MATH 486 - Fall 2025

Mathematical Theory of Games

Pennsylvania State University

Section 1: MWF 10:10am - 11:00am 011 Huck Life Sciences Bldg

Course Description: This one semester course provides an introduction to the mathematical theory of strategic interactions known as Game Theory, which I would define as *two or more individuals making decisions in the presence of the mutual uncertainty due to those decisions.* A relatively recent development in mathematics, Game Theory began in the 20th century, established by the efforts of John Von Neumann, John Nash, and others. It has become a useful approach for mathematical modeling in economics, biology, social phenomenon, and other areas. In this class we will mainly follow the required text, supplemented by additional reading. Assessments will include regular written homework assignments, quizzes, in-class work, three exams during the semester, and a final exam.

Instructor: Andrew Belmonte, Professor of Mathematics / Materials Science & Engineering
office: 322 McAllister Bldg email: belmonte@psu.edu.

Textbooks and Other Resources:

1. **(Required)** – Stephen Schecter and Herbert Gintis, *Game Theory in Action – An Introduction to Classical and Evolutionary Models* (Princeton University Press, 2016) ISBN-13: 9780691167657

2. **(Required)** – Kevin Leyton-Brown and Yoav Shoham, *Essentials of Game Theory: A Concise Multidisciplinary Introduction* (Princeton University Press, 2008) ISBN: 978-1598295931

3. **(Supplementary)** – Christopher H Griffin, *Game Theory Explained: A Mathematical Introduction With Optimization* (World Scientific, 2025) ISBN: 978-9811297212

4. Articles from the literature or other online articles, class notes, etc.

The required textbooks are also available to you electronically via the Penn State Libraries system; other materials will be provided on Canvas.

Material Covered During the Semester / Learning Goals: The course can be divided into four sections, which will include some or all of the following:

1. Introducing Games: Strategic Choices and Random Events
    - Elements of probability theory, random variables, expected values
    - Conditional probability, Bayes Theorem
    - Definition of a Game, Payoffs and Utility functions
    - Gambling as Games Against the House
2. Simultaneous Games - Discrete and Continuum Strategies
    - Normal form / matrix games
    - Common knowledge, rationalizable strategies
    - Best response, Dominance and Weak Dominance
    - Iterated dominance, Dominance solvable, Nash equilibrium (pure)
    - Prisoner's Dilemma, Snowdrift, other dilemmas
    - Zero-sum games, strictly competitive games
    - Security strategies, no-regret, risk
    - Social Utility, Pareto Optimality
    - Multiplayer games, Public Goods games
    - Mixed strategies and Mixed Nash equilibria
    - Cournot Duopoly and other continuum games, Best Response functions
    - Stackelberg games, introducing temporal sequence
3. Sequential Games
    - Decision trees / Extensive form games
    - Backwards induction, Ultimatum games
    - Equivalent normal form, Subgame Perfect Equilibrium
    - Conditional dominance, forward induction
    - Cooperative games, coalitions, Bargaining problems, Value of a Game
    - Fairness, Altruism, Trust, Punishment
4. Iterated Games, Evolutionary Games, Lack of Information
    - Repeated games, Tit-for-Tat, grim trigger strategy
    - Incomplete information, Bayesian normal form games
    - Evolutionarily Stable Strategies (ESS) / Evolutionary games

PREREQUISITES: MATH 220 (Matrices) and its prerequisite, MATH 140 / 140B / 110; or consent of the instructor.

HIGHLY RECOMMENDED: 2nd Semester Calculus (Math 141 or 141B), Multivariable Calculus (Math 230, 231, or 240).

ATTENDANCE AND PARTICIPATION: Your presence in class, contributing to discussions with questions or comments, are an essential aspect of how this course should run. I strongly encourage you to attend every class!

OFFICE HOURS: TBA, and by appointment.

GRADING: The grades for this course will be determined as follows:

1. WRITTEN HOMEWORK (10%) The assigned problems comprise an essential aspect of this course, requiring an investment of time and careful thought. An investment which will pay off! Your solutions to assigned problems should be written up carefully and in a well organized manner, and should involve clear explanations of reasoning and critical steps. Keep in mind that the reader of your solutions (me, the grader) cannot read your thoughts. You are encouraged to discuss the homework assignments with other students, however you must write up your solutions *individually*, and of course it must be your own work.

   Written Homework will be submitted through the online system Gradescope, and must be scanned or written in a legible way. The late policy for homework is: 50% of the remaining points will be docked for *every day* the assignment is turned in late.

2. QUIZZES (20%) These will be given throughout the semester, mostly **in-class** after each problem set, but occasionally as Canvas quizzes covering important concepts. The quizzes are an important intermediary between your course work at home (reading the texts, doing the HW problems, discussing, etc) and your individual responsibility for having learned the material as tested without books, notes, or the assistance of technology. Solutions to all quizzes will be provided for discussion and study.

3. MIDTERM EXAMS (40%) There will be three **in-class** assessments given during the semester: the first will be weighted as 10%, and the remaining two will each be 15%. No books, notes, or internet access may be used during the examinations. However, calculators will be allowed! You must bring your University ID card to all exams. The schedule will be posted here:

   Exam 1 TBA
   Exam 2 TBA
   Exam 3 TBA

If you are unable to attend the scheduled exams, you *must let me know in advance* - arrangements will be made for a makeup exam as needed. It is your responsibility to notify me of this conflict.

