914a802fc56c704ce87c42d7c92eb75e7 896bdc41ae88acfeffffff02a135ef01000000001976a914bc3b654dca7e 56b04dca18f2566cdaf02e8d9ada88ac99c3980000000001976a9141c4b c762dd5423e332166702cb75f40df79fea1288ac1943060001000000

Double-sha256 this to get the hash that's being signed (z)

010000001813f79011acb80925dfe69b3def355fe914bd1d96a3f5f71bf
8303c6a989c7d100000006b4830450221**00ed81ff192e75a3fd2304004d**cadb746fa5e24c5031ccfcf21320b0277457c98f02207a986d955c6e0cb3
5d446a89d3f56100f4d7f67801c31967743a9c8e10615bed01210349fc4e
631e3624a545de3f89f5d8684c7b8138bd94bdd531d2e213bf016b278afe
ffffff02a135ef01000000001976a914bc3b654dca7e56b04dca18f2566c
daf02e8d9ada88ac99c39800000000001976a9141c4bc762dd5423e33216
6702cb75f40df79fea1288ac19430600

We can get the signature from the scriptSig

010000001813f79011acb80925dfe69b3def355fe914bd1d96a3f5f71bf
8303c6a989c7d1000000006b483045022100ed81ff192e75a3fd2304004d
cadb746fa5e24c5031ccfcf21320b0277457c98f02207a986d955c6e0cb3
5d446a89d3f56100f4d7f67801c31967743a9c8e10615bed0121**0349fc4e**631e3624a545de3f89f5d8684c7b8138bd94bdd531d2e213bf016b278afe
ffffff02a135ef01000000001976a914bc3b654dca7e56b04dca18f2566c
daf02e8d9ada88ac99c39800000000001976a9141c4bc762dd5423e33216
6702cb75f40df79fea1288ac19430600

We can also get the public key from the scriptSig

Example

```
from binascii import unhexlify
from io import BytesIO
from tx import Tx
raw tx =
BytesIO(unhexlify('0100000001813f79011acb80925dfe69b3def355fe914bd1d96a3f5f71b
f8303c6a989c7d1000000006b483045022100ed81ff192e75a3fd2304004dcadb746fa5e24c503
1ccfcf21320b0277457c98f02207a986d955c6e0cb35d446a89d3f56100f4d7f67801c31967743
a9c8e10615bed01210349fc4e631e3624a545de3f89f5d8684c7b8138bd94bdd531d2e213bf016
b278afeffffff02a135ef01000000001976a914bc3b654dca7e56b04dca18f2566cdaf02e8d9ad
a88ac99c39800000000001976a9141c4bc762dd5423e332166702cb75f40df79fea1288ac19430
600'))
tx_obj = Tx.parse(raw_tx)
for i, tx_in in enumerate(tx_obj.tx_ins):
    print(tx_obj.verify_input(i))
```





```
tx.py:Tx:verify_input
tx.py:TxTest:test_verify_input1
```

Questions?

Pay to Script Hash

scriptPubKey (receiving) - 1 of 2

5141<mark>04fcf07bb1222f7925f2b7cc15183a40443c578e62ea17100aa3b44ba66905c95d4980aec4cd2f6eb426d1b1ec45d76724f26901099416b9265b76ba67c8b0b73d21</mark>0202be80a0ca69c0e000b97d507f45b98c49f58fec6650b64ff70e6ffccc3e6d00<mark>52</mark>ae

```
51 - OP_1
41 - Length of <pubkey1>
40fc...3d - <pubkey1>
21 - Length of <pubkey2>
0202...00 - <pubkey2>
52 - OP_2
ae - OP CHECKMULTISIG
```

scriptSig (spending) - 1 of 2

00483045022100e222a0a6816475d85ad28fbeb66e97c931081076dc9655 da3afc6c1d81b43f9802204681f9ea9d52a31c9c47cf78b71410ecae6188 d7c31495f5f1adfe0df5864a7401

```
00 - OP_0
48 - Length of <signature1>
3045...01 - <signature1>
```

scriptPubKey

m

<pub/>pubkey1>

<pub/>pubkey2>

<pub/>pubkey...>

<pub/>pubkeyn>

n

OP_CHECKMULTISIG

scriptSig

X

<signature1>

<signature2>

<signature...>

<signaturem>

Script

Χ

<signature1>

<signature...>

<signaturem>

m

<pub/>pubkey1>

<pub/>pubkey...>

<pub/>pubkeyn>

n

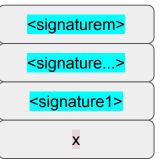
OP_CHECKMULTISIG

Script Stack Χ <signature1> <signature...> <signaturem> m <pub/>pubkey1> <pub/>pubkey...> <pub/>pubkeyn> n OP_CHECKMULTISIG

