Gravity from a Quantum Field Theory viewpoint

Nikoo Najafian

September 2018

Submitted in partial fulfilment of the requirements for the degree of Master of Science of Imperial College London.

Abstract

In this paper, a review is going to be given on some of the frameworks that have been developed to describe gravity by utilising the ideas from quantum field theory. Firstly, it is going to be shown that to have a truly Lorentz invariant Lagrangian describing mass-less particles, gauge invariance is a necessity. Secondly, charge conservation and equivalence principle of general relativity are going to be derived by developing a technique from quantum field theory, namely taking the soft photon and soft graviton limit. Thirdly, the Lorentz invariant action for the coupling of graviton to all other particles is going to be presented and the Einstein's tensor is going to be obtained as the equation of motion. Afterwards, the need for a modification of gravity is explained and massive graviton is illustrated as one of the ways to modify gravity. The complications associated with this method is also specified. Lastly, a binary system of black holes is considered and the gravitational radiation created at their spiral phase is outlined by the application of effective field theory.

Contents

1 Introduction

From the computation of a trajectory of a ball to the trajectory of a satellite, Newtonian theory of gravity has worked for countless times. Despite its many successes there are some observational and theoretical issues that signal to why this theory cannot be the correct description of the universe. One of the observational issues is that, the perihelion of Mercury is receding, which could not have been predicted using the Newtonian dynamics. The theoretical issue raised by Newton himself, is the idea of action at a distance. Meaning that an object at a distance can be moved without anything mediating. These issues may had motivated Einstein to formulate a geometric description of the universe, where space and time are intertwined and gravity is a consequence of curvature on the fabric of space-time[1].

On the other hand, quantum mechanics together with quantum field theory (QFT), represent the universe at the smallest energy scales. Formulation of QFT has given scientists a tool with which the particles of nature are described as excited states of an underlying field[2]. For long there has been attempts to write a quantum theory of gravity, however no complete theory has been written yet, due to mathematical problems that has been faced[3]. In this paper some of these attempts, together with their pros and cons are presented.

Gauge invariance of mass-less particles

Often gauge invariance is thought of a fundamental symmetry, however this is not true when studying relativistic particles. Lorentz transformation properties of mass-less particles implies that, it is not always possible to find the correct transformation that would boost these particles back to where they started. This fact brings some challenges to define the field operator associated with these particles. Eventually gauge invariance of such particles emerges as a consequence of these transformation properties. It can be said a Lagrangian describing relativistic particles is not truly Lorentz invariant, unless it is invariant under the specified gauge group[4].

Equivalence principle and conservation of charge

Charge conservation has always been taught as a principle. It is going to be shown that by utilising quantum field theory and making some general assumptions, such as Lorentz invarinace, zero mass and spin 1 of the photon, this so called principle can actually be derived[5].

Equivalence principle of general relativity states that, the inertial mass of an object is equal to its gravitational mass[1]. By making similar assumptions as for the charge conservation and again applying quantum field theory, this principle can also be derived.

Recovering GR from QFT

Turn of events has made the general theory of relativity to be written before the quantum description of the universe. This has made scientists to think of GR as the fundamental picture for the world. However, it is going to be shown that the same description as GR can be recovered by considering the coupling of gravitons to all other forms of matter[6].

The quest for a modification of gravity

For many times, GR has matched the observations made at solar system scale. The motivation to modify the already existing theory of gravity comes from observations made at galatic and cosmological scale. The unexpected velocity of spiral galaxies and accelerated expansion of universe has introduced dark matter and dark energy[7]. However, no significant advances has been made so far to depict this exotic state of matter and energy. These motivate physicists to find a modification to the gravity that would give an explanation for these phenomena. One way of modifying gravity is to consider a massive graviton, which has been discussed in this paper.

Application of EFT on gravitational waves

Gravitational waves are disturbances made by massive objects on the fabric of space-time. Since the start of Ligo and Virgo operations to make observations of these disturbances, theoretical frameworks have been developed to improve the accuracy of quantities computed for graviational waves. One of these techniques, proposed by Goldberger and Rothstein in 2005, is to apply effective field theory[8]. This enables for higher order terms to be calculated in a systematic way, so consequently boosts the accuracy of computations.

2 Gauge invariant theories for mass-less particles

In this section, the geometric description of gravity is forgotten and gravity is considered to be a phenomena due to the interaction of gravitons in flat space-time. It will be shown that gauge invariant theories describing photons and gravitons arise from the fact that, for relativistic particles a rest frame cannot be defined which returns complications for defining the spin of these particles[4]. For massive particles, the spin can always be defined in the rest frame of the particle and then boost this information to any other frame. However, this cannot be done for relativistic particles.

2.1 Spin properties of massive particles

In general, the single state of a particle with mass m and spin s is defined by its spin and momentum. In quantum mechanics this is written as $|\vec{p},\sigma\rangle$, where energy and momentum are related by the on-shell relation.

Under a Lorentz transformation this state transforms in the following way,

$$U(\Lambda)\,|\vec{p},\sigma\rangle = D(\Lambda)\,|\Lambda\vec{p},\sigma'\rangle\,, \tag{1}$$

where $U(\Lambda)$ is the unitary operator associated with the transformation and $D(\Lambda)$ is a matrix connecting σ and σ'. For a massive particle, by a suitable Lorentz transformation, the four momentum of the particle can be transformed to its rest frame, $p^\mu = (E,\vec{p}) \to k^\mu = (m,0)$. The subgroup of Lorentz transformations that leaves k^μ invariant is called its little group. This essentially defines the spin, by definition doing a transformation under the little group on the particle state, the momentum remains unchanged, so the only non-trivial part is how particles behave under spin translations. In this case the little group is the group of rotations $SO(3)$ [4].

2.2 Helicity for mass-less particles

In this case, an arbitrary four momentum is chosen in a way that is aligned with the z-axis, $k^\mu = (E',0,0,E')$. The little group for k^μ is $SO(2)$ around z-axis, which is isomorphic to the group $U(1)$, this has one dimensional irreducible representations, which are characterised by a phase, this phase is associated with the quantum number helicity. Therefore, the matrix $D_{\sigma\sigma'}(\Lambda)$ can be written as $e^{ih\theta(\Lambda)}$, where h is the charge for this $U(1)$ transformation and is considered to be the helicity. h in general is a continuous number, however taking into account the global properties of the Lorentz group this becomes quantized in multiples of $1/2$, $h = 0, \pm 1/2, \pm 1, \pm 2/3, ...$ It has to be noted that each value of helicity corresponds to an irreducible representation of $U(1)$, so pointing to a specific particle[4][9].

The little group for k^μ also contains two other transformations. These transformations are a combination of Lorentz boosts and rotations. First a Lorentz boost on the momentum k^μ then a rotation and a final boost which gives back the original four vector. The first Lorentz boost can be chosen to be along the x-axis or the y-axis, hence there are two additional members of the little group labelled as A and B.

Hence, the little group for the momentum k^μ contains three transformations, J_z, A and B with algebra,

$$[A,B] = 0, [J_z,A] = +iB, [J_z,B] = -iA.$$

This is the same algebra as the Euclidean group $ISO(2)$, so A and B can be thought of translation operators which get transformed into another by rotation J_z[4].

However, this will lead to divergent thermodynamics for a system with particle states as eigenstates of operators A and B. The divergence arises from the fact that these eigenstates with non-trivial eigenvalues can be rotated to any other state. This leads to continuous internal degrees of freedom for these particle states causing the divergence, because thermodynamic properties of a system are proportional to internal degrees of freedom[4].

In general, most of the systems in nature do not have this property, so it can be said that these particle states correspond to the states which have trivial eigenvalues for operators A and B. Therefore, essentially the little group of the four momentum of mass-less particles is the group of rotations. It can be concluded that mass-less particles are characterised by the quantum number helicity[4][9].

2.3 Local field operator of a massive spin 1 particle

To write the field operator for a particle, there are some ingredients that must be included such as, superposition of the momenta, creation and annihilation operators, plane waves and for particles with non-zero spin, polarisation vectors are also required.

The transformation properties of the polarisation vector are the same as the single particle state. In the rest frame of the particle ($\vec{p} = 0$) these can be written as,

$$\epsilon_\mu^{(0)} = (0,0,0,1), \quad \epsilon_\mu^{(1)} = (0,1,-i,0)/\sqrt{2}, \quad \epsilon_\mu^{(-1)} = (0,1,i,0)/\sqrt{2}.$$

The polarisation vectors associated with this particle in any other frame can be found by using the same Lorentz boost as the one on momentum[4][10].

Now the local field operator can be written in the following way as a sum over the spin variable σ,

$$V_\mu = \sum_{\sigma=0,\pm1} \int \frac{d^3p}{\sqrt{2E}} (\epsilon_\mu^\sigma(\vec{p}) a_{\vec{p}}^\sigma e^{ip.x} + h.c), \tag{2}$$

where $h.c$ stands for hermitian conjugate. In order to have a Lorentz invariant Lagrangian constructed out of this field operator, the following property must hold for V_μ,

$$U(\Lambda)V_\mu(x)U^{-1}(\Lambda) = (\Lambda^{-1})^\mu_\nu V^\nu(\Lambda.x), \tag{3}$$

where $U(\Lambda)$ is a unitary operator identifying a Lorentz transformation in the Hilbert space. Provided that, polarisation vectors and annihilation (creation) operators are in the same representation as the little group the above relation is obeyed. Therefore, the Lagrangian is going to give Lorentz invariant dynamics[10].

2.4 Origin of gauge invariance for mass-less particles

Considering mass-less particles with spin $s = 1$ and their four momentum chosen as $k^\mu \propto (1, 0, 0, 1)$. To write the polarisation vectors, it has to be noted that the these vectors must transform under the little group the same way as the single particle state. As said before, the little group consists of rotations around the z-axis (x-axis or y-axis, depending on the choice of k^μ), and the operators A and B. The polarisation vectors are written in the following way,

$$\epsilon^\mu_{\pm 1}(\vec{k}) = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0),$$

where ± 1 corresponds to the helicity of the particle state. By applying Lorentz transformation the polarisation vectors for any other four momentum can also be written,

$$\epsilon^\mu_{\pm 1}(\vec{p}) = \Lambda^\mu_\nu(k^\mu \to p^\mu)\epsilon^\nu_{\pm 1}(\vec{k}).$$

By rotating these vectors by an angle θ about the z-axis the following relation can be seen,

$$\Lambda^\mu_\nu(\theta)\epsilon^\mu_{\pm 1} = e^{i(\pm\theta)}\epsilon^\mu.$$

However, by applying Lorentz transformation associated with the operators A and B of the little group, these vectors acquire an additional term,

$$\Lambda^\mu_\nu(\alpha, \beta)\epsilon^\nu_{\pm 1}(\vec{k}) = \epsilon^\mu_{\pm 1}(\vec{k}) + (\alpha \pm i\beta)\frac{1}{\sqrt{2}k}k^\mu,$$

where α and β are parameters representing operators A and B respectively. It can be seen that ϵ^μ transforms non-trivially under these operators. This shows that the polarisation vectors are not in the same representation as the single particle state of the little group.

