EE2101 - Control Systems Assignment #1

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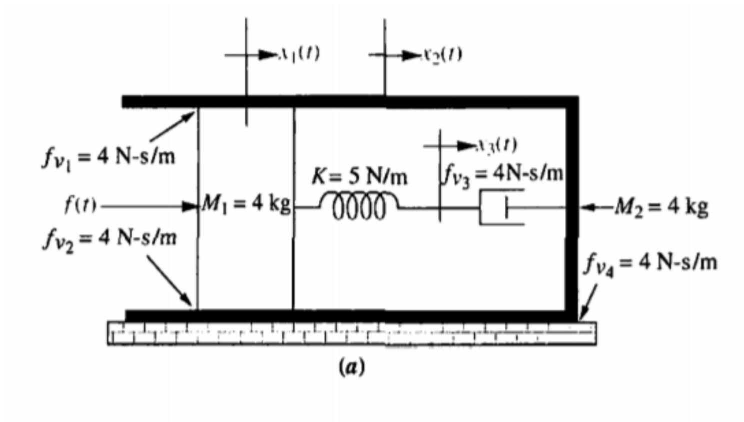
EE19BTECH11026

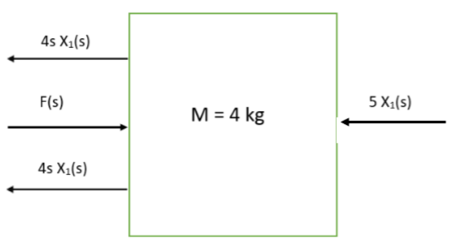
Problem 1

Q28) Find the Transfer function $\frac{X_3(s)}{F(s)}$ for each of the following:

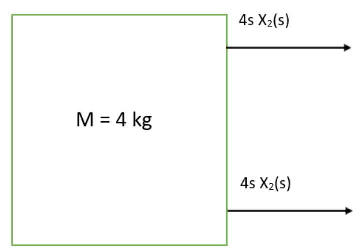


Part One

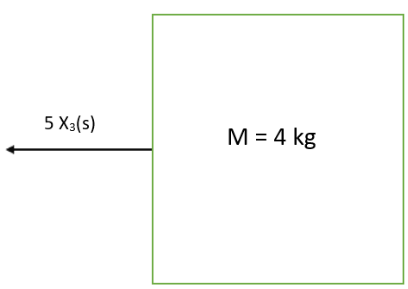
Solution Since there are three degrees of freedom, there must be 3 equations of motion Considering only M_1,



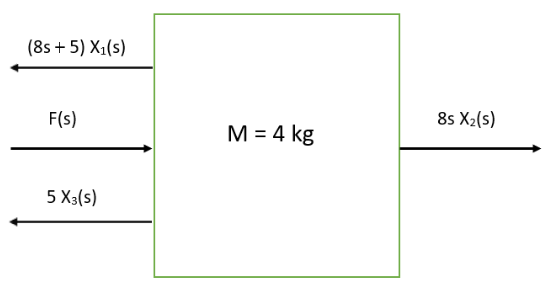
(a) Forces due to displacement $x_1(t)$



(b) Forces due to displacement $x_2(t)$



(c) Forces due to displacement $x_3(t)$

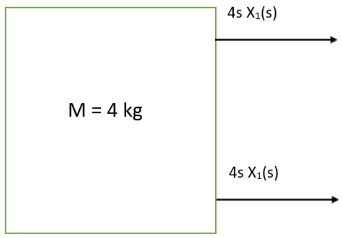


(d) Total Forces

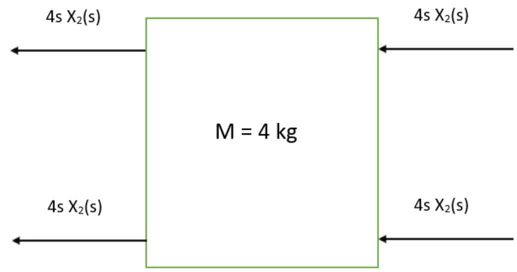
We get the equation:

