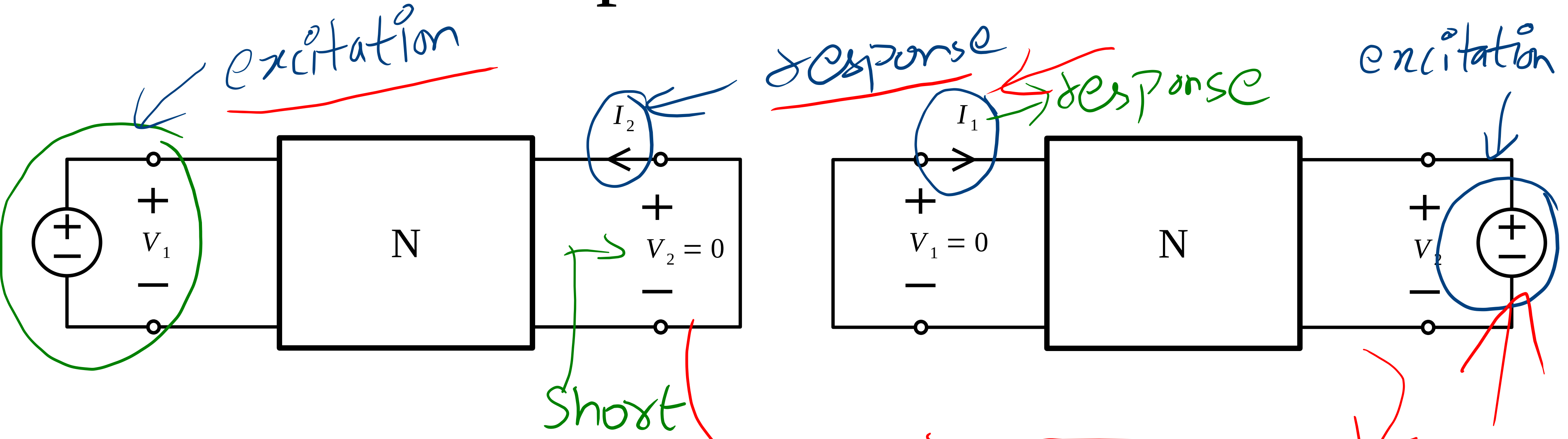
$$\boxed{y_{12} = y_{21}}$$

Reciprocal and Symmetrical Network

Reciprocal Network

- The *principle of reciprocity*, states that the ratio of response transform to the excitation transform remains identical even if the position of the response and excitation in the network are interchanged.
- Networks for which this condition holds are said to be *reciprocal*.

Reciprocal Network



For the network to be reciprocal

$$\left.\frac{I_2}{V_1}\right|_{V_2=0} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

Condition for **reciprocal** network

(in terms of **z parameter**)

Condition for **reciprocal** network (in terms of z parameter)

- Condition for reciprocal network

$$\left.\frac{I_2}{V_1}\right|_{V_2=0} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

- From the definition of z parameter:

$$\begin{cases} V_1 = z_{11} I_1 + z_{12} I_2 \\ V_2 = z_{21} I_1 + z_{22} I_2 \end{cases}$$

Condition for **reciprocal** network (in terms of z parameter)

- Finding I_2/V_1 for $V_2=0$, the expression of V_2 now becomes:

$$V_2 = z_{21} I_1 + z_{22} I_2$$

$$\left.0 = z_{21} I_1 + z_{22} I_2\right\} \rightarrow I_1 = -\frac{z_{22}}{z_{21}} I_2$$

- Putting the value of I_1 in the expression of V_1, we get:

$$V_1 = z_{11} I_1 + z_{12} I_2 = z_{11}\left(\frac{z_{22}}{-z_{21}} I_2\right) + z_{12} I_2$$

Condition for **reciprocal** network (in terms of z parameter)

$$V_1 = \frac{-z_{11}z_{22} + z_{12}z_{21}}{z_{21}} I_2 = -\frac{\Delta_z}{z_{21}} I_2$$