Processing

Script **Processing** Stack <signature1> <signature...> <signaturem> m <pub/>pubkey1> <pub/>pubkey...> <pub/>pubkeyn> n Χ OP_CHECKMULTISIG

Script Stack Processing



Script **Processing** Stack n <pub/>pubkeyn> <pub/>pubkey...> <pub/>pubkey1> m <signaturem> <signature...> <signature1> X

OP_CHECKMULTISIG

Script Stack Processing OP_CHECKMULTISIG -Checks if m of the n signatures are valid of <pub/>pubkeyn> the n public keys for <pub/>pubkey...> current transaction. <pub/>pubkey1> Puts 1 back if valid, 0 m otherwise. <signaturem> <signature...> <signature1> X OP CHECKMULTISIG

Script Stack

Processing

OP_CHECKMULTISIG Checks if m of the
signatures are valid of
the n public keys for
current transaction.
Puts 1 back if valid, 0
otherwise.

- The x can be anything. It's required because of a bug in OP_CHECKMULTISIG and would require a hard fork to fix.
- There's no way to make this an address. It's too long.
- Big Transaction Output for the UTXO set
- This was abused.

How is the whitepaper decoded from the blockchain (Tx with ~1000x m of n multisig outputs)



The whitepaper is apparently encoded at 54e48e5f5c656b26c3bca14a8c95aa583d07ebe84dde3b7dd4a78f4e4186e713, which is an *m* of *n multisig* Tx with **947** outputs (just under the scriptsig limit of 20kB!).



Using the Blocktrail Python SDK, I get a list of the outputs as hex using the following Python (2.7) code (NB, the APIKEY, APISECRET parameters are available if required from www.blocktrail.com):



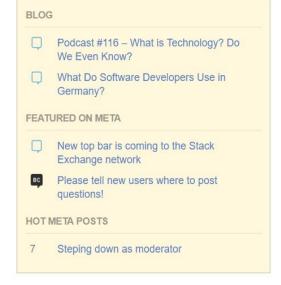
```
from blocktrail import APIClient
bt_client = APIClient(APIKEY, APISECRET, network='BTC')
txnObj = bt_client.transaction('54e48e5f5c656b26c3bca14a8c95aa583d07ebe84dde3b7dd4a78f4e4186
hashes = [(t['script_hex']) for t in (txnObj)['outputs']]
```

The resulting list is available here in full and is essentially all pay-to-pubkey-script Txns. An excerpt:

```
[u'5141e4cf0200067daf13255044462d312e340a25c3a4c3bcc3b6c39f0a322030206f626a0a3c3c2f4c656e677.....
u'514130206e200a303030303138323534302030303030206e200a747261696c65720a3c3c2f53697a65203638
u'51213e0a7374617274787265660a3138323732370a2525454f460a00000000000000051ae',
u'76a91462e907b15cbf27d5425399ebf6f0fb50ebb88f1888ac',
u'76a914031c79236ff3017496cf8d9a883f494458f245f288ac']
```

QUESTION: How is this array of hex data parsed into the bitcoin.pdf? Specific Python framed answers would be appreciated!

```
asked 2 years, 7 months ago
viewed 2,895 times
active 2 years, 6 months ago
```



P2SH - Really Flexible Addresses

- These are addresses that start with a "3"
- Flexible because part of the SCRIPT is kept by the creator of the address (RedeemScript)
- RedeemScript must be provided when spending
- RedeemScript is at first treated as an element,
 but then is interpreted as SCRIPT

P2SH - Really Flexible Addresses

Pay-to-Script-Hash scriptPubKey (receiving)

a91474d691da1574e6b3c192ecfb52cc8984ee7b6c5687

```
a9 - OP_HASH160
14 - Length of <hash>
74d6...56 - <hash>
87 - OP_EQUAL
```