The field operator for such particle states can be constructed in the usual way,

$$A^\mu(x) = \sum_{h=\pm 1} \int \frac{d^3p}{\sqrt{2E}}(\epsilon^\mu_h(\vec{p})a^h_{\vec{p}}e^{ip.x} + h.c). \tag{4}$$

In order to check that this field operator is a Lorentz invariant vector, unitary operator must be applied in a similar manner as the equation 2,

$$U(\Lambda)A^{\mu}(x)U^{-1}(\Lambda) = (\Lambda^{-1})^{\mu}_{\nu}A^{\nu}(\Lambda.x) + \partial^{\mu}\lambda_{\Lambda}(x), \tag{5}$$

it can be seen that this is not a Lorentz vector. The annihilation and creation operators in the field operator transform in the similar way as the single particle state, so transform trivially under the operators A and B. However, the polarisation vectors as shown above transform non-trivially under the little group. The additional term in the equation 5 is due to this non-trivial transformation. Therefore, using physical polarisation vectors for mass-less spin 1 particles, the local field operator A^{μ} transforms as a four vector plus a gauge transformation. The conclusion here is that, in order for mass-less particles to have Lorentz invariant interactions, they need to have a gauge invariant Lagrangian[4][10].

The same can be found for gravitons with helicity $h = \pm 2$. In this case the polarisation tensors can be defined as the tensor product of the polarisation vectors associated with helicity $h = \pm 1$[5],

$$\epsilon^{\mu\nu}_{\pm 2}(\vec{k}) = \epsilon^{\mu}_{\pm 1}(\vec{k})\epsilon^{\nu}_{\pm 1}(\vec{k}).$$

The local field operator for such particles can be constructed in the usual way,

$$h^{\mu\nu}(x) = \sum_{h=\pm 2} \int \frac{d^3p}{\sqrt{2E}}(\epsilon^{\mu\nu}_{h}(\vec{p})a^{h}_{\vec{p}}e^{ip.x} + h.c). \tag{6}$$

Similarly, it can be shown that this local field operator is not a Lorentz invariant tensor,

$$U(\Lambda)h^{\mu\nu}(x)U^{-1}(\Lambda) = (\Lambda^{-1})^{\mu}_{\alpha}(\Lambda^{-1})^{\nu}_{\beta}h^{\alpha\beta}(\Lambda.x) + \partial^{\mu}\zeta^{\nu}_{\Lambda}(x) + \partial^{\nu}\zeta^{\mu}_{\Lambda}(x), \tag{7}$$

where the additional terms are gauge transformations and are due to the non-trivial transformation of the polarisation tensors under the little group. Therefore, to construct a Lorentz invariant Lagrangian for gravitons it is necessary for the Lagrangian to also have gauge invariant properties[5][4].

2.5 Field strength tensor of A^{μ} and $h^{\mu\nu}$

In order to construct the polarisation tensor associated with the field strength tensor of A^{μ}, the following two terms are subtracted from each other,

$$\epsilon'^{\mu\nu}_{\pm 1}(\vec{k}) := \epsilon^{\mu}_{\pm 1}(\vec{k})k^{\nu} - \epsilon^{\nu}_{\pm 1}(\vec{k})k^{\mu},$$

this results in a polarisation tensor which is in the same representation as the single particle state. Hence, it transforms trivially under the little group operators A and B[5].

The field strength tensor is defined in terms of the partial derivatives of the field operator, $F^{\mu\nu} := \partial^\mu A^\nu - \partial^\nu A^\mu$. Using the equation 4 to expand A^μ in this definition, the Fourier transform of the above equation can be obtained.

In general relativity $h^{\mu\nu}$ can be interpreted as the perturbation of the metric around flat space-time. Riemann tensor about flat space-time is an analogue of $F^{\mu\nu}$ hence, it is the field strength tensor associated with $h^{\mu\nu}$. The polarisation tensor representing the Riemann tensor can be obtained by taking the tensor product of $\epsilon'^{\mu\nu}_{\pm 1}$ with itself, then substituting equation 6 into the obtained polarisation tensor, the Riemann tensor about flat space-time can be found[4],

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}(\partial_\mu\partial_\rho h_{\nu\sigma} - \partial_\nu\partial_\rho h_{\mu\rho} - \partial_\mu\partial_\sigma h_{\nu\rho} + \partial_\nu\partial_\sigma h_{\mu\rho}). \tag{8}$$

Therefore, it can be seen that gauge invariant Riemann tensor of GR emerges from the construction of polarisation tensors and local field operator associated with gravitons.

3 Equivalence Principle and Conservation of Charge derived from QFT

In the previous section, is was shown that in order to have a truly Lorentz invariant theory for mass-less particles, the theory must posses apparent Lorentz invariant properties (meaning indices must contract in the usual way) and also the Lagrangian needs to remain unchanged under gauge transformations. Under a gauge transformation the field operators associated with photons and gravitons transform in the following way,

$$A_\mu \to A_\mu + \partial_\mu\lambda \quad \text{and} \quad h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu\zeta_\nu + \partial_\nu\zeta_\mu. \tag{9}$$

In this section it will be shown how conservation of charge is not a fundamental fact rather a result that can be derived. Analogous to the charge conservation, the equality of gravitational and inertial mass will be derived.

3.1 Scattering of a generic particle from a photon

To start, a mass-less spin 1 particle is considered in a scattering process with a generic spin 0 particle, as shown in figure 1. The particle is assumed to have zero spin for the purpose of a simplified calculation. Fundamentally, this process cannot take place if not being part of a larger process, so the in-going particle is assumed to be on-shell[11]. The scattering amplitude is going to include a term coming from this vertex which it can be decomposed as,

$$M^{\mu}\epsilon_{\mu}^{*}.$$

Where the ϵ_{μ}^{*} is the polarisation vector representing the out-going photon.

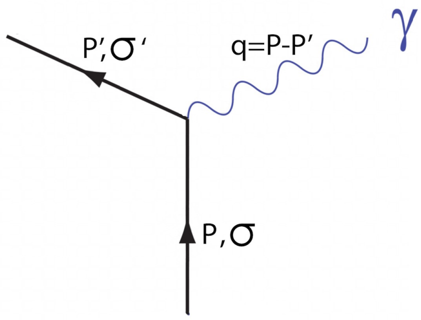

Figure 1: Feynman diagram representing the scattering of a spin zero particle ($\sigma = \sigma' = 0$) from a photon. P and P' represent momentum of the non-scattered and scattered particle respectively, also q is the momentum of the photon[12].

To find out the form of M^{μ}, it is known that this term depends on the momentum and the polarisation of the particles, $M^{\mu} = M^{\mu}(P, P', \sigma, \sigma')$. However, as the particles are considered to be scalars, therefore M^{μ} only depends on their momenta. Now, to focus on interactions at low energies, the photon is treated as a soft photon, so the limit where the momentum of the photon goes to zero, $q \to 0$[5]. By considering Lorentz invariant properties of M^{μ}, soft photon limit and the fact that in-going particle is assumed to be on-shell, M^{μ} can be written in the following way,

$$M^{\mu} = M^{\mu}(P) = P^{\mu} f(P^2) = ieP^{\mu}. \tag{10}$$

Where $f(P^2)$ is a function and considered to be a constant, because the particle is on-shell ($P^2 = m^2$). Also, this function can be defined to be ie for the benefit of future purposes. Hence, equation 10 is the most general form of the scattering amplitude for such processes at low energies[11].

3.2 Derivation of conservation of charge

Looking at the figure below, graph (a) represents the scattering process of some scalar particles. Graph (b) is the same process with the addition of a soft photon being emitted or absorbed. Graph (c) shows the decomposition of the same scattering process. The soft photon is attached to one of the out-going lines, dots are all the other possible ways that the soft photon can be attached to the in-going or the out-going lines. As the emitted (absorbed) photon is assumed to have a negligible or zero momentum, the final state β in the graphs (c) and (b) are approximately the same as the first one.

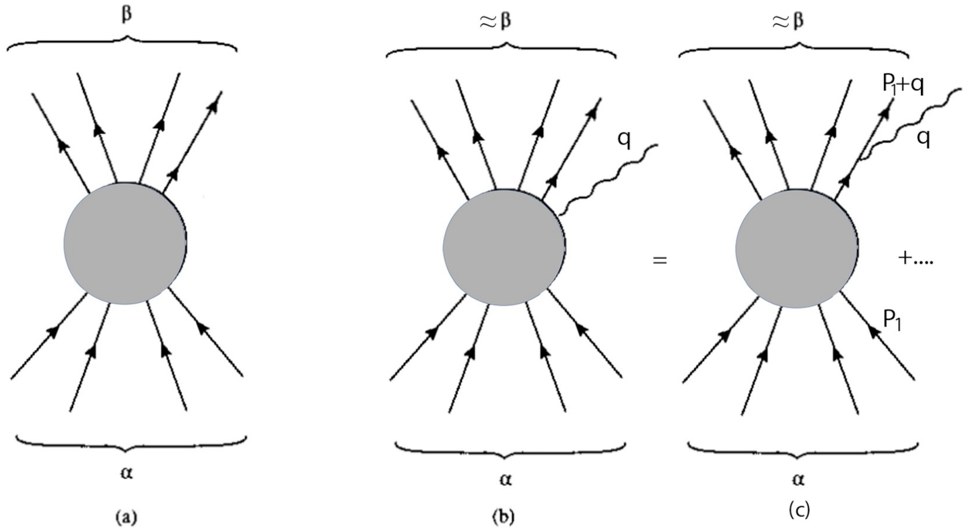

Figure 2: The graph (a) shows an arbitrary scattering process of scalar particles, $\alpha \to \beta$. Graphs (b) and (c) represent the same process with an addition of a soft photon. Straight lines represent scalar particles and wavy lines show the emission (absorption) of the soft photon[13].

The soft photon can be attached to in-going or out-going lines, for now consider that the photon is attached to one of the out-going lines as shown in the figure above. Using the analysis done in the previous part, together with the knowledge of Feynman rules, the contribution to the amplitude coming from this vertex is written in the following way,

$$
\begin{aligned}
iM &= -ie_1 P_1^\mu \frac{i}{(P_1+q)^2+m_1^2} iM_{\alpha\to\beta}\epsilon^*_\mu \\
&= iM_{\alpha\to\beta}\frac{e_1 P_1^\mu}{P_1.q}\epsilon^*_\mu.
\end{aligned} \tag{11}
$$

Note that, internal lines are not drawn but they are all the possible lines that can be included inside the circle in figure 2. External lines are the straight lines that can be in-going or out-going.

