Relativistic N-particle energy shift in finite volume:

Auxiliary

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1 Phaseshift Fits

The S-wave phase shift is determined by

$$\cot(\delta) = \frac{Z_{00}(1, q^2)}{\pi^{3/2} q} \qquad \text{with} \qquad q = \frac{Lk}{2\pi}. \tag{1}$$

Here, the variable k can be calculated from the correlator fit results for $E_1 \equiv M_\phi$ and E_2 according to

$$E_2(L) = 2\sqrt{k^2 + M_\phi(L)^2} \qquad \Leftrightarrow \qquad k = \sqrt{\left(\frac{E_2(L)}{2}\right)^2 - M_\phi(L)^2}. \tag{2}$$

The effective range expansion reads

$$k\, \cot(\delta) = -\frac{1}{a_0} + \frac{r_0\, k^2}{2} - P\, r_0^3\, k^4 + \mathcal{O}(k^6), \tag{3}$$

where a_0 denotes the S-wave scattering length, r_0 the S-wave effective range parameter and P the S-wave shape parameter. Considering this expansion, in the following four different models are fitted to the corresponding data provided by Eq. (1) and (2). Using $\cot(x) = \tan(x)^{-1}$ the different models read

(Ia)

$$\delta(k) = \arctan\left[\left(-\frac{1}{a_0\, k} + \frac{r_0\, k}{2}\right)^{-1}\right],$$

(Ib)

$$\Big[k\, \cot(\delta)\Big]\left(k^2\right) = -\frac{1}{a_0} + \frac{r_0\, k^2}{2},$$

(IIa)

$$\delta(k) = \arctan\left[\left(-\frac{1}{a_0\, k} + \frac{r_0\, k}{2} - P\, r_0^3\, k^3\right)^{-1}\right],$$

(IIb)

$$\Big[k\, \cot(\delta)\Big]\left(k^2\right) = -\frac{1}{a_0} + \frac{r_0\, k^2}{2} - P\, r_0^3\, \left(k^2\right)^2.$$

fit model	L	a_0	r_0	P	χ^2/ndof	p-value
	$[6, 24]$	0.407(17)	$-274(23)$	–	$14.63/17 = 0.86$	0.62
	$[7, 24]$	0.418(23)	$-300(42)$	–	$14.12/16 = 0.88$	0.59
	$[8, 24]$	0.505(36)	$-535(74)$	–	$3.44/15 = 0.23$	1.00
(Ia)	$[9, 24]$	0.507(41)	$-540(103)$	–	$3.44/14 = 0.25$	1.00
	$[10, 24]$	0.529(47)	$-614(132)$	–	$3.07/13 = 0.24$	1.00
	$[11, 24]$	0.528(49)	$-609(150)$	–	$3.07/12 = 0.26$	1.00
	$[12, 24]$	0.514(54)	$-545(216)$	–	$3.00/11 = 0.27$	0.99
	$[6, 24]$	0.419(16)	$-285(21)$	–	$28.20/17 = 1.66$	0.04
	$[7, 24]$	0.428(20)	$-309(36)$	–	$26.94/16 = 1.68$	0.04
	$[8, 24]$	0.504(38)	$-536(78)$	–	$4.95/15 = 0.33$	0.99
(Ib)	$[9, 24]$	0.507(45)	$-544(111)$	–	$4.94/14 = 0.35$	0.99
	$[10, 24]$	0.531(60)	$-627(158)$	–	$4.08/13 = 0.31$	0.99
	$[11, 24]$	0.531(66)	$-626(188)$	–	$4.08/12 = 0.34$	0.98
	$[12, 24]$	0.521(86)	$-581(303)$	–	$4.03/11 = 0.37$	0.97
	$[6, 24]$	0.483(36)	$-534(93)$	0.000049(19)	$10.13/16 = 0.63$	0.86
	$[7, 24]$	0.579(52)	$-890(138)$	0.0000329(92)	$4.09/15 = 0.27$	1.00
	$[8, 24]$	0.525(56)	$-633(249)$	0.00(0.12)	$3.37/14 = 0.24$	1.00
(IIa)	$[9, 24]$	0.542(64)	$-738(311)$	0.00(0.27)	$3.28/13 = 0.25$	1.00
	$[10, 24]$	0.497(66)	$-374(374)$	0.00(0.56)	$2.98/12 = 0.25$	1.00
	$[11, 24]$	0.373(16)	0(45)	0.00000(0.00012)	$9.95/11 = 0.90$	0.53
	$[12, 24]$	0.401(13)	0.0(8.6)	0.00000(0.00014)	$5.47/10 = 0.55$	0.86
	$[6, 24]$	0.474(33)	$-494(89)$	0.000052(35)	$19.96/16 = 1.25$	0.22
	$[7, 24]$	0.588(72)	$-918(173)$	0.000031(11)	$5.93/15 = 0.40$	0.98
	$[8, 24]$	0.531(78)	$-664(296)$	0.00(0.2)	$4.71/14 = 0.34$	0.99
(IIb)	$[9, 24]$	0.56(11)	$-820(407)$	0.00(0.3)	$4.43/13 = 0.34$	0.99
	$[10, 24]$	0.51(24)	$-462(521)$	0.00(0.26)	$4.03/12 = 0.34$	0.98
	$[11, 24]$	0.380(14)	0(34)	0.00000000(5.8e-07)	$21.35/11 = 1.94$	0.03
	$[12, 24]$	0.402(13)	0.0(1.7)	0.0000000049(0)	$9.91/10 = 0.99$	0.45

Table 1: Results from the different S-wave fits.

2 Mass Fit

fit intvl	M_∞	c_M	χ^2/ndof	p-value
$[6, 24]$	0.20441(15)	0.19079(84)	$134.17/17 = 7.89$	0.00
$[7, 24]$	0.20393(16)	0.2007(13)	$42.87/16 = 2.68$	2.93e−04
$[8, 24]$	**0.20368(17)**	0.2107(22)	$13.13/15 = 0.88$	0.59
$[9, 24]$	0.20352(18)	0.2201(48)	$8.20/14 = 0.59$	0.88
$[10, 24]$	0.20345(19)	0.2251(61)	$6.52/13 = 0.50$	0.93
$[11, 24]$	0.20341(20)	0.2284(92)	$6.28/12 = 0.52$	0.90
$[12, 24]$	0.20342(23)	0.227(15)	$6.27/11 = 0.57$	0.85
$[13, 24]$	0.20355(25)	0.208(22)	$4.73/10 = 0.47$	0.91
$[14, 24]$	0.20365(29)	0.188(34)	$4.16/9 = 0.46$	0.90
$[15, 24]$	0.20358(32)	0.205(47)	$3.91/8 = 0.49$	0.87
$[16, 24]$	0.20362(37)	0.195(71)	$3.87/7 = 0.55$	0.79

Table 2: Results obtained from the single particle mass fits.

3 Energy Shift Fits to Order L^{-5}

3.1 Fits without Exponentially Suppressed Corrections

shift	fit intvl	a_0	χ^2/ndof	p-value
	[8, 24]	0.2734(52)	156.67/16 = 9.79	0.00
	[9, 24]	0.3126(68)	72.16/15 = 4.81	1.84e−09
	[10, 24]	0.3358(81)	44.18/14 = 3.16	5.53e−05
	[11, 24]	0.3525(92)	30.17/13 = 2.32	4.45e−03
	[12, 24]	0.384(12)	15.29/12 = 1.27	0.23
	[13, 24]	0.400(14)	11.21/11 = 1.02	0.43
	[14, 24]	0.421(18)	7.42/10 = 0.74	0.69
ΔE_2	[15, 24]	0.426(20)	7.09/9 = 0.79	0.63
	[16, 24]	0.438(24)	6.34/8 = 0.79	0.61
	[17, 24]	0.430(30)	6.13/7 = 0.88	0.52
	[18, 24]	0.433(37)	6.11/6 = 1.02	0.41
	[19, 24]	0.458(45)	5.30/5 = 1.06	0.38
	[20, 24]	0.515(59)	3.26/4 = 0.81	0.52
	[21, 24]	0.503(84)	3.22/3 = 1.07	0.36
	[22, 24]	0.44(12)	2.49/2 = 1.25	0.29
	[8, 24]	0.2815(38)	208.62/16 = 13.04	0.00
	[9, 24]	0.3304(55)	71.73/15 = 4.78	2.19e−09
	[10, 24]	0.3424(63)	55.59/14 = 3.97	6.86e−07
	[11, 24]	0.3682(88)	36.96/13 = 2.84	4.20e−04
	[12, 24]	0.3755(92)	31.83/12 = 2.65	1.47e−03
	[13, 24]	0.400(12)	19.52/11 = 1.77	0.05
	[14, 24]	0.441(16)	4.66/10 = 0.47	0.91
ΔE_3	[15, 24]	0.448(18)	3.96/9 = 0.44	0.91
	[16, 24]	0.450(20)	3.88/8 = 0.49	0.87
	[17, 24]	0.455(27)	3.83/7 = 0.55	0.80
	[18, 24]	0.476(33)	2.62/6 = 0.44	0.85
	[19, 24]	0.488(40)	2.40/5 = 0.48	0.79
	[20, 24]	0.486(48)	2.40/4 = 0.60	0.66
	[21, 24]	0.521(73)	2.04/3 = 0.68	0.56
	[22, 24]	0.477(98)	1.56/2 = 0.78	0.46

Table 3: Results from the bare ΔE_2- and ΔE_3-fit to order L^{-5} **without** exponentially suppressed corrections.

shift	fit intvl	a_0	χ^2/ndof	p-value
	[8, 24]	0.2788(44)	191.19/16 = 11.95	0.00
	[9, 24]	0.3070(53)	118.50/15 = 7.90	0.00
	[10, 24]	0.3464(70)	45.07/14 = 3.22	3.97e−05
	[11, 24]	0.3571(81)	38.60/13 = 2.97	2.32e−04
	[12, 24]	0.379(10)	24.62/12 = 2.05	0.02
	[13, 24]	0.388(11)	21.52/11 = 1.96	0.03
	[14, 24]	0.425(15)	8.18/10 = 0.82	0.61
ΔE_4	[15, 24]	0.429(18)	7.92/9 = 0.88	0.54
	[16, 24]	0.428(19)	7.86/8 = 0.98	0.45
	[17, 24]	0.423(24)	7.77/7 = 1.11	0.35
	[18, 24]	0.456(29)	4.56/6 = 0.76	0.60
	[19, 24]	0.494(40)	2.72/5 = 0.54	0.74
	[20, 24]	0.511(50)	2.43/4 = 0.61	0.66
	[21, 24]	0.586(74)	0.70/3 = 0.23	0.87
	[22, 24]	0.69(15)	0.10/2 = 0.05	0.95
	[8, 24]	0.2625(43)	178.37/16 = 11.15	0.00
	[9, 24]	0.2711(46)	157.02/15 = 10.47	0.00
	[10, 24]	0.3445(79)	39.37/14 = 2.81	3.20e−04
	[11, 24]	0.3501(86)	36.57/13 = 2.81	4.83e−04
	[12, 24]	0.376(11)	21.24/12 = 1.77	0.05
	[13, 24]	0.380(11)	20.47/11 = 1.86	0.04
	[14, 24]	0.410(14)	8.77/10 = 0.88	0.55
ΔE_5	[15, 24]	0.415(19)	8.60/9 = 0.96	0.48
	[16, 24]	0.422(21)	8.19/8 = 1.02	0.42
	[17, 24]	0.412(26)	7.84/7 = 1.12	0.35
	[18, 24]	0.451(33)	4.18/6 = 0.70	0.65
	[19, 24]	0.489(42)	2.54/5 = 0.51	0.77
	[20, 24]	0.509(60)	2.29/4 = 0.57	0.68
	[21, 24]	0.593(90)	0.87/3 = 0.29	0.83
	[22, 24]	0.68(14)	0.25/2 = 0.12	0.88

Table 4: Results from the bare ΔE_4- and ΔE_5-fit to order L^{-5} **without** exponentially suppressed corrections.

3.2 Fits with Exponentially Suppressed Corrections

shift	fit intvl	a_0	χ^2/ndof	p-value
	[8, 24]	0.2911(58)	129.61/16 = 8.10	0.00
	[9, 24]	0.3283(74)	59.35/15 = 3.96	3.27e−07
	[10, 24]	0.3495(87)	37.23/14 = 2.66	6.82e−04
	[11, 24]	0.3650(98)	25.77/13 = 1.98	0.02
	[12, 24]	0.394(13)	13.51/12 = 1.13	0.33
	[13, 24]	0.408(15)	10.28/11 = 0.93	0.51
	[14, 24]	0.428(18)	7.17/10 = 0.72	0.71
ΔE_2	[15, 24]	0.432(21)	6.94/9 = 0.77	0.64
	[16, 24]	0.443(25)	6.32/8 = 0.79	0.61
	[17, 24]	0.434(31)	6.06/7 = 0.87	0.53
	[18, 24]	0.436(37)	6.05/6 = 1.01	0.42
	[19, 24]	0.462(46)	5.27/5 = 1.05	0.38
	[20, 24]	0.518(60)	3.27/4 = 0.82	0.51
	[21, 24]	0.505(84)	3.22/3 = 1.07	0.36
	[22, 24]	0.44(12)	2.49/2 = 1.25	0.29
	[8, 24]	0.3009(42)	167.19/16 = 10.45	0.00
	[9, 24]	0.3468(60)	57.03/15 = 3.80	8.12e−07
	[10, 24]	0.3572(68)	45.50/14 = 3.25	3.39e−05
	[11, 24]	0.3796(92)	32.14/13 = 2.47	2.29e−03
	[12, 24]	0.3863(97)	27.99/12 = 2.33	5.56e−03
	[13, 24]	0.409(12)	17.38/11 = 1.58	0.10
	[14, 24]	0.449(16)	4.29/10 = 0.43	0.93
ΔE_3	[15, 24]	0.454(18)	3.77/9 = 0.42	0.93
	[16, 24]	0.456(21)	3.74/8 = 0.47	0.88
	[17, 24]	0.460(28)	3.71/7 = 0.53	0.81
	[18, 24]	0.480(33)	2.59/6 = 0.43	0.86
	[19, 24]	0.491(40)	2.40/5 = 0.48	0.79
	[20, 24]	0.489(48)	2.39/4 = 0.60	0.66
	[21, 24]	0.523(73)	2.04/3 = 0.68	0.56
	[22, 24]	0.479(99)	1.56/2 = 0.78	0.46

Table 5: Results from the bare ΔE_2- and ΔE_3-fit to order L^{-5} **with** exponentially suppressed corrections.

shift	fit intvl	a_0	χ^2/ndof	p-value
	[8, 24]	0.2963(49)	158.02/16 = 9.88	0.00
	[9, 24]	0.3228(58)	98.63/15 = 6.58	2.38e−14
	[10, 24]	0.3598(74)	37.57/14 = 2.68	6.05e−04
	[11, 24]	0.3687(86)	33.28/13 = 2.56	1.54e−03
	[12, 24]	0.388(11)	22.01/12 = 1.83	0.04
	[13, 24]	0.396(12)	19.58/11 = 1.78	0.05
	[14, 24]	0.432(15)	7.90/10 = 0.79	0.64
ΔE_4	[15, 24]	0.435(18)	7.77/9 = 0.86	0.56
	[16, 24]	0.433(19)	7.66/8 = 0.96	0.47
	[17, 24]	0.428(24)	7.54/7 = 1.08	0.38
	[18, 24]	0.460(30)	4.44/6 = 0.74	0.62
	[19, 24]	0.498(40)	2.67/5 = 0.53	0.75
	[20, 24]	0.514(51)	2.39/4 = 0.60	0.66
	[21, 24]	0.589(75)	0.68/3 = 0.23	0.88
	[22, 24]	0.69(15)	0.10/2 = 0.05	0.95
	[8, 24]	0.2773(47)	154.39/16 = 9.65	0.00
	[9, 24]	0.2855(50)	136.75/15 = 9.12	0.00
	[10, 24]	0.3566(84)	33.86/14 = 2.42	2.16e−03
	[11, 24]	0.3612(91)	32.05/13 = 2.47	2.37e−03
	[12, 24]	0.385(11)	19.53/12 = 1.63	0.08
	[13, 24]	0.388(12)	18.99/11 = 1.73	0.06
	[14, 24]	0.418(15)	8.46/10 = 0.85	0.58
ΔE_5	[15, 24]	0.421(20)	8.38/9 = 0.93	0.50
	[16, 24]	0.427(22)	8.05/8 = 1.01	0.43
	[17, 24]	0.416(27)	7.65/7 = 1.09	0.36
	[18, 24]	0.455(33)	4.09/6 = 0.68	0.67
	[19, 24]	0.492(43)	2.50/5 = 0.50	0.78
	[20, 24]	0.513(61)	2.26/4 = 0.56	0.69
	[21, 24]	0.596(91)	0.86/3 = 0.29	0.84
	[22, 24]	0.68(14)	0.25/2 = 0.12	0.88

Table 6: Results from the bare ΔE_4- and ΔE_5-fit to order L^{-5} **with** exponentially suppressed corrections.

