

Assignment 2

P Ganesh Nikhil Madhav -CS20BTECH11036

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<https://github.com/Nik123-cpp/Assignment-2/blob/main/Assignment2.tex>

1 GATE PROBLEM 64

Let X_n denote the sum of points obtained when n fair dice are rolled together. The Expectation and Variance of X_n are

2 SOLUTION

X_n , which denotes the sum of points obtained when n fair dice are rolled together. We know, when one dice is rolled probability i.e $\Pr(X_1 = r)$ for all r in $\{1, 2, 3, 4, 5, 6\}$ is equal to $\frac{1}{6}$. Now I will calculate expectation value using below formula;

$$E(X_1) = \sum_{r=1}^6 r \cdot \Pr(X_1 = r) \quad (2.0.1)$$

$$= \frac{1}{6} \sum_{r=1}^6 r \quad (2.0.2)$$

$$= \frac{1}{6} \cdot \frac{6(7)}{2} = \frac{7}{2} \quad (2.0.3)$$

- 1) similarly when n dice are rolled, expectation value from each dice $= \frac{7}{2}$. So expectation of sum of points on n dice is n times the expectation value from each dice i.e

$$E(X_n) = n(E(X_1)) = \frac{7}{2}n \quad (2.0.4)$$

- 2) By Using the following formula and using (2.0.4) we can calculate variance of X_n

$$V(X_n) = (E(X_n)^2) - (E(X_n))^2 \quad (2.0.5)$$

$$= (E(X_n)^2) - \left(\frac{49}{4}n^2\right) \quad (2.0.6)$$

In (2.0.6), By substituting n as 1

$$V(X_1) = (E(X_1)^2) - \frac{49}{4} \quad (2.0.7)$$

Now calculating $E(X_1^2)$,

$$E(X_1^2) = \sum_{r=1}^6 r^2 \cdot \frac{1}{6} \quad (2.0.8)$$

$$= \frac{1}{6} \cdot \frac{6(6+1)(2(6)+1)}{6} \quad (2.0.9)$$

$$= \frac{1}{6} \cdot \frac{6 \cdot 7 \cdot 13}{6} \quad (2.0.10)$$

$$= \frac{7 \cdot 13}{6} \quad (2.0.11)$$

$$= \frac{91}{6} \quad (2.0.12)$$

From (2.0.7) and (2.0.12)

$$V(X_1) = \frac{91}{6} - \frac{49}{4} \quad (2.0.13)$$

$$= \frac{182 - 147}{12} \quad (2.0.14)$$

$$= \frac{35}{12} \quad (2.0.15)$$

Since the variance of a sum of independent events is the sum of their variances. So, When n dice are rolled the variance of X_n is n times the variance of the value when one dice is rolled i.e

$$V(X_n) = n \cdot V(X_1) = \frac{35}{12}n \quad (2.0.16)$$

Hence option(B) is correct.