$$F(s) = (4s^2 + 8s + 5)X_1(s) - 8sX_2(s) - 5X_3(s) \tag{1}$$

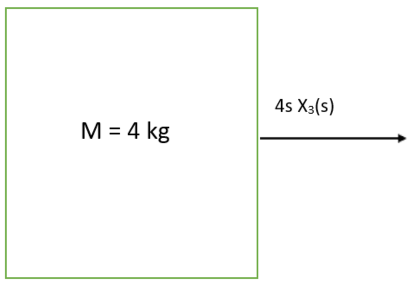
Similarly, doing this with M_2,



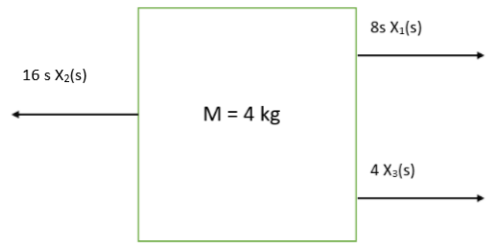
(a) Forces due to displacement $x_1(t)$



(b) Forces due to displacement $x_2(t)$



(c) Forces due to displacement $x_3(t)$



(d) Total Forces

$$8sX_1(s) - (4s^2 + 16s)X_2(s) + 4sX_3(s) = 0 \tag{2}$$

Now, for the 3rd equation, we consider the point of contact of the damper and spring. Since the mass of the point is 0, the sum of all forces at that point must be zero

$$5X_1(s) + 4sX_2(s) - (4s + 5)X_3(s) = 0 \tag{3}$$

Constructing a Matrix, we get

$$\begin{bmatrix} 4s^2 + 8s + 5 & -8s & -5 \\ 8s & -4s - 16s & 4s \\ 5 & +4s & -4s - 5 \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \\ X_3(s) \end{bmatrix} = \begin{bmatrix} F(s) \\ 0 \\ 0 \end{bmatrix}$$

$$\implies X_3 = \frac{\Delta_3}{\Delta}$$

$$\text{,where } \Delta = \begin{vmatrix} 4s^2 + 8s + 5 & -8s & -5 \\ 8s & -4s - 16s & 4s \\ 5 & +4s & -4s - 5 \end{vmatrix}$$

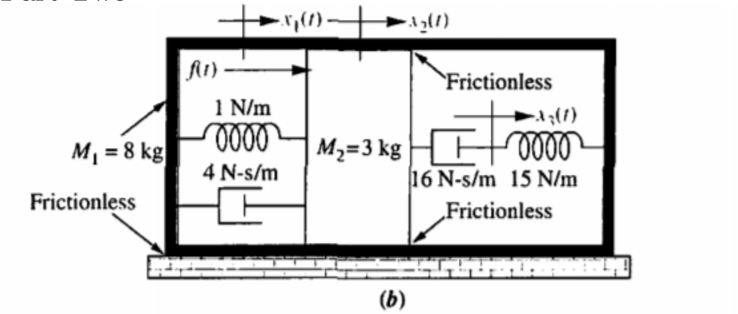
$$\Delta_3 = \begin{vmatrix} 4s^2 + 8s + 5 & -8s & F(s) \\ 8s & -4s - 16s & 0 \\ 5 & +4s & 0 \end{vmatrix}$$

Solving, we get

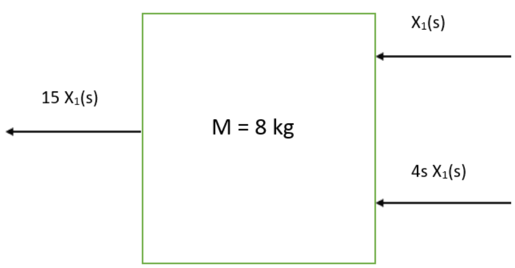
$$X_3(s) = \frac{F(s)(13s + 20)}{16s^4 + 100s^3 + 172s^2 + 60s} \tag{4}$$

$$\implies \frac{X_3(s)}{F(s)} = \frac{(13s + 20)}{16s^4 + 100s^3 + 172s^2 + 60s} \tag{5}$$

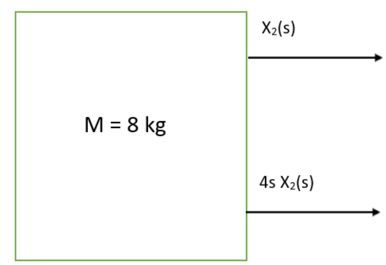
Part Two



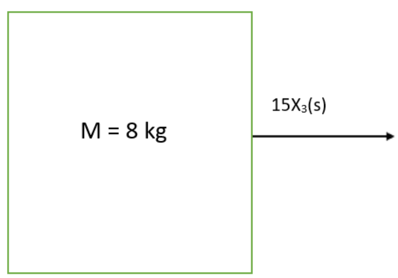
Solution Since there are three degrees of freedom, there must be 3 equations of motion Considering only M_1,