4 Bare Energy Shift Fits to Order L^{-6}

$$\begin{aligned}\Delta E_N =& \binom{N}{2}\frac{4\pi a_0}{ML^3}\left\{1-\left(\frac{a_0}{\pi L}\right)\mathcal{I}+\left(\frac{a_0}{\pi L}\right)^2\left[\mathcal{I}^2+(2N-5)\,\mathcal{J}\right]\right.\\ &-\left(\frac{a_0}{\pi L}\right)^3\left[(2N-7)\,\mathcal{I}\mathcal{J}+\mathcal{I}^3+(5N^2-41N+63)\,\mathcal{K}+8(N-2)\,(2\mathcal{Q}+\mathcal{R})\right]\\ &\left.+(4N-6)\,\frac{\pi\,a_0^2\,r_0}{L^3}+(4N-9)\,\frac{\pi a_0}{M^2L^3}-\frac{24Ma_0}{\sqrt{2\pi ML}}\,\exp\left(-ML\right)\right\}\\ &+\binom{N}{3}\left\{\frac{32\pi a_0^4}{ML^6}\,\left(3\sqrt{3}-4\pi\right)\,\left(\ln(ML)^2-\Gamma'(1)-\ln(4\pi)\right)-\frac{\bar{\mathcal{T}}}{6L^6}\right\}\end{aligned} \tag{4}$$

4.1 Non-Relativistic Fits without Exponentially Suppressed Corrections (rel. c. ×; exp. c. ×)

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3216(64)	−204(4)	–	79.12/17 = 4.65	5.49e−10
	[7, 24]	0.3580(95)	−229(6)	–	52.98/16 = 3.31	7.60e−06
	[8, 24]	**0.407(12)**	**−285(10)**	–	17.00/15 = 1.13	0.32
	[9, 24]	0.430(16)	−319(17)	–	12.31/14 = 0.88	0.58
	[10, 24]	0.461(21)	−367(23)	–	7.64/13 = 0.59	0.87
ΔE_2	[11, 24]	0.474(26)	−390(32)	–	6.73/12 = 0.56	0.87
	[12, 24]	0.482(34)	−410(75)	–	6.62/11 = 0.60	0.83
	[13, 24]	0.493(43)	−448(124)	–	6.45/10 = 0.64	0.78
	[14, 24]	0.482(60)	−400(439)	–	6.39/9 = 0.71	0.70
	[15, 24]	0.491(78)	−447(833)	–	6.36/8 = 0.80	0.61
	[16, 24]	0.46(10)	−227(6083)	–	6.18/7 = 0.88	0.52
	[6, 24]	0.3416(56)	−294(10)	−190877(10203)	95.81/18 = 5.32	1.29e−12
	[7, 24]	0.3751(76)	−282(10)	−228485(14076)	53.81/17 = 3.17	1.06e−05
	[8, 24]	0.4079(96)	−284(10)	−279971(20376)	23.49/16 = 1.47	0.10
	[9, 24]	0.412(12)	−283(10)	−282893(21419)	23.08/15 = 1.54	0.08
	[10, 24]	0.414(12)	−283(10)	−281992(21797)	22.67/14 = 1.62	0.07
ΔE_3	[11, 24]	0.414(12)	−283(10)	−282447(25286)	22.67/13 = 1.74	0.05
	[12, 24]	0.413(12)	−284(10)	−290703(27372)	22.06/12 = 1.84	0.04
	[13, 24]	0.411(12)	−284(10)	−345174(39018)	18.27/11 = 1.66	0.08
	[14, 24]	0.408(12)	−285(10)	−550405(71796)	6.72/10 = 0.67	0.75
	[15, 24]	0.408(12)	−285(10)	−618563(92961)	5.39/9 = 0.60	0.80
	[16, 24]	0.408(12)	−285(10)	−655672(120399)	5.15/8 = 0.64	0.74

Table 7: Results obtained from the **non-relativistic** bare ΔE_2- and ΔE_3-fit to order L^{-6} **without** exponentially suppressed corrections. In case of the ΔE_3-fit the a_0 and r_0 bootstrap samples obtained from the ΔE_2-fit within the interval $[8, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3406(60)	−297(10)	−174360(10144)	99.45/18 = 5.53	2.79e−13
	[7, 24]	0.3743(79)	−286(10)	−214137(13980)	57.98/17 = 3.41	2.26e−06
	[8, 24]	0.4037(96)	−282(10)	−255362(18407)	28.48/16 = 1.78	0.03
	[9, 24]	0.418(11)	−282(10)	−275474(21611)	21.60/15 = 1.44	0.12
	[10, 24]	0.415(12)	−282(10)	−274059(21252)	20.91/14 = 1.49	0.10
ΔE_4	[11, 24]	0.415(12)	−283(10)	−271798(21638)	20.39/13 = 1.57	0.09
	[12, 24]	0.413(12)	−283(10)	−281866(23119)	19.08/12 = 1.59	0.09
	[13, 24]	0.412(12)	−283(10)	−286356(25249)	18.89/11 = 1.72	0.06
	[14, 24]	0.408(12)	−285(10)	−377632(38257)	8.99/10 = 0.90	0.53
	[15, 24]	0.408(12)	−284(10)	−394578(51054)	8.74/9 = 0.97	0.46
	[16, 24]	0.408(12)	−284(10)	−390816(58799)	8.72/8 = 1.09	0.37
	[6, 24]	0.3108(52)	−305(10)	−130868(7181)	153.92/18 = 8.55	0.00
	[7, 24]	0.3506(71)	−289(10)	−173283(10735)	88.85/17 = 5.23	9.89e−12
	[8, 24]	0.4021(99)	−282(10)	−243520(17993)	33.06/16 = 2.07	0.01
	[9, 24]	0.417(11)	−283(10)	−271020(21287)	19.32/15 = 1.29	0.20
	[10, 24]	0.414(12)	−284(10)	−268722(20999)	18.64/14 = 1.33	0.18
ΔE_5	[11, 24]	0.415(12)	−284(10)	−267146(21229)	17.50/13 = 1.35	0.18
	[12, 24]	0.412(12)	−284(10)	−273415(21576)	16.19/12 = 1.35	0.18
	[13, 24]	0.412(12)	−284(10)	−273479(22036)	16.19/11 = 1.47	0.13
	[14, 24]	0.408(12)	−285(10)	−312905(26290)	9.09/10 = 0.91	0.52
	[15, 24]	0.408(12)	−285(10)	−318660(39175)	9.05/9 = 1.01	0.43
	[16, 24]	0.408(12)	−285(10)	−334283(46931)	8.69/8 = 1.09	0.37

Table 8: Results obtained from the **non-relativistic** bare ΔE_4- and ΔE_5-fit to order L^{-6} **without** exponentially suppressed corrections. Here, the a_0 and r_0 bootstrap samples obtained from the previous ΔE_2-fit within the interval $[8, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3216(64)	−204(4)	–	79.12/17 = 4.65	5.49e−10
	[7, 24]	0.3580(95)	−229(6)	–	52.98/16 = 3.31	7.60e−06
	[8, 24]	0.407(12)	−285(10)	–	17.00/15 = 1.13	0.32
	[9, 24]	**0.430(16)**	**−319(17)**	–	12.31/14 = 0.88	0.58
	[10, 24]	0.461(21)	−367(23)	–	7.64/13 = 0.59	0.87
ΔE_2	[11, 24]	0.474(26)	−390(32)	–	6.73/12 = 0.56	0.87
	[12, 24]	0.482(34)	−410(75)	–	6.62/11 = 0.60	0.83
	[13, 24]	0.493(43)	−448(124)	–	6.45/10 = 0.64	0.78
	[14, 24]	0.482(60)	−400(439)	–	6.39/9 = 0.71	0.70
	[15, 24]	0.491(78)	−447(833)	–	6.36/8 = 0.80	0.61
	[16, 24]	0.46(10)	−227(6083)	–	6.18/7 = 0.88	0.52
	[6, 24]	0.3406(57)	−313(16)	−204871(14605)	95.32/18 = 5.30	1.58e−12
	[7, 24]	0.3750(77)	−304(16)	−251921(20400)	54.39/17 = 3.20	8.58e−06
	[8, 24]	0.411(10)	−307(16)	−319505(29396)	22.63/16 = 1.41	0.12
	[9, 24]	0.426(14)	−314(16)	−354647(40323)	20.08/15 = 1.34	0.17
	[10, 24]	0.435(15)	−316(16)	−365906(43587)	18.09/14 = 1.29	0.20
ΔE_3	[11, 24]	0.440(16)	−320(17)	−367421(45888)	17.00/13 = 1.31	0.20
	[12, 24]	0.439(16)	−320(17)	−370407(46082)	16.81/12 = 1.40	0.16
	[13, 24]	0.436(16)	−319(17)	−406394(50686)	14.39/11 = 1.31	0.21
	[14, 24]	0.431(16)	−319(17)	−573465(74415)	5.37/10 = 0.54	0.86
	[15, 24]	0.431(16)	−318(17)	−623543(92977)	4.57/9 = 0.51	0.87
	[16, 24]	0.430(16)	−318(17)	−646686(118769)	4.47/8 = 0.56	0.81

Table 9: Results obtained from the **non-relativistic** bare ΔE_2- and ΔE_3-fit to order L^{-6} **without** exponentially suppressed corrections. In case of the ΔE_3-fit the a_0 and r_0 bootstrap samples obtained from the ΔE_2-fit within the interval $[9, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3395(61)	−297(16)	−172246(12688)	93.63/18 = 5.20	3.20e−12
	[7, 24]	0.3727(81)	−293(16)	−217909(18118)	57.14/17 = 3.36	3.09e−06
	[8, 24]	0.406(10)	−301(16)	−281178(26384)	28.71/16 = 1.79	0.03
	[9, 24]	0.432(13)	−315(16)	−348072(38974)	18.12/15 = 1.21	0.26
	[10, 24]	0.433(15)	−315(16)	−350498(41243)	18.08/14 = 1.29	0.20
ΔE_4	[11, 24]	0.440(15)	−320(17)	−362566(44203)	15.56/13 = 1.20	0.27
	[12, 24]	0.438(16)	−320(17)	−363139(43560)	15.34/12 = 1.28	0.22
	[13, 24]	0.437(16)	−320(17)	−363584(43959)	15.33/11 = 1.39	0.17
	[14, 24]	0.431(16)	−319(17)	−429817(49004)	8.14/10 = 0.81	0.62
	[15, 24]	0.431(16)	−318(17)	−435289(57780)	8.11/9 = 0.90	0.52
	[16, 24]	0.432(16)	−318(17)	−426839(64176)	8.01/8 = 1.00	0.43
	[6, 24]	0.3102(52)	−290(16)	−123179(8493)	135.43/18 = 7.52	0.00
	[7, 24]	0.3475(71)	−279(16)	−161810(12593)	79.74/17 = 4.69	4.27e−10
	[8, 24]	0.405(11)	−300(16)	−268580(27134)	33.50/16 = 2.09	0.01
	[9, 24]	0.431(13)	−316(17)	−341000(38374)	16.75/15 = 1.12	0.33
	[10, 24]	0.432(15)	−316(17)	−343608(40787)	16.71/14 = 1.19	0.27
ΔE_5	[11, 24]	0.437(15)	−320(17)	−354263(42981)	13.93/13 = 1.07	0.38
	[12, 24]	0.435(16)	−320(17)	−353067(42383)	13.76/12 = 1.15	0.32
	[13, 24]	0.436(16)	−320(17)	−352956(42551)	13.67/11 = 1.24	0.25
	[14, 24]	0.430(16)	−318(17)	−376327(42096)	8.40/10 = 0.84	0.59
	[15, 24]	0.431(16)	−318(17)	−373128(49734)	8.39/9 = 0.93	0.50
	[16, 24]	0.430(16)	−318(17)	−383985(55655)	8.20/8 = 1.03	0.41

Table 10: Results obtained from the **non-relativistic** bare ΔE_4- and ΔE_5-fit to order L^{-6} **without** exponentially suppressed corrections. Here, the a_0 and r_0 bootstrap samples obtained from the previous ΔE_2-fit within the interval $[9, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3216(64)	−204(4)	–	79.12/17 = 4.65	5.49e−10
	[7, 24]	0.3580(95)	−229(6)	–	52.98/16 = 3.31	7.60e−06
	[8, 24]	0.407(12)	−285(10)	–	17.00/15 = 1.13	0.32
	[9, 24]	0.430(16)	−319(17)	–	12.31/14 = 0.88	0.58
	[10, 24]	**0.461(21)**	**−367(23)**	–	7.64/13 = 0.59	0.87
ΔE_2	[11, 24]	0.474(26)	−390(32)	–	6.73/12 = 0.56	0.87
	[12, 24]	0.482(34)	−410(75)	–	6.62/11 = 0.60	0.83
	[13, 24]	0.493(43)	−448(124)	–	6.45/10 = 0.64	0.78
	[14, 24]	0.482(60)	−400(439)	–	6.39/9 = 0.71	0.70
	[15, 24]	0.491(78)	−447(833)	–	6.36/8 = 0.80	0.61
	[16, 24]	0.46(10)	−227(6083)	–	6.18/7 = 0.88	0.52
	[6, 24]	0.3403(57)	−357(20)	−240712(17783)	94.42/18 = 5.25	2.30e−12
	[7, 24]	0.3744(77)	−342(20)	−292538(24417)	54.57/17 = 3.21	8.04e−06
	[8, 24]	0.410(10)	−335(20)	−359427(33768)	23.64/16 = 1.48	0.10
	[9, 24]	0.428(14)	−340(20)	−399399(45144)	20.64/15 = 1.38	0.15
	[10, 24]	0.445(17)	−351(21)	−447974(58319)	17.10/14 = 1.22	0.25
ΔE_3	[11, 24]	0.474(20)	−373(23)	−532540(84004)	10.64/13 = 0.82	0.64
	[12, 24]	0.475(21)	−373(23)	−532930(84450)	10.62/12 = 0.89	0.56
	[13, 24]	0.471(21)	−372(23)	−542406(82349)	9.79/11 = 0.89	0.55
	[14, 24]	0.462(21)	−368(23)	−649609(89316)	4.03/10 = 0.40	0.95
	[15, 24]	0.462(21)	−367(23)	−675748(102020)	3.76/9 = 0.42	0.93
	[16, 24]	0.462(21)	−368(23)	−680174(123446)	3.75/8 = 0.47	0.88

