

Assignment 1

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Download all python codes from

<https://github.com/Nik123-cpp/Assignment-1/blob/main/assignment1.py>

and latex-tikz codes from

<https://github.com/Nik123-cpp/Assignment-1/blob/main/Assignment1.tex>

1 PROBLEM 3.4

The probability that a bulb produced by a factory will fuse after 150 days is 0.05. Find the probability that out of 5 such bulbs

- (i) none
 - (ii) not more than one
 - (iii) more than one
 - (iv) atleast one
- will fuse after 150 days of use.

Solution

Let X be random variable which denoting number of bulbs fuses after 150 days of use, among the 5 bulbs. Then by Binomial Distribution.

$$\Pr(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \quad (1.0.1) \quad 4)$$

$$k = 0, \dots, n \quad (1.0.2)$$

For given question $n = 5$, $p = 0.05$, $1 - p = 0.95$.

1) From (1.0.2)

$$\Pr(X = 0) = \binom{5}{0} (0.05)^0 (0.95)^5 = 0.77378094 \quad (1.0.3)$$

i.e the probability of all 5 bulbs working after 150 days of use

2) similarly

$$\Pr(X \leq 1) = \sum_{k=0}^1 \Pr(X = k) \quad (1.0.4)$$

$$= \sum_{k=0}^1 \binom{5}{k} (0.05)^k (0.95)^{5-k} \quad (1.0.5)$$

$$= 0.9774075025 \quad (1.0.6)$$

i.e the probability of either none of the bulbs or exactly one bulb will fuse after 150 days of use of 5 such bulbs.

3)

$$\Pr(X > 1) = \sum_{k=2}^5 \Pr(X = k) \quad (1.0.7)$$

Which is complement of second case i.e case(ii), So we can write

$$\Pr(X > 1) = 1 - \Pr(X \leq 1) \quad (1.0.8)$$

From (1.0.6)

$$\Pr(X > 1) = 1 - 0.9774075025 \quad (1.0.9)$$

$$= 0.0225924975 \quad (1.0.10)$$

$$\Pr(X \geq 1) = \sum_{k=1}^5 \Pr(X = k) \quad (1.0.11)$$

which is complement of first case i.e case(i), So we can write

$$\Pr(X \geq 1) = 1 - \Pr(X < 1) \quad (1.0.12)$$

From (1.0.3)

$$\Pr(X \geq 1) = 1 - 0.77378094 \quad (1.0.13)$$

$$= 0.22621906 \quad (1.0.14)$$

Below table is indicating probability of various events to occur

Number of bulbs fuses after 150days of use among 5 bulbs is	Probability for the event to occur
none	0.77378094
not more than 1	0.977407502
more than 1	0.022592497
atleast 1	0.22621906