Δ_z

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

Here,

$$z_{11}\,z_{22} - z_{12}\,z_{21} = \Delta_z$$

So,

$$V_1 = -\frac{\Delta_z}{z_{21}} I_2$$

$$\therefore \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{z_{21}}{\Delta_z}$$

Condition for **reciprocal** network (in terms of z parameter)

- Finding I_1/V_2 for $V_1=0$, the expression of V_1 now becomes:

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$0 = z_{11} I_1 + z_{12} I_2$$

$$I_2 = -\frac{z_{11}}{z_{12}} I_1$$

- Putting the value of I_2 in the expression of V_2, we get:

$$V_2 = z_{21} I_1 + z_{22} I_2 = z_{21} I_1 + z_{22}\left(\frac{z_{11}}{-z_{12}} I_1\right)$$

Condition for **reciprocal** network (in terms of z parameter)

$$\begin{cases} V_2 = \dfrac{z_{12}\, z_{21} - z_{11}\, z_{22}}{z_{12}} I_2 = -\dfrac{\Delta_z}{z_{12}} I_1 \end{cases}$$

Here,

$$z_{11}\, z_{22} - z_{12}\, z_{21} = \Delta_z$$

So,

$$\begin{cases} V_2 = -\dfrac{\Delta_z}{z_{12}} I_1 \end{cases} \qquad \therefore \left.\dfrac{I_1}{V_2}\right|_{V_1 = 0} = -\dfrac{z_{12}}{\Delta_z}$$

Condition for **reciprocal** network (in terms of z parameter)

- Condition for reciprocal network

$$\left.\frac{I_2}{V_1}\right|_{V_2=0} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

- In terms of z parameter, we have:

$$-\frac{z_{21}}{\cancel{\Delta_z}} = -\frac{z_{12}}{\cancel{\Delta_z}}$$

$$z_{21} = z_{12}$$

$\therefore$ For a network to be reciprocal $\quad z_{12} = z_{21}$

$z_{12} \neq z_{21}$

Condition for **reciprocal** network (in terms of **y parameter**)

Condition for **reciprocal** network (in terms of y parameter)

- Condition for reciprocal network

$$\left.\frac{I_2}{V_1}\right|_{V_2=0} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

- From the definition of y parameter:

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

Condition for **reciprocal** network (in terms of y parameter)

- Condition for reciprocal network

$$\left.\frac{I_2}{V_1}\right|_{V_2=0} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

y_{21} y_{12}

- In terms of y parameter, we have:

$$y_{21} = y_{12}$$

$\therefore$ For a network to be reciprocal $\quad y_{12} = y_{21}$

Condition for **reciprocal** network
(in terms of **transmission parameter**)
(or, **T parameter**)

Condition for **reciprocal** network (in terms of T parameter)

- Condition for reciprocal network

$$\left.\frac{I_2}{V_1}\right|_{V_2=0} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

- From the definition of T parameter:

$$\begin{cases} V_1 = A\,V_2 - B\,I_2 \\ I_1 = C\,V_2 - D\,I_2 \end{cases}$$

Condition for **reciprocal** network (in terms of T parameter)

- Finding I_2/V_1 for $V_2=0$, the expression of V_1 now becomes:

$$V_1 = A\,V_2 - B\,I_2$$

$$V_1 = -B\,I_2$$

$$\therefore \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{1}{B}$$

Condition for **reciprocal** network (in terms of T parameter)

- Finding I_1/V_2 for $V_1=0$, the expression of V_1 now becomes:

$$V_1 = A\,V_2 - B\,I_2$$

$$0 = A\,V_2 - B\,I_2$$

$$I_2 = \frac{A}{B}\,V_2$$

- Putting the value of I_2 in the expression of I_1, we get:

$$I_1 = C\,V_2 - D\,I_2 = C\,V_2 - D\left(\frac{A}{B}\,V_2\right)$$

Condition for **reciprocal** network (in terms of T parameter)

$$I_1 = \frac{BC - AD}{B} V_2 = -\frac{AD-BC}{B} V_2$$

Δ_T

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$\therefore \left. \frac{I_1}{V_2} \right|_{V_1 = 0} = -\frac{AD-BC}{B}$$