Table 11: Results obtained from the **non-relativistic** bare ΔE_2- and ΔE_3-fit to order L^{-6} **without** exponentially suppressed corrections. In case of the ΔE_3-fit the a_0 and r_0 bootstrap samples obtained from the ΔE_2-fit within the interval $[10, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3379(61)	−351(20)	−206495(16050)	95.20/18 = 5.29	1.66e−12
	[7, 24]	0.3715(82)	−344(20)	−260647(22655)	59.95/17 = 3.53	1.07e−06
	[8, 24]	0.406(10)	−340(20)	−327077(31423)	31.38/16 = 1.96	0.01
	[9, 24]	0.436(13)	−345(20)	−400171(43887)	18.96/15 = 1.26	0.22
	[10, 24]	0.444(17)	−349(21)	−425060(56013)	18.31/14 = 1.31	0.19
ΔE_4	[11, 24]	0.465(19)	−370(23)	−503380(74544)	12.07/13 = 0.93	0.52
	[12, 24]	0.468(20)	−371(23)	−508942(77066)	11.89/12 = 0.99	0.45
	[13, 24]	0.470(21)	−371(23)	−510122(78189)	11.68/11 = 1.06	0.39
	[14, 24]	0.461(21)	−367(23)	−536097(74180)	7.64/10 = 0.76	0.66
	[15, 24]	0.462(21)	−368(23)	−529910(78358)	7.57/9 = 0.84	0.58
	[16, 24]	0.463(21)	−367(23)	−516320(82405)	7.27/8 = 0.91	0.51
	[6, 24]	0.3066(53)	−345(21)	−144496(12038)	135.02/18 = 7.50	0.00
	[7, 24]	0.3444(75)	−341(21)	−199053(18379)	84.94/17 = 5.00	5.02e−11
	[8, 24]	0.407(12)	−346(21)	−322472(35259)	37.03/16 = 2.31	2.08e−03
	[9, 24]	0.436(13)	−349(21)	−398655(46294)	17.98/15 = 1.20	0.26
	[10, 24]	0.446(17)	−355(22)	−429180(59930)	17.15/14 = 1.23	0.25
ΔE_5	[11, 24]	0.460(18)	−368(23)	−479646(70534)	12.03/13 = 0.93	0.52
	[12, 24]	0.463(20)	−369(23)	−488859(74467)	11.81/12 = 0.98	0.46
	[13, 24]	0.467(21)	−369(23)	−493377(76231)	11.33/11 = 1.03	0.42
	[14, 24]	0.459(21)	−366(23)	−495369(71988)	8.06/10 = 0.81	0.62
	[15, 24]	0.460(21)	−367(23)	−484434(75430)	7.74/9 = 0.86	0.56
	[16, 24]	0.460(21)	−367(23)	−488475(78241)	7.70/8 = 0.96	0.46

Table 12: Results obtained from the **non-relativistic** bare ΔE_4- and ΔE_5-fit to order L^{-6} **without** exponentially suppressed corrections. Here, the a_0 and r_0 bootstrap samples obtained from the previous ΔE_2-fit within the interval $[10, 24]$ serve as priors.

4.2 Non-Relativistic Fits with Exponentially Suppressed Corrections (rel. c. ×; exp. c. ✓)

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3332(68)	−144(3)	–	67.89/17 = 3.99	4.96e−08
	[7, 24]	0.3654(98)	−168(5)	–	47.60/16 = 2.97	5.49e−05
	[8, 24]	**0.411(13)**	**−223(9)**	–	15.57/15 = 1.04	0.41
	[9, 24]	0.432(16)	−256(17)	–	11.69/14 = 0.83	0.63
	[10, 24]	0.461(21)	−305(24)	–	7.50/13 = 0.58	0.87
ΔE_2	[11, 24]	0.474(26)	−328(34)	–	6.71/12 = 0.56	0.88
	[12, 24]	0.480(34)	−347(79)	–	6.62/11 = 0.60	0.83
	[13, 24]	0.491(42)	−386(131)	–	6.45/10 = 0.65	0.78
	[14, 24]	0.480(59)	−335(451)	–	6.39/9 = 0.71	0.70
	[15, 24]	0.488(77)	−384(850)	–	6.36/8 = 0.80	0.61
	[16, 24]	0.46(10)	−156(6126)	–	6.19/7 = 0.88	0.52
	[6, 24]	0.3531(60)	−223(9)	−162697(10010)	79.80/18 = 4.43	9.30e−10
	[7, 24]	0.3828(79)	−217(9)	−189218(13240)	47.07/17 = 2.77	1.19e−04
	[8, 24]	0.4126(98)	−222(9)	−228435(18586)	21.76/16 = 1.36	0.15
	[9, 24]	0.416(12)	−222(9)	−229885(19225)	21.50/15 = 1.43	0.12
	[10, 24]	0.418(12)	−222(9)	−228319(19703)	21.05/14 = 1.50	0.10
ΔE_3	[11, 24]	0.419(12)	−222(9)	−226558(23721)	21.03/13 = 1.62	0.07
	[12, 24]	0.418(13)	−222(9)	−234163(26068)	20.55/12 = 1.71	0.06
	[13, 24]	0.415(13)	−223(9)	−286015(38262)	17.15/11 = 1.56	0.10
	[14, 24]	0.412(13)	−223(9)	−484985(71481)	6.33/10 = 0.63	0.79
	[15, 24]	0.412(13)	−223(9)	−548752(92544)	5.15/9 = 0.57	0.82
	[16, 24]	0.412(13)	−223(9)	−582235(120132)	4.96/8 = 0.62	0.76

Table 13: Results obtained from the **non-relativistic** bare ΔE_2- and ΔE_3-fit to order L^{-6} **with** exponentially suppressed corrections. In case of the ΔE_3-fit the a_0 and r_0 bootstrap samples obtained from the ΔE_2-fit within the interval $[8, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3510(63)	−226(9)	−145791(9651)	84.90/18 = 4.72	1.17e−10
	[7, 24]	0.3811(82)	−220(9)	−174100(12789)	51.80/17 = 3.05	2.21e−05
	[8, 24]	0.4082(98)	−220(9)	−204848(16509)	26.52/16 = 1.66	0.05
	[9, 24]	0.422(11)	−221(9)	−220069(19188)	20.39/15 = 1.36	0.16
	[10, 24]	0.418(12)	−221(9)	−219590(18796)	19.77/14 = 1.41	0.14
ΔE_4	[11, 24]	0.420(12)	−222(9)	−216756(19277)	19.06/13 = 1.47	0.12
	[12, 24]	0.417(12)	−222(9)	−226172(21177)	18.03/12 = 1.50	0.11
	[13, 24]	0.417(12)	−222(9)	−229960(23748)	17.91/11 = 1.63	0.08
	[14, 24]	0.413(12)	−223(9)	−318549(37504)	8.73/10 = 0.87	0.56
	[15, 24]	0.413(12)	−223(9)	−332677(50457)	8.56/9 = 0.95	0.48
	[16, 24]	0.413(13)	−223(9)	−327498(58405)	8.53/8 = 1.07	0.38
	[6, 24]	0.3211(55)	−229(9)	−108658(6797)	135.07/18 = 7.50	0.00
	[7, 24]	0.3576(74)	−220(9)	−139511(9740)	80.78/17 = 4.75	2.79e−10
	[8, 24]	0.406(10)	−219(9)	−193476(15960)	30.95/16 = 1.93	0.01
	[9, 24]	0.421(11)	−222(9)	−215524(18845)	18.42/15 = 1.23	0.24
	[10, 24]	0.417(12)	−222(9)	−214315(18472)	17.83/14 = 1.27	0.21
ΔE_5	[11, 24]	0.419(12)	−223(9)	−212373(18739)	16.51/13 = 1.27	0.22
	[12, 24]	0.416(12)	−223(9)	−218524(19409)	15.50/12 = 1.29	0.22
	[13, 24]	0.416(12)	−223(9)	−218254(20061)	15.50/11 = 1.41	0.16
	[14, 24]	0.413(12)	−223(9)	−257011(24851)	8.87/10 = 0.89	0.54
	[15, 24]	0.413(12)	−223(9)	−260276(38275)	8.86/9 = 0.98	0.45
	[16, 24]	0.412(13)	−223(9)	−274800(46319)	8.55/8 = 1.07	0.38

Table 14: Results obtained from the **non-relativistic** bare ΔE_4- and ΔE_5-fit to order L^{-6} **with** exponentially suppressed corrections. Here, the a_0 and r_0 bootstrap samples obtained from the previous ΔE_2-fit within the interval $[8, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3332(68)	−144(3)	–	67.89/17 = 3.99	4.96e−08
	[7, 24]	0.3654(98)	−168(5)	–	47.60/16 = 2.97	5.49e−05
	[8, 24]	0.411(13)	−223(9)	–	15.57/15 = 1.04	0.41
	[9, 24]	**0.432(16)**	**−256(17)**	–	11.69/14 = 0.83	0.63
	[10, 24]	0.461(21)	−305(24)	–	7.50/13 = 0.58	0.87
ΔE_2	[11, 24]	0.474(26)	−328(34)	–	6.71/12 = 0.56	0.88
	[12, 24]	0.480(34)	−347(79)	–	6.62/11 = 0.60	0.83
	[13, 24]	0.491(42)	−386(131)	–	6.45/10 = 0.65	0.78
	[14, 24]	0.480(59)	−335(451)	–	6.39/9 = 0.71	0.70
	[15, 24]	0.488(77)	−384(850)	–	6.36/8 = 0.80	0.61
	[16, 24]	0.46(10)	−156(6126)	–	6.19/7 = 0.88	0.52
	[6, 24]	0.3523(60)	−239(15)	−175774(15149)	79.61/18 = 4.42	1.00e−09
	[7, 24]	0.3828(80)	−236(15)	−210664(20219)	47.56/17 = 2.80	1.00e−04
	[8, 24]	0.415(10)	−242(15)	−263800(28118)	21.01/16 = 1.31	0.18
	[9, 24]	0.428(14)	−251(16)	−289575(36963)	19.18/15 = 1.28	0.21
	[10, 24]	0.436(15)	−253(17)	−297661(39778)	17.36/14 = 1.24	0.24
ΔE_3	[11, 24]	0.442(16)	−258(17)	−297604(42158)	16.15/13 = 1.24	0.24
	[12, 24]	0.441(16)	−258(17)	−300545(42524)	15.99/12 = 1.33	0.19
	[13, 24]	0.438(16)	−257(17)	−335753(47843)	13.77/11 = 1.25	0.25
	[14, 24]	0.433(16)	−256(17)	−499910(72898)	5.17/10 = 0.52	0.88
	[15, 24]	0.433(16)	−255(17)	−547289(91690)	4.45/9 = 0.49	0.88
	[16, 24]	0.433(16)	−255(17)	−568097(117834)	4.37/8 = 0.55	0.82

Table 15: Results obtained from the **non-relativistic** bare ΔE_2- and ΔE_3-fit to order L^{-6} **with** exponentially suppressed corrections. In case of the ΔE_3-fit the a_0 and r_0 bootstrap samples obtained from the ΔE_2-fit within the interval $[9, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3498(64)	−223(15)	−142299(12837)	80.14/18 = 4.45	8.11e−10
	[7, 24]	0.3795(84)	−225(15)	−176390(17505)	51.04/17 = 3.00	2.91e−05
	[8, 24]	0.410(10)	−236(16)	−226596(24728)	26.74/16 = 1.67	0.04
	[9, 24]	0.434(13)	−252(16)	−280922(35690)	17.63/15 = 1.18	0.28
	[10, 24]	0.435(15)	−252(17)	−282105(37265)	17.61/14 = 1.26	0.23
ΔE_4	[11, 24]	0.441(15)	−258(17)	−292488(40101)	15.00/13 = 1.15	0.31
	[12, 24]	0.440(16)	−257(17)	−293457(39593)	14.83/12 = 1.24	0.25
	[13, 24]	0.440(16)	−257(17)	−293591(40242)	14.83/11 = 1.35	0.19
	[14, 24]	0.434(16)	−255(17)	−359433(46218)	8.04/10 = 0.80	0.62
	[15, 24]	0.433(16)	−255(17)	−363268(55570)	8.02/9 = 0.89	0.53
	[16, 24]	0.434(16)	−255(17)	−353838(62340)	7.91/8 = 0.99	0.44
	[6, 24]	0.3203(55)	−213(15)	−98774(8627)	118.75/18 = 6.60	1.11e−16
	[7, 24]	0.3544(74)	−207(15)	−126723(12202)	72.59/17 = 4.27	7.69e−09
	[8, 24]	0.409(11)	−235(16)	−213982(25081)	31.33/16 = 1.96	0.01
	[9, 24]	0.433(13)	−253(17)	−274134(35092)	16.41/15 = 1.09	0.36
	[10, 24]	0.434(15)	−253(17)	−275553(36798)	16.40/14 = 1.17	0.29
ΔE_5	[11, 24]	0.439(15)	−258(17)	−284761(38891)	13.57/13 = 1.04	0.40
	[12, 24]	0.437(16)	−257(17)	−284265(38322)	13.45/12 = 1.12	0.34
	[13, 24]	0.438(16)	−257(17)	−283820(38548)	13.33/11 = 1.21	0.27
	[14, 24]	0.432(16)	−254(17)	−308153(38474)	8.32/10 = 0.83	0.60
	[15, 24]	0.433(16)	−255(17)	−303422(46884)	8.29/9 = 0.92	0.51
	[16, 24]	0.432(16)	−255(17)	−313601(53267)	8.13/8 = 1.02	0.42

Table 16: Results obtained from the **non-relativistic** bare ΔE_4- and ΔE_5-fit to order L^{-6} **with** exponentially suppressed corrections. Here, the a_0 and r_0 bootstrap samples obtained from the previous ΔE_2-fit within the interval $[9, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3332(68)	−144(3)	–	67.89/17 = 3.99	4.96e−08
	[7, 24]	0.3654(98)	−168(5)	–	47.60/16 = 2.97	5.49e−05
	[8, 24]	0.411(13)	−223(9)	–	15.57/15 = 1.04	0.41
	[9, 24]	0.432(16)	−256(17)	–	11.69/14 = 0.83	0.63
	[10, 24]	**0.461(21)**	**−305(24)**	–	7.50/13 = 0.58	0.87
ΔE_2	[11, 24]	0.474(26)	−328(34)	–	6.71/12 = 0.56	0.88
	[12, 24]	0.480(34)	−347(79)	–	6.62/11 = 0.60	0.83
	[13, 24]	0.491(42)	−386(131)	–	6.45/10 = 0.65	0.78
	[14, 24]	0.480(59)	−335(451)	–	6.39/9 = 0.71	0.70
	[15, 24]	0.488(77)	−384(850)	–	6.36/8 = 0.80	0.61
	[16, 24]	0.46(10)	−156(6126)	–	6.19/7 = 0.88	0.52
	[6, 24]	0.3519(60)	−278(20)	−210647(18886)	79.05/18 = 4.39	1.25e−09
	[7, 24]	0.3822(80)	−269(20)	−249056(24789)	47.80/17 = 2.81	9.23e−05
	[8, 24]	0.415(10)	−268(20)	−300847(33144)	21.90/16 = 1.37	0.15
	[9, 24]	0.430(14)	−274(20)	−330188(42293)	19.71/15 = 1.31	0.18
	[10, 24]	0.446(17)	−286(22)	−370086(53777)	16.58/14 = 1.18	0.28
ΔE_3	[11, 24]	0.474(20)	−312(24)	−442540(77546)	10.34/13 = 0.80	0.67
	[12, 24]	0.475(21)	−313(24)	−442768(77966)	10.33/12 = 0.86	0.59
	[13, 24]	0.471(21)	−310(24)	−453179(76207)	9.55/11 = 0.87	0.57
	[14, 24]	0.463(21)	−306(24)	−561291(84805)	3.96/10 = 0.40	0.95
	[15, 24]	0.462(21)	−305(24)	−586184(98219)	3.72/9 = 0.41	0.93
	[16, 24]	0.462(21)	−305(24)	−589326(120505)	3.71/8 = 0.46	0.88