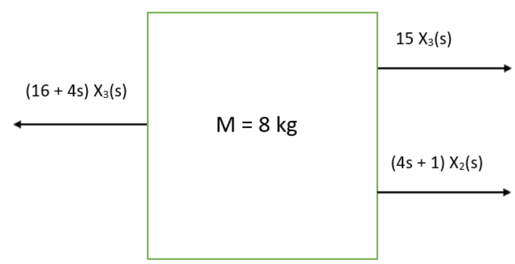
(a) Forces due to displacement $x_1(t)$



(b) Forces due to displacement $x_2(t)$



(c) Forces due to displacement $x_3(t)$

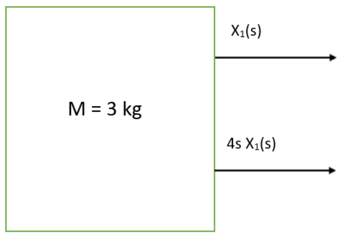


(d) Total Forces

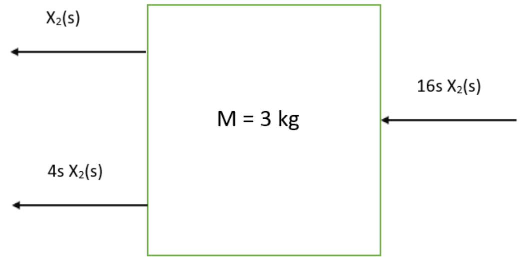
We get the equation:

$$(8s^2 + 4s + 16)X_1(s) - (4s + 1)X_2(s) - 15X_3(s) = 0 \tag{6}$$

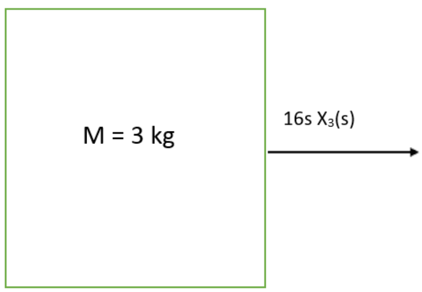
Similarly, doing this with M_2,



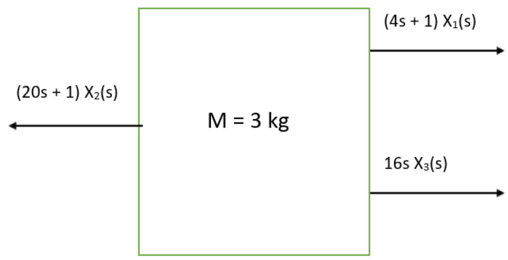
(a) Forces due to displacement $x_1(t)$



(b) Forces due to displacement $x_2(t)$



(c) Forces due to displacement $x_3(t)$



(d) Total Forces

$$-(4s+1)X_1(s) + (3s^2+20s+1)X_2(s) + 16sX_3(s) = F(s) \tag{7}$$

Now, for the 3rd equation, we consider the point of contact of the damper and spring. Since the mass of the point is 0, the sum of all forces at that point must be zero

$$15X_1(s) + 16sX_2(s) - (16s+15)X_3(s) = 0 \tag{8}$$

Constructing a Matrix, we get

$$\begin{bmatrix} 8s^2+4s+16 & -(4s+1) & -15 \\ -(4s+1) & +(3s^2+20s+1) & 16s \\ 15 & 16s & -(16s+15) \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \\ X_3(s) \end{bmatrix} = \begin{bmatrix} 0 \\ F(s) \\ 0 \end{bmatrix}$$

$$\implies X_3 = \frac{\Delta_3}{\Delta}$$

$$\text{,where } \Delta = \begin{vmatrix} 8s^2+4s+16 & -(4s+1) & -15 \\ -(4s+1) & +(3s^2+20s+1) & 16s \\ 15 & 16s & -(16s+15) \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} 8s^2+4s+16 & -(4s+1) & 0 \\ -(4s+1) & +(3s^2+20s+1) & F(s) \\ 15 & 16s & \end{vmatrix}$$

Solving, we get

$$X_3(s) = \frac{F(s)(128s^3+64s^2+316s+15)}{384s^5+1064s^4+3476s^3+165s^2} \tag{9}$$

$$\implies \frac{X_3(s)}{F(s)} = \frac{(128s^3+64s^2+316s+15)}{384s^5+1064s^4+3476s^3+165s^2} \tag{10}$$