Condition for **reciprocal** network (in terms of T parameter)

- Condition for reciprocal network

$$\left.\frac{I_2}{V_1}\right|_{V_2=0} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

- In terms of T parameter, we have:

$$-\frac{1}{B} = -\frac{AD-BC}{B}$$

$$AD - BC = 1$$

$\therefore$ For a network to be reciprocal $\quad AD - BC = 1$

Condition for **reciprocal** network (in terms of **inverse transmission parameter** or, **T' parameter**)

Condition for **reciprocal** network (in terms of T' parameter)

- Condition for reciprocal network in terms of **T' parameters** is similar to the condition for reciprocal network in terms of **T parameters**.

So,

$\therefore$ For a network to be reciprocal $\quad A'D' - B'C' = 1$

$AD - BC = 1$

Condition for **reciprocal** network (in terms of **h parameter**)

Condition for **reciprocal** network (in terms of h parameter)

- Condition for reciprocal network

$$\left.\frac{I_2}{V_1}\right|_{V_2=0} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

- From the definition of h parameter:

$$\begin{cases} V_1 = h_{11} I_1 + h_{12} V_2 \\ I_2 = h_{21} I_1 + h_{22} V_2 \end{cases}$$

Condition for **reciprocal** network (in terms of h parameter)

- Finding I_2/V_1 for $V_2=0$, the expression of V_1 now becomes:

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$V_1 = h_{11} I_1$$

$$I_1 = \frac{1}{h_{11}} V_1$$

- Putting the value of I_1 in the expression of I_2, we get:

$$I_2 = h_{21} I_1 + h_{22} V_2 = h_{21}\left(\frac{1}{h_{11}} V_1\right) = \frac{h_{21}}{h_{11}} V_1$$

Condition for **reciprocal** network (in terms of h parameter)

$$I_2 = \frac{h_{21}}{h_{11}} V_1$$

$$\therefore \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{h_{21}}{h_{11}}$$

Condition for **reciprocal** network (in terms of h parameter)

- Finding I_1/V_2 for $V_1=0$, the expression of V_1 now becomes:

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$0 = h_{11} I_1 + h_{12} V_2$$

$$I_1 = -\frac{h_{12}}{h_{11}} V_2$$

$$\therefore \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\frac{h_{12}}{h_{11}}$$

Condition for **reciprocal** network (in terms of h parameter)

- Condition for reciprocal network

$$\left.\frac{I_2}{V_1}\right|_{V_2=0} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

- In terms of h parameter, we have:

$$\frac{h_{21}}{h_{11}} = -\frac{h_{12}}{h_{11}}$$

$$h_{21} = -h_{12}$$

$\therefore$ For a network to be reciprocal $\quad h_{12} = -h_{21}$

Condition for **reciprocal** network (in terms of **g parameter**)

Condition for **reciprocal** network (in terms of g parameter)

- Condition for reciprocal network in terms of **g parameters** is similar to the condition for reciprocal network in terms of **h parameters**.

So,

$h_{12} = -h_{21}$

$\therefore$ For a network to be reciprocal $g_{12} = -g_{21}$

Condition for **reciprocal** network (in summary)