Table 17: Results obtained from the **non-relativistic** bare ΔE_2- and ΔE_3-fit to order L^{-6} **with** exponentially suppressed corrections. In case of the ΔE_3-fit the a_0 and r_0 bootstrap samples obtained from the ΔE_2-fit within the interval $[10, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3484(64)	−273(20)	−176777(16598)	81.88/18 = 4.55	4.01e−10
	[7, 24]	0.3785(85)	−270(20)	−217938(22378)	53.62/17 = 3.15	1.14e−05
	[8, 24]	0.410(11)	−272(20)	−270054(30144)	29.12/16 = 1.82	0.02
	[9, 24]	0.438(13)	−280(20)	−328753(40999)	18.40/15 = 1.23	0.24
	[10, 24]	0.445(17)	−285(22)	−346908(51020)	17.95/14 = 1.28	0.21
ΔE_4	[11, 24]	0.465(19)	−308(24)	−415730(68236)	11.93/13 = 0.92	0.53
	[12, 24]	0.468(20)	−309(24)	−420323(70454)	11.75/12 = 0.98	0.47
	[13, 24]	0.470(21)	−310(24)	−420714(71591)	11.52/11 = 1.05	0.40
	[14, 24]	0.461(21)	−304(24)	−449250(68151)	7.64/10 = 0.76	0.66
	[15, 24]	0.462(21)	−305(24)	−442134(72936)	7.55/9 = 0.84	0.58
	[16, 24]	0.463(21)	−305(24)	−427745(77444)	7.23/8 = 0.90	0.51
	[6, 24]	0.3169(56)	−263(21)	−121765(12501)	119.26/18 = 6.63	1.11e−16
	[7, 24]	0.3516(77)	−265(21)	−164277(18164)	77.46/17 = 4.56	1.08e−09
	[8, 24]	0.411(12)	−277(21)	−264948(33368)	34.49/16 = 2.16	4.66e−03
	[9, 24]	0.438(13)	−284(21)	−326923(43197)	17.53/15 = 1.17	0.29
	[10, 24]	0.446(17)	−290(23)	−349872(54547)	16.92/14 = 1.21	0.26
ΔE_5	[11, 24]	0.459(18)	−305(23)	−394213(64431)	11.97/13 = 0.92	0.53
	[12, 24]	0.463(20)	−307(24)	−402136(67825)	11.73/12 = 0.98	0.47
	[13, 24]	0.467(20)	−307(24)	−405545(69505)	11.24/11 = 1.02	0.42
	[14, 24]	0.459(21)	−303(24)	−410176(65382)	8.07/10 = 0.81	0.62
	[15, 24]	0.461(21)	−304(24)	−398108(69372)	7.71/9 = 0.86	0.56
	[16, 24]	0.460(21)	−304(24)	−401879(72627)	7.68/8 = 0.96	0.47

Table 18: Results obtained from the **non-relativistic** bare ΔE_4- and ΔE_5-fit to order L^{-6} **with** exponentially suppressed corrections. Here, the a_0 and r_0 bootstrap samples obtained from the previous ΔE_2-fit within the interval $[10, 24]$ serve as priors.

4.3 Relativistic Fits without Exponentially Suppressed Corrections (rel. c. ✓; exp. c. ×)

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3216(64)	−166(3)	–	79.12/17 = 4.65	5.49e−10
	[7, 24]	0.3580(95)	−195(6)	–	52.98/16 = 3.31	7.60e−06
	[8, 24]	**0.407(12)**	**−256(10)**	–	17.00/15 = 1.13	0.32
	[9, 24]	0.430(16)	−291(17)	–	12.31/14 = 0.88	0.58
	[10, 24]	0.461(21)	−341(24)	–	7.64/13 = 0.59	0.87
ΔE_2	[11, 24]	0.474(26)	−365(33)	–	6.73/12 = 0.56	0.87
	[12, 24]	0.482(34)	−385(76)	–	6.62/11 = 0.60	0.83
	[13, 24]	0.493(43)	−424(126)	–	6.45/10 = 0.64	0.78
	[14, 24]	0.482(60)	−375(441)	–	6.39/9 = 0.71	0.70
	[15, 24]	0.491(78)	−423(836)	–	6.36/8 = 0.80	0.61
	[16, 24]	0.46(10)	−201(6088)	–	6.18/7 = 0.88	0.52
	[6, 24]	0.3416(56)	−260(10)	−132748(8976)	95.81/18 = 5.32	1.29e−12
	[7, 24]	0.3751(76)	−251(10)	−157680(12197)	53.81/17 = 3.17	1.06e−05
	[8, 24]	0.4079(96)	−254(10)	−196009(17503)	23.49/16 = 1.47	0.10
	[9, 24]	0.412(12)	−254(10)	−197260(18125)	23.08/15 = 1.54	0.08
	[10, 24]	0.414(12)	−254(10)	−195403(18654)	22.67/14 = 1.62	0.07
ΔE_3	[11, 24]	0.414(12)	−254(10)	−195883(23049)	22.67/13 = 1.74	0.05
	[12, 24]	0.413(12)	−254(10)	−204590(25589)	22.06/12 = 1.84	0.04
	[13, 24]	0.411(12)	−255(10)	−259939(38155)	18.27/11 = 1.66	0.08
	[14, 24]	0.408(12)	−256(10)	−466492(71683)	6.72/10 = 0.67	0.75
	[15, 24]	0.408(12)	−255(10)	−534520(92804)	5.39/9 = 0.60	0.80
	[16, 24]	0.408(12)	−255(10)	−571768(120461)	5.15/8 = 0.64	0.74

Table 19: Results obtained from the **relativistic** bare ΔE_2- and ΔE_3-fit to order L^{-6} **without** exponentially suppressed corrections. In case of the ΔE_3-fit the a_0 and r_0 bootstrap samples obtained from the ΔE_2-fit within the interval $[8, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3406(60)	−263(10)	−116442(8599)	99.45/18 = 5.53	2.79e−13
	[7, 24]	0.3743(79)	−254(10)	−143610(11724)	57.98/17 = 3.41	2.26e−06
	[8, 24]	0.4037(96)	−253(10)	−173118(15414)	28.48/16 = 1.78	0.03
	[9, 24]	0.418(11)	−253(10)	−187307(17963)	21.60/15 = 1.44	0.12
	[10, 24]	0.415(12)	−253(10)	−187341(17553)	20.91/14 = 1.49	0.10
ΔE_4	[11, 24]	0.415(12)	−254(10)	−184706(18068)	20.39/13 = 1.57	0.09
	[12, 24]	0.413(12)	−254(10)	−195727(20274)	19.08/12 = 1.59	0.09
	[13, 24]	0.412(12)	−254(10)	−200611(23120)	18.89/11 = 1.72	0.06
	[14, 24]	0.408(12)	−255(10)	−293464(37332)	8.99/10 = 0.90	0.53
	[15, 24]	0.408(12)	−255(10)	−310433(50399)	8.74/9 = 0.97	0.46
	[16, 24]	0.408(12)	−255(10)	−306618(58437)	8.72/8 = 1.09	0.37
	[6, 24]	0.3108(52)	−268(10)	−83175(5956)	153.92/18 = 8.55	0.00
	[7, 24]	0.3506(71)	−255(10)	−111715(8813)	88.85/17 = 5.23	9.89e−12
	[8, 24]	0.4021(99)	−252(10)	−161937(14794)	33.06/16 = 2.07	0.01
	[9, 24]	0.417(11)	−254(10)	−183149(17620)	19.32/15 = 1.29	0.20
	[10, 24]	0.414(12)	−254(10)	−182433(17193)	18.64/14 = 1.33	0.18
ΔE_5	[11, 24]	0.415(12)	−255(10)	−180396(17459)	17.50/13 = 1.35	0.18
	[12, 24]	0.412(12)	−255(10)	−187955(18356)	16.19/12 = 1.35	0.18
	[13, 24]	0.412(12)	−255(10)	−188032(19129)	16.19/11 = 1.47	0.13
	[14, 24]	0.408(12)	−255(10)	−228718(24271)	9.09/10 = 0.91	0.52
	[15, 24]	0.408(12)	−255(10)	−234508(37990)	9.05/9 = 1.01	0.43
	[16, 24]	0.408(12)	−256(10)	−250368(46174)	8.69/8 = 1.09	0.37

Table 20: Results obtained from the **relativistic** bare ΔE_4- and ΔE_5-fit to order L^{-6} **without** exponentially suppressed corrections. Here, the a_0 and r_0 bootstrap samples obtained from the previous ΔE_2-fit within the interval $[8, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3216(64)	−166(3)	–	79.12/17 = 4.65	5.49e−10
	[7, 24]	0.3580(95)	−195(6)	–	52.98/16 = 3.31	7.60e−06
	[8, 24]	0.407(12)	−256(10)	–	17.00/15 = 1.13	0.32
	[9, 24]	**0.430(16)**	**−291(17)**	–	12.31/14 = 0.88	0.58
	[10, 24]	0.461(21)	−341(24)	–	7.64/13 = 0.59	0.87
ΔE_2	[11, 24]	0.474(26)	−365(33)	–	6.73/12 = 0.56	0.87
	[12, 24]	0.482(34)	−385(76)	–	6.62/11 = 0.60	0.83
	[13, 24]	0.493(43)	−424(126)	–	6.45/10 = 0.64	0.78
	[14, 24]	0.482(60)	−375(441)	–	6.39/9 = 0.71	0.70
	[15, 24]	0.491(78)	−423(836)	–	6.36/8 = 0.80	0.61
	[16, 24]	0.46(10)	−201(6088)	–	6.18/7 = 0.88	0.52
	[6, 24]	0.3406(57)	−279(16)	−147549(13669)	95.32/18 = 5.30	1.58e−12
	[7, 24]	0.3750(77)	−273(16)	−181518(18822)	54.39/17 = 3.20	8.58e−06
	[8, 24]	0.411(10)	−277(16)	−234325(26802)	22.63/16 = 1.41	0.12
	[9, 24]	0.426(14)	−286(17)	−262938(35766)	20.08/15 = 1.34	0.17
	[10, 24]	0.435(15)	−288(17)	−270584(38578)	18.09/14 = 1.29	0.20
ΔE_3	[11, 24]	0.440(16)	−292(17)	−269739(40736)	17.00/13 = 1.31	0.20
	[12, 24]	0.439(16)	−293(17)	−273149(41183)	16.81/12 = 1.40	0.16
	[13, 24]	0.436(16)	−292(17)	−310500(46915)	14.39/11 = 1.31	0.21
	[14, 24]	0.431(16)	−291(17)	−479805(72661)	5.37/10 = 0.54	0.86
	[15, 24]	0.431(16)	−290(17)	−529933(91628)	4.57/9 = 0.51	0.87
	[16, 24]	0.430(16)	−290(17)	−553173(117915)	4.47/8 = 0.56	0.81

Table 21: Results obtained from the **relativistic** bare ΔE_2- and ΔE_3-fit to order L^{-6} **without** exponentially suppressed corrections. In case of the ΔE_3-fit the a_0 and r_0 bootstrap samples obtained from the ΔE_2-fit within the interval $[9, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3395(61)	−263(16)	−115141(11606)	93.63/18 = 5.20	3.20e−12
	[7, 24]	0.3727(81)	−261(16)	−148317(16266)	57.14/17 = 3.36	3.09e−06
	[8, 24]	0.406(10)	−271(16)	−198111(23469)	28.71/16 = 1.79	0.03
	[9, 24]	0.432(13)	−287(16)	−253866(34474)	18.12/15 = 1.21	0.26
	[10, 24]	0.433(15)	−287(17)	−255727(36048)	18.08/14 = 1.29	0.20
ΔE_4	[11, 24]	0.440(15)	−293(17)	−265106(38697)	15.56/13 = 1.20	0.27
	[12, 24]	0.438(16)	−292(17)	−266494(38180)	15.34/12 = 1.28	0.22
	[13, 24]	0.437(16)	−292(17)	−267034(38909)	15.33/11 = 1.39	0.17
	[14, 24]	0.431(16)	−291(17)	−335881(45391)	8.14/10 = 0.81	0.62
	[15, 24]	0.431(16)	−290(17)	−341427(54994)	8.11/9 = 0.90	0.52
	[16, 24]	0.432(16)	−290(17)	−332821(61905)	8.01/8 = 1.00	0.43
	[6, 24]	0.3102(52)	−255(16)	−76066(7771)	135.43/18 = 7.52	0.00
	[7, 24]	0.3475(71)	−245(16)	−101723(11298)	79.74/17 = 4.69	4.27e−10
	[8, 24]	0.405(11)	−270(16)	−185932(23730)	33.50/16 = 2.09	0.01
	[9, 24]	0.431(13)	−288(17)	−247376(33891)	16.75/15 = 1.12	0.33
	[10, 24]	0.432(15)	−288(17)	−249380(35557)	16.71/14 = 1.19	0.27
ΔE_5	[11, 24]	0.437(15)	−293(17)	−257903(37546)	13.93/13 = 1.07	0.38
	[12, 24]	0.435(16)	−292(17)	−257592(36889)	13.76/12 = 1.15	0.32
	[13, 24]	0.436(16)	−292(17)	−257067(37096)	13.67/11 = 1.24	0.25
	[14, 24]	0.430(16)	−290(17)	−282872(37250)	8.40/10 = 0.84	0.59
	[15, 24]	0.431(16)	−290(17)	−279611(45994)	8.39/9 = 0.93	0.50
	[16, 24]	0.430(16)	−290(17)	−290663(52568)	8.20/8 = 1.03	0.41

Table 22: Results obtained from the **relativistic** bare ΔE_4- and ΔE_5-fit to order L^{-6} **without** exponentially suppressed corrections. Here, the a_0 and r_0 bootstrap samples obtained from the previous ΔE_2-fit within the interval $[9, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3216(64)	−166(3)	–	79.12/17 = 4.65	5.49e−10
	[7, 24]	0.3580(95)	−195(6)	–	52.98/16 = 3.31	7.60e−06
	[8, 24]	0.407(12)	−256(10)	–	17.00/15 = 1.13	0.32
	[9, 24]	0.430(16)	−291(17)	–	12.31/14 = 0.88	0.58
	[10, 24]	**0.461(21)**	**−341(24)**	–	7.64/13 = 0.59	0.87
ΔE_2	[11, 24]	0.474(26)	−365(33)	–	6.73/12 = 0.56	0.87
	[12, 24]	0.482(34)	−385(76)	–	6.62/11 = 0.60	0.83
	[13, 24]	0.493(43)	−424(126)	–	6.45/10 = 0.64	0.78
	[14, 24]	0.482(60)	−375(441)	–	6.39/9 = 0.71	0.70
	[15, 24]	0.491(78)	−423(836)	–	6.36/8 = 0.80	0.61
	[16, 24]	0.46(10)	−201(6088)	–	6.18/7 = 0.88	0.52
	[6, 24]	0.3403(57)	−324(20)	−184233(16997)	94.42/18 = 5.25	2.30e−12
	[7, 24]	0.3744(77)	−311(20)	−223002(23068)	54.57/17 = 3.21	8.04e−06
	[8, 24]	0.410(10)	−306(20)	−274894(31592)	23.64/16 = 1.48	0.10
	[9, 24]	0.428(14)	−312(20)	−307041(40897)	20.64/15 = 1.38	0.15
	[10, 24]	0.445(17)	−324(21)	−348017(52459)	17.10/14 = 1.22	0.25
ΔE_3	[11, 24]	0.474(20)	−348(23)	−419199(75850)	10.64/13 = 0.82	0.64
	[12, 24]	0.475(21)	−348(23)	−419363(76236)	10.62/12 = 0.89	0.56
	[13, 24]	0.471(21)	−346(24)	−430611(74623)	9.79/11 = 0.89	0.55
	[14, 24]	0.462(21)	−342(24)	−541724(83896)	4.03/10 = 0.40	0.95
	[15, 24]	0.462(21)	−341(24)	−568068(97565)	3.76/9 = 0.42	0.93
	[16, 24]	0.462(21)	−341(24)	−572527(120098)	3.75/8 = 0.47	0.88

