TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
# Examination Control Division
**2080 Chaitra**

| Exam. | Regular | | |
|---|---|---|---|
| **Level** | BE | **Full Marks** | 80 |
| **Programme** | BEL, BEI, BEX, BCT, BGE | **Pass Marks** | 32 |
| **Year / Part** | II / II | **Time** | 3 hrs. |

## *Subject:* - Applied Mathematics (*SH 551*)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt* ***All*** *questions.*
- ✓ *The figures in the margin indicate* ***Full Marks****.*
- ✓ *Assume suitable data if necessary.*

1. Define harmonic function of a complex variable. Show that $u(x,y) = y^3 - 3x^2y$ is harmonic and find its corresponding analytic function. [5]
2. Find the linear transformation which maps the points $z = 2, i, -2$ into the points $w = 1, i, -1$. [5]
3. Evaluate $\int_C \frac{\cos \pi z^2}{(z-1)(z-2)} dz$ where $C: |z| = 3$ by using Cauchy's integral formula. [5]
4. Obtain the Laurent's series expansion of function $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in the region $2 < |z| < 3$ . [5]
5. Define zeros and poles of order m for function of a complex variable. Find poles and residues of $f(z) = \frac{z(z-2)}{(z+1)^2(z^2+1)}$ [5]
6. Evaluate integral $\int_0^{2\pi} \frac{1}{2+\cos\theta} d\theta$ by contour integration in the complex plane. [5]
7. Find the Z-transform of;$(i) t^2 e^{-at}$ $, t \geq 0$ $(ii) \sin h\, k\theta$ $, k \geq 0$ [2.5+2.5]
8. Obtain the Z-transform of $(1 - e^{-at}), t \geq 0$ and hence evaluate $x(\infty)$ by using the final value theorem. [5]
9. Find the inverse Z-transform of function $X(z) = \frac{z^2}{(z+1)(z-1)^2}$ . [5]
10. Solve the difference equation $x(k+2) + 5x(k+1) + 6x(k) = 2^k$ given that $x(0) = 0,\ x(1) = 1.$ [5]
11. Solve the one-dimensional wave equation for a tightly stretched string of length $\ell$ fixed at both ends if the initial deflection is $u(x,0) = \ell x - x^2$ and the initial velocity is zero. [10]
12. Derive one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ and find it's all possible solutions. [10]
13. Find Fourier cosine integral of $f(x) = e^{-x}$ and hence show that $\int_0^\infty \frac{\cos \omega x}{1+\omega^2} d\omega = \frac{\pi}{2} e^{-x}$. [5]
14. Find the Fourier transform of $f(x) = \begin{cases} 1 & for\ |x| < 1 \\ 0 & for\ |x| > 1 \end{cases}$ and then evaluate $\int_0^\infty \frac{\sin x}{x} dx$ [5]

***

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
**Examination Control Division**
**2079 Chaitra**

| | | | |
|---|---|---|---|
| **Exam.** | **Regular** | | |
| **Level** | BE | **Full Marks** | 80 |
| **Programme** | BEL, BEI, BEX, BCT, BGE | **Pass Marks** | 32 |
| **Year / Part** | II / II | **Time** | 3 hrs. |

## *Subject:* - Applied Mathematics (*SH 551*)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt* ***All*** *questions.*
- ✓ *The figures in the margin indicate* ***Full Marks****.*
- ✓ *Assume suitable data if necessary.*

1. Define harmonic function. Determine the analytical function $f(z) = u + iv$ if $u = 3x^2y - y^3$. [1+4]
2. Find the linear fractional transformations that maps $z = 0, -i, 2i$ into the points $w = 5i, \infty, -i/3$ respectively. [5]
3. State cauchy's integral theorem. Apply cauchy's integral formula to evaluate $$\int_c \frac{e^z}{(z-1)(z-3)}dz \quad \text{where } C: |z| = 2.$$ [1+4]
4. Expand the laurent's series of the function $$f(z) = \frac{1}{z^2 - 3z + 2} \text{ in the region } |<|z| < 2.$$ [5]
5. Evaluate $\int_c \frac{2z-1}{z(z+1)(z-3)}dz$ where C is the circle $|z| = 2$ by residue method. [5]
6. Evaluate $\int_0^{2\pi} \frac{2d\theta}{2+\cos\theta}$ by contour integration. [5]
7. Find the z-transform of $a^k$ for $k \geqq 0$, and hence obtain the z-transform of $a^k \sin k\theta$. [2+3]
8. State and prove initial value theorem. [5]
9. Find the inverse $Z$ – transform of $\frac{z}{(z-1)^2(z-2)}$ by inversion integral method. [5]
10. Solve the difference equation [5]
    $x(k+2) - 4x(k+1) + 4x(k) = 0$ with given conditions $x(0) = 0, x(1) = 1$.
11. Derive one dimensional wave equation and find its all possible solutions. [10]
12. Solve the one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ with boundry and initial condition $$u(0,t) = 0, u(\int \iota, t) = 0 \text{ and } u(x,0) = \frac{100x}{\iota}$$ [10]
13. Find that the Fourier Cosine integral representation of [5]
    $f(x) = e^{-kx}$ $(x > 0, k > 0)$ and
    Hence show that $\int_0^{\infty} \frac{\cos\omega x}{k^2+\omega^2}d\omega = \frac{\pi}{2k}e^{-kx}, x > 0, k > 0.$
14. Find the Fourier transform of the function $e^{-x^2}$, also verify that convolution theorem for the functions $f(x) = e^{-x^2}$ and $g(x) = e^{-x^2}$ [5]

***

TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

# Examination Control Division

**2078 Chaitra**

| **Exam.** | | **Regular** | |
|---|---|---|---|
| **Level** | BE | **Full Marks** | 80 |
| **Programme** | BEL,BEX,BCT BEI,BGE | **Pass Marks** | 32 |
| **Year / Part** | II / II | **Time** | 3 hrs. |

## *Subject:* - Applied Mathematics *(SH 551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Define an analytic function f(z) of complex variable z at a point. If $f(z) = u(x,y) + i\,v(x,y)$ is analytic, show that $u_x = v_y$ and $u_y = -v_x$. [1+4]
2. Define conformal mapping. Find the linear transformation which maps the points $z = 0,1,\infty$ in to the points $w = -3, -1, 1$ respectively. [5]
3. State and proof Cauchy's Integral theorem. [5]
4. Obtain the Taylor's series expansion of the complex function $f(z) = \frac{z+1}{(z-3(z-4)}$ about the center $z = 2$ up to four term. [5]
5. State Cauchy residue theorem. Apply it to evaluate $\int_c \frac{4-3z}{z(z-1)(z-2)}dz$ where C is the circle $|z| = \frac{3}{2}$. [1+4]
6. Evaluate integrals $\int_0^{\pi} \frac{1}{3+2\cos\theta}d\theta$ by contour integration. [5]
7. If $x(t) = 0$ for $t < 0, Z[x(t)] = X(z)$ for $t \geq 0$, then prove that $Z[e^{-at}x(t)] = X(ze^{aT})$. [5]
8. Obtain the Z- transform of (i) $te^{-at}$ (ii) sin at [2.5+2.5]
9. Obtain the inverse Z- transform of $X(z) = \frac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$ where T is the sampling period. [5]
10. Solve the difference equation $y_{n+2} - 4y_{n+1} + 4y_n = 0$ with given condition $y_0 = 0, y_1 = 1$. [5]
11. A tightly stretched string with fixed ends $x = 0$ and $x = \ell$ is initially in position given by $u(x,0) = u_0 \sin^3\frac{\pi x}{\ell}$ If it is released from rest in this position, find the displacement at any time t at any distance x from one end. [10]
12. Derive one dimensional heat equation and solve it completely. [10]
13. Obtain the fourier sine and cosine integral of $f(x) = x$ for $0 < x < a$, [5]

