Cl §5

№4 (1,2,3)

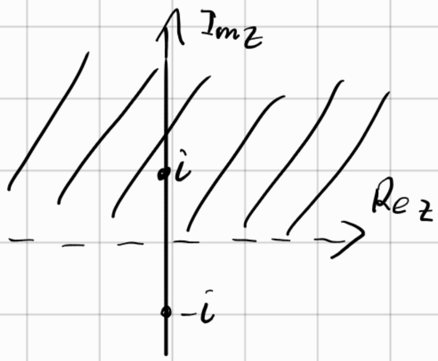
1) $i^{17}+i^{18}+i^{19}+i^{20}=i^{16}(i+i^2+i^3+i^4)=1(i-1-i+1)=0$

2)
$$2i\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=2i\left(-\frac{3}{4}-\frac{1}{4}\right)=-2i$$

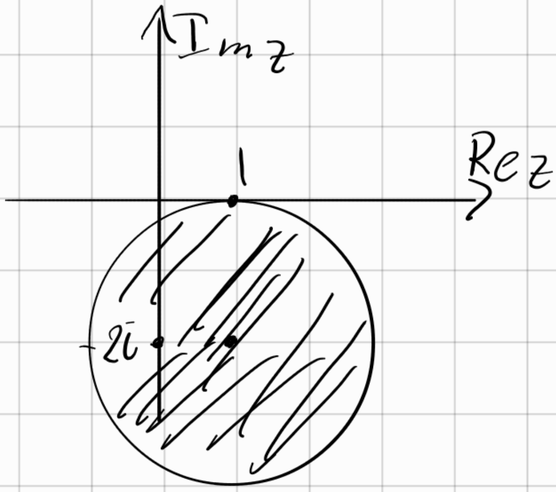
3)
$$\frac{1+i}{1-i}+\frac{1-i}{1+i}=\frac{(1+i)^2+(1-i)^2}{(1+i)(1-i)}=\frac{1+2i-1+1-2i-1}{1+1}=0$$

№15 (2,3,5)

2) $|z-i|<|z+i|$

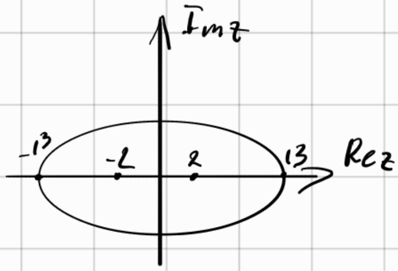


3) $|z+2i-1|\leq 2$



5)

$|z-2|+|z+2|=26$



№30(5)

$$z=\frac{(1+i)^{9}}{(1-i)^{7}}=\frac{(1+i)^{16}}{2^{7}}=\frac{4^{4}}{2^{7}}=\frac{2^{8}}{2^{7}}=2$$

$1+i$
$(1+i)^2=2i$
$(1+i)^3=2i-2$
$(1+i)^4=2(i-1)(i+1)=4$
$(1+i)^5=4+4i$
$(1+i)^6=4(1+i)^2=$
$\vdots$

№31(2)

$$\left(\frac{1+\sqrt{3}i}{i-1}\right)^6=\left(\frac{2e^{i\frac{\pi}{3}}}{\sqrt{2}e^{i\frac{3\pi}{4}}}\right)^6=\left(\sqrt{2}\,e^{-i\frac{5}{12}\pi}\right)^6=8\,e^{-i\frac{5}{2}\pi}=8\left(\cos\left(-\frac{5\pi}{2}\right)+i\sin\left(-\frac{5\pi}{2}\right)\right)=8\left(\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}\right)=-8i$$

№32(4,7,8)

4) $z^8=1+i=\sqrt{2}\,e^{i\frac{\pi}{4}}$

$z^8=|z|^8e^{i8\varphi}=\sqrt{2}\,e^{i\left(\frac{\pi}{4}+2\pi k\right)}$

$z=2^{\frac{1}{16}}e^{i\left(\frac{\pi}{32}+\frac{\pi k}{4}\right)}\quad k\in[0;7)$

7) $z^6+64=0$

$z^6=-64=64e^{i(\pi+2\pi k)}$

$z=2\,e^{i\left(\frac{\pi}{6}+\frac{\pi k}{3}\right)}\quad k\in\{0\ldots5\}$

8) $z^2=(\bar{z})^3$

$z=|z|\,e^{i\varphi}$

$|\bar{z}|=|z|\,e^{-i\varphi}$

$|z|^2e^{i2\varphi}=|z|^3e^{-i3\varphi}$

$\Downarrow\; z\neq0$

$e^{i2\varphi}=|z|\,e^{i3\varphi}$

$1=|z|\,e^{-i5\varphi}$

$|z|=1$

$-5\varphi=2\pi k$

$\varphi=-\frac{2}{5}\pi k\quad k\in\{0\ldots4\}$

Ответ: $0;\ e^{-i\frac{2}{5}\pi k}$

T1 $z^3+7z^2+24z+18=0$

$$\begin{array}{l|l} z^3+7z^2+24z+18 & z+1 \\ \hline -(z^3+z^2) & z^2+6z+18 \\ \quad 6z^2+24z & \\ \quad -(6z^2+6z) & \\ \qquad 18z+18 & \\ \qquad -(18z+18) & \\ \qquad\quad 0 & \end{array}$$