Table 23: Results obtained from the **relativistic** bare ΔE_2- and ΔE_3-fit to order L^{-6} **without** exponentially suppressed corrections. In case of the ΔE_3-fit the a_0 and r_0 bootstrap samples obtained from the ΔE_2-fit within the interval $[10, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3379(61)	−318(20)	−150569(14972)	95.20/18 = 5.29	1.66e−12
	[7, 24]	0.3715(82)	−312(20)	−192088(20819)	59.95/17 = 3.53	1.07e−06
	[8, 24]	0.406(10)	−311(20)	−244364(28656)	31.38/16 = 1.96	0.01
	[9, 24]	0.436(13)	−317(20)	−304354(39574)	18.96/15 = 1.26	0.22
	[10, 24]	0.444(17)	−322(21)	−325533(49790)	18.31/14 = 1.31	0.19
ΔE_4	[11, 24]	0.465(19)	−344(23)	−394147(66903)	12.07/13 = 0.93	0.52
	[12, 24]	0.468(20)	−346(23)	−398404(68982)	11.89/12 = 0.99	0.45
	[13, 24]	0.470(21)	−346(23)	−398541(70033)	11.68/11 = 1.06	0.39
	[14, 24]	0.461(21)	−341(23)	−428780(66854)	7.64/10 = 0.76	0.66
	[15, 24]	0.462(21)	−342(24)	−422321(71779)	7.57/9 = 0.84	0.58
	[16, 24]	0.463(21)	−341(24)	−408253(76397)	7.27/8 = 0.91	0.51
	[6, 24]	0.3066(53)	−310(21)	−99095(11152)	135.02/18 = 7.50	0.00
	[7, 24]	0.3444(75)	−308(21)	−140660(16755)	84.94/17 = 5.00	5.02e−11
	[8, 24]	0.407(12)	−316(21)	−239395(31719)	37.03/16 = 2.31	2.08e−03
	[9, 24]	0.436(13)	−322(21)	−302882(41764)	17.98/15 = 1.20	0.26
	[10, 24]	0.446(17)	−328(22)	−328922(53324)	17.15/14 = 1.23	0.25
ΔE_5	[11, 24]	0.460(18)	−342(23)	−373103(63166)	12.03/13 = 0.93	0.52
	[12, 24]	0.463(20)	−343(23)	−380545(66356)	11.81/12 = 0.98	0.46
	[13, 24]	0.467(21)	−343(23)	−383572(67932)	11.33/11 = 1.03	0.42
	[14, 24]	0.459(21)	−339(23)	−389139(63918)	8.06/10 = 0.81	0.62
	[15, 24]	0.460(21)	−341(24)	−377494(68000)	7.74/9 = 0.86	0.56
	[16, 24]	0.460(21)	−341(24)	−381698(71389)	7.70/8 = 0.96	0.46

Table 24: Results obtained from the **relativistic** bare ΔE_4- and ΔE_5-fit to order L^{-6} **without** exponentially suppressed corrections. Here, the a_0 and r_0 bootstrap samples obtained from the previous ΔE_2-fit within the interval $[10, 24]$ serve as priors.

4.4 Relativistic Fits with Exponentially Suppressed Corrections (rel. c. ✓; exp. c. ✓)

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3332(68)	−108(3)	–	67.89/17 = 3.99	4.96e−08
	[7, 24]	0.3654(98)	−135(5)	–	47.60/16 = 2.97	5.49e−05
	[8, 24]	**0.411(13)**	**−194(9)**	–	15.57/15 = 1.04	0.41
	[9, 24]	0.432(16)	−228(18)	–	11.69/14 = 0.83	0.63
	[10, 24]	0.461(21)	−279(25)	–	7.50/13 = 0.58	0.87
ΔE_2	[11, 24]	0.474(26)	−303(35)	–	6.71/12 = 0.56	0.88
	[12, 24]	0.480(34)	−322(81)	–	6.62/11 = 0.60	0.83
	[13, 24]	0.491(42)	−361(132)	–	6.45/10 = 0.65	0.78
	[14, 24]	0.480(59)	−310(454)	–	6.39/9 = 0.71	0.70
	[15, 24]	0.488(77)	−359(854)	–	6.36/8 = 0.80	0.61
	[16, 24]	0.46(10)	−130(6131)	–	6.19/7 = 0.88	0.52
	[6, 24]	0.3531(60)	−190(9)	−100393(8906)	79.80/18 = 4.43	9.30e−10
	[7, 24]	0.3828(79)	−186(9)	−115428(11568)	47.07/17 = 2.77	1.19e−04
	[8, 24]	0.4126(98)	−193(9)	−142521(15920)	21.76/16 = 1.36	0.15
	[9, 24]	0.416(12)	−193(9)	−142619(16311)	21.50/15 = 1.43	0.12
	[10, 24]	0.418(12)	−193(9)	−140021(17053)	21.05/14 = 1.50	0.10
ΔE_3	[11, 24]	0.419(12)	−193(9)	−138157(22038)	21.03/13 = 1.62	0.07
	[12, 24]	0.418(13)	−193(9)	−146184(24842)	20.55/12 = 1.71	0.06
	[13, 24]	0.415(13)	−194(9)	−198920(37830)	17.15/11 = 1.56	0.10
	[14, 24]	0.412(13)	−194(9)	−399251(71625)	6.33/10 = 0.63	0.79
	[15, 24]	0.412(13)	−194(9)	−462900(92588)	5.15/9 = 0.57	0.82
	[16, 24]	0.412(13)	−194(9)	−496514(120356)	4.96/8 = 0.62	0.76

Table 25: Results obtained from the **relativistic** bare ΔE_2- and ΔE_3-fit to order L^{-6} **with** exponentially suppressed corrections. In case of the ΔE_3-fit the a_0 and r_0 bootstrap samples obtained from the ΔE_2-fit within the interval $[8, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3510(63)	−192(9)	−84152(8188)	84.90/18 = 4.72	1.17e−10
	[7, 24]	0.3811(82)	−189(9)	−100960(10698)	51.80/17 = 3.05	2.21e−05
	[8, 24]	0.4082(98)	−191(9)	−120785(13719)	26.52/16 = 1.66	0.05
	[9, 24]	0.422(11)	−193(9)	−130328(15767)	20.39/15 = 1.36	0.16
	[10, 24]	0.418(12)	−192(9)	−131255(15431)	19.77/14 = 1.41	0.14
ΔE_4	[11, 24]	0.420(12)	−193(9)	−127947(16079)	19.06/13 = 1.47	0.12
	[12, 24]	0.417(12)	−193(9)	−138258(18843)	18.03/12 = 1.50	0.11
	[13, 24]	0.417(12)	−193(9)	−142376(22199)	17.91/11 = 1.63	0.08
	[14, 24]	0.413(12)	−194(9)	−232583(37020)	8.73/10 = 0.87	0.56
	[15, 24]	0.413(12)	−193(9)	−246736(50142)	8.56/9 = 0.95	0.48
	[16, 24]	0.413(13)	−193(9)	−241481(58351)	8.53/8 = 1.07	0.38
	[6, 24]	0.3211(55)	−194(9)	−57587(5655)	135.07/18 = 7.50	0.00
	[7, 24]	0.3576(74)	−187(9)	−75390(7984)	80.78/17 = 4.75	2.79e−10
	[8, 24]	0.406(10)	−190(9)	−110136(12957)	30.95/16 = 1.93	0.01
	[9, 24]	0.421(11)	−194(9)	−126138(15357)	18.42/15 = 1.23	0.24
	[10, 24]	0.417(12)	−193(9)	−126441(14949)	17.83/14 = 1.27	0.21
ΔE_5	[11, 24]	0.419(12)	−194(9)	−123968(15270)	16.51/13 = 1.27	0.22
	[12, 24]	0.416(12)	−194(9)	−131310(16668)	15.50/12 = 1.29	0.22
	[13, 24]	0.416(12)	−194(9)	−130990(17715)	15.50/11 = 1.41	0.16
	[14, 24]	0.413(12)	−194(9)	−171046(23401)	8.87/10 = 0.89	0.54
	[15, 24]	0.413(12)	−194(9)	−174333(37505)	8.86/9 = 0.98	0.45
	[16, 24]	0.412(13)	−194(9)	−189087(45926)	8.55/8 = 1.07	0.38

Table 26: Results obtained from the **relativistic** bare ΔE_4- and ΔE_5-fit to order L^{-6} **with** exponentially suppressed corrections. Here, the a_0 and r_0 bootstrap samples obtained from the previous ΔE_2-fit within the interval $[8, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3332(68)	−108(3)	–	67.89/17 = 3.99	4.96e−08
	[7, 24]	0.3654(98)	−135(5)	–	47.60/16 = 2.97	5.49e−05
	[8, 24]	0.411(13)	−194(9)	–	15.57/15 = 1.04	0.41
	[9, 24]	**0.432(16)**	**−228(18)**	–	11.69/14 = 0.83	0.63
	[10, 24]	0.461(21)	−279(25)	–	7.50/13 = 0.58	0.87
ΔE_2	[11, 24]	0.474(26)	−303(35)	–	6.71/12 = 0.56	0.88
	[12, 24]	0.480(34)	−322(81)	–	6.62/11 = 0.60	0.83
	[13, 24]	0.491(42)	−361(132)	–	6.45/10 = 0.65	0.78
	[14, 24]	0.480(59)	−310(454)	–	6.39/9 = 0.71	0.70
	[15, 24]	0.488(77)	−359(854)	–	6.36/8 = 0.80	0.61
	[16, 24]	0.46(10)	−130(6131)	–	6.19/7 = 0.88	0.52
	[6, 24]	0.3523(60)	−206(15)	−114190(14333)	79.61/18 = 4.42	1.00e−09
	[7, 24]	0.3828(80)	−205(15)	−137183(18832)	47.56/17 = 2.80	1.00e−04
	[8, 24]	0.415(10)	−213(15)	−176736(25749)	21.01/16 = 1.31	0.18
	[9, 24]	0.428(14)	−222(17)	−196974(32676)	19.18/15 = 1.28	0.21
	[10, 24]	0.436(15)	−225(17)	−201597(35088)	17.36/14 = 1.24	0.24
ΔE_3	[11, 24]	0.442(16)	−231(17)	−198986(37370)	16.15/13 = 1.24	0.24
	[12, 24]	0.441(16)	−231(17)	−202320(38045)	15.99/12 = 1.33	0.19
	[13, 24]	0.438(16)	−229(17)	−238880(44603)	13.77/11 = 1.25	0.25
	[14, 24]	0.433(16)	−228(17)	−405296(71610)	5.17/10 = 0.52	0.88
	[15, 24]	0.433(16)	−227(17)	−452729(90724)	4.45/9 = 0.49	0.88
	[16, 24]	0.433(16)	−227(17)	−473628(117290)	4.37/8 = 0.55	0.82

Table 27: Results obtained from the **relativistic** bare ΔE_2- and ΔE_3-fit to order L^{-6} **with** exponentially suppressed corrections. In case of the ΔE_3-fit the a_0 and r_0 bootstrap samples obtained from the ΔE_2-fit within the interval $[9, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3498(64)	−190(15)	−81439(11913)	80.14/18 = 4.45	8.11e−10
	[7, 24]	0.3795(84)	−193(15)	−104130(15875)	51.04/17 = 3.00	2.91e−05
	[8, 24]	0.410(10)	−207(16)	−141826(22044)	26.74/16 = 1.67	0.04
	[9, 24]	0.434(13)	−224(17)	−185831(31390)	17.63/15 = 1.18	0.28
	[10, 24]	0.435(15)	−224(17)	−186677(32327)	17.61/14 = 1.26	0.23
ΔE_4	[11, 24]	0.441(15)	−231(17)	−194262(34827)	15.00/13 = 1.15	0.31
	[12, 24]	0.440(16)	−230(17)	−195955(34529)	14.83/12 = 1.24	0.25
	[13, 24]	0.440(16)	−230(17)	−196113(35605)	14.83/11 = 1.35	0.19
	[14, 24]	0.434(16)	−228(17)	−264587(43177)	8.04/10 = 0.80	0.62
	[15, 24]	0.433(16)	−227(17)	−268475(53329)	8.02/9 = 0.89	0.53
	[16, 24]	0.434(16)	−227(17)	−258868(60591)	7.91/8 = 0.99	0.44
	[6, 24]	0.3203(55)	−177(15)	−48328(8090)	118.75/18 = 6.60	1.11e−16
	[7, 24]	0.3544(74)	−174(15)	−64095(11184)	72.59/17 = 4.27	7.69e−09
	[8, 24]	0.409(11)	−205(16)	−129769(21892)	31.33/16 = 1.96	0.01
	[9, 24]	0.433(13)	−225(17)	−179652(30784)	16.41/15 = 1.09	0.36
	[10, 24]	0.434(15)	−225(17)	−180669(31800)	16.40/14 = 1.17	0.29
ΔE_5	[11, 24]	0.439(15)	−230(17)	−187689(33671)	13.57/13 = 1.04	0.40
	[12, 24]	0.437(16)	−230(17)	−187950(33115)	13.45/12 = 1.12	0.34
	[13, 24]	0.438(16)	−230(17)	−187028(33429)	13.33/11 = 1.21	0.27
	[14, 24]	0.432(16)	−226(17)	−213806(34107)	8.32/10 = 0.83	0.60
	[15, 24]	0.433(16)	−227(17)	−208980(43690)	8.29/9 = 0.92	0.51
	[16, 24]	0.432(16)	−227(17)	−219345(50725)	8.13/8 = 1.02	0.42

Table 28: Results obtained from the **relativistic** bare ΔE_4- and ΔE_5-fit to order L^{-6} **with** exponentially suppressed corrections. Here, the a_0 and r_0 bootstrap samples obtained from the previous ΔE_2-fit within the interval $[9, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3332(68)	−108(3)	–	67.89/17 = 3.99	4.96e−08
	[7, 24]	0.3654(98)	−135(5)	–	47.60/16 = 2.97	5.49e−05
	[8, 24]	0.411(13)	−194(9)	–	15.57/15 = 1.04	0.41
	[9, 24]	0.432(16)	−228(18)	–	11.69/14 = 0.83	0.63
	[10, 24]	**0.461(21)**	**−279(25)**	–	7.50/13 = 0.58	0.87
ΔE_2	[11, 24]	0.474(26)	−303(35)	–	6.71/12 = 0.56	0.88
	[12, 24]	0.480(34)	−322(81)	–	6.62/11 = 0.60	0.83
	[13, 24]	0.491(42)	−361(132)	–	6.45/10 = 0.65	0.78
	[14, 24]	0.480(59)	−310(454)	–	6.39/9 = 0.71	0.70
	[15, 24]	0.488(77)	−359(854)	–	6.36/8 = 0.80	0.61
	[16, 24]	0.46(10)	−130(6131)	–	6.19/7 = 0.88	0.52
	[6, 24]	0.3519(60)	−245(20)	−149859(18239)	79.05/18 = 4.39	1.25e−09
	[7, 24]	0.3822(80)	−239(20)	−176363(23665)	47.80/17 = 2.81	9.23e−05
	[8, 24]	0.415(10)	−239(20)	−214347(31266)	21.90/16 = 1.37	0.15
	[9, 24]	0.430(14)	−246(20)	−237014(38470)	19.71/15 = 1.31	0.18
	[10, 24]	0.446(17)	−259(22)	−269834(48305)	16.58/14 = 1.18	0.28
ΔE_3	[11, 24]	0.474(20)	−287(24)	−329167(69678)	10.34/13 = 0.80	0.67
	[12, 24]	0.475(21)	−287(25)	−329157(70065)	10.33/12 = 0.86	0.59
	[13, 24]	0.471(21)	−285(25)	−341303(68917)	9.55/11 = 0.87	0.57
	[14, 24]	0.463(21)	−280(25)	−453305(80074)	3.96/10 = 0.40	0.95
	[15, 24]	0.462(21)	−279(25)	−478396(94430)	3.72/9 = 0.41	0.93
	[16, 24]	0.462(21)	−279(25)	−481562(117757)	3.71/8 = 0.46	0.88