    $= 0$ for $x > a$.
14. Find the fourier cosine transform of $f(x) = e^{-x}, x > 0$ and hence parseval's identity, show that $\int_0^{\infty} \frac{1}{(1+x^2)^2}dx = \frac{\pi}{4}$. [5]

***

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
**Examination Control Division**
**2077 Chaitra**

| **Exam.** | **Regular** | | |
|---|---|---|---|
| **Level** | BE | **Full Marks** | 80 |
| **Programme** | BEL, BEX, BCT, BEI, BGE | **Pass Marks** | 32 |
| **Year / Part** | II / II | **Time** | 3 hrs. |

## ***Subject:*** - Applied Mathematics *(SH 551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) State Cauchy-Riemann equation in polar form. Prove that f(z) = |z| is not an analytic function. [1+4]

   b) Find the linear transformation which maps the points z = 0, 1, ∝ into the points w = -3, -1, 1 respectively. Find also fixed point of the transformation. [4+1]

2. a) State and prove Cauchy's Integral Formula. [1+4]

   b) Find the Laurent's series of $f(x) = \dfrac{z^2 - 1}{(z+2)(z+3)}$ in the region 2 < |z| < 3. [5]

3. a) State Cauchy Residue theorem and hence evaluate the integral $\int_C \dfrac{z-1}{(z+1)^2(z-2)} dz$ where C : |z-i| = 2. [1+4]

   b) Using counter Integration, evaluate $\int_0^{2\pi} \dfrac{1}{2+\cos\theta} d\theta$ in the complex plane. [5]

4. a) State and prove initial value theorem of z-transform. [5]

   b) Find the z-transform of the following sequence for t ≥ 0. [2.5+2.5]

   (i) $te^{-at}$ (ii) sinat

5. a) Find the inverse z-transform of the function $\dfrac{2z^3 + z}{(1-2)^2(z-1)}$. [5]

   b) Solve the difference equation $x(k+2) - 4x(k+1) + 4x(k) = 0$ with conditions x(0) = 1, x(1) = 0. [5]

6. Derive one dimensional heat equation and solve it completely. [5+5]

7. A string is stretched and fastened to two points apart. Motion is started by displacing the string in the form $u(n, 0) = u_0 \sin\dfrac{\pi x}{l}$ from which it is released at time t=0. Show that the displacement at any point at a distance x from one end at a time t is given by $u(x, t) = u_0 \sin\dfrac{\pi x}{l} \cos\dfrac{\pi ct}{l}$. [10]

8. a) Define the complex form of Fourier integral of a given function with usual notation. Find the Fourier integral representation of the function $f(x) = \begin{pmatrix} 1 & if & |x| < 1 \\ 0 & if & |x| > 1 \end{pmatrix}$ and hence evaluate $\int_0^{\infty} \dfrac{\sin w}{w} dw$. [1+3+1]

   b) Find the Fourier sine transformation of $f(x)=e^{-|x|}$ and hence evaluate the integral $\int_0^{s} \dfrac{s \sin sx}{s^2+1} ds$. [5]

***

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
# Examination Control Division
**2076 Baisakh**

| Exam. | | Back | |
|---|---|---|---|
| **Level** | BE | **Full Marks** | 80 |
| **Programme** | BEL, BEX, BCT, BGE | **Pass Marks** | 32 |
| **Year / Part** | II / II | **Time** | 3 hrs. |

## *Subject:* - Applied Mathematics *(SH 551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Obtain polar form Cauchy-Riemann equations for function of complex variable. [5]
2. Find the linear fractional transformation which maps the points z=0,1,∞ into the points w= -3,-1,1 respectively. [5]
3. Define Complex integration. How does it differ from real integration? Derive Cauchy integral formula for function f(z). [1+1+3]
4. Define Laurent's Series for the function of complex variable. Obtain Taylor's series for function [1+4]

$$f(z)=\frac{z}{z^2+4} \text{ about } z=i$$

5. State Cauchy residue theorem. Apply it to evaluate $\oint_c \tan z\,dz$, where c is the region |z|=2. [1+4]
6. Evaluate integral $\int_0^{2\pi}\frac{d\theta}{a+\sin\theta}$; a>1 by contour integration in complex plane. [5]
7. Obtain z-transform of sin ωt and hence obtain z-transform of $e^{at}\sin\omega t$. [3+2]
8. Obtain the inverse z-transform of $X(z)=\frac{z^2}{(z-1)^2(z-e^{-aT})}$ [5]
9. State and prove shifting to the right theorem for z-transform. [5]
10. Solve the difference equation: [5]

    $x(k+2)-x(k+1)+0.25x(k)=u(k)$ where x(0)=1 and x(1)=2 and u(k) is a unit step function; by z-transform method.
11. Find Fourier integral of the function [5]

$$f(x)=\begin{cases}0 & \text{if } x<0\\ e^{-x} & \text{if } x<0\end{cases}$$

12. Find the Fourier Sine transform of $e^{-x}$, x≥0 and hence show that $\int_0^{\infty}\frac{x^2}{(1+x^2)^2}dx=\frac{\pi}{4}$ [5]
13. Solve the wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$ with the given boundary conditions; u(0,t)=0, u(π,t)=0, u(x,0)=0 and $\left(\frac{\partial u}{\partial t}\right)_{t=0}=3(Lx-x^2)$. [10]
14. Derive one dimensional heat equation and solve it completely. [10]

***

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
## Examination Control Division
**2076 Bhadra**

| Exam. | Regular | | |
|---|---|---|---|
| Level | BE | Full Marks | 80 |
| Programme | BEL, BEX, BCT, BGE | Pass Marks | 32 |
| Year / Part | II / II | Time | 3 hrs. |

### *Subject:* - Applied Mathematics *(SH 551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) If $u=(x-1)^3-3xy^2+3y^2$, then find v and construct an analytic function $f(z)=u+iv$. [5]

   b) Define conformal mapping. Find mobius transformation which maps the points z = 0, 1, ∞ into the points w = -3, -1, 1 respectively. [1+4]

2. a) Define Complex integral. Using Cauchy's integral formula evaluate, $\int_C \frac{\sin z}{z^6}dz$ where C : $|z|=1$. [1+4]

   b) Obtain the Laurrent's Series expansion of $f(z)=\frac{z^2-1}{(z+2)(z+3)}$ in the region $2<|z|<3$ [5]

