Assignment 2

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1 GATE PROBLEM 64

Let X_n denote the sum of points obtained when n fair dice are rolled together.The Expectation and Variance of X_n are

2 SOLUTION

We know, when one dice is rolled probability i.e $\Pr(X_1 = r)$ for all r in {1,2,3,4,5,6} is equal to p

$$p = \frac{1}{6} \qquad (2.0.1)$$

Let Y_i denote the value obtained on ith dice when n dices are rolled , therefore

$$X_n = \sum_{i=1}^{n} Y_i \qquad (2.0.2)$$

Now i will calculate expectation value of value obtained when one dice is rolled using below formula;

$$E(Y_i) = E(X_1) = \sum_{r=1}^{6} (r \times p) \qquad (2.0.3)$$

$$= \frac{1}{6} \times \sum_{r=1}^{6} r \qquad (2.0.4)$$

$$= \frac{7}{2}. \qquad (2.0.5)$$

1) Since the Expectation value of a sum of independent events is the sum of their expectation. So,

$$E(X_n) = \sum_{i=1}^{n} E(Y_i) \qquad (2.0.6)$$

$$= \sum_{i=1}^{n} \frac{7}{2} = \frac{7}{2}n \qquad (2.0.7)$$

2) By Using the following formula ,we can calculate variance of X_1 ,

$$V(X_1) = (E(X_1)^2) - (E(X_1))^2 \qquad (2.0.8)$$

$$\sum_{i=1}^{k} r^2 = \frac{k \times (k+1) \times (2(k)+1)}{6} \qquad (2.0.9)$$

Now calculating $E(X_1^2)$,by using (2.0.9)

$$E(X_1^2) = \sum_{r=1}^{6} (r^2 \times p) \qquad (2.0.10)$$

$$= \frac{1}{6} \times \sum_{r=1}^{6} r^2 \qquad (2.0.11)$$

$$= \frac{91}{6} \qquad (2.0.12)$$

By using (2.0.5),(2.0.8) and (2.0.12)

$$V(X_1) = \frac{35}{12} \qquad (2.0.13)$$

Since the variance of a sum of independent events is the sum of their variances. So,

$$V(X_n) = \sum_{i=1}^{n} V(Y_i) \qquad (2.0.14)$$

$$= \frac{35}{12}n \qquad (2.0.15)$$

Hence option(B) is correct.