To take into account all the other possible ways that the photon can attach to the this process, the scattering amplitude becomes a sum over all the external lines and takes the form below,

$$
iM = iM_{\alpha\to\beta}\sum_i \eta_i \frac{e_i P_i^\mu}{P_i.q}\epsilon^*_\mu, \tag{12}
$$

where η_i is negative for in-going lines and positive for out-going lines[11][5]. It should be noted that, the contributions coming from attaching the photon to internal lines are ignored, because they are less divergent than the soft photon limit $(q \to 0)$[11].

In section 2, it was shown that the polarisation vectors associated with mass-less particles do not transform trivially under the little group operators A and B, whereas the particle state is invariant,

$$
\epsilon_\mu \to \epsilon_\mu + Cq_\mu, \quad |\vec{q}, \pm 1\rangle \to |\vec{q}, \pm 1\rangle \,, \tag{13}
$$

where C is a constant. In order to have a consistent theory describing the particle state, ϵ_μ and its transformed form must represent the same physical state. This statement is the same as gauge invariance but at the level of polarisation vectors and scattering amplitudes[11]. Hence, the scattering amplitude written in equation 12 must be insensitive to transformations under the little group operators A and B[11]. The following equation shows the transformation of the studied amplitude under these operators,

$$
iM \to iM + CM^\mu q_\mu.
$$

For this amplitude to be invariant hence, ϵ_μ and $\epsilon_\mu \to \epsilon_\mu + Cq_\mu$ describing the same photon state then, it is essential to have, $M^\mu q_\mu = 0$. Which means if ϵ^*_μ is replaced with q_μ in the amplitude the result is zero. This claim leads to the statement of charge conservation,

$$
M_{\alpha\to\beta}\sum_i \eta_i \frac{e_i P_i.q}{P_i.q} + ...(\text{less divergent}) = 0,
$$

therefore,

$$\sum_i e_i \eta_i = 0 \quad \to \quad \sum_{\text{out-going}} e_i = \sum_{\text{in-going}} e_i. \tag{14}$$

Hence, in order to have true Lorentz invariance through gauge invariance, these coupling constants must be conserved between in-going and out-going states[5][11].

3.3 Derivation of equivalence principle

The same approach can be applied to derive the equivalence of gravitational and inertial mass. The figure below is the same scattering process as figure 1, with the replacement of the photon with a graviton.

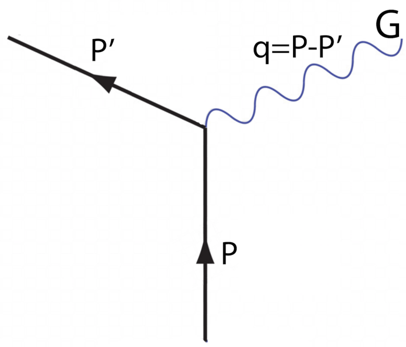

Figure 3: Feynman diagram showing the scattering of a spin zero particle from a graviton. P and P' are momenta of the non-scattered and scattered particle respectively. The wavy line is the scattering of the graviton G with momentum q [12].

Assuming soft graviton limit ($q \to 0$), the contribution to the scattering amplitude coming from this vertex is,

$$\lim_{q\to 0} M^{\mu\nu}(P.P')\epsilon^*_{\mu\nu}(q) = P^\mu P^\nu f(P^2) = igP^\mu P^\nu, \tag{15}$$

where, $f(P^2)$ is a generic function which is a constant, as P^2 for on-shell particles is m^2. Therefore, $f(P^2)$ is defined to be equal to ig, where g is a coupling constant and analogue of charge.

Now, the same scattering process as in figure 2 is considered, with the difference of a soft graviton being emitted or absorbed. The scattering amplitude for this process is written analogous to the equation 11,

$$iM = iM_{\alpha\to\beta}\sum_i \eta_i g_i \frac{P_i^\mu P_i^\nu}{P_i.q}\epsilon^*_{\mu\nu}, \tag{16}$$

where g_i are coupling constants for each particle and η_i has the same properties as the previous part[11].

As before, the transformation properties of the polarisation vectors and the particle states under the little group operators A and B are investigated,

$$|\vec{q},\pm 2\rangle \to |\vec{q},\pm 2\rangle\,, \quad \text{and} \quad \epsilon_{\mu\nu}(q) \to \epsilon_{\mu\nu}(q) + C_\mu q_\nu + D_\nu q_\mu, \tag{17}$$

where C and D are constant vectors. Therefore, for the amplitude to stay the same under these transformations, in other words, to have gauge invariance at the level of amplitudes, it is essential to have, $M^{\mu\nu}q_\mu = 0$. Which leads to the following result,

$$M_{\alpha\to\beta}\sum_i \eta_i P_i^\nu \frac{g_i P_i.q}{P_i.q} + ...(\text{less divergent}) = 0,$$

hence,

$$\sum_{\text{out-going}} g_i P_i^\mu = \sum_{\text{in-going}} g_i P_i^\mu. \tag{18}$$

Now, a vector F^μ is defined as,

$$F^\mu \equiv \sum_i g_i P_i^\mu, \quad \text{so} \quad F^\mu_{out} = F^\mu_{in}.$$

It can be shown that four momentum is the only possible vector that can be conserved in such scattering processes, known as the Coleman–Mandula theorem[14][11].

Consider an arbitrary $2 \to 2$ elastic scattering, where $\vec{P}_1$, $\vec{P}_2$ are incoming momenta and $\vec{P}_1'$, $\vec{P}_2'$ are out-going momenta. By moving to the centre of mass frame and also considering the result obtained in equation 18, the following relations for the incoming and out-going momenta can be written,

$$\vec{P}_1 + \vec{P}_2 = 0 = \vec{P}_1' + \vec{P}_2' \tag{19}$$

$$g_1\vec{P}_1 + g_2\vec{P}_2 = g_1\vec{P}_1' + g_2\vec{P}_2'. \tag{20}$$

Combining equations 19 and 20,

$$(g_1 - g_2)(\vec{p}_1 - \vec{p}_1') = 0, \tag{21}$$

so,

$$\vec{p}_1 = \vec{p}_1' \quad \text{or} \quad g_1 = g_2. \tag{22}$$

The first result where the momenta are the same is the trivial scattering where the particles just go through. The second possibility is of our interest, where the particles do collide and their momenta change. The result obtained is that, the coupling constants must be the same. Therefore,

$$F^{\mu} = g \sum_{i} P_{i}^{\mu}, \tag{23}$$

which means F^{μ} is the sum of the four momentum. This result shows that g is the same for all particles, so graviton couples universally to all particles. This is the statement of equivalence principle[11][5].

4 Recovering the non-linear structure of GR

4.1 Construction of GR action in flat space-time in a gauge invarinat manner

The full action describing the interactions of gravitons ($h_{\mu\nu}$) and all other particles is decomposed as below,

$$S[h, \psi] = S_{g}^{free}[h, .] + S_{matter}[\psi] + S_{int}[h, \psi], \tag{24}$$

where $S_{g}^{free}[h, .]$ is associated with the free propagation of gravitons. $S_{matter}[\psi]$ describes free and interacting particles (excluding gravitons) and $S_{int}[h, \psi]$ is expressing the interaction of matter with gravitons including graviton self-interactions[11].

In order for this action to have true Lorentz invariant properties it must posses apparent Lorentz and also gauge invariance. Gauge invariance of the above action can be studied term by term. Firstly, as $S_{matter}[\psi]$ does not depend on h then it must be gauge invariant.

Note that, to construct $S_{g}^{free}[h, .]$ in a gauge invariant way, the Hamiltonian describing the propagation of free particles must be taken into account. This is because, the free action must be in such a way that, if the field expansion is plugged into and the Hamiltonian associated with it is written, then the Hamiltonian must become a sum over all momenta of harmonic oscillators[11]. The Hamiltonian describing the free propagation of gravitons is the following,

$$H = \sum_{h=\pm 2} \int \frac{d^3p}{(2\pi)^3} (a_h^{\dagger}(\vec{p}) a_h(\vec{p}) p^0 + \text{vacuum energy}). \tag{25}$$

Now, by considering the above Hamiltonian, below is the general structure expected for the free action,

$$S_g^{free} = -\frac{1}{2}\int d^4x(\partial_\alpha h_{\mu\nu}\partial^\alpha h^{\mu\nu} - 2(\partial_\mu h^\mu_\nu)^2 + 2\partial_\mu h^\mu_\nu \partial^\nu h - (\partial_\mu h)^2), \qquad (26)$$

where, $-2(\partial_\mu h^\mu_\nu)^2 + 2\partial_\mu h^\mu_\nu \partial^\nu h - (\partial_\mu h)^2$ are there to make sure the action is invariant under gauge transformation. This is exactly the action of GR expanded around flat space up to quadratic order[11].