3. a) Evaluate $\int_0^{2\pi}\frac{d\theta}{1+\frac{1}{2}\cos\theta}$, applying contour integration in the complex plane. [5]

   b) Define the pole of order m of a function of a complex variable. Find the residue of $f(z)=\frac{z^2}{(z-1)^2(z+2)}$ at its poles. [1+4]

4. a) Obtain the Fourier integral of the function $f(x)=\begin{cases}1, & \text{for } |x|<1,\\ 0, & \text{for } |x|>1\end{cases}$

   and hence evaluate $\int_0^{\infty}\frac{\sin x}{x}dx=\frac{\pi}{2}$ [3+2]

   b) State and prove the convolution theorem for Fourier transform. [5]

5. a) Find the z-transform of x(t) = Sin$\omega$t, for $t \geq 0$. [5]

   b) Find the inverse z-transform of:

   (i) $\frac{z^2}{z^2-3z+2}$ by partial fraction method [2]

   (ii) $\frac{z}{(z+2)^2(z-1)}$ by inversion integral method [3]

6. a) State and prove shifting to the right theorem for z-transform. [5]

   b) Solve the difference equation $x(k+2)+3x(k+1)+2x(k)=0$, with the conditions x(0) = 0, x(1) = 1 [5]

7. Solve the wave equation for a tightly stretched string of length L fixed at both ends if the initial deflection is $u(x, 0) = Lx - x^2$ and the initial velocity is zero. [10]

8. Change the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ into polar form. [10]

***

25 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

# Examination Control Division

**2075 Bhadra**

| **Exam.** | **Regular** | | |
|---|---|---|---|
| **Level** | BE | **Full Marks** | 80 |
| **Programme** | BEL,BEX, BCT, BGE | **Pass Marks** | 32 |
| **Year / Part** | II / II | **Time** | 3 hrs. |

## *Subject:* - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Define harmonic function of complex variable. Show that $u(x,y)=y^3-3x^2y$ is harmonic and find corresponding analytic function. [1+4]
2. Define conformal mapping for function of complex variable. Show that function of complex variable $w=iz$ is transformed through an angle $\frac{\pi}{2}$ in w-plane. [1+4]
3. State and prove Cauchy's integral theorem. [5]
4. Define Laurent's Series for the function of complex variable. Find Laurent's series of the function $f(z)=\frac{z}{(z+2)(z+3)}$ in the region $2<|z|<3$. [1+4]
5. Define pole of order m for function of complex variable. Find residues of $f(z)=\frac{z^2-2z}{(z+1)^2(z^2+1)}$ at its poles. [1+4]
6. Evaluate $\int_{-\infty}^{\infty}\frac{x^2}{(x^2+1)(x^2+4)}dx$ by contour integration in the complex plane. [5]
7. Find the Z-transform of: [3+2]
   i) $t^2e^{at}$
   ii) $e^{-at}\cos wt$
8. Find the inverse Z-transform of: [2.5+2.5]
   i) $X(z)=\frac{2z^2-5z}{(z-2)(z-3)}$ (By partial fraction method)
   ii) $X(z)=\frac{z^{-2}}{(1-z^{-1})^3}$ (By inversion integral method)
9. State final value theorem for Z-transform. Obtain Z-transform of $(1-e^{-at})$; a>0 and hence evaluate $x(\infty)$ by using final value theorem. [1+4]
10. Solve the difference equation: [5]
    $x(k+2)-3x(k+1)+2x(k)=0$; given that $x(0)=0$ and $x(1)=1$ by using z-transform method.
11. Find the Fourier integral of the function: [5]
    $$f(x)=\begin{cases}1, & \text{for } 0<x<\pi\\ 0, & \text{for } x>\pi\end{cases}$$

12. Find the Fourier transform of $e^{-x^2}$. Also verify the convolution theorem for $f(x)=e^{-x^2}$ and $g(x)=e^{-x^2}$ [5]

13. Derive one dimensional wave equation and solve it completely. [10]

14. Solve completely the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ under the conditions: [10]

$$u(0,y)=u(l,y)=u(x,o)=0, u(x,\infty)=\sin\left(\frac{n\pi x}{l}\right)$$

***

14 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

**Examination Control Division**

**2075 Baisakh**

| Exam. | Back | | |
|---|---|---|---|
| Level | BE | Full Marks | 80 |
| Programme | BGE, BEL, BEX, BCT | Pass Marks | 32 |
| Year / Part | II / II | Time | 3 hrs. |

***Subject:*** - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *The figures in the margin indicate Full Marks.*
- ✓ *Assume suitable data if necessary.*

1. a) Define harmonic function. Is V = arg(z) is harmonic? If yes, find a corresponding harmonic conjugate. [1+1+3]

   b) Define conformal mapping. Find the bilinear transformation which maps the points z = 0, 1, ∞ into the points w = -3, -1, 1 respectively. [1+4]

2. a) Distinguish between Cauchy integral Theorem and Cauchy integral formula. Using Cauchy integral formula evaluate $\int_c \frac{e^z}{(z+1)(z-2)}dz$ where C is the circle $|z-1|=3$. [1+4]

   b) State and Prove Taylor's series for function of complex variable. [5]

3. a) Define an isolated pole. Using Cauchy's residue theorem evaluate $\int_c \frac{z-1}{(z+1)^2(z-2)}dz$ where C is the circle $|z-i|=2$. [5]

   b) Evaluate the integral by contour integration: [5]

   $$\int_{-\infty}^{\infty} \frac{x^2}{(1+x^2)(x^2+4)}dx$$

4. a) Obtain the z-transform of $(1-e^{-at})$, a > 0 and hence evaluate $x(\infty)$ by using final value theorem. [2+3]

   b) Obtain the inverse z-transform of:

   $$X(z)=\frac{2z^3+z}{(z-2)^2(z-1)}$$ by using partial fraction method. [5]

5. a) Define z-transform of function f(t). Find the z-transform of following sequences: [1+2+2]

   (i) $f(k)=\left\{\begin{matrix}15,10,7,4,1,-1,3,6\\ \quad\quad\quad\quad\uparrow\end{matrix}\right\}$

   (ii) $f(k)=\begin{cases}5^k & ; \quad k<0\\ 2^k & ; \quad k\geq 0\end{cases}$

   b) Solve the difference equation by the application of z-transform:

   $x(k+2)+3x(k+1)+2x(k)=0$ with conditions x(0) = 0, x(1) =1. [5]

6. a) A tightly stretched string with fixed ends at $x = 0$ and $x = l$ is initially at rest in its equilibrium position. Find the deflection $u(x, t)$ if it is set vibrating by giving to each of its points a velocity $3(lx-x^2)$. [10]

b) Derive two dimensional heat equation. [10]