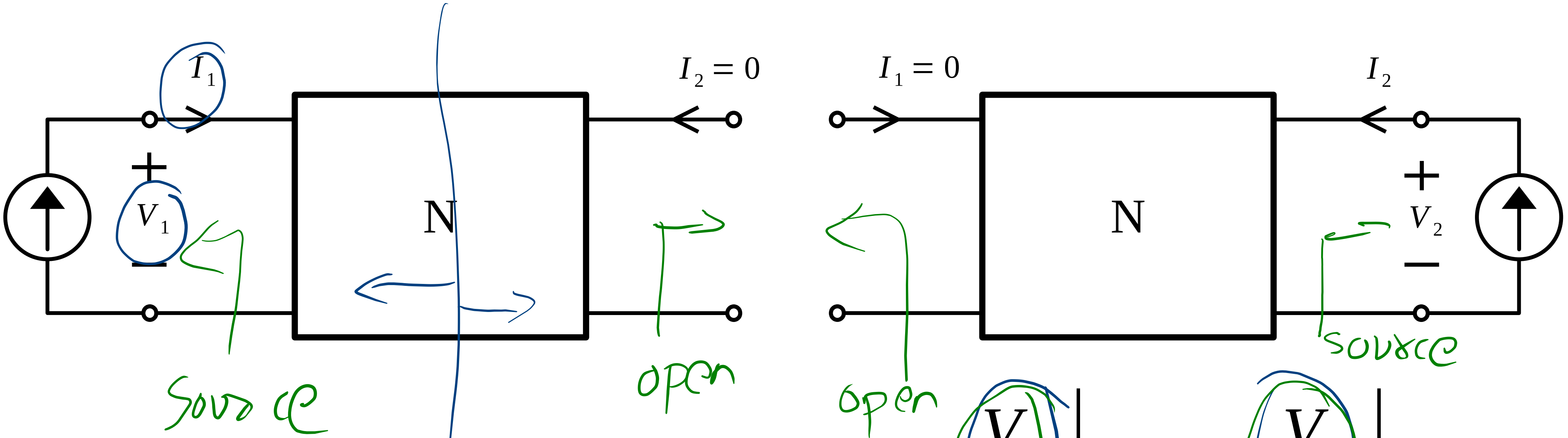
Condition for **reciprocal** network

Parameter	Condition
z	$z_{12} = z_{21}$
y	$y_{12} = y_{21}$
T	$AD - BC = 1$
T'	$A'D' - B'C' = 1$
h	$h_{12} = - h_{21}$
g	$g_{12} = - g_{21}$