To construct $S_{int}[h,\psi]$, note that it's gauge transformation is,

$$S_{int}[h,\psi] \to S_{int}[\psi, h_{\mu\nu} + \partial_\mu\zeta_\nu + \partial_\nu\zeta_\mu].$$

Expanding this to first order in the variation of $h_{\mu\nu}$ it becomes,

$$S_{int}[h,\psi] + \int d^4x(2\frac{\delta S_{int}}{\delta h_{\mu\nu}}\partial_\mu\zeta_\nu) + ...,$$

hence, for $S_{int}[h,\psi]$ to admit gauge invariance the following must be true,

$$-2\int d^4x(\partial_\mu \frac{\delta S_{int}}{\delta h_{\mu\nu}}\zeta_\nu) = 0 \quad \Rightarrow \quad \partial_\mu \frac{\delta S_{int}}{\delta h_{\mu\nu}} = 0.$$

The above is a functional derivative which can be called a conserved current,

$$J^{\mu\nu}[\psi, h] \equiv \frac{\delta S_{int}}{\delta h_{\mu\nu}}, \qquad (27)$$

using this, a global charge can be defined which is a constant in time,

$$\begin{aligned} Q^\mu &\equiv \int d^3x J^{0\mu} \frac{d}{dt}Q^\mu \\ &= \int d^3x \partial_0 J^{0\mu} \\ &= -\int d^3x \partial_i J^{i\mu} = 0. \end{aligned} \qquad (28)$$

In the previous chapter, "it was argued that the only possible four vector that can be conserved in a non-trivial interacting system is the four momentum up to an overall constant"[11] hence,

$$Q^\mu = gP^\mu \quad \text{where} \quad P^\mu = \int d^3x T^{0\mu},$$

then,

$$Q^\mu - gP^\mu = 0 \quad \to \quad \int d^3x(J^{0\mu} - gT^{0\mu}) = 0.$$

For the integral above to be zero, the integrand must be a total spacial derivative,

$$J^{0\mu} - gT^{0\mu} = \partial_i(...),$$

so calling $\Delta^{\mu\nu} \equiv J^{\mu\nu} - gT^{\mu\nu}$, $\Delta^{\mu\nu}$ must be equal to the total derivative of some other tensor,

$$\Delta^{\mu\nu} = \partial_\alpha \Sigma^{\alpha\mu\nu} = \partial_\alpha \partial_\beta \Phi^{([\alpha\mu][\beta\nu])}.$$

Where the above equation with the specified symmetric properties is obtained using cohomology in differential geometry[15], the proof however, is not shown in this paper. Therefore, when $\mu = 0$ then $\alpha \neq 0$ so it must be a spatial index $\alpha = i$,

$$\Delta^{0\nu} = \partial_i \Sigma^{i0\nu}. \tag{29}$$

To summarise, it was shown that the derivative of the interacting action with respect to the garviton field is equal to a tensor $J^{\mu\nu}$. This tensor is equivalent to the stress energy tensor multiplied by a constant g, up to terms which are derivatives. It can be argued that, as these derivatives correspond to momenta, compared to the stress energy tensor they cannot be relevant for low energy processes. Therefore, the requirement for $S_{int}[h,\psi]$ to be gauge invariant is that the tensor, $J^{\mu\nu} \equiv \frac{\delta S_{int}[h,\psi]}{\delta h_{\mu\nu}}$ is a conserved tensor $\partial_\mu J^{\mu\nu} = 0$. Also, as shown, this tensor can be written as $J^{\mu\nu} = gT^{\mu\nu}+$higher derivatives. These higher derivatives are spatial derivatives and when integrating over three dimensional space they become zero, so can be neglected[11][16].

4.2 Action of GR in curved space-time

Assuming $g_{\mu\nu}$ as the metric of the curved space-time, below is the action of GR,

$$S_{GR}[g,\psi] = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \tilde{S}_{matter}[\psi, g], \tag{30}$$

where $\tilde{S}_{matter}[\psi, g]$ is the action associated with matter in curved space-time. This has the same form as $S_{matter}[\psi, g]$, with the difference that partial derivatives are changed to covariant derivatives[17].

Varying this action with respect to the metric $g_{\mu\nu}$, Einstein's equation is obtained,

$$G_{\mu\nu} = 8\pi G \tilde{T}^{matter}_{\mu\nu}, \tag{31}$$

$\tilde{T}^{matter}_{\mu\nu}$ is the stress energy tensor for matter in curved space-time. The definition for this tensor is,

$$\tilde{T}^{matter}_{\mu\nu}[\psi, g] \equiv \frac{2\delta\tilde{S}_{matter}}{\sqrt{-g}\delta g_{\mu\nu}}. \tag{32}$$

The curved metric of space-time can be decomposed into a flat and a geometric part,

$$g_{\mu\nu} = \eta_{\mu\nu} + H_{\mu\nu}. \tag{33}$$

Using this expansion of metric, Einstein's equation can be re-written as,

$$G^{(1)}_{\mu\nu} = 8\pi G(\tilde{T}^{matter}_{\mu\nu} + t_{\mu\nu}), \tag{34}$$

where $G^{(1)}_{\mu\nu}$ is the Einstein's tensor to first order in $H_{\mu\nu}$ and $t_{\mu\nu} \equiv \frac{1}{8\pi G}(G^{(1)}_{\mu\nu} - G_{\mu\nu})$, so by definition it includes all the non-linear terms in the Einstein's tensor. Defining the tensor, $\tau_{\mu\nu} \equiv \tilde{T}^{matter}_{\mu\nu} + t_{\mu\nu}$ then, $G^{(1)}_{\mu\nu} = 8\pi G\tau_{\mu\nu}$. The full Einstein's tensor obeys the Bianchi identity $\nabla_\mu G^{\mu\nu} = 0$ and the linear part of the Einstein's tensor obeys the linear Bianchi identity $\partial_\mu G^{(1)}_{\mu\nu} = 0$ hence, it can be seen that, $\partial_\mu \tau^{\mu\nu} = 0$ must also be true[17][16].

Going back to the action for flat space-time, $S = S^{free}_g + S_{matter} + S_{int}$, the variation of this action gives the equation of motion for $h_{\mu\nu}$,

$$\frac{\delta S^{free}_g}{\delta h_{\mu\nu}} = -\frac{\delta S_{int}}{\delta h_{\mu\nu}}.$$

After substituting the form found for S^{free}_g and S_{int}, the equation of motion is found to be the linear Einstein's tensor with the difference that $H_{\mu\nu}$ is replaced with $h_{\mu\nu}$[17][16],

$$G^{(1)}_{\mu\nu}[H \to h] = -\alpha T_{\mu\nu}[h, \psi]. \tag{35}$$

Note that, here the original constant g is replaced with α to avoid confusion with the metric.

4.3 Equivalence of Einstein's tensor derived from QFT and GR

In order for the two theories to match namely, to match the equations 35 and 34, it is required to have $T_{\mu\nu} = \tau_{\mu\nu}$. This claim can be proven in a non-linear manner or in perturbation theory. The proof however, is not shown here, but it can be found on the paper by David G.Boulware and S.Deser, which is titled as Classical General Relativity Derived from Quantum Gravity[16]. To find the relative normalisation, S^{free}_g must match with S_{GR},

$$S^{free}_g = \int d^4x(-1/2\partial_\alpha h_{\mu\nu}\partial^\alpha h^{\mu\nu} + ...(\text{terms to have gauge invariance})) \tag{36}$$

$$S_{GR} \supset \int d^4x \frac{1}{16\pi G}\sqrt{-g}R \quad ,$$

expanding the GR action in terms of the metric,

$$\rightarrow \quad = \frac{-1}{64\pi G}\int d^4x \partial_\alpha H_{\mu\nu}\partial^\alpha H^{\mu\nu} + ..., \tag{37}$$

where dots correspond to non-linear terms in $H_{\mu\nu}$ and terms due to gauge invariance. After comparing equation 36 with 37, the following can be concluded,

$$h_{\mu\nu} = \frac{1}{\sqrt{32\pi G}} H_{\mu\nu}. \tag{38}$$

Therefore, the canonical re-normalised graviton field in quantum field theory is identified with the geometric gravitation used in general relativity[17][16]. The metric can also be re-written as, $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{32\pi G} h_{\mu\nu}$. The Einstein's tensor in curved space-time is $G_{\mu\nu} = 8\pi G\tilde{T}_{\mu\nu}$ hence, the constant in equation 35 can be identified as, $\alpha = \sqrt{8\pi G}$.

4.4 Self interaction of gravitons

Einstein-Hilbert action involves only gravitons and describes graviton's self-interactions,

$$\begin{aligned} S_{EH} &= \frac{1}{16\pi G}\int d^4x\sqrt{-g}R[g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{32\pi G}h_{\mu\nu}] \\ &= \frac{1}{16\pi G}\int d^4x\sqrt{-det(\eta+\kappa h)}R[\eta+\kappa h] \\ &= S_g^{free} + S_{int}[h], \end{aligned} \tag{39}$$

where $h_{\mu\nu}$ is the canonically normalised graviton field as before and $\kappa \equiv \sqrt{32\pi G}$[18]. The metric also has been expanded around flat space-time. The above action is written schematically meaning that, index contractions are neglected and the focus is on the power of fields and derivatives. The schematic form for S_g^{free} and S_{int} are,

$$S_g^{free} = \int d^4x \partial h\partial h$$

$$\begin{aligned} S_{int} &\sim \frac{1}{\kappa^2}\int d^4x(\sum_{n=3}^{\infty}\partial^2\kappa^n h^n) \\ &\sim \sum_{n=3}^{\infty}\int d^4x\kappa^{n-2}\partial^2 h^n, \end{aligned}$$

by definition quadratic and lower power terms are all included in S_g^{free}[17][16].

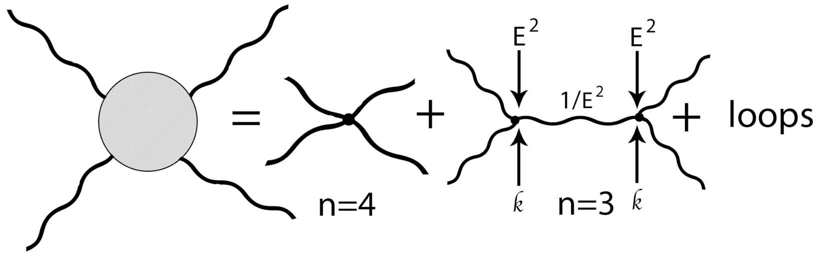

Figure 4: Shows an arbitrary $2 \to 2$ scattering of gravitons, the process has been expanded into Feynman diagrams up to fourth order in h.

Consider an arbitrary $2 \to 2$ scattering process of two gravitons. Figure 4 shows the decomposition of such process into possible Feynman diagrams, up to the fourth order in the graviton field. Using the interacting action, the scattering amplitude for such a process is of the form, $M \sim \kappa^2 E^2$. Using dimensional analysis, κ has dimension, $[\kappa] = (mass)^{-1}$. Planck mass is also, $M_{pc} = \frac{1}{\sqrt{8\pi G}} \sim 2 \times 10^{18} Gev$. So it can be written that, $\kappa = \frac{2}{Mpc}$[17]. Hence, the scattering amplitude is proportional to,

$$M \sim \kappa^2 E^2 \sim (\frac{E}{Mpc})^2. \tag{40}$$

The unitarity of the S matrix element $SS^{\dagger} = I$, makes sure that the sum of the probabilities stays as one, $\sum prob = 1$. In addition, the unitarity implies that, the amplitude for a $2 \to 2$ scattering at a fixed angle, must not grow more than the logarithm of the energy, as the energy goes to infinity[17][19],

$$M_{2\to 2} \quad <_{E\to\infty} \quad \text{up to } logE. \tag{41}$$

Looking back at the scattering amplitude found for the studied $2 \to 2$ process of gravitons, the amplitude grows quadratically with energy. As explained this is a problem and is a signal of the the break down of perturbation theory.

For those canonically normalised fields with coupling constants that have inverse mass dimension, the interactions are called non-renormalisable. The problem experienced above is a general phenomena that is seen to mostly happen for non-renormalisable interactions[17]. As explained, for the studied process the coupling constant κ has inverse mass dimension, so the above problem was expected to be seen.