7. a) Obtain the Fourier sine integral representation of $e^{-x}\cos x$ and hence show that

$$\int_0^{\infty}\frac{\omega^3 \sin \omega x}{\omega^4+4}d\omega = \frac{\pi}{2}e^{-x}\cos x, \quad x>0.$$ [5]

b) Find the Fourier Cosine transform of $f(x) = e^{-x}, x > 0$ and hence by Parseval's identity, show that [5]

$$\int_0^{\infty}\frac{1}{\left(1+x^2\right)^2}dx = \frac{\pi}{4}.$$

14 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
# Examination Control Division
**2074 Bhadra**

| Exam. | | Regular | |
|---|---|---|---|
| Level | BE | Full Marks | 80 |
| Programme | BGE, BEL, BEX, BCT | Pass Marks | 32 |
| Year / Part | II / II | Time | 3 hrs. |

## *Subject*: - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) Define an analytic function for a function of complex variable. Derive Cauchy Riemann equations in Cartesian form. [1+4]

   b) Define linear fractional mapping. Find bilinear mapping which maps the points z = 0, 1, -1 to w = i, 2, 4. [1+4]

2. a) State and Prove Cauchy integral theorem. [5]

   b) Point out difference between Taylor's series and Laurent's series. Find Laurent' series of function $f(z) = \frac{\sin z}{z^6}$ , $0 < |z| < TR$ [1+4]

3. a) Define pole of order m. Using Cauchy's residue theorem evaluate $\int_c \cot z \, dz$; where C is $|z| = 1$. [1+4]

   b) Using Counter integration evaluate, [5]

   $$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^2}.$$

4. a) Find the z-transform of: [2+3]

   (i) cosat (ii) $te^{-at}$

   b) State final value theorem. If x(t) = 0 for t < 0 and Z[x(t)] = X(z) for t ≥ 0 then prove that: [1+4]

   $$Z[x(t+nT)] = z^n\left[X(z) - \sum_{k=0}^{n-1} x(kT)z^k\right].$$

5. a) Obtain inverse Z-transform of $\frac{z(3z^2-6z+4)}{(z-1)^2(z-2)}$. [5]

   b) Solve the difference equation by the application of z-transform:

   x (k+2) -4x (k+1) + 4x(k) = 0; with conditions x(0) = 1; x(1) = 0. [5]

6. a) Derive one dimensional wave equation and solve it completely. [5+5]

   b) A uniform rod of length $\ell$ has its end maintained at a temperature 0°C and the initial temperature of the rod is:

   $$u(x,0) = 3\sin\frac{\pi x}{\ell} \quad \text{for } 0<x<\ell.$$

   Find the temperature u(x, t). [10]

7. a) Find Fourier integral of the function [5]

   $$f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$

   b) Verify the convolution theorem for Fourier transform for the functions $f(x) = g(x) = e^{-x^2}$. [5]

***

14 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
**Examination Control Division**
**2073 Bhadra**

| Exam. | Regular | | |
|---|---|---|---|
| Level | BE | Full Marks | 80 |
| Programme | BEL, BEX, BCT, BGE | Pass Marks | 32 |
| Year / Part | II / II | Time | 3 hrs. |

***Subject:*** - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) Define harmonic function of complex variable. Determine the analytical function

   $f(z) = u + iv$ if $u = y^3 - 3x^2y$ [1+4]

   b) Derive Cauchy-Reimann equations if function of complex variable $f(z) = u + iv$ is analytic in cartesian form. [5]

2. a) What do you mean by conformal mapping? Find the linear transformation which maps points $z_1 = 1, z_2 = i, z_3 = -1$ into the points $w_1 = 0, w_2 = 1, w_3 = \infty$. [1+4]

   b) State and prove Cauchy's integral formula. [5]

3. a) State Taylor's theorem. Find the Laurent's series representation of the function

   $f(z) = \dfrac{z}{(z+1)(z+2)}$ in the annular region between $|z| = 1$ and $|z| = 2$. [1+4]

   b) Define zero of order m of function of complex variable .Determine the poles and residue at poles of the functions $f(z) = \dfrac{1+z}{(z+2)(1-z)^2}$. [1+4]

   ***OR***

   Evaluate the real integral $\int_{-\infty}^{\infty} \dfrac{x^2}{(1+x^2)^3} dx$ by contour integration in the complex plane. [5]

4. a) Define z-transform. How does it differ from Fourier transform? Obtain z-transform of

   (i) $t^2a^t$ (ii) $\cos at$ [1+1+1.5+1.5]

   b) State initial value theorem for z transform. Find the initial value x(0) and x(1) for the function. [1+4]

   $$X(z) = \frac{(1-e^{T})z^{-1}}{(1-z^{-1})(1-e^{-T}z^{-1})}$$

5. a) Obtain the inverse z-transform of $X(z) = \dfrac{3z^3 + 2z}{(z-3)^2(z-2)}$ by using inversion integral method. [5]

   b) Apply method of z-transform to solve the difference equation [5]

   $x(k+2) - 4x(k+1) + 4x(k) = 0; x(0) = 0, x(1) = 1$

6. Solve completely one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions: [10]

$u(0,t) = 0, u(l,t) = 0, u(x,0) = 0$ and $\left(\frac{\partial u}{\partial t}\right)_{\text{at } t=0} = 3(lx - x^2)$

7. Derive one dimensional heat equation and solve it completely. [10]

8. a) State convolution theorem for Fourier transform. Give its importance with suitable example. [2+3]

b) Find the Fourier cosine integral of the function $f(x) = e^{-kx} (x > 0, k > 0)$ and hence show that $\int_0^\infty \frac{\cos\omega x d\omega}{k^2 + \omega^2} = \frac{\pi}{2k} e^{-kx}; x > 0, k > 0$ [5]

***

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
**Examination Control Division**
**2073 Magh**

| Exam. | New Back (2066 & Later Batch) | | |
|---|---|---|---|
| Level | BE | Full Marks | 80 |
| Programme | BEL, BEX, BCT, BGE | Pass Marks | 32 |
| Year / Part | II / II | Time | 3 hrs. |

***Subject:*** - Applied Mathematics *(SH551)*

- Candidates are required to give their answers in their own words as far as practicable.
- *Attempt **All** questions.*
- *The figures in the margin indicate **Full Marks**.*
- *Assume suitable data if necessary.*

1. a) Define analytical function of complex-variable. Determine the analytic function $f(z) = u + iv$ if $u = \log\sqrt{x^2+y^2}$. [1+4]

   b) Express Cauchy Riemann equations $U_x = V_y$ and $U_y = -V_x$ into polar form. [5]

2. a) Define bilinear transformation. Obtain the linear transformation which maps points $z_1 = -i, z_2 = 0, z_3 = i$ into $w_1 = -1, w_2 = i, w_3 = 1$ [1+4]

   b) Evaluate $\int_c \frac{e^{2z}}{z^2-3z+2}dz$ in the circle $|z| = 3$ by using Cauchy integral formula. [5]

3. a) State Laurent's Theorem. Obtain the Taylor's series expansion of $f(z) = \frac{1}{z^2+4}$ about the point $z = i$. [1+4]

   b) Define residue at poles. Evaluate $\oint_c \frac{\sin z}{z^6}dz$ , $C:|z| = 1$ by residue method. [1+4]

   ***OR***

   Evaluate real integral $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta} d\theta$ by contour integration in the complex plane. [5]