P2SH

scriptSig (spending)

```
00483045022100dc92655fe37036f47756db8102e0d7d5e28b3beb83a8fe f4f5dc0559bddfb94e02205a36d4e4e6c7fcd16658c50783e00c34160997 7aed3ad00937bf4ee942a8993701483045022100da6bee3c93766232079a 01639d07fa869598749729ae323eab8eef53577d611b02207bef15429dca dce2121ea07f233115c6f09034c0be68db99980b9a6c5e75402201475221 022626e955ea6ea6d98850c994f9107b036b1334f18ca8830bfff1295d21 cfdb702103b287eaf122eea69030a0e9feed096bed8045c8b98bec453e1ffac7fbdbd4bb7152ae
```

```
00 - OP_0
48 - Length of <signaturex>
3045...01 - <signaturex>
47 - Length of redeemScript
```

P2SH

scriptPubKey

OP_HASH160

<hash>

OP_EQUAL

scriptSig

0P_0

<signature1>

<signature2>

<redeemScript>

Script

0P_0

<signature1>

<signature2>

<redeemScript>

OP_HASH160

<hash>

OP_EQUAL

P2SH

Script

Stack

Processing

0P_0

<signature1>

<signature2>

<redeemScript>

OP_HASH160

<hash>

OP_EQUAL

Script Stack <signature1> <signature2> <redeemScript> OP_HASH160 <hash> OP_EQUAL

Processing

OP_0 - Puts a 0 on the stack

0P_0

Script Stack **Processing** OP_0 - Puts a 0 on the stack <signature1> <signature2> <redeemScript> OP_HASH160 <hash> OP_EQUAL 0

Script

Stack

Processing

OP_HASH160
<hash>
OP_EQUAL

<redeemScript>
<signature2>
<signature1>
0

Script Stack Processing OP_HASH160-Does a SHA256 and then a <redeemScript> RIPEMD160 to <signature2> the top element. <hash> <signature1> OP_EQUAL OP_HASH160 0

Script Stack Processing OP_HASH160-Does a SHA256 and then a <hash> RIPEMD160 to <signature2> the top element. <hash> <signature1> OP_EQUAL 0

Script Stack Processing OP_EQUAL -Checks that the top two elements <hash> are equal. If so, <hash> put a 1 on stack, <signature2> if not put a 0 on <signature1> stack OP_EQUAL 0

Script Stack Processing <signature2> <signature1> 0

OP_EQUAL Checks that the top two elements are equal. If so, put a 1 on stack, if not put a 0 on stack

Script Stack <signature2> <signature1> 0

Processing

Special rule:

OP_HASH160<hash>
OP_EQUAL evaluates
to true means that the

<redeemScript> is
now evaluated as
Script elements

redeemScript

5221<mark>022626e955ea6ea6d98850c994f9107b036b1334f18ca8830bfff129</mark>
5d21cfdb70<mark>21</mark>03b287eaf122eea69030a0e9feed096bed8045c8b98bec45
3e1ffac7fbdbd4bb7152ae

```
52 - OP_2
21 - Length of <pubkeyx>
0...01 - <pubkeyx>
52 - OP_2
ae - OP_CHECKMULTISIG
```

Script Stack 0P_2 <pub/>pubkey1> <pub/>pubkey2> <signature2> 0P_2 <signature1> OP_CHECKMULTISIG 0

Processing

Special rule:

OP_HASH160<hash>
OP_EQUAL evaluates
to true means that the

<redeemScript> is
now evaluated as
Script elements

Script

Stack

Processing

<pub/>pubkey1>

<pub/>pubkey2>

0P_2

OP_CHECKMULTISIG

2

<signature2>

<signature1>

0

Script Stack <pub/>pubkey2> <pub/>pubkey1> <signature2> <signature1> 0

Processing

OP_CHECKMULTISIG

Script Stack <pub/>pubkey2> <pub/>pubkey1> <signature2> <signature1> 0

Processing

OP_CHECKMULTISIG Checks if m of the
signatures are valid of
the n public keys for
current transaction.
Puts 1 back if valid, 0
otherwise.

OP_CHECKMULTISIG

Script Stack Processing

Checks if m of the signatures are valid of the n public keys for current transaction.
Puts 1 back if valid, 0

otherwise.

OP_CHECKMULTISIG-

1

P2SH - Really Flexible Addresses

- RedeemScript was made new as a part of the introduction of p2sh (BIP0016)
- RedeemScript will be added to the processing queue only if the hash matches the hash in ScriptPubKey
- RedeemScript substitution is a bit hacky and was a huge controversy at the time.