These interactions are called non-renormalisable, because infinite number of parameters are needed to re-normalise the theory, so infinite number of experiments must be carried out to input these parameters to the theory. This is one of the reasons why quantum gravity is thought of as an complicated theory. This is where Effective Field Theory comes into play. In EFT it is assumed that, at low energies the effect of these infinitely many parameters on the amplitude becomes negligible[17].

4.5 Examples of non-renormalisable theories described by EFT

1) General theory of relativity as described in the previous section, where the coupling constant has inverse mass dimension, $\kappa \sim \frac{1}{Mpc} \sim (10^{19}Gev)$. 2)Fermi theory of weak interactions, where the coupling constant is of the order inverse squared mass of the W bosons, $G_F \sim M_W^{-2} \sim (100Gev)^{-2}$. To see where EFT plays a part, consider the beta decay as shown in figure 5.

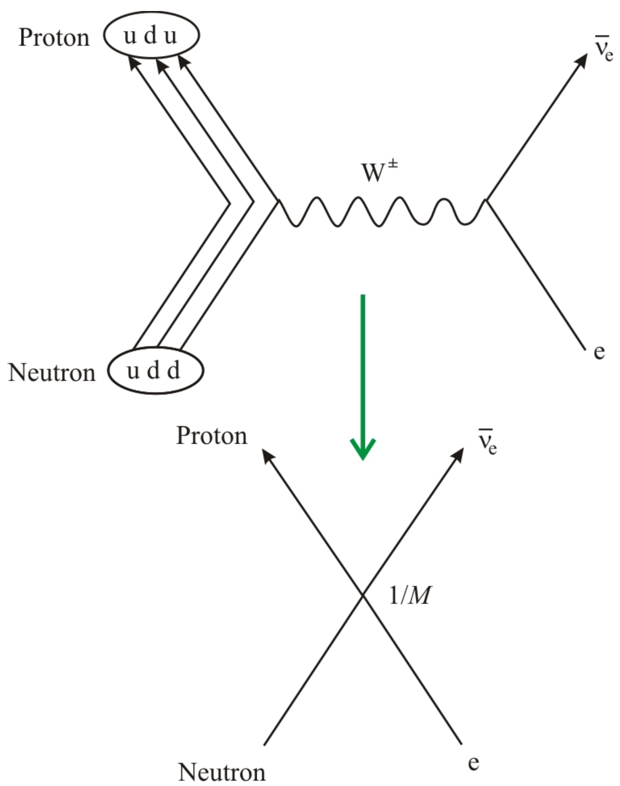

Figure 5: The above is the Feynman diagram of beta decay. Which is the decay of a neutron to a proton and emission an electron and an anti electron neutrino mediated by a W boson. Below is the same process at the level of first order in energy expansion. Hence, at the level of Fermi theory[20].

The propagator in this Feynman diagram can be written as $\frac{1}{P^2-M_w^2}$. Since, the energy difference between the up and down quark is small compared to the mass of the W boson, the propagator can be expanded in powers of the energy,

$$\frac{1}{P^2-M_w^2} \quad \rightarrow \quad -\frac{1}{M_w^2}-\frac{P^2}{M_w^4}-\frac{P^4}{M_w^6}+\ldots \quad . \tag{42}$$

By only considering the first term in the above energy expansion and ignoring higher order terms, the theory effectively becomes the Fermi theory. Hence, the Fermi theory of weak interactions is a theory in first order expansion of the energy. By adding weakly coupled massive particles, the Fermi theory completes and Standard Model of particle physics emerges[17].

To understand EFT, it must be thought of in terms of a derivative expansion of the Lagrangian. To describe a theory in EFT, the most general Lagrangian allowed by the symmetries of the theory must be written. The Lagrangian must then be organised in ascending powers of the derivatives. The idea is that at low energies only the terms with lowest order derivatives are relevant.

3)Chiral Lagrangian for Pions, $\vec{\pi}$. After canonically re-normalising the pion field, the Lagrangian schematically is,

$$L \sim -1/2(\partial\vec{\pi})^2+\frac{1}{f_\pi}\vec{\pi}^2(\partial\vec{\pi})^2+\ldots \quad .$$

The coupling constant f_π is of the same order of Λ in QCD. Therefore, $f_\pi \sim \Lambda_{QCD} \sim 200Mev$, it has mass dimension one, $[f_\pi]=mass$. Hence, the coupling constant in the Lagrangian has dimension inverse mass. The UV completion of this theory is QCD[17].

5 Infrared modifications of gravity

The need for modifying the theory of gravity comes from some observations made at galatic and cosmological scales, these do not meet the expected behaviour found using general relativity. Observations at galactic scale showed that the velocity of the spiral galaxies does not match the expected theoretical velocity. To explain this behaviour dark matter is introduced, however no significant progress is made in describing this exotic matter. At cosmological scale the accelerated expansion of the universe, rather than a decelerating expansion is a puzzle. Einstein himself introduced the cosmological constant into his equations in order to have an accelerating expansion of the universe[21][18].

In QFT cosmological constant is known as the vacuum energy. Nowadays, the main question is that why the expansion of the universe is so slow compared to the value of vacuum energy. In fact the observed value of vacuum energy is sixty orders of magnitude smaller than the value found from QFT. To explain this, Dark Energy has been introduced and also some physicist try to find an answer by modifying the theory of gravity at large distances. The cosmological constant problem has been a motivation for modifying gravity however, no real progress has been made. This is an area where it has a large potential for further research[21][18].

One of the possible ways for infrared modification of gravity is to consider a massive graviton. The curved metric as before can be expanded into a flat and a geometric part, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. Here, $h_{\mu\nu}$ is the non-canonically renormalised field. The actions is,

$$S = S_{EH} + S_{mass} + S_{matter}[\psi, g_{\mu\nu}], \tag{43}$$

where $S_{matter}[\psi, g_{\mu\nu}]$ is the action describing interactions of matter in curved space-time. Einstein-Hilbert action is,

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R, \tag{44}$$

expanding in $h_{\mu\nu}$ it becomes,

$$= -1/2M_{pc}^2\int d^4x(\partial_\alpha h_{\mu\nu}\partial^\alpha h^{\mu\nu} + \text{terms due to gauge invariance} + O(h^3)). \tag{45}$$

The action for the mass term is,

$$S_{mass} = -1/2M_{pc}^2\int d^4x(m_1^2 h_{\mu\nu}h^{\mu\nu} + m_2^2 h^2), \tag{46}$$

note that there are two mass parameters and $h_\alpha^\alpha = h$ is the trace[21][18].

The action describing the interactions of matter is,

$$S_{matter} = 1/2\int d^4x(h_{\mu\nu}T^{\mu\nu}_{matter}[\psi] + O(h^2)), \tag{47}$$

where this action shows at leading order, how matter sources the linear $h_{\mu\nu}$ and there is linear coupling between $h_{\mu\nu}$ and the stress-energy tensor. The equation of motion following from this action is,

$$M_{pc}^2(G^{(1)}_{\mu\nu} + m_1^2 h_{\mu\nu} + m_2^2 h\eta_{\mu\nu}) = T^{\mu\nu}_{matter}. \tag{48}$$

Consider a massive particle with spin 1. The equation of motion in this case is,

$$\partial_\mu F^{\mu\nu} + m^2 w^\nu = J^\nu. \tag{49}$$

Taking the divergence of this equation with respect to ν,

$$\partial_\nu \partial_\mu F^{\mu\nu} + m^2 \partial_\nu w^\nu = \partial_\nu J^\nu \quad \rightarrow \quad m^2 \partial_\nu w^\nu = 0, \tag{50}$$

where $F^{\mu\nu}$ is anti-symmetric so, its double derivative becomes zero and J^ν is a conserved vector. Possible plane wave solution for w^ν is the following,

$$w^\nu = \epsilon^\nu e^{ik.x} + c.c,$$

substituting this plane wave solution into $m^2 \partial_\nu w^\nu = 0$, it can be seen that ϵ^ν must be orthogonal to k_ν, $m^2 \epsilon^\nu k_\nu = 0$.

The same process can be done for gravitons to find the possible constrains on its polarisation vector. The divergence of the equation of motion, equation 48 is,

$$\begin{aligned} m_1^2 \partial_\mu h_{\mu\nu} + m_2^2 \partial_\mu h \eta_{\mu\nu} = \partial_\mu T^{\mu\nu} = 0, \\ m_1^2 \partial_\mu h_{\mu\nu} + m_2^2 \partial_\nu h = 0. \end{aligned} \tag{51}$$

Note that, Einstein's tensor obeys the bianchi identity $\partial_\mu G^{(1)}_{\mu\nu} = 0$. The plane wave expansion of the graviton field is,

$$h_{\mu\nu} = \epsilon^{\mu\nu} e^{ik.x} + c.c,$$

substituting this expansion into equation 51, the expression below is obtained,

$$m_1^2 k_\mu \epsilon^{\mu\nu} h_{\mu\nu} + m_2^2 k^\nu \epsilon^\alpha_\alpha = 0. \tag{52}$$

Hence, it can be seen that there are four constrains on the polarisation vector of the plane wave[21].

Starting with the symmetric tensor $\epsilon^{\mu\nu}$, this has 10 independent components, considering the four constrains just discovered the numebr of degrees of freedom reduces to 6. However, the field for a massive spin 2 particle has 5 degrees of freedom, $2s + 1 = 5$. Meaning it has 5 independent polarisation states. Therefore, there is an extra degree of freedom for the studied polarisation vector, which is a problem.

5.1 Fierz-Pauli tuning

To fix this problem an extra constrain is required on the polarisation vector. To see where this constrain can come from, consider again the equation of motion of equation 48. The double divergence of this equation is,

$$m_1^2 \partial_\mu \partial_\nu h_{\mu\nu} + m_2^2 \Box h = 0, \tag{53}$$

if making the choice that $m_1^2 = -m_2^2 \equiv m^2$, then the equation above becomes the trace of the Einstein's tensor in leading order. Hence, the assumption to make is that Einstein's tensor is trace-less on the equation of motion,

$$G_{\mu}^{(1)\mu} = \partial_\mu \partial_\nu h^{\mu\nu} - \Box h = 0. \tag{54}$$

Now, looking at propagation of gravitons in empty space, $T_{\mu\nu} = 0$ and then taking the trace of equation of motion (equation 48), the following result is obtained,

$$m^2(h - 4h) = 0 \quad \rightarrow \quad h = 0, \quad \epsilon_\mu^\mu = 0. \tag{55}$$

Therefore, the polarisation tensor is trace-less. This gives the required constrain and the number of independent polarisation tensors reduces to 5. In addition, looking back at equation 52, it can be seen that $\epsilon_{\mu\nu}$ must be transverse, $k_\mu \epsilon_{\mu\nu} = 0$. Hence, the polarisation tensor is trace-less and transverse, which means, it has the same representation as the group of rotations SO(3)[21]. This tuning of mass term is known as the Fierz-Pauli tuning. The action describing the mass term, known as the Fierz-Pauli mass term is,

$$S_{mass}^{FP} = -1/2 M_{pc}^2 m^2 \int d^4x (h_{\mu\nu} h^{\mu\nu} - h^2). \tag{56}$$

This massive theory of gravity is going to certainly modify the behaviour of gravity at cosmological scales. If the mass parameter is to be chosen to be the value of Hubble parameter today, $m \sim H_0 \sim (10^{28} cm)^{-1}$, then gravity becomes much weaker. In addition, it is desirable to recover general theory of relativity at smaller scales, such as at the size of the solar system. However, this is not achieved with the described massive theory of gravitons[21].