4. a) Define Z-transform and its region of convergence. Find the Z-transform of [1+1+1.5+1.5]

   i) $t^2e^{-at}$ ii) sinat

   b) State and prove final value theorem for Z-transform. [1+4]

5. a) Find the inverse Z-transform of $f(z) = \frac{z-4}{(z-1)(z-2)^2}$ by partial fraction method. [5]

   b) Use the method of Z-transform to solve the difference equation. [5]

   $x(k+2) + 2x(k+1) + 3x(k) = 0 : x(0) = 0, x(1) = 2$

6. Derive one dimensional wave equation and solve it completely. [10]

7. Solve completely the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ under the conditions: [10]

   $$u(o,y) = u(l,y) = u(x,0) = 0, u(x,\infty) = \sin\left(\frac{n\pi x}{1}\right)$$

8. a) Define Fourier transform of a function. How does it differ from Fourier series? Support your answer with suitable example. [1.5+1.5+2]

   b) Find the Fourier Sine transform of $e^{-x}$ , $x \geq 0$ and hence show that

   $$\int_0^\infty \frac{x^2}{(1+x^2)^2}dx = \frac{\pi}{4}$$ [5]

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
**Examination Control Division**
**2072 Ashwin**

| Exam. | Regular | | |
|---|---|---|---|
| Level | BE | Full Marks | 80 |
| Programme | BEL, BEX, BCT, BGE | Pass Marks | 32 |
| Year / Part | II / II | Time | 3 hrs. |

***Subject:*** - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) If $u=(x-1)^3-3xy^2+3y^2$, determine V so that u + iv is an analytic function of x+iy. [5]

   b) Define an analytic function. Express Cauchy Riemann equations $u_x=v_y$ and $u_y=-v_x$ in polar from. [5]

2. a) Find the bilinear transformation which maps points $z_1=1, z_2=i, z_3=-1$ into the points $w_1=i, w_2=-1, w_3=-i$ respectively. [5]

   b) Evaluate $\int_0^{1+i}(x^2+iy)dz$ along the path $y=x^2$ [5]

3. a) Express $f(z)=\dfrac{1}{(z^2-3z+2)}$ as Laurent's series in the region $1<|z|<2$. [5]

   b) Evalute $\int_0^{2\pi}\dfrac{1}{5-4\sin\theta}d\theta$ by contour integration method in complex plane. [5]

4. a) Find z-transform of: [5]
   i) $te^{-at}$
   ii) sinat

   b) State and prove final value theorem for z- transform. [5]

5. a) Find the inverse z-transform of $\dfrac{2z^2-5z}{(z-2)(z-3)}$ by using partial fraction method. [5]

   b) Solve difference equation $x(k+2)-3x(k+1)+2x(k)=4^k$ for $x(0)=0$ and $x(1)=1$. [5]

6. Derive one dimensional wave equation and obtain its solution. [10]

7. Solve one dimensional heat equation: [10]

   $\dfrac{\partial u}{\partial t}=c^2\dfrac{\partial^2 u}{\partial x^2}$ under the conditions:

   i) u is not infinite as $t\to\infty$

   *ii)* $\dfrac{\partial u}{\partial x}=0$ for $x=0$ and $x=l$

   iii) $u(x,0)=lx-x^2$ for $t=0$; between $x=0$ and $x=l$

8. a) Find Fourier integral representation of $f(x)=e^{-x}, x>0$ and hence evaluate $\int_0^{\infty}\dfrac{\cos(sx)}{s^2+1}ds$ [5]

   b) Find the Fourier cosine transform of $f(x)=e^{-|x|}$ and hence, by Parseval's identity, shown that $\int_0^{\infty}\dfrac{1}{(1+x^2)^2}dx=\dfrac{\pi}{4}$ [5]

***

05 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
**Examination Control Division**
**2072 Magh**

| Exam. | New Back (2066 & Later Batch) | | |
|---|---|---|---|
| Level | BE | Full Marks | 80 |
| Programme | All (Except B.Arch) | Pass Marks | 32 |
| Year / Part | I / II | Time | 3 hrs. |

***Subject:*** - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $u = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$ prove that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0$

2. Find the extreme value of $x^2+y^2+z^2$ connected by the relation $x+y+z=3a$

3. Evaluate: $\iint r \sin\theta \, dr.d\theta$ over the area of the cardioid $r = a(l + \cos\theta)$ above the initial line.

4. Evaluate: $\int_0^a \int_0^{\sqrt{(a^2-x^2)}} y^2.\sqrt{(x^2+y^2)}\ dy.dx$ by changing polar coordinates.

***OR***

Evaluate: $\iiint x^{l-1}.y^{m-1}.z^{n-1} dx.dy.dz$

Evaluate: x, y, z are all positive but $\left(\dfrac{x}{a}\right)^p + \left(\dfrac{y}{b}\right)^q + \left(\dfrac{z}{c}\right)^r \le 1$

5. Find the length of perpendicular from the point (3, -1, 11) to the line $\dfrac{x}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$.

   Also obtain the equation of the perpendicular.

6. Find the magnitude of the line of the shortest distance between the lines $\dfrac{X}{4} = \dfrac{Y+1}{3} = \dfrac{Z-2}{2}$, $5x-2y-3z+6=0,\ x-3y+2z-3=0$

7. Find the centre and radius of the circle in which the sphere $x^2+y^2+z^2-8x+4y+8z-45=0$ is cut by the phane $x-2y+2z=3$

8. The plane through OX and OY include an angle $\alpha$, show that their line of intersection lies on the cone $z^2(x^2+y^2+z^2) = x^2y^2\tan^2\alpha$

9. Solve by power series method of the differential equation y''-y=0

10. Express $f(x) = x^3 - 5x^2 + x + 2$ in terms of Legendre's polynomials.

11. Prove the Bessel's function: $J_{3/2}(x) = \sqrt{\dfrac{2}{\pi x}}\left(\dfrac{\sin x}{x} - \cos x\right)$

12. If $\vec{a},\vec{b},\vec{c}$ and $\vec{a}',\vec{b}',\vec{c}'$ are the reciprocal system of vectors then prove that

$$\vec{a}\times\vec{b}'+\vec{b}\times\vec{c}'+\vec{c}\times\vec{a}'=\frac{\vec{a}+\vec{b}+\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]},\ \left[\vec{a}\,\vec{b}\,\vec{c}\right]\neq 0$$

13. If $\vec{r}=a\cos t\,\vec{i}+a\sin t\,\vec{j}+at\tan\alpha\,\vec{k}$, find $\left[\vec{r}\ \frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dt^2}\right]$

14. Find the directional derivative of $\phi(x,y,z)=xy^2+yz^3$ at the point (2, -1, 1) in the direction of victor $\vec{i}+2\vec{j}+2\vec{k}$

*OR*

If $\vec{r}$ be the position vector and $\vec{a}$ is constant vector then prove that $\nabla.\left(\frac{\vec{a}\times\vec{r}}{r}\right)=0$

