Assignment 2

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https://github.com/Nik123−cpp/Assignment−2/blob/main/Assignment2.tex

1 Gate Problem 64

Let X_n denote the sum of points obtained when n fair dice are rolled together.The Expectation and Variance of X_n are

2 Solution

X_n ,which denotes the sum of points obtained when n fair dice are rolled together We know, when one dice is rolled probability i.e $\Pr(X_1 = r)$ for all r in {1,2,3,4,5,6} is equal to p

$$p = \frac{1}{6} \tag{2.0.1}$$

,Now i will calculate expectation value using below formula;

$$E(X_1) = \sum_{r=1}^{6} r.\Pr(X_1 = r) \tag{2.0.2}$$

$$= \sum_{r=1}^{6} r.p \tag{2.0.3}$$

$$= p.\sum_{r=1}^{6} .r \tag{2.0.4}$$

$$= p.\frac{6(7)}{2} \tag{2.0.5}$$

$$= \frac{1}{6}.\frac{6.7}{2} = \frac{7}{2}. \tag{2.0.6}$$

1) similarly when n dice are rolled ,expectation value from each dice = $\frac{7}{2}$. So expectation of sum of points on n dice is n times the expectation value from each dice i.e

$$E(X_n) = n(E(X_1)) = \frac{7}{2}n \tag{2.0.7}$$

2) By Using the following formula and using (2.0.7) we can calculate variance of X_n

$$V(X_n) = (E(X_n)^2) - (E(X_n))^2 \tag{2.0.8}$$

$$= (E(X_n)^2) - (\frac{49}{4}n^2) \tag{2.0.9}$$

In (2.0.9) ,By substituting n as 1

$$V(X_1) = (E(X_1)^2) - \frac{49}{4} \tag{2.0.10}$$

Now calculating E(X_1^2),

$$E(X_1^2) = \sum_{r=1}^{6} r^2.p \tag{2.0.11}$$

$$= p.\sum_{r=1}^{6} r^2 \tag{2.0.12}$$

$$= p.\frac{6(6+1)(2(6)+1)}{6} \tag{2.0.13}$$

$$= \frac{1}{6}.\frac{6.7.13}{6} \tag{2.0.14}$$

$$= \frac{7.13}{6} \tag{2.0.15}$$

$$= \frac{91}{6} \tag{2.0.16}$$

From (2.0.10) and (2.0.16)

$$V(X_1) = \frac{91}{6} - \frac{49}{4} \tag{2.0.17}$$

$$= \frac{182 - 147}{12} \tag{2.0.18}$$

$$= \frac{35}{12} \tag{2.0.19}$$

Since the variance of a sum of independent events is the sum of their variances. So, When n dice are rolled the variance of X_n is n times the variance of the value when one dice is rolled i.e

$$V(X_n) = n.V(X_1) = \frac{35}{12}n \tag{2.0.20}$$

Hence option(B) is correct.