$z=-1$

$(z+1)(z^2+6z+18)$

$D=-36\qquad \frac{-6\pm6i}{2}=-3\pm3i$

Ответ: $-1;\ -3\pm3i$

C2 §2

№ 3(2,4)

2) $\int \frac{x^2+2}{(x-1)(x+1)^2} dx$

$$\frac{x^2+2}{(x-1)(x+1)^2} = \frac{A}{(x-1)} + \frac{B}{x+1} + \frac{C}{(x+1)^2} = \frac{A(x+1)^2 + B(x-1)(x+1) + C(x-1)}{(x+1)^2(x-1)}$$

$$\begin{aligned} &A+B=1 \\ &2A+C=0 \\ &A-B-C=2 \end{aligned} \qquad 4A=3 \quad A=\frac{3}{4} \quad B=\frac{1}{4} \quad C=-\frac{3}{2}$$

$$\int \frac{3}{4(x-1)} + \frac{1}{4(x+1)} - \frac{3}{2(x+1)^2} dx = \frac{3}{4}\ln|x-1| + \frac{1}{4}|x+1| + \frac{3}{2}\frac{1}{(x+1)} + C$$

4) $\int \frac{x^2+1}{x(x-1)^3} dx$

$$\frac{x^2+1}{x(x-1)^3} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3} = \frac{A(x-1)^3 + Bx(x-1)^2 + Cx(x-1) + Dx}{x(x-1)^3}$$

$$\begin{aligned} &D=2 \\ &A=-1 \\ &B=1 \\ &C=0 \end{aligned} \qquad \begin{aligned} &A+B=0 \\ &3A+B-C+D=0 \end{aligned}$$

$$\int \frac{-1}{x} + \frac{1}{x-1} + \frac{2}{(x-1)^3} dx = -\ln|x| + \ln|x-1| - \frac{1}{(x-1)^2} + C$$

№4(2,5)

2) $\int \frac{dx}{x^3+1} = \int \frac{dx}{(x+1)(x^2-x+1)} = \int \frac{1}{3(x+1)} - \frac{1}{3}\frac{x-2}{x^2-x+1} dx = \frac{1}{3}\ln|x+1| - \frac{1}{6}\int \frac{2x-4}{x^2-x+1} dx =$

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} = \frac{A(x^2-x+1) + (Bx+C)(x+1)}{x^3+1}$$

$$\begin{aligned} &A+B=0 \\ &-A+B+C=0 \\ &A+C=1 \end{aligned}$$

$-A-A+1-A=0 \quad A=\frac{1}{3}$

$B=-\frac{1}{3} \quad C=\frac{2}{3}$

$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\int \frac{2x-1}{x^2-x+1} dx + \frac{1}{2}\int \frac{dx}{x^2-x+1} =$

к арктангенсу

$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\ln(x^2-x+1) + \frac{1}{2}\int \frac{dx}{x^2-x+\frac{1}{4}+\frac{3}{4}}$

$$\frac{1}{2}\int \frac{dx}{(x-\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} = \frac{1}{\sqrt{3}} arctg \frac{2x-1}{\sqrt{3}} + C$$

5)

$$\int \frac{x^3+x^2+x+3}{(x+3)(x^2+x+1)}\,dx$$

$$\frac{x^3+x^2+x+3}{(x+3)(x^2+x+1)} = 1 - \frac{3x^2+3x}{(x+3)(x^2+x+1)}$$

$$\frac{3x^2+3x}{(x+3)(x^2+x+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+x+1} = \frac{Ax^2+Ax+A+Bx^2+3Bx+Cx+3C}{(x+3)(x^2+x+1)}$$

$$\begin{cases} A+B=3 \\ A+3C=0 \\ A+3B+C=3 \end{cases} \quad B=\frac{3}{7} \quad A=\frac{18}{7} \quad C=-\frac{6}{7}$$

$$\int 1-\frac{18}{7(x+3)} - \frac{3x-6}{7(x^2+x+1)}\,dx = x - \frac{18}{7}\ln|x+3| - \int\frac{3x-6}{7(x^2+x+1)}dx =$$

$$= x - \frac{18}{7}\ln|x+3| - \int\left(\frac{1}{7}\cdot\frac{3}{2}\frac{2x+1}{x^2+x+1} - \frac{3}{14}\frac{5}{x^2+x+1}\right)dx =$$

$$= x - \frac{18}{7}\ln|x+3| - \frac{3}{14}\ln|x^2+x+1| + \frac{15}{14}\int\frac{dx}{x^2+x+1} =$$

$$\frac{15}{14}\int\frac{dx}{(x+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} = \frac{5\sqrt{3}}{7}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}$$

$$x - \frac{18}{7}\ln|x+3| - \frac{3}{14}\ln|x^2+x+1| + \frac{5\sqrt{3}}{7}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}$$

C2 §3

№1(4)

$$\int\frac{\sqrt{x-1}-\sqrt{x+1}}{\sqrt{x-1}+\sqrt{x+1}}\,dx = \int\frac{(\sqrt{x-1}-\sqrt{x+1})^2}{x-1-x-1}\,dx = -\frac{1}{2}\int x-1+x+1-2\sqrt{x^2-1}\,dx =$$

$$= \int\sqrt{x^2-1}-x\,dx = -\frac{x^2}{2} + \int\sqrt{x^2-1}\,dx$$