15. Determine whether the series $\sum\frac{n}{1+n\sqrt{n+1}}$ is convergent or divergent.

16. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty}\frac{(-1)^n x^n}{n.2^n}$

***

35 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
**Examination Control Division**
2071 Bhādra

| Exam. | Regular / Back | | |
|---|---|---|---|
| Level | BE | Full Marks | 80 |
| Programme | BEL, BEX, BCT, BOE | Pass Marks | 32 |
| Year / Part | II / II | Time | 3 hrs. |

## Subject: - Applied Mathematics (SH551)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *All questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. Determine the analytic function f(z) = u + iv if $u = \log\sqrt{x^2+y^2}$.
2. State and prove Cauchy's integral formula.
3. Find the Taylor's series of $f(z) = \frac{1}{1-z}$ about z = 3i.
4. Evaluate the integral: $\oint_C \frac{z^2}{(z+1)(z+3)}$ where C: |z| = 4, using residue theorem.
5. Define conformal mapping, show that $w = \frac{az+b}{cz+d}$ is invariant to
$$\left(\frac{w-w_1}{w-w_3}\right)\times\left(\frac{w_2-w_3}{w_2-w_1}\right)=\left(\frac{z-z_1}{z-z_3}\right)\times\left(\frac{z_2-z_3}{z_2-z_1}\right)$$
6. Using contour integration, evaluate real integral: $\int_{-\infty}^{+\infty}\frac{x^2dx}{(x^2+a^2)(x^2+b^2)}$
7. Find the z-transform of x(z) = cosh t sinh t.
8. State and prove "final value theorem" for the z-transform.
9. Find the inverse z-transform of $x(z) = \frac{z}{z^2+7z+10}$.
10. Using z-transform solve the difference equation:
$x(K+2) + 6x(K+1) + 9x(K) = 2^K$; $x_0 = x_1 = 0$.
11. Derive one-dimensional heat equation.
12. Solve the wave equation for a tightly stretched string of length 'l' fixed at both ends if the initial deflection in $y(x, 0) = lx - x^2$ and the initial velocity is zero.
13. Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ under the conditions $u(0, y) = u(l, y) = u(x, 0) = 0$, $u(x, a) = \sin\left(\frac{n\pi x}{l}\right)$
14. Derive the wave equation (vibrating of a string).
15. Find the Fourier cosine transform of $f(x) = e^{-m|x|}$ and hence show that $\int_0^\infty \frac{\cos py}{y^2+\beta^2}dy = \frac{\pi}{2\beta}e^{-p\beta}$.
16. Find the Fourier integral representation of the function $f(x) = e^{-x}$, $x \ge 0$ with $f(-x) = f(x)$.
Hence evaluate $\int_0^\infty \frac{\cos(sx)}{s^2+1}ds$.

***

12 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
**Examination Control Division**
2a 2071 Magh

| Exam. | New Back (2066 & Later Batch) | | |
|---|---|---|---|
| Level | BE | Full Marks | 80 |
| Programme | BEL, BEX, BCT, BGE | Pass Marks | 32 |
| Year / Part | II / II | Time | 3 hrs. |

***Subject:*** - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt* ***All*** *questions.*
- ✓ *The figures in the margin indicate* ***Full Marks.***
- ✓ *Assume suitable data if necessary.*

1. a) Determine the analytic function $f(z) = u + iv$ if $u = 3x^2y - y^3$. [5]

   b) Find the linear transformation which maps the points $z = 0, 1, \infty$ into the points $w = -3, -1, 1$ respectively. Find also fixed points of the transformation. [5]

2. a) State and prove Cauchy's integral formula. [5]

   b) Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$. [5]

3. a) Find the first four terms of the Taylor's series expansion of the complex function $f(z) = \frac{z+1}{(z-3)(z-4)}$ about the centre $z = 2$. [5]

   b) Evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = \frac{3}{2}$. [5]

   ***OR***

   Evaluate $\int_0^{2\pi} \frac{1}{\cos\theta + 2} d\theta$ by contour integration in the complex plane.

4. Derive one dimensional heat equation $u_t = c^2u_{xx}$ and solve it completely. [10]

5. Find all possible solution of Laplace equation $u_{xx} + u_{yy} = 0$. Using this, hence solve $u_{xx} + u_{yy} = 0$, under the conditions $u(0, y) = 0$, $u(x, y) = 0$ when $y \to \infty$ and $u(x, 0) = \sin x$. [10]

6. a) Find the z-transform of $\sin K\theta$. Use it to find the $z[a^K \sin K\theta]$. [5]

   b) If $z[x(K)] = \frac{2z^2 + 3z + 12}{(z-1)^4}$, find the value of $x(2)$ and $x(3)$. [5]

7. a) Find the inverse z-transform of $x(z) = \frac{3z^3 + 2z}{(z-3)^2(z-2)}$ by using inversion integral method. [5]

   b) Using z-transform solve the difference equation $x(K+2) - 4x(K+1) + 4x(K) = 2^K$ given that $x(0) = 0$, $x(1) = 1$. [5]

8. a) Find the Fourier sine integral of the function $f(x) = e^{-Kx}$ and hence show that [5]

   $$\int_0^\infty \frac{\lambda \sin \lambda x}{\lambda^2 + \beta^2} d\lambda = \frac{\pi}{2} e^{-Kx}, \quad x > 0, K > 0$$

   b) Find the Fourier sine transform of $e^{-x}$, $x \geq 0$ and hence show that [5]

   $$\int_0^\infty \frac{x \sin mx}{x^2 + 1} dx = \frac{\pi}{2} e^{-m}, \quad m > 0$$

***

25 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
**Examination Control Division**
**2070 Bhadra**

| Exam. | Regular | | |
|---|---|---|---|
| Level | BE | Full Marks | 80 |
| Programme | BEL, BEX, BCT | Pass Marks | 32 |
| Year / Part | II / II | Time | 3 hrs. |

***Subject:*** - Applied Mathematics ***(SH551)***

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Show that $u(x, y) = x^2 + 2xy - y^2$ is a harmonic function and determine $v(x, y)$ in such a way that $f(z) = u(x, y) + iv(x, y)$ is analytic. [5]
2. Define complex integral. State and prove Cauchy integral formula. [5]

***OR***

Obtain bilinear transformation which maps $-i$, o, $i$ to $-1$, $i$, 1. [5]

3. Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ where C is $|z| = 3$ using Cauchy's integral formula. [5]
4. Obtain the Laurent series which represents the function $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ $2 < |z| < 3$. [5]
5. Find the Laurent series of $f(z) = \frac{1}{4+z^2}$ about the point $z = i$. [5]
6. State and prove Taylor series of a function f(z). [5]
7. Derive one dimensional wave equation $u_{tt} = c^2 u_{xx}$ and solve it completely. [10]
8. Solve one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the boundary condition $\frac{\partial u}{\partial x} = 0$ when $x = 0$ and $x = L$ and initial condition $u(x, 0) = x$ for $0 < x < L$. [10]
9. Find Z transform of (a) $te^{-at}$ and (b) sin at. [5]
10. Find the inverse z-transform (a) $\frac{z-4}{(z-1)(z-2)^2}$ (b) $\frac{z}{z^2-3z+2}$. [5]
11. Obtain the Z transform of $x(t) = (1 - e^{-at})$, $a > 0$ and hence evaluate $x(\infty)$ by using final value theorem. [5]
12. Solve using z-transform the difference equation $x(K+2) + 2x(K+1) + 3x(K) = 0$. [5]
13. Find the Fourier sine transform of $f(x) = e^{-x}$, $x \geq 0$ and hence evaluate $\int_0^\infty \frac{x \sin x}{(1+x^2)} dx$. [5]
14. State and prove convolution theorem of Fourier transform. [5]

