

Assignment 3

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Download all python codes from

<https://github.com/Nik123-cpp/Assignment-3/blob/main/assignment3.py>

and latex-tikz codes from

<https://github.com/Nik123-cpp/Assignment-1/blob/main/Assignment3.tex>

1 GATE PROBLEM 64 (2012 /MA)

A and B are friends. They decide to meet between 1PM and 2PM on a given day. There is a condition that whoever arrives first will not wait for the other more than 15 minutes. The probability that they will meet on that day is

(A) $\frac{1}{4}$ (B) $\frac{1}{16}$ (C) $\frac{7}{16}$ (D) $\frac{9}{16}$

2 SOLUTION

Let A arrive x minutes past 1PM.

Let B arrive y minutes past 1PM.

The condition on x,y is

$$x, y \in [0, 60] \quad (2.0.1)$$

- 1) If A and B meet on that day then they should satisfy the following condition along with (2.0.1)

$$|x - y| \leq 15 \quad (2.0.2)$$

Let A1 implies area occupied by $|x - y| \leq 15$ under the conditions (2.0.1)

Let A2 be the total area occupied under the condition (2.0.1).So

$$\Pr(|x - y| \leq 15) = \frac{A1}{A2} \quad (2.0.3)$$

A2 is the area of square with 60 units as side length,So

$$A2 = (60).(60) \quad (2.0.4)$$

$$A2 = 3600. \quad (2.0.5)$$

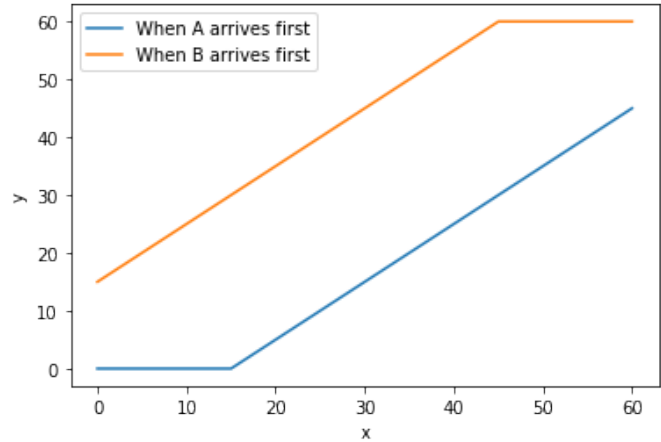


Fig. 1: Represents the possibility that they meet

A1 can be calculated by removing two triangles in the square of side 60 units. The area of two triangles are same and equal to triangle of base 45 units and height 45 units. Which can be seen in Fig 1, So

$$A1 = A2 - 2(\text{Area of triangle}) \quad (2.0.6)$$

$$= A2 - 2\left(\frac{1}{2} \cdot 45 \cdot 45\right) \quad (2.0.7)$$

$$= A2 - 45^2 \quad (2.0.8)$$

using equation (2.0.5) in above equation

$$A1 = 3600 - 45^2 \quad (2.0.9)$$

$$= 60^2 - 45^2 \quad (2.0.10)$$

$$= (60 - 45)(60 + 45) \quad (2.0.11)$$

$$= 15 \cdot 105 = 1575. \quad (2.0.12)$$

therefore, by using equations (2.0.3), (2.0.5) and (2.0.12)

$$\Pr(|x - y| \leq 15) = \frac{A1}{A2} \quad (2.0.13)$$

$$= \frac{1575}{3600} = \frac{7}{16} \quad (2.0.14)$$

Therefore option(c) is correct.