Symmetrical Network

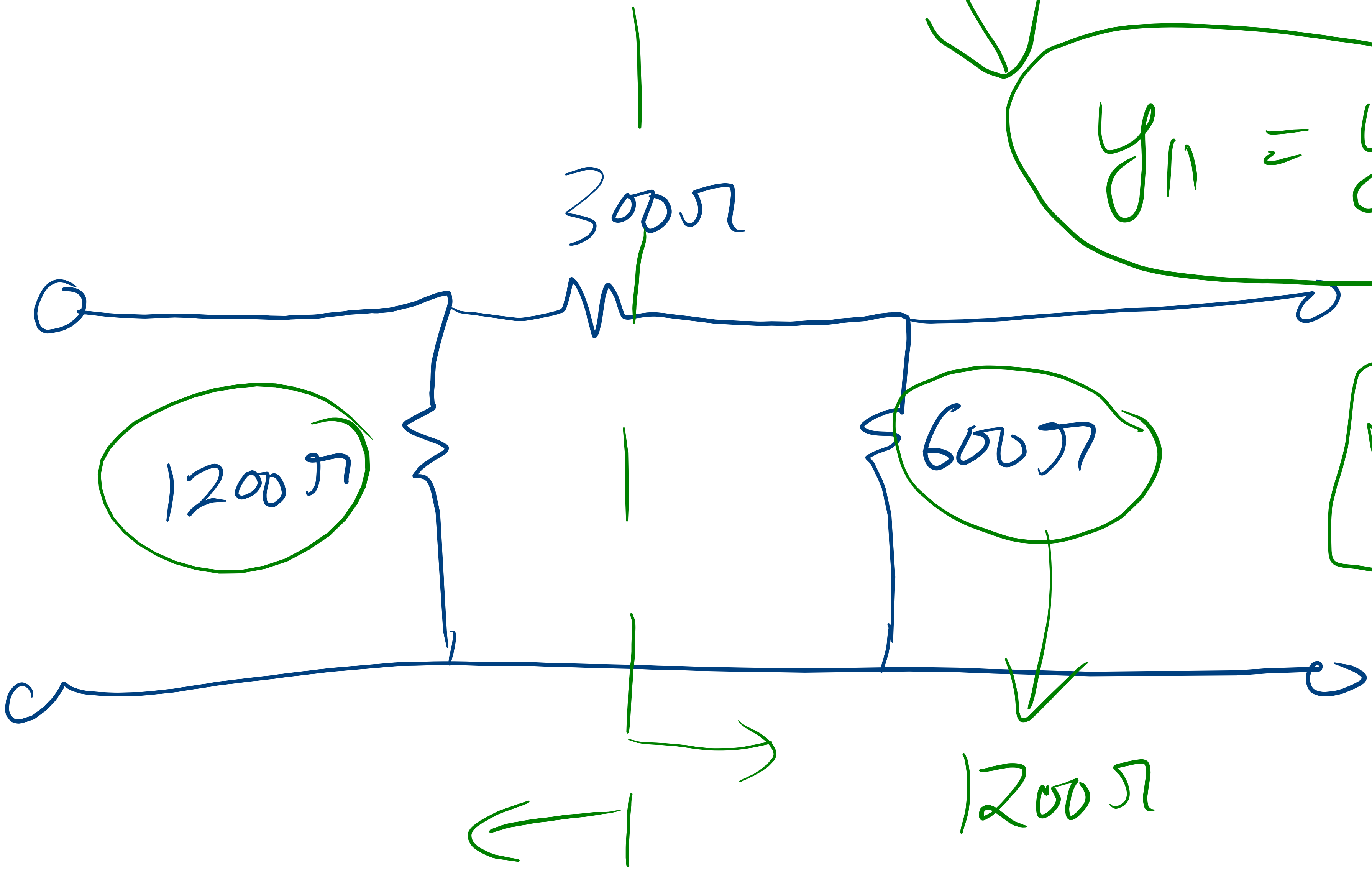
- A two port network is termed as symmetrical if the input and output ports can be exchanged without altering the port voltages and currents.

Symmetrical Network



For the network to be symmetrical

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$



$y_{11} = y_{22}$

$y_{11} \neq y_{22}$

Condition for **symmetrical** network (in terms of z parameter)

Condition for **symmetrical** network (in terms of z parameter)

- Condition for symmetrical network

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

- From the definition of z parameter:

$$\begin{cases} V_1 = z_{11} I_1 + z_{12} I_2 \\ V_2 = z_{21} I_1 + z_{22} I_2 \end{cases}$$

Condition for **symmetrical** network (in terms of z parameter)

- Condition for symmetrical network

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

- In terms of z parameter, we have:

$$z_{11} = z_{22}$$

z

$\therefore$ For a network to be symmetrical $z_{11} = z_{22}$

Condition for **symmetrical** network (in terms of y parameter)

Condition for **symmetrical** network (in terms of y parameter)

- Condition for symmetrical network

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

- From the definition of y parameter:

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

Condition for **symmetrical** network (in terms of y parameter)

- Finding V_1/I_1 for $I_2=0$, the expression of I_2 now becomes:

$$I_2 = y_{21} V_1 + y_{22} V_2$$
$$0 = y_{21} V_1 + y_{22} V_2$$

$$V_2 = -\frac{y_{21}}{y_{22}} V_1$$

- Putting the value of V_2 in the expression of I_1, we get:

$$I_1 = y_{11} V_1 + y_{12} V_2 = y_{11} V_1 + y_{12}\left(-\frac{y_{21}}{y_{22}} V_1\right)$$