5.2 Stuckelberg trick

Consider massive spin 2 particles, they have 5 independent polarisation states. These states are in the same representation of the little group of the single particle state. It means that, by a suitable Lorentz transformation these states can be transformed into one another. The idea behind this trick is that, at high energies it becomes harder for some of these states to transform. This mimics the behaviour of photons, where helicity is a Lorentz invariant quantity[21]. The high energy scale for this system is set by momentum and mass, $P >> m$. Hence, at high energies it is sensible to decompose the polarisation states into the representation of the little group for mass-less particles,

$$5 = 2(h = \pm 2) + 2(h = \pm 1) + 1(h = 0). \tag{57}$$

Therefore, the projection of the angular momentum of the studied particle onto its momentum, can have these possible values, $h = \pm 2, \pm 1, 0$. In the high energy limit it is convenient to study each of these helicity states separately, rather than having one tensor to describe all of them.

The field operator associated with each helicity state and their corresponding gauge transformations are[18],

$$h = \pm 2 \quad \rightarrow \quad h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu \zeta_\nu + \partial_\nu \zeta_\mu$$

$$h = \pm 1 \quad \rightarrow \quad A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$$

$$h = 0 \quad \rightarrow \quad \phi(scalar).$$

To execute this trick at the level of Lagrangian, $h_{\mu\nu}$ is replaced with the following tensor,

$$H_{\mu\nu} \equiv h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu + 2\partial_\mu \partial_\nu \phi. \tag{58}$$

This tensor is invariant under the gauge transformations described above, provided that the following is also carried out,

$$A_\mu \rightarrow A_\mu - \zeta_\mu$$

$$\phi \rightarrow \phi - \Lambda.$$

Note that, by choosing the unitary gauge $A_\mu = 0$, $\phi = 0$ the original action can be recovered[21]. It means that, this is just a rewriting of the original action and no new physics has been introduced.

The action evaluated at the new tensor $H_{\mu\nu}$ is,

$$S[H] = S_{EH}[h] + S_{matter}[h] + S_{mass}[H], \tag{59}$$

note that $S_{EH}[h] = S_{EH}[H]$ and $S_{matter}[h] = S_{matter}[H]$, because these terms are invariant under the gauge transformation of $h_{\mu\nu}$. Focusing only on the terms dependent on ϕ, the mass term becomes,

$$S_{mass}[H] \supset -1/2 M_{pc}^2 m^2 \int d^4x \quad 4(\partial_\mu \partial_\nu \phi \partial^\mu \partial^\nu \phi - (\Box \phi)^2). \tag{60}$$

Suppose for now, the Fierz-Pauli tuning is not imposed, so considering different mass parameters, the action becomes,

$$\begin{aligned} S_{mass}[H] &\supset -2M_{pc}^2 \int d^4x (m_1^2 \partial_\mu \partial_\nu \phi \partial^\mu \partial^\nu \phi + m_2^2 (\Box \phi)^2) \quad \text{(integration by parts)} \\ &\supset -2M_{pc}^2 \int ((m_1^2 + m_2^2)(\Box \phi)^2). \end{aligned}$$

Looking at the above result and considering $(\Box\phi)^2$ as the kinetic energy, it can be seen that this object has four derivatives. It means that classically we would have needed four initial conditions to impose on the equation of motion. At the level of Lagrangian, four initial conditions is associated with two degrees of freedom. It can be shown that, whenever a four derivative kinetic energy leads to two degrees of freedom, these will have opposite energies. At the quantum level these theories have unstable vacuum against the production of these particles with opposite energies[21].

Fierz-Pauli tuning solves this problem of unstable vacuum by setting $m_1^2 = -m_2^2$. This way the problematic kinetic energy $\Box\phi^2$ cancels out. However, this does not mean that field ϕ has no kinetic energy. After the application of this tuning the mass term and the Einstein's-Hilbert action are,

$$S_{mass}[H] \supset -1/2M_{pc}^2 m^2 \int d^4x \quad 2\partial_\mu\partial_\nu\phi(h^{\mu\nu} - h\eta^{\mu\nu}), \tag{61}$$

$$S_{EH}[H] \supset -1/2M_{pc}^2 \int d^4x(\partial_\alpha h^{\mu\nu}\partial^\alpha h^{\mu\nu} + ...), \tag{62}$$

where dots in the above equation are the terms to enforce gauge invariance. Looking at equation 61 it can be seen that $S_{mass}[H]$ includes a mixture of ϕ and $h^{\mu\nu}$. To find the eigenmodes, this action must become diagonalised. To do this, the field $h^{\mu\nu}$ is redefined as $h^{\mu\nu} \equiv \hat{h}^{\mu\nu} + m^2\phi\eta_{\mu\nu}$. Using this field redefinition the Einstein-Hilbert action becomes,

$$S_{EH}[h] = S_{EH}[\hat{h}] + \int d^4x(\partial\hat{h}\partial\phi + \partial\phi\partial\phi), \tag{63}$$

where in the integral above the contractions are omitted and the focus is on the general form. It can be seen that, this integral is of the same form as the off diagonal term of the action (equation 61) with opposite sign, so the off diagonal term can be cancelled. Hence,

$$S_{EH} + S_{mass} \supset -3M_{pc}^2 m^4 \int d^4x(\partial\phi)^2, \tag{64}$$

which is the kinetic term for the field ϕ[21].

Looking back at equation 64, it can be seen that the kinetic energy for the field ϕ contains the mass parameter m^4. In canonical re-normalisation of the field the mass parameter is going to be absorbed into the field ϕ. Unless the interaction terms are also suppressed by m^4, then the interactions become very large, they will be enhanced by $1/m^4$ term.

5.3 Self interactions

In the Stuckelberg trick as explained before the graviton field is decomposed into a helicity 1, helicity 2 and a scalar field. To study self interactions in this theory, in high energy limit these interactions also decompose for each field. By studying the Lagrangian it can be found that the most dominant self interactions are due to the scalar field[21]. As said before, in unitary gauge the action is,

$$S = S_{EH}[h] + S_{mass}[h] + S_{matter}[h, \psi]. \tag{65}$$

The best way to apply this trick and separate interactions is to incorporate non-linear gauge transformation of GR. This procedure is sensible as S_{EH} and S_{matter} are invariant under this non-linear gauge transformation. To do this, the field replacement has to also contain the non-linear piece in a gauge transformation[21],

$$H_{\mu\nu} \equiv h_{\mu\nu} + \partial_\mu \pi_\nu + \partial_\nu \pi_\mu - \partial_\mu \pi^\alpha \partial^\nu \pi^\alpha, \tag{66}$$

where,

$$\pi_\mu \equiv A_\mu + \partial_\mu \phi. \tag{67}$$

The self interactions involving ϕ are going to come from the mass term S_{mass},

$$S^{FP}_{mass}[h + 2\partial\partial\phi - \partial\partial\phi.\partial\partial\phi] \sim M^2_{pc} m^4 h^2.$$

The part of the interacting action showing the self interactions of the field ϕ becomes,

$$S_{int}[\phi] = M^2_{pc} m^4 [(\partial^2\phi)^3 + (\partial^2\phi)^4].$$

Note that, the kinetic energy term for the scalar field was of the form, $M^2_{pc} m^4 (\partial_\phi)^2$, so the canonically re-normalised field must be, $\phi^c = m^2 M_{pc}.\phi$. Then in canonical re-normalisation the self interactions become,

$$S_{int} \to \frac{(\partial^2\phi^c)^3}{\Lambda_5^5} + \frac{(\partial^2\phi^c)^4}{\Lambda_4^8}, \tag{68}$$

where,

$$\Lambda_5 \equiv (m^4 M_{pc})^{1/5}$$

$$\Lambda_4 \equiv (m^3 M_{pc})^{1/4}$$

$$\Lambda_5 \ll \Lambda_4.$$

Hence, the self interactions are suppressed by some constants that have inverse mass dimension. As explained before these coupling constants give rise to non-renormalisable interactions[21].

In order to look at these interactions from an effective field theory view point, the range of validity for this theory is from energy E=0 to the energy scale of $\Lambda_5 \sim (10^{20}cm)^{-1}$. Therefore, the region of validity is too narrow which is a sign of a problematic theory. To improve this, non-linear terms corresponding to potential interactions can be added to S^{FP}_{mass}, which widens the energy scale to $\Lambda_3 = (m^2 M_{pc})^{1/3} \sim (1000^k m)^{-1}$[21].

6 Application of Effective Field Theory on Gravitational Waves

In this section, it is going to be shown how effective field theory ideas can be applied to classical gravity. Namely, to help to understand the emission of gravitational waves by astrophysical systems, such as two black holes in a binary system.

Consider two black holes with comparable masses orbiting each other as shown in figure 6. The virial theorem for such systems states that the kinetic

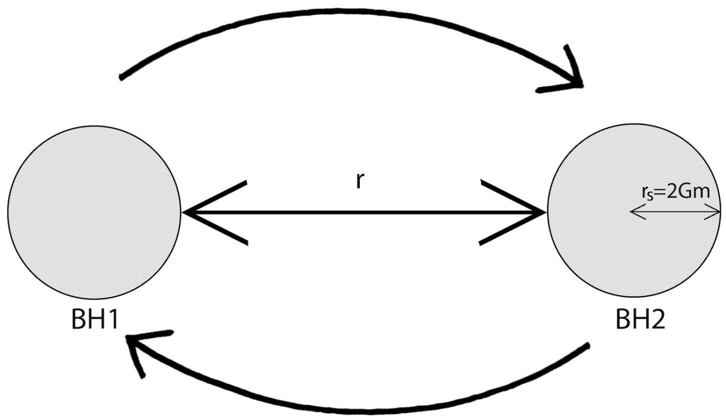

Figure 6: Binary system of two black holes with comparable masses orbiting each other. The distance between the two is r and their radius is $r_s = 2GM$.

energy must be of the same order as the potential energy. In this case it means $mv^2 \sim \frac{Gm}{r}$. Which leads to,

$$v^2 = \frac{G}{r} = \frac{r_s}{r},$$

where $r_s \ll r \quad \rightarrow \quad v \ll 1$. This means that the black holes are orbiting each other relatively slow, so they must be considered to be in a non-relativistic system.