***

25

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
**Examination Control Division**
**2070 Magh**

| Exam. | New Back (2066 & Later Batch) | | |
|---|---|---|---|
| Level | BE | Full Marks | 80 |
| Programme | BEL, BEX, BCT | Pass Marks | 32 |
| Year / Part | II / II | Time | 3 hrs. |

***Subject:*** - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Define analytic function. Show that the function $f(z) = \frac{1}{z^4}$ is analytic except $z = 0$ [5]

2. Define complex integral. Evaluate $\int_c \log z\,dz; c: |z| = 1$ [5]

***OR***

Obtain a bilinear transformation which maps -i, 0, i to -1, i, 1.

3. Evaluate $\int_0^{1+i} (x^2 + iy)\,dz$ along the path $y = x$. [5]

4. Find the Taylor series of $f(z) = \frac{1}{4+z^2}$ about the point $z = i$. [5]

5. Evaluate the integrals by residue theorem $\int_c \frac{1-\cos z}{z^3}dz$ [5]

6. State Cauchy's Residue theorem and use it to evaluate $\int_c \frac{z^2}{3+4z+z^2}dz$ where C is $|z|=2$ [5]

***OR***

Evaluate $\int_0^{2\pi} \frac{d\theta}{\cos\theta + 2}$ by contour integration in complex plane.

7. Derive the one dimensional wave equation. [10]

8. A rod of length L has its ends A and B maintained at 0° and 100°respectively until steady state prevails. If the changes are made by reducing the temperature of end B to 85° and increasing that of end A to 15°, then find the temperature distribution in the rod at a time t. [10]

9. Find the z-transform of (i) $e^{-at}\sin wt$ (ii) $\cos at$ [5]

10. Obtain inverse Z-transform of (i) $\frac{z+2}{(z-2)(z-3)}$, (ii) $\frac{z}{(z-2)(z-1)}$ [5]

11. If $x(k) = 0$ for $k < 0$ and $Z\{x(k)\} = X(z)$ for $k > 0$ then prove that $Z\{x(k+n)\} = z^n X(z) - z^n \sum_{k=0}^{n-1} \chi(k) z^{-k}$ where n = 0, 1, 2.... [5]

12. Solve the difference equation $x(k+2) - 4x(k+1) + 4x(k) = 0$ with conditions, $x(0) = 0, x(1) = 1$ [5]

13. Find the cosine transform of $f(x) = e^{-mx}$ $m > 0$ show that $\int_0^\infty \frac{\cos pr}{r^2 + B^2} = \frac{\Pi}{2B} e^{-PB}$ [5]

14. Find the Fourier transform of $g(x) = \begin{cases} 1 - x^2 & \text{if } -1 < x < 1; \\ 0, & \text{if otherwise.} \end{cases}$ [5]

and hence use it to evaluate $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right)\cos(x/2)dx$

***

| 25 TRIBHUVAN UNIVERSITY | Exam. | Regular (2066 & Later Batch) | | |
|---|---|---|---|---|
| INSTITUTE OF ENGINEERING | Level | BE | Full Marks | 80 |
| Examination Control Division | Programme | BEL, BEX, BCT | Pass Marks | 32 |
| 2069 Bhadra | Year / Part | II / II | Time | 3 hrs. |

*Subject:* - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *All questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. Determine the analytic function $f(z) = u(x,y) + iv(x,y)$ if $u(x,y) = x^2-y^2$.

2. Define complex integral. Evaluate: $\oint_C (z+1)dz$ where C is the square with vertices at $z = 0$, $z = 1$, $z = 1+i$ and $z = i$.

*OR*

Find linear fractional transformation mapping of: $-2 \mapsto \infty, 0 \mapsto \frac{1}{2}, 2 \mapsto \frac{3}{4}$.

3. a) State Cauchy's integral formula and evaluate the integral $\oint_C \frac{4-3z}{z(z-1)(z-2)}dz$, where C is circle $|z| = \frac{3}{2}$.

   b) Obtain the Laurent series which represents the function $f(z) = \frac{1}{(1+z^2)(z+2)}$ when $1 < |z| < 2$.

4. a) Find the Taylor's series expansion of $f(z) = \frac{1}{z^2+4}$ about the point $z = i$.

   b) Evaluate $\int_C \tan z \, dz$ where C is a circle $|z| = 2$ by Cauchy's residue theorem.

*OR*

Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta} d\theta$ by contour integration in the complex plane.

5. Find the z-transforms of: (i) $\cosh(at)\sin(bt)$ (ii) $n.(n-1)$; $n = k$

6. Find the inverse z-transforms of: (i) $\frac{Z}{Z^2-3Z+2}$ (ii) $\frac{Z}{(Z+1)^2(Z-1)}$.

7. a) State and prove convolution theorem for z-transform.

   b) Solve by using z-transform the difference equation $x(k+2) + 2x(k+1) + 3x(k) = 0$ given that $x(0) = 0$ and $x(1) = 2$

8. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ given that u = 0 as t→∞ as well as u= 0 at x = 0 and x = $l$

9. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, which satisfies the condition u(0,y) = u(L,y) = u(x,0) = 0 and $u(x,a)=\sin\left(\frac{n\pi x}{L}\right)$.

***OR***

The diameter of a semi-circular plate of radius a is kept at 0°C and the temperature at the semi-circular boundary is $u_0$. Find the steady state temperature in the plate.

10. Find the Fourier integral representation of the function $f(x) = e^{-x}$, $x \geq o$ with $f(-x) = f(x)$.

Hence evaluate $\int_0^{\infty} \frac{\cos(sx)}{s^2+1} ds$.

11. Find the Fourier transform of:

$$f(x) = 1-x^2, \ |x|<1$$
$$= \ 0, \ |x|>1$$ and hence evaluate

$$\int_0^{\infty} \left(\frac{x\cos x - \sin x}{x^3}\right) \cos\frac{x}{2} dx.$$

***

25 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
**Examination Control Division**
2069 Poush

| Exam. | New Back (2066 & Later Batch) | | |
|---|---|---|---|
| Level | BE | Full Marks | 80 |
| Programme | BEL, BEX, BCT | Pass Marks | 32 |
| Year / Part | II / II | Time | 3 hrs. |

***Subject:*** - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *All questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $f(z) = u + iv$ is any analytic function of complex variable z and $u-v = e^x(\cos y - \sin y)$, find f(z) in terms of z.
2. State and prove Cauchy's integral theorem.
3. Using Cauchy integral formal to evaluate: $\oint_c \frac{z\,dz}{(z-1)(z-3)}$ where $C: |z| = 3/2$.
4. Find the Laurent's series expansion of $f(z) = \frac{1-\cos z}{z^3}, 0 < |z| < R$.
5. Define singular points and poles. compute the residue of $f(z) = \frac{z^2}{(z-2)(z^2+1)}$ at its pole(s).

