Assignment 2

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Download latex-tikz codes from

https://github.com/Nik123−cpp/Assignment−2/blob/main/Assignment2.tex

1 Gate Problem 64

Let X_n denote the sum of points obtained when n fair dice are rolled together.The Expectation and Variance of X_n are

2 Solution

We know, when one dice is rolled probability i.e $\Pr(X_1 = r)$ for all r in {1,2,3,4,5,6} is equal to p

$$p = \frac{1}{6} \tag{2.0.1}$$

Let Y_i denote the value obtained on ith dice when n dices are rolled , therefore

$$X_n = \sum_{i=1}^{n} Y_i \tag{2.0.2}$$

Now i will calculate expectation value of value obtained when one dice is rolled using below formula;

$$E(Y_i) = E(X_1) = \sum_{r=1}^{6} (r \times p) \tag{2.0.3}$$

$$= \frac{1}{6} \times \sum_{r=1}^{6} r \tag{2.0.4}$$

$$= \frac{7}{2}. \tag{2.0.5}$$

1) Since the Expectation value of a sum of independent events is the sum of their expectation. So,

$$E(X_n) = \sum_{i=1}^{n} E(Y_i) \tag{2.0.6}$$

$$= \sum_{i=1}^{n} \frac{7}{2} = \frac{7}{2} n \tag{2.0.7}$$

2) By Using the following formula ,we can calculate variance of X_1 ,

$$V(X_1) = (E(X_1)^2) - (E(X_1))^2 \tag{2.0.8}$$

$$\sum_{i=1}^{k} r^2 = \frac{k \times (k+1) \times (2(k)+1)}{6} \tag{2.0.9}$$

Now calculating E(X_1^2),by using (2.0.9)

$$E(X_1^2) = \sum_{r=1}^{6} (r^2 \times p) \tag{2.0.10}$$

$$= \frac{1}{6} \times \sum_{r=1}^{6} r^2 \tag{2.0.11}$$

$$= \frac{91}{6} \tag{2.0.12}$$

By using (2.0.5),(2.0.8) and (2.0.12)

$$V(X_1) = V(Y_i) \tag{2.0.13}$$

$$= \frac{35}{12} \tag{2.0.14}$$

Variance of sum can be calculated by using following formula,

$$V(X_n) = V(\sum_{i=0}^{n} Y_i) \tag{2.0.15}$$

$$= \sum_{i=1}^{n} V(Y_i) + \sum_{1 \le i \ne j \le n} \mathrm{Cov}(Y_i, Y_j) \tag{2.0.16}$$

Since Co-variance of independent random variables is zero.So,

$$V(X_n) = \sum_{i=1}^{n} V(Y_i) + 0 \tag{2.0.17}$$

$$= \frac{35}{12} n \tag{2.0.18}$$

Hence option(B) is correct.