4. Final Exams (30%) The final exam for this course will be cumulative / comprehensive:

Final Exam TBA (scheduled by the University during finals week)

Active Learning Activities: We will occasionally spend some class time focused on solving game theory related exercises in small groups; you will have the option to work by yourself. The instructor will be available during this time to clarify or discuss these exercises with you. Your work will be collected, and graded equivalently to an assigned problem set.

Math 486 Game Night: With sufficient student interest, a weekly game night will be held - reserved room TBA. A certain number of card decks and chess boards will be provided. All registered students in Math 486 this semester will be welcome to attend.

Tutors and Math Center: Free mathematics tutoring is available at Penn State Learning, located in 220 Boucke Building. More information can be found here.

Final Grades: Final course grades will be assigned as follows:

Grade	Percent Score
A	93% - 100%
A-	90% - 92%
B+	87% - 89%
B	83% - 86%
B-	80% - 82%
C+	77% - 79%
C	70% - 76%
D	60% - 69%
F	0% - 59%

Deferred Grade: Students who are currently passing a course but are unable to complete the course because of illness or emergency may be granted a deferred grade which will allow the student to complete the course within the first six weeks of the following semester. Note that deferred grades are limited to those students who can verify and document a valid reason for not being able to take the final examination. For more information see DF grade.

LATE-DROP: Students may add/drop a course without academic penalty within the first ten calendar days of the semester. A student may late drop a course within the first twelve weeks of the semester but accrues late drop credits equal to the number of credits in the dropped course. A baccalaureate student is limited to 16 late drop credits. The late drop deadline for Fall 2025 is Friday November 14, 2025, at 11:59pm (ET).

ACADEMIC INTEGRITY: All Penn State policies regarding ethics and honorable behavior apply to this course (see links below for policy statements). Academic integrity is the pursuit of scholarly activity free from fraud and deception and is an educational objective of this institution. All University policies regarding academic integrity apply to this course. Academic dishonesty includes, but is not limited to, cheating, plagiarizing, fabricating of information or citations, facilitating acts of academic dishonesty by others, having unauthorized possession of examinations, submitting work of another person or work previously used without informing the instructor, or tampering with the academic work of other students. For any material or ideas obtained from other sources, such as text you find on the web, in the library, etc., a source reference must be given. Direct quotes from any source must be identified as such. All exam answers must be your own, and you must not provide any assistance to other students during exams. Any instances of academic dishonesty **WILL** be pursued under the regulations concerning academic integrity of the University and the Eberly College of Science.

Academic dishonesty is not limited to simply cheating on an exam or assignment. The following is quoted directly from the "PSU Faculty Senate Policies for Students" regarding academic integrity and academic dishonesty:

> *Academic integrity is the pursuit of scholarly activity free from fraud and deception and is an educational objective of this institution. Academic dishonesty includes, but is not limited to, cheating, plagiarizing, fabricating of information or citations, facilitating acts of academic dishonesty by others, having unauthorized possession of examinations, submitting work of another person or work previously used without informing the instructor, or tampering with the academic work of other students.*

All University and Eberly College of Science policies regarding academic integrity/academic dishonesty apply to this course and the students enrolled in this course. Refer to the following URL for further details on the academic integrity policies of the Eberly College of Science: https://science.psu.edu/current-students/integrity. Each student in this course is expected to work entirely on her/his own while taking any exam, to complete assignments on her/his own effort without the assistance of others unless directed otherwise by the instructor, and to abide by University and Eberly College of Science policies about academic integrity and academic dishonesty. Academic dishonesty can result in assignment of "F" by the course instructors or "XF" by Judicial Affairs as the final grade for the student.

In an examination setting, unless the instructor gives explicit prior instructions to the contrary, violations of academic integrity shall consist of any attempt to receive assistance from

written or printed aids, from any person or papers or electronic devices, or of any attempt to give assistance, whether the student doing so has completed their own work or not. Other violations include, but are not limited to, any attempt to gain an unfair advantage with regard to an examination, such as tampering with a graded exam or claiming another's work to be one's own. Failure to comply will lead to sanctions against the student in accordance with the Policy on Academic Dishonesty in the Eberly College of Science.

Students with Disabilities: Penn State welcomes students with disabilities into the University's educational programs. Every Penn State campus has an office for students with disabilities. Student Disability Resources (SDR) website provides contact information for every Penn State campus: http://equity.psu.edu/sdr/disability-coordinator. For further information, please visit Student Disability Resources website: http://equity.psu.edu/sdr.

Counseling and Psychological Services: Many students at Penn State face personal challenges or have psychological needs that may interfere with their academic progress, social development, or emotional wellbeing. The university offers a variety of confidential services to help you through difficult times, including individual and group counseling, crisis intervention, consultations, online chats, and mental health screenings. These services are provided by staff who welcome all students and embrace a philosophy respectful of clients' cultural and religious backgrounds, and sensitive to differences in race, ability, gender identity and sexual orientation:

1. Counseling and Psychological Services at University Park (CAPS):
   (http://studentaffairs.psu.edu/counseling/): 814-863-0395
2. Counseling and Psychological Services at Commonwealth Campuses:
   (http://senate.psu.edu/faculty/counseling-services-at-commonwealth-campuses/)
3. Penn State Crisis Line (24 hours/7 days/week): 877-229-6400
   Crisis Text Line (24 hours/7 days/week): Text LIONS to 741741

Educational Equity and Bias Response: Consistent with University Policy AD29, students who believe they have experienced or observed a hate crime, an act of intolerance, discrimination, or harassment that occurs at Penn State are urged to report these incidents as outlined on the University's Bias Response webpage: https://equity.psu.edu/bias-response