Non-relativistic motion of these black holes imposes some simplifications on the analysis. The most important implication is that, since $\frac{r_s}{r}$ is also small, non-linearity in gravity is going to become negligible by this parameter. Physically it means that, as the two black holes are at a large distance from each other, non-linear solutions of Einstein's equation have no effect on their interactions. In this approximation the black holes can be considered as point particles which are coupled to linear gravity[7].

As these black holes orbit each other, they emit gravitational waves and so lose energy. When their energy reduces to a certain point, the black holes start to spiral into each other, when they start to mix, non-linearity becomes important. In the merging phase a larger vibrating black hole is formed from an asymmetric shape. These asymmetric configurations can be thought of as modes for this black hole. These modes vibrate and emit gravitational waves, so the black hole loses energy and starts to settle down. This phase is known as the ring down phase, where the black hole settles down with some small perturbations[7]. Hence, there are three phases where gravitational waves are emitted by such systems,

-Spiral phase, treated in perturbation theory.

-Merging phase, treated as a full non-linear process. (can be done with numerical relativity)

-Ring down, treated in perturbation theory of a spherical object with small vibrating modes.

let us focus on the spiral phase, as non-linearity can be ignored and black holes are treated as point particles. The frequency of rotation associated with this system and so the frequency of the emitted gravitational waves is of the order, $w \sim \frac{v}{r}$. Then, the wavelength of these gravitational waves is $\lambda \sim \frac{1}{w} \sim \frac{r}{v} \gg r$. Hence, there is a hierarchy of scales in this system, $r_s \ll r \ll \lambda$ [7].

The usual way of computing gravitational waves in such systems starts with moving all the non-linear terms of the gravitational filed to the right hand side of the Einstein's equation, as described in section 4,

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad \rightarrow \quad G^{(1)}_{\mu\nu} = 8\pi G \tau_{\mu\nu}.$$

Then this system is solved perturbatively. Using Newtonian dynamics, the zeroth order $\tau_{\mu\nu}$ is computed. Substituting this into the Einstein's equation the gravitational field at first order is found. Then, the first order solution can be plugged back into equation as first order $\tau_{\mu\nu}$ to solve for second order solution and so on[7]. This method is not practical, as it takes a long time to compute high enough orders to make practical predictions. The reason is that, with this theory the full structure of the black holes must be considered, which is not desirable as these are complicated objects in GR. Also, it is not clear how the small parameter $v \ll 1$ can be used in this way of computations[7][22].

In 2005, Goldberger and Rothstein proposed that, effective field theory must be used for such systems. They suggested that, to describe a system where there is a hierarchy of some scale, it is sensible to apply effective field theory. To describe the dynamics of the binary system of black holes, an EFT can be written where it treats the black holes as point particles and is valid between r_s and λ. Afterwards, as the wavelength of the gravitational waves is much larger than the size of the binary r, the binary system itself can be treated as a point particle coupled to the gravitational waves[7][22].

6.1 Path integral formulation for effective field theory

For instance take the action of standard model, $S_{SM}[\psi, w, z, h]$, where h stands for the Higgs field. Consider the correlation function for low energy particles ψ,

$$< \psi\psi\psi\psi >= \int D\psi Dw Dz Dh \quad \psi\psi\psi\psi \quad e^{iS_{SM}[\psi,w,z,h]}. \tag{69}$$

At this level, EFT dictates to do the path integral in two steps, doing the path integral over the heavy fields first as a functional of the light fields and then, do the path integral over the light fields,

$$< \psi\psi\psi\psi >= \int D\psi \quad \psi\psi\psi\psi \quad e^{iS_{eff}[\psi]}, \tag{70}$$

where,

$$e^{iS_{eff}[\psi]} \equiv \int Dw Dz Dh \quad e^{iS_{SM}[\psi,w,z,h]}. \tag{71}$$

This way of computation is impractical most of the times as this approach is only possible if the full structure of the theory is known. Often, the effective action must be guessed. To guess S_{eff}, the symmetries of the system must be considered, so effective action must have the correct degrees of freedom. Also, the transformation properties of the system under these symmetries must be studied. Then, the effective action is written with all the possible terms compatible with the symmetries, organised as a perturbative series with free coefficients. These coefficients can be found by studying the full theory or can be measured from experiments[7].

6.2 Black hole coupling to an external gravitational field

Consider the coupling of a black hole to some external gravitational field, as shown in figure 7. In our approximation, the black hole is replaced with a point particle. In order to study the finite size effects, higher order couplings must be included for the point particle. Higher order derivatives of the field are a measure of how different the gravitational field at point A is compared to the point B. The degrees of freedom for this system are the gravitational field $h_{\mu\nu}(x)$ and the world line of the point particle $x_a^\mu(\zeta)$, where $a = 1, ..., N$ labels the black holes in this system (N is the number of black holes)[7].

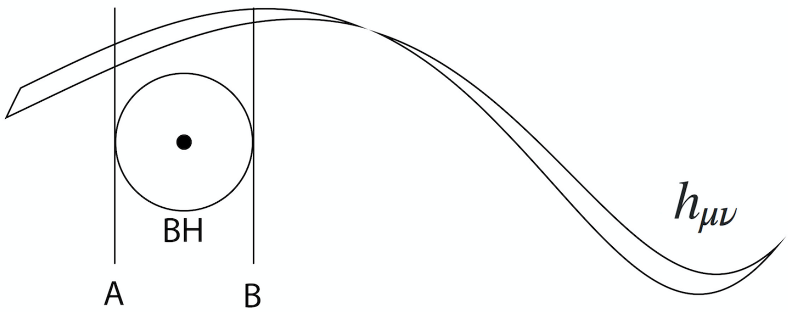

Figure 7: This figure shows a black hole in an external gravitational field. The dot in the middle represents the point particle approximation and A and B shows the finite size of this black hole. $h_{\mu\nu}$ is the external gravitational field.

The symmetries associated with this system are diffeomorphism invariance, $x^\mu \rightarrow x'^\mu(x)$ and reparameterization of the world line parameter, $\zeta \rightarrow \zeta'(\zeta)$. The effective action for this system is,

$$S_{eff}[x_a^\mu, g_{\mu\nu} \equiv \eta_{\mu\nu} + h_{\mu\nu}] = S_{EH}[g] + \sum_{a=1}^{N} S_{pp}[x_a^\mu, g], \tag{72}$$

where $S_{pp}[x_a^\mu, g]$ is the action for point particles and the structure for it has to be find out. This is the description of an effective field theory for black holes coupled to a large wavelength gravitational field. With the approximation that the black hole is treated as a point particle, as the size of it is negligible compared to the wavelength of the gravitational field[7][22].

The point particle action can be written as,

$$S_{pp} = -m_a \int d\tau_a, \tag{73}$$

where $d\tau_a$ is the infinitesimal proper time interval,

$$d\tau_a = \sqrt{g_{\mu\nu} x_a(\zeta) dx_a^\mu dx_a^\nu} = d\zeta \sqrt{g_{\mu\nu} x_a(\zeta) \dot{x}_a^\mu \dot{x}_a^\nu} \quad \text{where,} \quad \dot{x}_a^\mu \equiv \frac{dx_a^\mu}{d\zeta}. \tag{74}$$

In order to take into account the finite size effects of the black hole, higher order derivatives of the gravitational field must also be considered for the action. These terms must be compatible with the symmetries of the system, so only covariant objects such as curvature tensors can be added[7][18],

$$S_{pp} = -m_a \int d\tau_a \quad + C_R^a \int d\tau_a R(x_a) + C_v^a \int d\tau_a R_{\mu\nu}(x_a) \frac{dx_a^\mu}{d\tau_a} \frac{dx_a^\nu}{d\tau_a}. \tag{75}$$

In the above equation m_a, C_R^a and C_v^a are free coefficients and $R(x_a)$ is the Ricci scalar and $R_{\mu\nu}(x_a)$ is the Ricci tensor. By dimensional analysis it can be seen that , C_R^a and C_v^a are of the order mr_s^2, so indeed these additional terms are zero as r_s goes to zero. By a correct field redefinition of the metric these terms can be removed from the action, in QFT they are called redundant as they cannot effect any observable quantity[7][22].

In QFT there is a theorem stating that, if the zeroth order equation of motion of the metric sets, certain terms in the theory to zero, these terms can be removed by a field redefinition of the metric, and their effects can be included in the higher derivative terms[7][2]. In this case the zeroth order equation of motion of the metric is $G_{\mu\nu} = 0$ which means, $R_{\mu\nu} = R = 0$. Therefore, the additional terms added to the action can be removed and their effects moved to higher order terms. However, the Riemann tensor is not set to zero so, can be included in the action and encodes the finite size effects of the black hole[7],

$$S_{pp} = -m_a \int d\tau_a + \quad C_E^a \int d\tau_a E_{\mu\nu}(x^a) E^{\mu\nu}(x^a) + C_B^a \int d\tau_a B_{\mu\nu}(x^a) B^{\mu\nu}(x^a), \tag{76}$$

where,

$$E_{\mu\nu} \equiv R_{\mu\alpha\nu\beta} \frac{dx_a^\alpha}{d\tau_a} \frac{dx_a^\beta}{d\tau_a},$$

$$B_{\mu\nu} \equiv \epsilon_{\mu\alpha\beta\sigma} \frac{dx_a^\sigma}{d\tau_a} R^{\alpha\beta}{}_{\rho\nu} \frac{dx_a^\rho}{d\tau_a}.$$

$E_{\mu\nu}$ and $B_{\mu\nu}$ are as electric and magnetic components of the Riemann tensor.

The additional terms added to the point particle action are quadratic in $h_{\mu\nu}$. Meaning that, in the absence of an external field they do not source any new field however, in the presence of an external gravitational field these terms create corrections to it. They also include four derivatives of the metric therefore, the coefficients C_E^a and C_B^a must be of the order mr_s^4[22].