P2SH Transaction

010000001868278ed6ddfb6c1ed3ad5f8181eb0c7a385aa0836f01d5e4789e6bd304d87221a00
0000 db00483045022100dc92655fe37036f47756db8102e0d7d5e28b3beb83a8fef4f5dc0559bd
dfb94e02205a36d4e4e6c7fcd16658c50783e00c341609977aed3ad00937bf4ee942a899370148
3045022100da6bee3c93766232079a01639d07fa869598749729ae323eab8eef53577d611b0220
7bef15429dcadce2121ea07f233115c6f09034c0be68db99980b9a6c5e75402201475221022626
e955ea6ea6d98850c994f9107b036b1334f18ca8830bfff1295d21cfdb702103b287eaf122eea6
9030a0e9feed096bed8045c8b98bec453e1ffac7fbdbd4bb7152ae
ffffffff04d3b1140000000
001976a914904a49878c0adfc3aa05de7afad2cc15f483a56a88ac
7f40090000000001976a914
418327e3f3dda4cf5b9089325a4b95abdfa0334088ac
722c0c00000000001976a914ba35042cfe
9fc66fd35ac2224eebdafd1028ad2788ac
dc4ace0200000000
17a91474d691da1574e6b3c192ec
fb52cc8984ee7b6c568700000000

```
01000000 - version
01 - # of inputs
04 - # of outputs
0882...22 - previous tx hash
1a000000 - previous tx index
1976...ac - p2pkh scriptPubKey
17a9...87 - p2sh scriptPubKey
17a9...
```

Creating a p2sh address

a91474d691da1574e6b3c192ecfb52cc8984ee7b6c5687

<hash> = 74d691da1574e6b3c192ecfb52cc8984ee7b6c56

BIP0013 defines how to turn this into an address For mainnet prepend byte $b' \x05'$, for testnet byte $b' \x00'$

Example

```
from binascii import unhexlify
from helper import h160_to_p2sh_address

print(h160_to_p2sh_address(unhexlify('74d691da1574e6b3c192ec fb52cc8984ee7b6c56'), testnet=False))
print(h160_to_p2sh_address(unhexlify('74d691da1574e6b3c192ec fb52cc8984ee7b6c56'), testnet=True))
```





helper.py:h160_to_p2sh_address helper.py:HelperTest

Questions?

scriptSig (spending)

00483045022100dc92655fe37036f47756db8102e0d7d5e28b3beb83a8fef4f5dc0559bddfb94e
02205a36d4e4e6c7fcd16658c50783e00c341609977aed3ad00937bf4ee942a899370148304502
2100da6bee3c93766232079a01639d07fa869598749729ae323eab8eef53577d611b02207bef15
429dcadce2121ea07f233115c6f09034c0be68db99980b9a6c5e75402201475221022626e955ea
6ea6d98850c994f9107b036b1334f18ca8830bfff1295d21cfdb702103b287eaf122eea69030a0
e9feed096bed8045c8b98bec453e1ffac7fbdbd4bb7152ae

```
00 - OP_0
48 - Length of <signaturex>
3045...01 - <signaturex>
47 - Length of redeemScript
5221...ae - <redeemScript>
```

```
Sig1 = 3045022100dc92655fe37036f47756db8102e0d7d5e28b3beb83a8fef4f5dc0559bddfb94e02205a36d4e4e6c7fcd16658c50783e00c341609977aed3ad00937bf4ee942a8993701
```

Sig2 =

3045022100da6bee3c93766232079a01639d07fa869598749729ae323eab 8eef53577d611b02207bef15429dcadce2121ea07f233115c6f09034c0be 68db99980b9a6c5e75402201

What did this sign and how do we verify?

010000001868278ed6ddfb6c1ed3ad5f8181eb0c7a385aa0836f01d5e4789e6bd304d87221a000000000fffffffff04d3b11400000000000001976a914904a49878c0adfc3aa05de7afad2cc15f483a56a88ac7f400900000000001976a914418327e3f3dda4cf5b9089325a4b95abdfa0334088ac722c0c0000000000001976a914ba35042cfe9fc66fd35ac2224eebdafd1028ad2788acdc4ace020000000017a91474d691da1574e6b3c192ecfb52cc8984ee7b6c568700000000

We replace the scriptSig for all inputs with 00, as before

010000001868278ed6ddfb6c1ed3ad5f8181eb0c7a385aa0836f01d5e4789e6bd304d87221a000000475221022626e955ea6ea6d98850c994f9107b036b1334f18ca8830bfff1295d21cfdb702103b287eaf122eea69030a0e9feed096bed8045c8b98bec453e1ffac7fbdbd4bb7152aeffffffff04d3b114000000001976a914904a49878c0adfc3aa05de7afad2cc15f483a56a88ac7f4009000000001976a914418327e3f3dda4cf5b9089325a4b95abdfa0334088ac722c0c0000000001976a914ba35042cfe9fc66fd35ac2224eebdafd1028ad2788acdc4ace020000000017a91474d691da1574e6b3c192ecfb52cc8984ee7b6c568700000000

4752...ae - RedeemScript

We replace the p2sh spending input with the RedeemScript instead of the scriptPubKey.