Condition for **symmetrical** network (in terms of y parameter)

$$I_1 = y_{11} V_1 + y_{12}\left(-\frac{y_{21}}{y_{22}} V_1\right) = \frac{y_{11}\, y_{22} - y_{12}\, y_{21}}{y_{22}} V_1$$

Here,

$$y_{11}\, y_{22} - y_{12}\, y_{21} = \Delta_y$$

So,

$$I_1 = \frac{\Delta_y}{y_{22}} V_1 \qquad \therefore \left.\frac{V_1}{I_1}\right|_{I_2 = 0} = \frac{y_{22}}{\Delta_y}$$

Condition for **symmetrical** network (in terms of y parameter)

- Finding V_2/I_2 for $I_1=0$, the expression of I_1 now becomes:

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$0 = y_{11} V_1 + y_{12} V_2$$

$$V_1 = -\frac{y_{12}}{y_{11}} V_2$$

- Putting the value of V_1 in the expression of I_2, we get:

$$I_2 = y_{21} V_1 + y_{22} V_2 = y_{21}\left(-\frac{y_{12}}{y_{11}} V_2\right) + y_{22} V_2$$

Condition for **symmetrical** network (in terms of y parameter)

$$I_2 = y_{21}\left(-\frac{y_{12}}{y_{11}}V_2\right) + y_{22}V_2 = \frac{y_{11}y_{22} - y_{12}y_{21}}{y_{11}}V_2$$

Here,

$$y_{11}y_{22} - y_{12}y_{21} = \Delta_y$$

So,

$$I_2 = \frac{\Delta_y}{y_{11}}V_2 \qquad \therefore \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{y_{11}}{\Delta_y}$$

Condition for **symmetrical** network (in terms of y parameter)

- Condition for symmetrical network

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

- In terms of y parameter, we have:

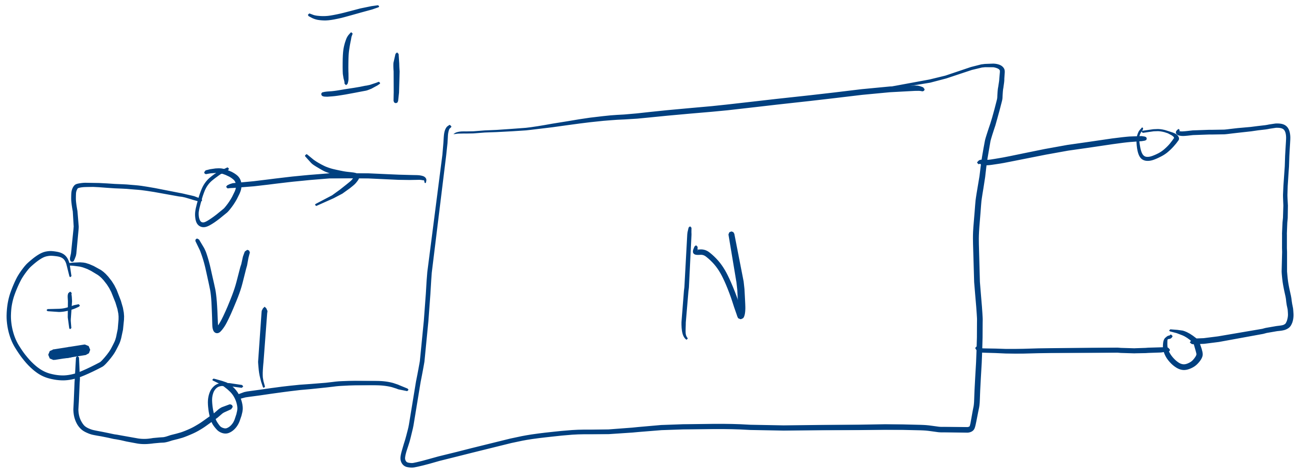
$$\frac{y_{22}}{\cancel{\Delta_y}} = \frac{y_{11}}{\cancel{\Delta_y}}$$

$$y_{22} = y_{11}$$

$\therefore$ For a network to be symmetrical $y_{11} = y_{22}$

$$y_{11} = y_{22}$$

$$\left.\frac{I_1}{V_1}\right|_{V_2=0} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