   *OR*

   Using contour integration to evaluate $\int_0^{2\pi} \frac{d\theta}{2+\sin\theta}$.
6. Find the z-transform of the following: (any two)

   i) $K^2a^k$ for $K \geq 0$ (ii) $a^k \cos k\pi$ for $k \geq 0$ (iii) $e^{at}\cos wt$ for $t \geq 0$.
7. State and prove final value theorem for the z-transform.
8. Solve the difference equation: $y_{n+2} + 2y_{n+1} + y_n = n$ where $y_0 = y_1 = 0$, and n = k
9. A tightly stretched string with fixed end points x = 0 and x = l is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement y (x, t).
10. Change the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in to polar form.
11. Define Fourier series in complex form. Verify the convolution theorem for $f(x) = g(x) = e^{-x^2}$.
12. Find the Fourier cosine transform of $f(x) = e^{-x}$, x>0 and hence by Parseral's identity, show that $\int_0^\infty \frac{1}{(1+x^2)^2}dx = \frac{\pi}{4}$.

***

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
**Examination Control Division**
**2068 Bhadra**

| Exam. | | Regular | |
|---|---|---|---|
| Level | BE | Full Marks | 80 |
| Programme | BEL, BEX, BCT | Pass Marks | 32 |
| Year / Part | II / II | Time | 3 hrs. |

***Subject:*** - Applied Mathematics

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt __All__ questions.*
- ✓ *__All__ questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. a) State necessary conditions for a function f(z) to be analytic. Show that the function f(z) = log z is analytic everywhere except at the origin.

   b) Find the linear fractional transformation that maps the points $z_1 = -i$, $z_2 = 0$ and $z_3 = i$ into points $w_1 = -1$, $w_2 = i$, $w_3 = 1$ respectively.

2. a) State and prove Cauchy's integral formula.

   b) Write the statement of Cauchy's integral formula. Use it to evaluate the integral $\oint_C \frac{e^z}{(z-1)(z-3)} dz$ where C is the circle $|z| = 2$.

3. a) Write the statement of Taylor's theorem. Find the Laurent series for the function $f(z) = \frac{1}{z^2 - 3z + 2}$ in the region $1 < |z| < 2$.

   b) State Cauchy-residue theorem. Using it evaluate $\oint_C \frac{\sin z}{z^6} dz$ where C: $|z| = 1$.

   ***OR***

   Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta} d\theta$ by contour integration in the complex plane.

4. a) Show that the Z-transform of cos kθ is $\frac{z(z-\cos\theta)}{z^2 - 2z\cos\theta + 1}$. Use this result to find Z-transform of $a^k$ cos kθ.

   b) Obtain the inverse Z-transform of $\frac{2z^3 + z}{(z-2)^2(z-1)}$, using partial fraction method.

5. a) Solve the difference equation x(k + 2) – x( k + 1) + 0.25x(k) = u(k) where x(0) = 1 and x(1) = 2 and u(k) is unit step function.

   b) State and prove shifting theorem of z-transform.

6. Derive one-dimensional wave equation governing transverse vibration of string and solve it completely.

7. Solve the one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions:

a) u is not infinite as $t \to \infty$

b) $\frac{\partial u}{\partial x} = 0$ for $x = 0$ and $x = l$ and

c) $u(x,0) = lx - x^2$ for $t = 0$ between $x = 0$ and $x = l$

***OR***

The diameter of a semi circular plate of radius a is kept at 0°C and temperature at the semi circular boundary is T°C. Show that the steady temperature in the plate is given by $u(r, \theta) \frac{4T}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \left(\frac{r}{a}\right)^{2n-1} \sin (2n-1)\theta$

8. a) Find the Fourier cosine integral representation of the function $f(x) = e^{-kx}$ $(x > 0, k > 0)$ and hence show that

$$\int_0^{\infty} \frac{\cos \omega x}{k^2 + \omega^2} d\omega = \frac{\pi}{2k} e^{-kx} \quad (x > 0, K > 0)$$

b) Obtain Fourier sine transform of $e^{-x}$, $(x > 0)$ and hence evaluate $\int_0^{\infty} \frac{x^2}{(1+x^2)^2} dx$.

***

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
**Examination Control Division**
**2068 Magh**

| | | | |
|---|---|---|---|
| Exam. | New Back (2066 & Later Batch) | | |
| Level | BE | Full Marks | 80 |
| Programme | BEL, BEX, BCT | Pass Marks | 32 |
| Year / Part | II / II | Time | 3 hrs. |

***Subject:*** - Applied Mathematics

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) Define harmonic function and show that the function $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic and hence determine its harmonic conjugate function. [5]

   b) Define conformal mapping. Find the linear fractional transformation that maps three points 2i, −2, −2i onto three given points −2, −2 i, 2. [5]

2. a) State and prove Cauchy integral theorem. [5]

   b) Evaluate the following integral using Cauchy integral formula $\int_c \frac{4-3z}{z(z-1)(z-2)}$ where c is the circle $|z| = 3/2$. [5]

3. a) Find the Taylor's series expansion of $f(z) = \frac{2z^3+1}{z^2+z}$ about the point z = 1. [5]

   b) Determine the poles and the residue at each pole of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$. [5]

   ***OR***

   Evaluate $\int_0^{2\pi} \frac{d\theta}{\sqrt{2} - \cos\theta}$ by contour integration in complex plane.

4. a) Find the z-transform of: (any two) [5]

   i) $a^{|k|}$  ii) $\sin\alpha k$  iii) cosh*at* sin*bt*

   b) Find inverse z- transform of $\frac{z-4}{(z-1)(z-2)^2}$ by using inversion integral method. [5]

5. a) State and prove final value theorem for z transform. [5]

   b) Solve by using z transform $y_{k+2} - 4y_{k+1} + 4y_k = 0$ where $y_0 = 1$ and $y_1 = 0$. [5]

6. A tightly stretched string with fixed end points x = 0 and x = 1 is initially in the position given by $y = y_0 \sin^3\frac{\pi x}{L}$. If y (o, t) = y (L, t) = 0, then find the displacement y(x, t) where initial velocity is zero. [10]

7. Deduce the two dimensional Laplace equation into polar form. [10]

   ***OR***

   Derive one dimensional heat equation for the flow of heat along a metallic rod by conduction and solve it completely.

8. a) State and prove convolution theorem for Fourier transform. [5]

   b) Find the Fourier cosine transform of $f(x) = e^{-mx}$ $(m > 0)$ and hence show that $\int_0^\infty \frac{\cos py}{y^2+\beta^2}dy = \frac{\pi}{2\beta}e^{-p\beta}$ $(\beta > 0, p > 0)$. [5]