010000001868278ed6ddfb6c1ed3ad5f8181eb0c7a385aa0836f01d5e47
89e6bd304d87221a000000475221022626e955ea6ea6d98850c994f9107b
036b1334f18ca8830bfff1295d21cfdb702103b287eaf122eea69030a0e9
feed096bed8045c8b98bec453e1ffac7fbdbd4bb7152aeffffffff04d3b1
140000000001976a914904a49878c0adfc3aa05de7afad2cc15f483a56a
88ac7f4009000000001976a914418327e3f3dda4cf5b9089325a4b95ab
dfa0334088ac722c0c00000000001976a914ba35042cfe9fc66fd35ac222
4eebdafd1028ad2788acdc4ace02000000017a91474d691da1574e6b3c1
92ecfb52cc8984ee7b6c56870000000001000000

01000000 - Sighash (SIGHASH_ALL)

Now we append the sigHash

We now take the double_sha256 of the whole thing

```
from binascii import unhexlify
from helper import double_sha256

sha = double_sha256(unhexlify('0100...1000000'))
z = int.from_bytes(sha, 'big')
print(hex(z))
```

scriptSig (spending)

00483045022100dc92655fe37036f47756db8102e0d7d5e28b3beb83a8fef4f5dc0559bddfb94e02205a36d4e4e6c7fcd16658c50783e00c341609977aed3ad00937bf4ee942a8993701483045022100da6bee3c93766232079a01639d07fa869598749729ae323eab8eef53577d611b02207bef15429dcadce2121ea07f233115c6f09034c0be68db99980b9a6c5e75402201475221022626e955ea6ea6d98850c994f9107b036b1334f18ca8830bfff1295d21cfdb702103b287eaf122eea69030a0e9feed096bed8045c8b98bec453e1ffac7fbdbd4bb7152ae

```
00 - OP_0
48 - Length of <signaturex>
3045...01 - <signaturex>
47 - Length of redeemScript
5221...ae - <redeemScript>
```

We derive the r and s from the first signature

30<mark>450221</mark>00dc92655fe37036f47756db8102e0d7d5e28b3beb83a8fef4f5 dc0559bddfb94e<mark>0220</mark>5a36d4e4e6c7fcd16658c50783e00c341609977aed 3ad00937bf4ee942a89937<mark>01</mark>

```
30 - Marker
45 - Length of sig
02 - Marker for r value
21 - r value length
00dc...4e - r value
02 - Marker for s value
20 - s value length
5a36...37 - s value
01 - sighash
```

 $r = 0 \times 00 dc 92655 fe 37036 f47756 db 8102 e 0 d7 d5 e 28 b3 beb 83 a 8 fe f4 f5 dc 0559 b dd fb 94 e$

s = 0x5a36d4e4e6c7fcd16658c50783e00c341609977aed3ad00937bf4ee942a89937

redeemScript

5221022626e955ea6ea6d98850c994f9107b036b1334f18ca8830bfff129 5d21cfdb702103b287eaf122eea69030a0e9feed096bed8045c8b98bec45 3e1ffac7fbdbd4bb7152ae

redeemScript

5221<mark>022626e955ea6ea6d98850c994f9107b036b1334f18ca8830bfff129</mark>
5d21cfdb70<mark>21</mark>03b287eaf122eea69030a0e9feed096bed8045c8b98bec45
3e1ffac7fbdbd4bb7152ae

```
52 - OP_2
21 - Length of <pubkeyx>
0....01 - <pubkeyx>
52 - OP_2
ae - OP_CHECKMULTISIG
```

022626e955ea6ea6d98850c994f9107b036b1334f18ca8830bfff1295d21 cfdb70

We derive the pubkey from the first pubkey

Example

```
from binascii import unhexlify
from io import BytesIO
from tx import Tx

raw_tx = BytesIO(unhexlify('010000000...00000000'))
tx_obj = Tx.parse(raw_tx)
for i, tx_in in enumerate(tx_obj.tx_ins):
    print(tx_obj.verify_input(i))
```





```
tx.py:Tx:verify_input
tx.py:TxTest:test_verify_input2
```

Questions?