Table 29: Results obtained from the **relativistic** bare ΔE_2- and ΔE_3-fit to order L^{-6} **with** exponentially suppressed corrections. In case of the ΔE_3-fit the a_0 and r_0 bootstrap samples obtained from the ΔE_2-fit within the interval $[10, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
	[6, 24]	0.3484(64)	−240(20)	−117036(15652)	81.88/18 = 4.55	4.01e−10
	[7, 24]	0.3785(85)	−239(20)	−146605(20752)	53.62/17 = 3.15	1.14e−05
	[8, 24]	0.410(11)	−243(20)	−185555(27652)	29.12/16 = 1.82	0.02
	[9, 24]	0.438(13)	−252(20)	−232127(37005)	18.40/15 = 1.23	0.24
	[10, 24]	0.445(17)	−258(22)	−247225(45129)	17.95/14 = 1.28	0.21
ΔE_4	[11, 24]	0.465(19)	−282(24)	−306571(60843)	11.93/13 = 0.92	0.53
	[12, 24]	0.468(20)	−284(24)	−309836(62652)	11.75/12 = 0.98	0.47
	[13, 24]	0.470(21)	−284(24)	−309136(63765)	11.52/11 = 1.05	0.40
	[14, 24]	0.461(21)	−278(24)	−341895(61363)	7.64/10 = 0.76	0.66
	[15, 24]	0.462(21)	−279(25)	−334470(66988)	7.55/9 = 0.84	0.58
	[16, 24]	0.463(21)	−279(25)	−319581(72114)	7.23/8 = 0.90	0.51
	[6, 24]	0.3169(56)	−229(21)	−72994(11706)	119.26/18 = 6.63	1.11e−16
	[7, 24]	0.3516(77)	−232(21)	−103226(16698)	77.46/17 = 4.56	1.08e−09
	[8, 24]	0.411(12)	−248(21)	−180241(30054)	34.49/16 = 2.16	4.66e−03
	[9, 24]	0.438(13)	−257(21)	−230396(38923)	17.53/15 = 1.17	0.29
	[10, 24]	0.446(17)	−263(23)	−249534(48225)	16.92/14 = 1.21	0.26
ΔE_5	[11, 24]	0.459(18)	−279(24)	−287732(57307)	11.97/13 = 0.92	0.53
	[12, 24]	0.463(20)	−281(24)	−293852(59988)	11.73/12 = 0.98	0.47
	[13, 24]	0.467(20)	−281(24)	−295735(61503)	11.24/11 = 1.02	0.42
	[14, 24]	0.459(21)	−276(24)	−303915(57704)	8.07/10 = 0.81	0.62
	[15, 24]	0.461(21)	−278(25)	−291088(62454)	7.71/9 = 0.86	0.56
	[16, 24]	0.460(21)	−278(25)	−295008(66367)	7.68/8 = 0.96	0.47

Table 30: Results obtained from the **relativistic** bare ΔE_4- and ΔE_5-fit to order L^{-6} **with** exponentially suppressed corrections. Here, the a_0 and r_0 bootstrap samples obtained from the previous ΔE_2-fit within the interval $[10, 24]$ serve as priors.

5 Global Energy Shift Fits to Order L^{-6} without Priors

5.1 Global Fits without Exponentially Suppressed Corrections

fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
$[6, 24]$	0.3116(39)	−198(2)	−79077(2852)	$289.02/73 = 3.96$	0.00
$[7, 24]$	0.3489(57)	−217(5)	−121069(6913)	$177.26/69 = 2.57$	1.79e−11
$[8, 24]$	0.3968(82)	−247(8)	−199650(14697)	$107.05/65 = 1.65$	7.96e−04
$[9, 24]$	0.428(11)	−291(14)	−301721(33767)	$66.18/61 = 1.08$	0.30
$[10, 24]$	0.438(15)	−320(21)	−362327(52137)	$52.83/57 = 0.93$	0.63
$[11, 24]$	0.475(19)	−393(27)	−567872(90850)	$39.60/53 = 0.75$	0.91
$[12, 24]$	0.493(24)	−420(45)	−674072(154147)	$37.27/49 = 0.76$	0.89
$[13, 24]$	0.499(31)	−428(81)	−709696(242840)	$35.55/45 = 0.79$	0.84
$[14, 24]$	0.463(42)	−265(304)	−346307(430995)	$32.73/41 = 0.80$	0.82
$[15, 24]$	0.490(57)	−417(422)	−639768(673014)	$30.71/37 = 0.83$	0.76
$[16, 24]$	0.505(72)	−490(727)	−827472(965137)	$27.13/33 = 0.82$	0.75

Table 31: Results from the **non-relativistic** global energy shift fit (ΔE_N-fit with $N = 2, 3, 4, 5$) **without** exponentially suppressed corrections.

fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
$[6, 24]$	0.3116(39)	−159(2)	−30068(1789)	$289.02/73 = 3.96$	0.00
$[7, 24]$	0.3489(57)	−183(5)	−59655(5182)	$177.26/69 = 2.57$	1.79e−11
$[8, 24]$	0.3968(82)	−217(8)	−120209(11999)	$107.05/65 = 1.65$	7.96e−04
$[9, 24]$	0.428(11)	−263(15)	−209301(29700)	$66.18/61 = 1.08$	0.30
$[10, 24]$	0.438(15)	−292(21)	−265422(46637)	$52.83/57 = 0.93$	0.63
$[11, 24]$	0.475(19)	−368(28)	−454224(82982)	$39.60/53 = 0.75$	0.91
$[12, 24]$	0.493(24)	−396(46)	−551507(143457)	$37.27/49 = 0.76$	0.89
$[13, 24]$	0.499(31)	−404(82)	−583974(228596)	$35.55/45 = 0.79$	0.84
$[14, 24]$	0.463(42)	−239(306)	−238156(412970)	$32.73/41 = 0.80$	0.82
$[15, 24]$	0.490(57)	−393(424)	−518404(646454)	$30.71/37 = 0.83$	0.76
$[16, 24]$	0.505(72)	−466(730)	−698698(930182)	$27.13/33 = 0.82$	0.75

Table 32: Results from the **relativistic** global energy shift fit (ΔE_N-fit with $N = 2, 3, 4, 5$) **without** exponentially suppressed corrections.

5.2 Global Fits with Exponentially Suppressed Corrections

fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
$[6, 24]$	0.3227(41)	−138(2)	−59994(2405)	$264.13/73 = 3.62$	0.00
$[7, 24]$	0.3564(59)	−156(4)	−90257(6000)	$166.68/69 = 2.42$	4.68e−10
$[8, 24]$	0.4013(84)	−186(7)	−151441(13007)	$103.75/65 = 1.60$	1.60e−03
$[9, 24]$	0.430(11)	−228(15)	−235725(30825)	$65.31/61 = 1.07$	0.33
$[10, 24]$	0.439(15)	−255(22)	−286929(47814)	$52.29/57 = 0.92$	0.65
$[11, 24]$	0.474(19)	−331(28)	−472297(84213)	$39.52/53 = 0.75$	0.92
$[12, 24]$	0.491(24)	−359(48)	−565633(144682)	$37.28/49 = 0.76$	0.89
$[13, 24]$	0.497(31)	−366(85)	−594921(229840)	$35.57/45 = 0.79$	0.84
$[14, 24]$	0.461(42)	−195(314)	−245971(414270)	$32.74/41 = 0.80$	0.82
$[15, 24]$	0.488(57)	−353(434)	−526506(648243)	$30.72/37 = 0.83$	0.76
$[16, 24]$	0.503(71)	−429(743)	−707541(932637)	$27.14/33 = 0.82$	0.75

Table 33: Results from the **non-relativistic** global energy shift fit (ΔE_N-fit with $N = 2, 3, 4, 5$) **with** exponentially suppressed corrections.

fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
$[6, 24]$	0.3227(41)	−100(2)	−7432(1352)	$264.13/73 = 3.62$	0.00
$[7, 24]$	0.3564(59)	−122(4)	−26164(4300)	$166.68/69 = 2.42$	4.68e−10
$[8, 24]$	0.4013(84)	−156(7)	−70172(10438)	$103.75/65 = 1.60$	1.60e−03
$[9, 24]$	0.430(11)	−200(15)	−142405(26874)	$65.31/61 = 1.07$	0.33
$[10, 24]$	0.439(15)	−227(22)	−189799(42507)	$52.29/57 = 0.92$	0.65
$[11, 24]$	0.474(19)	−305(29)	−359011(76572)	$39.52/53 = 0.75$	0.92
$[12, 24]$	0.491(24)	−334(48)	−443857(134298)	$37.28/49 = 0.76$	0.89
$[13, 24]$	0.497(31)	−342(86)	−470216(215938)	$35.57/45 = 0.79$	0.84
$[14, 24]$	0.461(42)	−169(316)	−138700(396605)	$32.74/41 = 0.80$	0.82
$[15, 24]$	0.488(57)	−328(437)	−406206(622156)	$30.72/37 = 0.83$	0.76
$[16, 24]$	0.503(71)	−405(746)	−579916(898269)	$27.14/33 = 0.82$	0.75

Table 34: Results from the **relativistic** global energy shift fit (ΔE_N-fit with $N = 2, 3, 4, 5$) **with** exponentially suppressed corrections.

6 Global Energy Shift Fits to Order L^{-6} with Priors

6.1 Non-Relativistic Global Fits

fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
$[6, 24]$	0.3269(41)	−204(6)	−95417(3132)	$403.21/56 = 7.20$	0.00
$[7, 24]$	0.3649(54)	−229(7)	−148708(6039)	$200.08/53 = 3.78$	0.00
$[8, 24]$	0.4087(77)	−246(8)	−216929(10870)	$97.02/50 = 1.94$	7.61e−05
$[9, 24]$	0.4252(92)	−273(9)	−273327(18454)	$56.58/47 = 1.20$	0.16
$[10, 24]$	0.420(11)	−279(10)	−275576(19856)	$46.18/44 = 1.05$	0.38
$[11, 24]$	0.421(11)	−280(10)	−273865(20343)	$44.45/41 = 1.08$	0.33
$[12, 24]$	0.419(11)	−281(10)	−275764(20984)	$43.93/38 = 1.16$	0.23
$[13, 24]$	0.421(12)	−280(10)	−275037(21834)	$43.02/35 = 1.23$	0.17
$[14, 24]$	0.416(12)	−282(10)	−305285(26175)	$36.78/32 = 1.15$	0.26
$[15, 24]$	0.414(12)	−283(10)	−283120(37431)	$33.60/29 = 1.16$	0.25
$[16, 24]$	0.414(12)	−284(10)	−286338(43291)	$28.58/26 = 1.10$	0.33

Table 35: Results from the **non-relativistic** global energy shift fit (ΔE_N-fit with $N = 3, 4, 5$) **without** exponentially suppressed corrections. Here, the a_0 and r_0 bootstrap samples obtained from the corresponding ΔE_2-fit within the interval $[8, 24]$ serve as priors.

6.2 Relativistic Global Fits

fit intvl	a_0	r_0	$\widetilde{\mathcal{T}}$	χ^2/ndof	p-value
$[6, 24]$	0.3278(40)	−167(5)	−41906(2577)	$406.64/56 = 7.26$	0.00
$[7, 24]$	0.3653(53)	−196(7)	−81823(4916)	$200.50/53 = 3.78$	0.00
$[8, 24]$	0.4086(76)	−216(8)	−132615(8843)	$97.00/50 = 1.94$	7.64e−05
$[9, 24]$	0.4251(91)	−244(9)	−182054(15429)	$56.59/47 = 1.20$	0.16
$[10, 24]$	0.420(11)	−250(9)	−186411(16514)	$46.18/44 = 1.05$	0.38
$[11, 24]$	0.421(11)	−251(10)	−184347(16908)	$44.45/41 = 1.08$	0.33
$[12, 24]$	0.419(11)	−252(10)	−187053(17956)	$43.93/38 = 1.16$	0.23
$[13, 24]$	0.421(12)	−251(10)	−185414(18939)	$43.02/35 = 1.23$	0.17
$[14, 24]$	0.416(12)	−253(10)	−217906(24094)	$36.78/32 = 1.15$	0.26
$[15, 24]$	0.414(12)	−254(10)	−196530(36034)	$33.60/29 = 1.16$	0.25
$[16, 24]$	0.414(12)	−255(10)	−200051(42206)	$28.58/26 = 1.10$	0.33

Table 36: Results from the **relativistic** global energy shift fit (ΔE_N-fit with $N = 3, 4, 5$) **without** exponentially suppressed corrections. Here, the a_0 and r_0 bootstrap samples obtained from the corresponding ΔE_2-fit within the interval $[8, 24]$ serve as priors.

fit intvl	a_0	r_0	$\widetilde{\mathcal{T}}$	χ^2/ndof	p-value
$[6, 24]$	0.3221(45)	−73.0(60)	+7389(3311)	$211.00/55 = 3.84$	0.00
$[7, 24]$	0.3568(63)	−103.0(82)	−12623(6750)	$130.87/52 = 2.52$	9.77e−09
$[8, 24]$	0.4044(88)	−134.6(88)	−48680(11602)	$67.30/49 = 1.37$	0.04
$[9, 24]$	0.420(10)	−144(20)	−65054(27260)	$48.60/46 = 1.06$	0.37
$[10, 24]$	0.420(12)	−164(25)	−92376(34204)	$43.01/43 = 1.00$	0.47
$[11, 24]$	0.421(12)	−132(43)	−49080(54062)	$40.14/40 = 1.00$	0.46
$[12, 24]$	0.419(12)	−107(56)	−24559(66091)	$39.30/37 = 1.06$	0.37
$[13, 24]$	0.418(12)	+30(88)	+130486(93976)	$33.23/34 = 0.98$	0.51
$[14, 24]$	0.413(12)	+432(188)	+549652(193558)	$21.67/31 = 0.70$	0.89
$[15, 24]$	0.413(12)	+490(224)	+614552(230324)	$21.26/28 = 0.76$	0.81
$[16, 24]$	0.413(12)	+506(277)	+615738(282634)	$20.08/25 = 0.80$	0.74

Table 37: Results from the **relativistic** global energy shift fit (ΔE_N-fit with $N = 3, 4, 5$) **with** exponentially suppressed corrections. Here, the a_0 bootstrap samples obtained from the corresponding ΔE_2-fit within the interval $[8, 24]$ serve as priors.

fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
$[6, 24]$	0.3386(42)	−104(5)	−13477(2589)	$353.82/56 = 6.32$	0.00
$[7, 24]$	0.3728(54)	−134(6)	−42446(4623)	$185.00/53 = 3.49$	1.11e−16
$[8, 24]$	0.4133(76)	−156.2(71)	−79302(7975)	$93.43/50 = 1.87$	1.91e−04
$[9, 24]$	0.4290(92)	−184.9(86)	−122668(13554)	$55.16/47 = 1.17$	0.19
$[10, 24]$	0.424(11)	−190.1(90)	−128566(14540)	$44.41/44 = 1.01$	0.45
$[11, 24]$	0.425(11)	−191.8(92)	−126127(14964)	$42.36/41 = 1.03$	0.41
$[12, 24]$	0.423(11)	−192.0(92)	−128598(16386)	$41.95/38 = 1.10$	0.30
$[13, 24]$	0.426(12)	−191.6(93)	−126233(17501)	$40.97/35 = 1.17$	0.22
$[14, 24]$	0.420(12)	−193(9)	−158770(23172)	$35.26/32 = 1.10$	0.32
$[15, 24]$	0.418(12)	−193(9)	−136895(35439)	$32.16/29 = 1.11$	0.31
$[16, 24]$	0.418(12)	−193.9(93)	−140303(41783)	$27.39/26 = 1.05$	0.39

Table 38: Results from the **relativistic** global energy shift fit (ΔE_N-fit with $N = 3, 4, 5$) **with** exponentially suppressed corrections. Here, the a_0 and r_0 bootstrap samples obtained from the corresponding ΔE_2-fit within the interval $[8, 24]$ serve as priors.

fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
$[6, 24]$	0.3223(46)	−73.0(59)	+7398(3312)	$209.30/55 = 3.81$	0.00
$[7, 24]$	0.3563(63)	−101.7(83)	−11629(6746)	$131.06/52 = 2.52$	9.19e−09
$[8, 24]$	0.4055(89)	−134.9(87)	−49381(11637)	$66.90/49 = 1.37$	0.05
$[9, 24]$	0.430(12)	−165(20)	−97673(32183)	$45.22/46 = 0.98$	0.50
$[10, 24]$	0.435(14)	−190(24)	−136303(40043)	$38.98/43 = 0.91$	0.65
$[11, 24]$	0.441(15)	−187(39)	−129511(61114)	$35.22/40 = 0.88$	0.68
$[12, 24]$	0.440(15)	−175(49)	−112189(71398)	$34.91/37 = 0.94$	0.57
$[13, 24]$	0.439(16)	−77(77)	+16881(97263)	$30.09/34 = 0.89$	0.66
$[14, 24]$	0.431(16)	+240(166)	+396798(189789)	$20.78/31 = 0.67$	0.92
$[15, 24]$	0.432(16)	+265(197)	+435601(225450)	$20.45/28 = 0.73$	0.85
$[16, 24]$	0.431(16)	+256(240)	+412585(276592)	$19.38/25 = 0.78$	0.78

Table 39: Results from the **relativistic** global energy shift fit (ΔE_N-fit with $N = 3, 4, 5$) **with** exponentially suppressed corrections. Here, the a_0 bootstrap samples obtained from the corresponding ΔE_2-fit within the interval $[9, 24]$ serve as priors.

fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
$[6, 24]$	0.3350(41)	−90(5)	−3644(2847)	$274.43/56 = 4.90$	0.00
$[7, 24]$	0.3686(54)	−119.3(63)	−28962(5310)	$156.78/53 = 2.96$	3.40e−12
$[8, 24]$	0.4125(79)	−148.4(73)	−69957(9137)	$84.57/50 = 1.69$	1.62e−03
$[9, 24]$	0.4375(96)	−198(13)	−149418(21051)	$53.77/47 = 1.14$	0.23
$[10, 24]$	0.439(12)	−214(15)	−173460(26515)	$41.02/44 = 0.93$	0.60
$[11, 24]$	0.444(13)	−226(16)	−189012(31201)	$36.62/41 = 0.89$	0.67
$[12, 24]$	0.444(14)	−226(17)	−189029(31547)	$36.42/38 = 0.96$	0.54
$[13, 24]$	0.447(14)	−225(17)	−187358(31721)	$35.17/35 = 1.00$	0.46
$[14, 24]$	0.440(15)	−223(17)	−206721(33392)	$31.26/32 = 0.98$	0.50
$[15, 24]$	0.439(15)	−225(17)	−182800(42332)	$28.27/29 = 0.97$	0.50
$[16, 24]$	0.437(15)	−225(17)	−184307(47916)	$24.18/26 = 0.93$	0.57

Table 40: Results from the **relativistic** global energy shift fit (ΔE_N-fit with $N = 3, 4, 5$) **with** exponentially suppressed corrections. Here, the a_0 and r_0 bootstrap samples obtained from the corresponding ΔE_2-fit within the interval $[9, 24]$ serve as priors.

fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
$[6, 24]$	0.3225(45)	−73(6)	+7342(3310)	$208.67/55 = 3.79$	0.00
$[7, 24]$	0.3567(62)	−102(8)	−12235(6765)	$129.92/52 = 2.50$	1.31e−08
$[8, 24]$	0.4053(88)	−135(9)	−49718(11680)	$66.81/49 = 1.36$	0.05
$[9, 24]$	0.431(12)	−168(21)	−101922(33253)	$44.99/46 = 0.98$	0.51
$[10, 24]$	0.443(15)	−207(25)	−165483(45346)	$37.19/43 = 0.86$	0.72
$[11, 24]$	0.464(18)	−253(38)	−254958(78870)	$30.04/40 = 0.75$	0.87
$[12, 24]$	0.469(19)	−255(43)	−259927(88164)	$29.55/37 = 0.80$	0.80
$[13, 24]$	0.468(20)	−200(65)	−167499(112832)	$26.37/34 = 0.78$	0.82
$[14, 24]$	0.457(21)	+15(142)	+151859(196141)	$20.06/31 = 0.65$	0.93
$[15, 24]$	0.459(21)	+2(163)	+151645(228949)	$19.54/28 = 0.70$	0.88
$[16, 24]$	0.459(21)	−30(197)	+95391(278601)	$18.53/25 = 0.74$	0.82

Table 41: Results from the **relativistic** global energy shift fit (ΔE_N-fit with $N = 3, 4, 5$) **with** exponentially suppressed corrections. Here, the a_0 bootstrap samples obtained from the corresponding ΔE_2-fit within the interval $[10, 24]$ serve as priors.

fit intvl	a_0	r_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value
$[6, 24]$	0.3374(41)	−91(5)	−4682(2910)	$322.47/56 = 5.76$	0.00
$[7, 24]$	0.3736(54)	−122(6)	−33412(5500)	$189.67/53 = 3.58$	0.00
$[8, 24]$	0.4201(77)	−147(7)	−72893(9508)	$94.67/50 = 1.89$	1.40e−04
$[9, 24]$	0.4452(96)	−200(14)	−161335(22782)	$56.17/47 = 1.20$	0.17
$[10, 24]$	0.450(12)	−234(18)	−216760(31039)	$42.05/44 = 0.96$	0.56
$[11, 24]$	0.468(16)	−277(22)	−302075(53509)	$30.91/41 = 0.75$	0.87
$[12, 24]$	0.472(18)	−281(23)	−309272(57798)	$30.21/38 = 0.80$	0.81
$[13, 24]$	0.476(17)	−278(23)	−307148(55184)	$28.67/35 = 0.82$	0.77
$[14, 24]$	0.468(18)	−276(23)	−313208(53834)	$26.29/32 = 0.82$	0.75
$[15, 24]$	0.468(19)	−279(23)	−285287(60349)	$23.51/29 = 0.81$	0.75
$[16, 24]$	0.465(20)	−277(24)	−280075(64591)	$20.46/26 = 0.79$	0.77

Table 42: Results from the **relativistic** global energy shift fit (ΔE_N-fit with $N = 3, 4, 5$) **with** exponentially suppressed corrections. Here, the a_0 and r_0 bootstrap samples obtained from the corresponding ΔE_2-fit within the interval $[10, 24]$ serve as priors.

7 Energy Shift Fits to Order L^{-6} with Substituted Effective Range Parameter

In a relativistic approach it can be shown that the one-loop value of the effective range parameter reads

$$r_0 = -\frac{1}{M^2\, a_0} + \frac{20}{3\pi\, M}. \tag{5}$$

Substituting this expression into Eq. (4) provides the N-particle ground state model only depending on two fit-parameters (scattering length a_0 and threshold amplitude $\bar{\mathcal{T}}$):

$$\begin{aligned}
\Delta E_N =& \binom{N}{2} \frac{4\pi a_0}{ML^3} \Bigg\{ 1 - \left(\frac{a_0}{\pi L}\right) \mathcal{I} + \left(\frac{a_0}{\pi L}\right)^2 \left[\mathcal{I}^2 + (2N-5)\,\mathcal{J}\right] \\
&- \left(\frac{a_0}{\pi L}\right)^3 \left[(2N-7)\,\mathcal{I}\mathcal{J} + \mathcal{I}^3 + (5N^2 - 41N + 63)\,\mathcal{K} + 8(N-2)\,(2\mathcal{Q} + \mathcal{R})\right] \\
&- (4N-6)\,\frac{\pi a_0^2}{L^3} \left(\frac{1}{M^2\, a_0} - \frac{20}{3\pi\, M}\right) + (4N-9)\,\frac{\pi a_0}{M^2 L^3} \\
&- \frac{24 M a_0}{\sqrt{2\pi ML}}\, \exp\left(-ML\right) \Bigg\} \\
&+ \binom{N}{3} \left\{ \frac{32\pi a_0^4}{ML^6} \left(3\sqrt{3} - 4\pi\right) \left(\ln(ML)^2 - \Gamma'(1) - \ln(4\pi)\right) - \frac{\bar{\mathcal{T}}}{6L^6} \right\}.
\end{aligned} \tag{6}$$

8 Bare Energy Shift Fits to Order L^{-6} with Substituted Effective Range Parameter

shift	fit intvl	a_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value	$r_{0,\,\mathrm{ptb}}$	$\bar{\mathcal{T}}_{\mathrm{ptb}}$
	$[6, 24]$	0.2748(39)	–	$185.50/18 = 10.31$	0.00	−77(1)	25403(724)
	$[7, 24]$	0.3007(51)	–	$110.84/17 = 6.52$	8.88e−16	−70(1)	30417(1035)
	$[8, 24]$	0.3210(68)	–	$86.06/16 = 5.38$	1.32e−11	−65(2)	34662(1471)
	$[9, 24]$	0.3504(83)	–	$42.83/15 = 2.86$	1.67e−04	−58(2)	41302(1959)
	$[10, 24]$	0.3672(94)	–	$29.20/14 = 2.09$	0.01	−55(2)	45358(2325)
ΔE_2	$[11, 24]$	0.381(10)	–	$21.03/13 = 1.62$	0.07	−53(2)	48831(2566)
	$[12, 24]$	0.406(13)	–	$11.79/12 = 0.98$	0.46	−49(2)	55450(3554)
	$[13, 24]$	0.419(16)	–	$9.42/11 = 0.86$	0.58	−47(2)	59058(4513)
	$[14, 24]$	0.436(19)	–	$6.95/10 = 0.70$	0.73	−45(2)	63947(5576)
	$[15, 24]$	0.440(21)	–	$6.81/9 = 0.76$	0.66	−44(3)	65126(6219)
	$[16, 24]$	0.450(25)	–	$6.31/8 = 0.79$	0.61	−43(3)	68120(7571)
	$[6, 24]$	0.3523(60)	+20983(1801)	$77.77/18 = 4.32$	2.09e−09	−58(1)	41752(1426)
	$[7, 24]$	0.3818(80)	+40520(4030)	$47.69/17 = 2.81$	9.57e−05	−53(1)	49036(2059)
	$[8, 24]$	0.412(10)	+68969(7223)	$24.28/16 = 1.52$	0.08	−48(1)	57101(2776)
	$[9, 24]$	0.413(13)	+70092(14910)	$24.27/15 = 1.62$	0.06	−48(2)	57378(3615)
	$[10, 24]$	0.413(13)	+71786(18213)	$24.25/14 = 1.73$	0.04	−48(2)	57378(3615)
ΔE_3	$[11, 24]$	0.413(13)	+61388(22298)	$23.60/13 = 1.82$	0.04	−48(2)	57378(3615)
	$[12, 24]$	0.415(13)	+56601(22577)	$21.77/12 = 1.81$	0.04	−48(2)	57935(3633)
	$[13, 24]$	0.413(13)	+556(36544)	$17.97/11 = 1.63$	0.08	−48(2)	57378(3615)
	$[14, 24]$	0.408(13)	−209768(72409)	$6.67/10 = 0.67$	0.76	−49(2)	55997(3571)
	$[15, 24]$	0.408(13)	−276794(93120)	$5.36/9 = 0.60$	0.80	−49(2)	55997(3571)
	$[16, 24]$	0.407(13)	−313596(120148)	$5.13/8 = 0.64$	0.74	−49(2)	55723(3562)

Table 43: Results obtained from the **relativistic** bare ΔE_2- and ΔE_3-fit to order L^{-6} **with** exponentially suppressed corrections, using Eq. (6) as fit model for different fit intervals. In case of the ΔE_3-fit the a_0 bootstrap samples obtained from the ΔE_2-fit within the interval $[12, 24]$ serve as priors.

shift	fit intvl	a_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value	$r_{0,\,\mathrm{ptb}}$	$\bar{\mathcal{T}}_{\mathrm{ptb}}$
	[6, 24]	0.3500(63)	+16957(1129)	78.64/18 = 4.37	1.48e−09	−58(1)	41208(1487)
	[7, 24]	0.3784(82)	+29327(2616)	49.72/17 = 2.92	4.67e−05	−53(1)	48167(2091)
	[8, 24]	0.4047(99)	+46063(4510)	27.66/16 = 1.73	0.03	−49(1)	55095(2699)
	[9, 24]	0.419(12)	+58779(7407)	22.79/15 = 1.52	0.09	−47(2)	59058(3386)
	[10, 24]	0.413(13)	+49133(11358)	21.55/14 = 1.54	0.09	−48(2)	57378(3615)
ΔE_4	[11, 24]	0.414(13)	+54184(13944)	21.15/13 = 1.63	0.07	−48(2)	57656(3624)
	[12, 24]	0.415(13)	+39707(16725)	18.71/12 = 1.56	0.10	−48(2)	57935(3633)
	[13, 24]	0.414(13)	+33680(21955)	18.53/11 = 1.68	0.07	−48(2)	57656(3624)
	[14, 24]	0.409(13)	−64177(38413)	8.92/10 = 0.89	0.54	−49(2)	56272(3580)
	[15, 24]	0.409(13)	−80886(52065)	8.70/9 = 0.97	0.47	−49(2)	56272(3580)
	[16, 24]	0.409(13)	−76764(60158)	8.68/8 = 1.08	0.37	−49(2)	56272(3580)
	[6, 24]	0.3202(55)	+11913(647)	119.82/18 = 6.66	0.00	−65(1)	34490(1188)
	[7, 24]	0.3544(73)	+20617(1474)	72.22/17 = 4.25	8.93e−09	−58(1)	42251(1744)
	[8, 24]	0.402(10)	+41940(3799)	31.37/16 = 1.96	0.01	−50(1)	54362(2708)
	[9, 24]	0.418(11)	+50836(4739)	20.13/15 = 1.34	0.17	−47(2)	58776(3097)
	[10, 24]	0.412(13)	+42442(9161)	18.99/14 = 1.36	0.17	−48(2)	57101(3606)
ΔE_5	[11, 24]	0.414(13)	+47606(10497)	17.95/13 = 1.38	0.16	−48(2)	57656(3624)
	[12, 24]	0.412(13)	+34552(14114)	16.05/12 = 1.34	0.19	−48(2)	57101(3606)
	[13, 24]	0.412(13)	+34494(16480)	16.05/11 = 1.46	0.14	−48(2)	57101(3606)
	[14, 24]	0.408(13)	−11595(23962)	9.07/10 = 0.91	0.53	−49(2)	55997(3571)
	[15, 24]	0.408(13)	−17279(38847)	9.03/9 = 1.00	0.43	−49(2)	55997(3571)
	[16, 24]	0.407(13)	−33644(47460)	8.67/8 = 1.08	0.37	−49(2)	55723(3562)

Table 44: Results obtained from the **relativistic** bare ΔE_4- and ΔE_5-fit to order L^{-6} **with** exponentially suppressed corrections, using Eq. (6) as fit model for different fit intervals. Here, the a_0 bootstrap samples obtained from the ΔE_2-fit within the interval $[12, 24]$ serve as priors.