6.3 Optical theorem

Now consider the full action where the gravitational field has been integrated out, this creates the effective action for point particles,

$$\int DhS[x_a^\mu, g_{\mu\nu}] = e^{iS_{eff}[x_a^\mu]}. \tag{77}$$

The optical theorem at the level of the action states that, after integrating out a specific field from an action, the imaginary part of the obtained effective action is going to be proportional to the total rate of production of the field, integrated over the energy E and the solid angle Ω[23][7]. In this case, this statement can be translated as,

$$Im(S_{eff})\Big|_{\text{over a time T}} = T1/2 \int dEd\Omega \frac{d\Gamma}{d\Omega dE}, \tag{78}$$

where, $\frac{d\Gamma}{d\Omega dE}$ is the rate of production of gravitons. Hence, this procedure allows for the total power associated with the emission of gravitons to be calculated,

$$p = \int dEd\Omega E \frac{d\Gamma}{d\Omega dE}. \tag{79}$$

Note that, varying the real of the effective action with respect to the point particle's position, gives the effective equation of motion for the point particle. This is going to help to find corrections for the orbits of circulating particles[7].

6.4 Black hole coupling to a gravitational field and emission of gravitational waves

As explained before there are two hierarchies in this system, $r_s \ll r \ll \lambda$, in order to make use of this hierarchy, the integral of equation 77, can be done in two steps. First, the integral over the fields with wavelengths of order r is taken. Which results in an integral over the long wavelength fields. This process gives an EFT for the gravitational waves coupled to point particles.

The field $h_{\mu\nu}$ can be decomposed as,

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + H_{\mu\nu}, \tag{80}$$

where, $H_{\mu\nu}$ is known as the potential mode (off-shell) and is the part of the gravitational field which is keeping the system together. It's wavelength is of the order r (the size of the system), and it oscillates with the same frequency, $w \sim v/r$, with wave-number, $k \sim 1/r$. $\bar{h}_{\mu\nu}$ is known as the radiation mode (on-shell) and expands to infinity. This part of the gravitational field is responsible for the gravitational waves. Its typical frequency is of the order $w \sim v/r$, and the wave-number is of the same order $k \sim v/r$[7].

Now, with this decomposition, short-wavelength fields $H_{\mu\nu}$ can be integrated out, then an EFT is obtained for long-wavelength gravitational fields $\bar{h}_{\mu\nu}$ coupled to point sources.

In order to help with the power counting of the action, the powers of v must be explicit. v is the expansion parameter for this system and is of the order $v^2 \sim r_s/r$. To do this Goldberger and Rothstein propose to use the Fourier transform of the field $H_{\mu\nu}$[22],

$$H_{\mu\nu}(x) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}.\vec{x}} H_{\vec{k}\mu\nu}(x_0). \tag{81}$$

The reason is that, now all the derivatives acting on the potential or radiation modes are of the order v/r, $(\partial\bar{h}, \partial H) \sim v/r(\bar{h}, H)$.

The effective action for the radiation modes coupled to point particles can be written as,

$$e^{i(S^{(2)}_{eff}[\bar{h},x_a])} = \int DH_{\vec{k}}(t) e^{i(S_{EH} + \sum_a S_{pp}(\bar{h}+H,x_a))}. \tag{82}$$

The kinetic energy term for the potential mode $H_{\mu\nu}$ is included in the Einstein-Hilbert action,

$$S_{EH} \supset -1/2 \int dt \frac{d^3k}{(2\pi)^3} (k^2 H_{\vec{k},\mu\nu} H^{\mu\nu}_{-\vec{k}} - \frac{\vec{k}^2}{2} H_{\vec{k}} H_{-\vec{k}}). \tag{83}$$

The expansion parameter v is treated as a coupling constant, so any term that is suppressed by some power of v is counted as an interaction term. In a Feynman diagram they are associated with a vertex. Equation 83 is the leading order action for the field $H_{\mu\nu}$ which is at zeroth order in v. Hence, the propagator associated with the potential mode must be obtained from this term[7][22]. The Feynman propagator associated with the field $H_{\mu\nu}$ is,

$$< T(H^{\mu\nu}_{\vec{k}}(t) H^{\alpha\beta}_{\vec{q}}(0)) >= \frac{-i}{\vec{k}^2} (2\pi)^3 \delta^3(\vec{k} + \vec{q}) P^{\mu\nu,\alpha\beta} \delta(t). \tag{84}$$

Now, the goal is to expand both sides of equation 82 and match each term depending on how they scale with respect to the expansion parameter v. By considering the Feynman diagrams associated with each term and following the power counting rules, the right hand side perturbative expansion can be organised in powers of v, in a systematic way[7][24].

For example, the effective action for radiation modes and point particles at order v^2 is,

$$S^{(2)}_{eff}[\bar{h}, x_a] = 1/8 \sum_a m_a v_a^4 + \sum_{a \neq b} \frac{G m_a m_b}{|x_a - x_b|} \Big[3(v_a^2 + v_b^2) - 7(v_a.v_b) \tag{85}$$

$$- \frac{v_a(x_a - x_b) v_b (x_a - x_b)}{|x_a - x_b|} - \frac{G^2 m_a m_b (m_a + m_b)}{2|x_a - x_b|} \Big]. \tag{86}$$

Note that, in this equation the parts that include the radiation modes are ignored and the focus is on the positions of the point particles. Hence, this is the order v^2 correction of the Newtonian potential between particles[7].

To continue, the same procedure with the power counting and utilising the Feynman diagrams can be applied to the radiation modes, the effective action obtained at second order in v and first order in $\bar{h}$ is [24],

$$S^{(2)}_{eff}[\bar{h}, x_a] \supset - \frac{1}{2M_{pc}} \sum_{a \neq b} \Big[\bar{h}_{00} 1/2 m_a v_a^2 - \frac{G m_a m_b}{|x_a - x_b|} \Big] \tag{87}$$

$$- \frac{1}{2M_{pc}} \epsilon_{ijk} L_k \partial_j \bar{h}_{0i} + \frac{1}{2M_{pc}} \sum_a m_a x_a^i x_a^j R_{0i0j}. \tag{88}$$

The first term in expression 87 describes the interaction of the Newtonian potential with the Newtonian energy, this statement shows that in GR both energy and mass gravitate. The second term which is in expression 88 shows the coupling between the total angular momentum and the external gravitational field. The last term creates a Newtonian potential by considering the total energy of the system. This term is responsible for creating the gravitational waves. The reason the first two term cannot emit gravitational waves is that they are associated with the energy of the system and total angular momentum where both are conserved quantities. Whereas, the last term includes positions coupled to the Riemann tensor associated with the external radiation mode, this can create oscillations[7][22].

In order to pursue with our initial goal, so to calculate the total power emitted by the gravitational waves, the imaginary part of the above action must be considered. Then, the procedure described by the Optical Theorem can be followed which results in the radiation formula,

$$P = \frac{G}{5} < \dddot{Q}_{ij} \dddot{Q}_{ij} >_{\text{time average}} . \tag{89}$$

By computing arbitrary higher order terms in the expansion parameter v and utilising the power counting rules and studying Feynman diagrams, this computation can become more and more accurate. This is an area where there is a huge potential for further research, as this technique allows for more higher order terms to be taken into account compared to other methods of computations.

7 Summary

This paper showed some of the techniques that has been developed to describe gravity from a quantum field theory viewpoint. It represented, why gauge invariance is necessary for theories describing relativistic particles. Charge conservation and equivalence principle was shown as a result rather than a principle. Later, Einstein's tensor was derived by taking a purely QFT approach. It was discussed why there is a need for a modification of theory of gravity and massive gravity represented as one of the ways to modify gravity. Afterwards, a framework was illustrated to understand gravitational waves emission from an effective field theory viewpoint.

References

[1] Dr. Eugene A. Lim. General relativity. pages 5–10, 2018. Available at `http://www.damtp.cam.ac.uk//user/eal40/teach/GR/GR.pdf`.

[2] Micheal E. Peskin and Daniel V. Schroeder. An introduction to quantum field theory. page Preface, 1995.

[3] Steven Weinberg. The quantum theory of fields. pages 1–2, 1995.

[4] A Nicolis. Effective field theories for gravity. Available at `https://www.youtube.com/watch?v=xVYmBwh6zh8`.

[5] Steven Weinberg. Derivation of charge conservation and equality of gravitational and inertial mass. 1964.

[6] Steven Weinberg. Derivation of maxwell's and einstein's equations. page Introduction, 1965.

[7] A Nicolis. Effective field theories for gravity. Available at `https://www.youtube.com/watch?v=yWDOwW_pkGw`.

[8] Walter D. Goldberger and Ira Z. Rothstein. An effective field theory of gravity for extended objects. pages 1–3, 2005.

[9] Prof. Daniel Waldram. Particle symmetries. pages 34–56, 2018. Available at `https://www.imperial.ac.uk/media/imperial-college/research-centres-and-groups/theoretical-physics/msc/current/partsymm/waldram-notes/ps-notes-2.pdf`.

[10] Steven Weinberg. Derivation of gauge invarinace and the equivalence principle from lorentz invariance of the s-matrix. 1964.

[11] A Nicolis. Effective field theories for gravity. https://www.youtube.com/watch?v=wmckfDAukGo.

[12] Light compton scattering feynman diagram photon - light. https://www.kisspng.com/png-light-compton-scattering-feynman-diagram-photon-1429551/.

[13] Hua Xing Zhu. Soft factorization of gauge theory amplitudes beyond one loop. https://www.kisspng.com/png-light-compton-scattering-feynman-diagram-photon-1429551/.

[14] Prof. Jeffrey E. Mandula. Coleman-mandula theorem. http://www.scholarpedia.org/article/Coleman-Mandula-theorem.

[15] Mikio Nakahara. Geometry, topology and physics.

[16] David G. Boulware and S. Deser. Classical general relativity derived from quantum gravity.

[17] A Nicolis. Effective field theories for gravity. https://www.youtube.com/watch?v=ai90DPMwtCQ.

[18] Junpu Wang. The effective field theory approach to fluid dynamics, modified gravity theories, and cosmology.

[19] John F. Donoghue. Introduction to the effective field theory description of gravity.

[20] Sabine Hossenfelder. The paradigm shift you didn't notice. 2014. Available at `http://backreaction.blogspot.com/2014/11/the-paradigm-shift-you-didnt-notice.html`.

[21] A Nicolis. Effective field theories for gravity. 2017. Available at `https://www.youtube.com/watch?v=yWDOwW_pkGw&list=PLDxsZU4NC6Z4wQ_qaekc8u6LacOZbakDR&index=4`.

[22] Walter D. Goldberger and Ira Z. Rothstein. An effective field theory of gravity for extended objects. 2005.

[23] R. G. Newton. Optical theorem and beyond. 1976.

[24] John F. Donoghue and Tibor Torma. On the power counting of loop diagrams in general relativity. 2013.