9 Global Energy Shift Fits to Order L^{-6} with Substituted Effective Range Parameter

9.1 Global Energy Shift Fits without Priors

fit intvl	a_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value	$r_{0,\,\mathrm{ptb}}$	$\bar{\mathcal{T}}_{\mathrm{ptb}}$
$[6, 24]$	0.2859(27)	8067(298)	$393.35/74 = 5.32$	0.00	−73.89(81)	27496(524)
$[7, 24]$	0.3126(36)	12978(675)	$247.91/70 = 3.54$	0.00	−66.69(90)	32872(762)
$[8, 24]$	0.3412(52)	22099(1858)	$181.81/66 = 2.75$	9.10e−13	−60(1)	39162(1198)
$[9, 24]$	0.3698(63)	30446(2326)	$106.15/62 = 1.71$	4.11e−04	−55(1)	46002(1572)
$[10, 24]$	0.3753(76)	21753(6300)	$77.17/58 = 1.33$	0.05	−54(1)	47381(1923)
$[11, 24]$	0.3900(89)	29203(7439)	$65.04/54 = 1.20$	0.14	−51(1)	51165(2339)
$[12, 24]$	0.412(11)	42559(12695)	$50.38/50 = 1.01$	0.46	−48(2)	57101(3052)
$[13, 24]$	0.429(13)	47838(15430)	$41.32/46 = 0.90$	0.67	−46(2)	61910(3755)
$[14, 24]$	0.444(17)	36432(24748)	$32.93/42 = 0.78$	0.84	−44(2)	66315(5081)
$[15, 24]$	0.448(18)	59668(38784)	$31.28/38 = 0.82$	0.77	−43(2)	67515(5428)
$[16, 24]$	0.453(21)	67913(47179)	$27.66/34 = 0.81$	0.77	−43(2)	69031(6403)

Table 45: Results from the **relativistic** global energy shift fit (ΔE_N-fit with $N = 2, 3, 4, 5$) **with** exponentially suppressed corrections.

9.2 Global Energy Shift Fits with Priors

fit intvl	a_0	$\bar{\mathcal{T}}$	χ^2/ndof	p-value	$r_{0,\,\mathrm{ptb}}$	$\bar{\mathcal{T}}_{\mathrm{ptb}}$
$[6, 24]$	0.3193(38)	11703(448)	$212.35/56 = 3.79$	0.00	−65.07(91)	34296(821)
$[7, 24]$	0.3378(46)	17367(877)	$150.86/53 = 2.85$	2.50e−11	−60.94(98)	38385(1050)
$[8, 24]$	0.3594(58)	28857(2229)	$114.49/50 = 2.29$	5.74e−07	−57(1)	43451(1407)
$[9, 24]$	0.3908(73)	40486(2967)	$64.72/47 = 1.38$	0.04	−51(1)	51376(1924)
$[10, 24]$	0.3883(87)	31085(7311)	$57.19/44 = 1.30$	0.09	−52(1)	50720(2276)
$[11, 24]$	0.409(11)	45287(9172)	$44.86/41 = 1.09$	0.31	−49(2)	56272(3030)
$[12, 24]$	0.414(11)	40462(13274)	$40.89/38 = 1.08$	0.34	−48(2)	57656(3067)
$[13, 24]$	0.421(12)	41721(15102)	$37.17/35 = 1.06$	0.37	−47(2)	59623(3402)
$[14, 24]$	0.417(13)	5289(23168)	$32.73/32 = 1.02$	0.43	−47(2)	58495(3650)
$[15, 24]$	0.415(13)	22959(36083)	$30.49/29 = 1.05$	0.39	−48(2)	57935(3633)
$[16, 24]$	0.413(13)	15858(42520)	$26.54/26 = 1.02$	0.43	−48(2)	57378(3615)

Table 46: Results from the **relativistic** global energy shift fit (ΔE_N-fit with $N = 3, 4, 5$) **with** exponentially suppressed corrections. Here, the a_0 bootstrap samples obtained from the corresponding ΔE_2-fit (also substituted r_0, see Table 43) within the interval $[12, 24]$ serve as priors.

10 Bare Energy Shift Fits to Order L^{-6} with Substituted Threshold Amplitude

shift	fit intvl	a_0	r_0	χ^2/ndof	p-value	$r_{0,\,\mathrm{ptb}}$	$\bar{\mathcal{T}}_{\mathrm{ptb}}$
	$[6, 24]$	0.3332(68)	−108(3)	$67.89/17 = 3.99$	4.96e−08	−62(1)	37347(1527)
	$[7, 24]$	0.3654(98)	−135(5)	$47.60/16 = 2.97$	5.49e−05	−56(2)	44914(2412)
	$[8, 24]$	0.411(13)	−194(9)	$15.57/15 = 1.04$	0.41	−48(2)	56824(3598)
	$[9, 24]$	0.432(16)	−228(18)	$11.69/14 = 0.83$	0.63	−45(2)	62779(4653)
	$[10, 24]$	0.461(21)	−279(25)	$7.50/13 = 0.58$	0.87	−42(2)	71491(6516)
ΔE_2	$[11, 24]$	0.474(26)	−303(35)	$6.71/12 = 0.56$	0.88	−40(3)	75579(8294)
	$[12, 24]$	0.480(34)	−322(81)	$6.62/11 = 0.60$	0.83	−40(4)	77505(10982)
	$[13, 24]$	0.491(42)	−361(132)	$6.45/10 = 0.65$	0.78	−39(4)	81098(13876)
	$[14, 24]$	0.480(59)	−310(454)	$6.39/9 = 0.71$	0.70	−40(6)	77505(19054)
	$[15, 24]$	0.488(77)	−359(854)	$6.36/8 = 0.80$	0.61	−39(8)	80110(25281)
	$[16, 24]$	0.46(10)	−130(6131)	$6.19/7 = 0.88$	0.52	−42(11)	71181(30949)
	$[6, 24]$	0.3521(60)	−35(1)	$77.89/18 = 4.33$	2.00e−09	−58(1)	41704(1425)
	$[7, 24]$	0.3814(80)	−46(2)	$47.78/17 = 2.81$	9.29e−05	−53(1)	48934(2056)
	$[8, 24]$	0.413(10)	−57(2)	$22.69/16 = 1.42$	0.12	−48(1)	57378(2782)
	$[9, 24]$	0.425(14)	−65(6)	$21.33/15 = 1.42$	0.13	−46(2)	60761(4006)
	$[10, 24]$	0.439(17)	−76(8)	$18.86/14 = 1.35$	0.17	−44(2)	64830(5024)
ΔE_3	$[11, 24]$	0.496(25)	−122(11)	$9.65/13 = 0.74$	0.72	−38(2)	82758(8345)
	$[12, 24]$	0.514(29)	−131(12)	$8.14/12 = 0.68$	0.77	−36(3)	88874(10031)
	$[13, 24]$	0.505(32)	−122(23)	$7.73/11 = 0.70$	0.74	−37(3)	85789(10874)
	$[14, 24]$	0.485(33)	−67(50)	$3.47/10 = 0.35$	0.97	−39(3)	79128(10770)
	$[15, 24]$	0.484(33)	−61(58)	$3.41/9 = 0.38$	0.95	−39(3)	78802(10748)
	$[16, 24]$	0.485(33)	−66(68)	$3.39/8 = 0.42$	0.91	−39(3)	79128(10770)

Table 47: Results obtained from the **relativistic** bare ΔE_2- and ΔE_3-fit to order L^{-6} **with** exponentially suppressed corrections. In case of the ΔE_2-fit the model does not depend on the threshold amplitude $\bar{\mathcal{T}}$ since this parameter only occurs for $N > 2$ (thus no substitution by the expression from perturbation theory). In case of the ΔE_3-fit the a_0 bootstrap samples obtained from the ΔE_2-fit within the interval $[12, 24]$ serve as priors.

shift	fit intvl	a_0	r_0	χ^2/ndof	p-value	$r_{0,\,\mathrm{ptb}}$	$\bar{\mathcal{T}}_{\mathrm{ptb}}$
	$[6,24]$	0.3491(64)	−26.06(76)	$77.45/18 = 4.30$	2.39e−09	−59(1)	40996(1507)
	$[7,24]$	0.3784(85)	−33(1)	$50.41/17 = 2.97$	3.64e−05	−53(1)	48167(2167)
	$[8,24]$	0.410(11)	−42(2)	$27.02/16 = 1.69$	0.04	−48(2)	56548(3038)
	$[9,24]$	0.436(14)	−49(3)	$17.65/15 = 1.18$	0.28	−45(2)	63947(4110)
	$[10,24]$	0.441(18)	−52(6)	$17.50/14 = 1.25$	0.23	−44(2)	65422(5343)
ΔE_4	$[11,24]$	0.471(22)	−68(7)	$11.68/13 = 0.90$	0.55	−41(2)	74626(6974)
	$[12,24]$	0.487(28)	−78(12)	$10.81/12 = 0.90$	0.55	−39(3)	79782(9176)
	$[13,24]$	0.495(30)	−84(13)	$10.16/11 = 0.92$	0.52	−38(3)	82425(9993)
	$[14,24]$	0.477(32)	−55(32)	$7.76/10 = 0.78$	0.65	−40(3)	76539(10271)
	$[15,24]$	0.481(32)	−67(37)	$7.43/9 = 0.83$	0.59	−40(3)	77828(10357)
	$[16,24]$	0.485(33)	−80(39)	$6.89/8 = 0.86$	0.55	−39(3)	79128(10770)
	$[6,24]$	0.3162(57)	−22.76(45)	$112.09/18 = 6.23$	1.33e−15	−66(1)	33633(1216)
	$[7,24]$	0.3503(78)	−27.73(79)	$72.54/17 = 4.27$	7.83e−09	−58(2)	41279(1841)
	$[8,24]$	0.408(12)	−38(1)	$33.20/16 = 2.07$	0.01	−49(2)	55997(3297)
	$[9,24]$	0.436(14)	−41(1)	$17.26/15 = 1.15$	0.30	−45(2)	63947(4110)
	$[10,24]$	0.443(19)	−43(5)	$16.99/14 = 1.21$	0.26	−44(2)	66017(5665)
ΔE_5	$[11,24]$	0.464(21)	−52(5)	$12.07/13 = 0.93$	0.52	−42(2)	72424(6558)
	$[12,24]$	0.480(28)	−60(10)	$11.31/12 = 0.94$	0.50	−40(3)	77505(9044)
	$[13,24]$	0.489(30)	−65(10)	$10.37/11 = 0.94$	0.50	−39(3)	80439(9872)
	$[14,24]$	0.474(31)	−48(20)	$8.27/10 = 0.83$	0.60	−40(3)	75579(9888)
	$[15,24]$	0.482(32)	−66(26)	$7.46/9 = 0.83$	0.59	−40(3)	78152(10379)
	$[16,24]$	0.482(33)	−66(32)	$7.46/8 = 0.93$	0.49	−40(3)	78152(10703)

Table 48: Results obtained from the **relativistic** bare ΔE_4- and ΔE_5-fit to order L^{-6} **with** exponentially suppressed corrections. Here, the a_0 bootstrap samples obtained from the ΔE_2-fit within the interval $[12,24]$ serve as priors.

11 Global Energy Shift Fits to Order L^{-6} with Substituted Threshold Amplitude

11.1 Global Energy Shift Fits without Priors

fit intvl	a_0	r_0	χ^2/ndof	p-value	$r_{0,\,\mathrm{ptb}}$	$\bar{\mathcal{T}}_{\mathrm{ptb}}$
$[6, 24]$	0.2494(22)	−22.18(76)	$680.67/74 = 9.20$	0.00	−86.23(87)	20924(373)
$[7, 24]$	0.2933(34)	−26(1)	$324.62/70 = 4.64$	0.00	−71.77(96)	28938(675)
$[8, 24]$	0.3332(53)	−35(2)	$207.30/66 = 3.14$	1.11e−16	−62(1)	37347(1192)
$[9, 24]$	0.3654(64)	−36(2)	$113.01/62 = 1.82$	8.17e−05	−56(1)	44914(1577)
$[10, 24]$	0.3711(83)	−25(7)	$82.64/58 = 1.42$	0.02	−55(1)	46326(2076)
$[11, 24]$	0.3873(94)	−29(7)	$67.66/54 = 1.25$	0.10	−52(2)	50459(2453)
$[12, 24]$	0.412(12)	−37(10)	$51.07/50 = 1.02$	0.43	−48(2)	57101(3329)
$[13, 24]$	0.428(14)	−35(11)	$41.60/46 = 0.90$	0.66	−46(2)	61622(4034)
$[14, 24]$	0.441(18)	−21(18)	$33.00/42 = 0.79$	0.84	−44(2)	65422(5343)
$[15, 24]$	0.448(20)	−39(29)	$31.30/38 = 0.82$	0.77	−43(2)	67515(6031)
$[16, 24]$	0.454(23)	−43(34)	$27.66/34 = 0.81$	0.77	−43(3)	69336(7027)

Table 49: Results from the **relativistic** global energy shift fit (ΔE_N-fit with $N = 2, 3, 4, 5$) **with** exponentially suppressed corrections.

11.2 Global Energy Shift Fits with Priors

fit intvl	a_0	r_0	χ^2/ndof	p-value	$r_{0,\,\mathrm{ptb}}$	$\bar{\mathcal{T}}_{\mathrm{ptb}}$
$[6, 24]$	0.3013(36)	−22.55(44)	$260.11/56 = 4.64$	0.00	−69.58(96)	30538(734)
$[7, 24]$	0.3244(44)	−27.19(83)	$180.40/53 = 3.40$	8.88e−16	−64(1)	35400(964)
$[8, 24]$	0.3531(61)	−38(2)	$129.16/50 = 2.58$	6.28e−09	−58(1)	41941(1453)
$[9, 24]$	0.3872(74)	−40(2)	$68.48/47 = 1.46$	2.20e−02	−52(1)	50433(1932)
$[10, 24]$	0.3861(95)	−34(7)	$62.24/44 = 1.41$	0.04	−52(2)	50147(2471)
$[11, 24]$	0.409(11)	−42(7)	$47.58/41 = 1.16$	0.22	−49(2)	56272(3030)
$[12, 24]$	0.415(13)	−40(12)	$43.89/38 = 1.15$	0.24	−48(2)	57935(3633)
$[13, 24]$	0.438(14)	−43(11)	$29.87/35 = 0.85$	0.71	−45(2)	64535(4129)
$[14, 24]$	0.465(20)	−44(17)	$20.07/32 = 0.63$	0.95	−41(2)	72737(6260)
$[15, 24]$	0.468(22)	−59(24)	$19.33/29 = 0.67$	0.91	−41(2)	73678(6929)
$[16, 24]$	0.465(23)	−54(30)	$18.46/26 = 0.71$	0.86	−41(3)	72737(7198)

Table 50: Results from the **relativistic** global energy shift fit (ΔE_N-fit with $N = 3, 4, 5$) **with** exponentially suppressed corrections. Here, the a_0 bootstrap samples obtained from the corresponding ΔE_2-fit (see Table 47) within the interval $[12, 24]$ serve as priors.

12 Global Energy Shift Fits to Order L^{-6} with Substituted Effective Range Parameter as well as Threshold Amplitude

12.1 Global Energy Shift Fits without Priors

fit intvl	a_0	χ^2/ndof	p-value	$r_{0,\,\mathrm{ptb}}$	$\bar{\mathcal{T}}_{\mathrm{ptb}}$
$[6, 24]$	0.2112(14)	$8248.17/75 = 109.98$	0.00	−103.71(78)	15005(202)
$[7, 24]$	0.3250(36)	$2235.11/71 = 31.48$	0.00	−63.75(83)	35532(792)
$[8, 24]$	0.3835(59)	$408.93/67 = 6.10$	0.00	−52.44(97)	49474(1527)
$[9, 24]$	0.4002(69)	$183.45/63 = 2.91$	1.06e−13	−50(1)	53877(1863)
$[10, 24]$	0.4052(78)	$117.82/59 = 2.00$	8.47e−06	−49(1)	55231(2131)
$[11, 24]$	0.4108(92)	$86.32/55 = 1.57$	4.45e−03	−48(1)	56769(2547)
$[12, 24]$	0.423(11)	$61.93/51 = 1.21$	0.14	−47(1)	60190(3134)
$[13, 24]$	0.433(12)	$49.48/47 = 1.05$	0.37	−45(2)	63070(3499)
$[14, 24]$	0.458(15)	$27.68/43 = 0.64$	0.97	−42(2)	70563(4625)
$[15, 24]$	0.458(18)	$26.56/39 = 0.68$	0.94	−42(2)	70563(5549)
$[16, 24]$	0.460(20)	$24.89/35 = 0.71$	0.90	−42(2)	71181(6192)

Table 51: Results from the **relativistic** global energy shift fit (ΔE_N-fit with $N = 2, 3, 4, 5$) **with** exponentially suppressed corrections.