Condition for **symmetrical** network (in terms of **transmission parameter**) (or, **T parameter**)

Condition for **symmetrical** network (in terms of T parameter)

- Condition for symmetrical network

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

- From the definition of T parameter:

$$\begin{cases} V_1 = A\,V_2 - B\,I_2 \\ I_1 = C\,V_2 - D\,I_2 \end{cases}$$

Condition for **symmetrical** network (in terms of T parameter)

- Finding V_1/I_1 for $I_2=0$, the expression of V_1 now becomes:

$$V_1 = A\,V_2 - B\,I_2$$

$$V_1 = A\,V_2$$

$$V_2 = \frac{1}{A}V_1$$

- Putting the value of V_2 in the expression of I_1, we get:

$$I_1 = C\,V_2 - D\,I_2 = C\left(\frac{1}{A}V_1\right) = \frac{C}{A}V_1$$

Condition for **symmetrical** network (in terms of T parameter)

$$I_1 = \frac{C}{A} V_1$$

$$\therefore \left\{ \left. \frac{V_1}{I_1} \right|_{I_2 = 0} = \frac{A}{C} \right\},$$

Condition for **symmetrical** network (in terms of T parameter)

- Finding V_2/I_2 for $I_1=0$, the expression of I_1 now becomes:

$$I_1 = C\,V_2 - D\,I_2$$

$$0 = C\,V_2 - D\,I_2$$

$$C\,V_2 = D\,I_2$$

$$\therefore \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{D}{C}$$

Condition for **symmetrical** network (in terms of T parameter)

- Condition for symmetrical network

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

- In terms of T parameter, we have:

$$\frac{A}{C} = \frac{D}{C}$$

$$A = D$$

$\therefore$ For a network to be symmetrical $A = D$

Condition for **symmetrical** network (in terms of **inverse transmission parameter** or, **T' parameter**)

Condition for **symmetrical** network (in terms of A'B'C'D' parameter)

- Condition for symmetrical network in terms of **T' parameters** is similar to the condition for symmetrical network in terms of **T parameters**.

So,

$A = D$

$\therefore$ For a network to be symmetrical $\quad A' = D'$

Condition for **symmetrical** network (in terms of **h parameter**)

Condition for **symmetrical** network (in terms of h parameter)

- Condition for symmetrical network

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

- From the definition of h parameter:

$$V_1 = h_{11}\, I_1 + h_{12}\, V_2$$
$$I_2 = h_{21}\, I_1 + h_{22}\, V_2$$

Condition for **symmetrical** network (in terms of h parameter)

- Finding V_1/I_1 for $I_2=0$, the expression of I_2 now becomes:

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$0 = h_{21} I_1 + h_{22} V_2$$

$$V_2 = -\frac{h_{21}}{h_{22}} I_1$$

- Putting the value of V_2 in the expression of V_1, we get:

$$V_1 = h_{11} I_1 + h_{12} V_2 = h_{11} I_1 + h_{12}\left(-\frac{h_{21}}{h_{22}} I_1\right)$$

Condition for **symmetrical** network (in terms of h parameter)

$$V_1 = h_{11} I_1 + h_{12}\left(-\frac{h_{21}}{h_{22}} I_1\right) = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} I_1$$

Here,

$$h_{11} h_{22} - h_{12} h_{21} = \Delta_h$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

So,

$$V_1 = \frac{\Delta_h}{h_{22}} I_1 \rightarrow \therefore \left.\frac{V_1}{I_1}\right|_{I_2 = 0} = \frac{\Delta_h}{h_{22}}$$

Condition for **symmetrical** network (in terms of h parameter)

- Finding V_2/I_2 for $I_1=0$, the expression of I_2 now becomes:

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$I_2 = h_{22} V_2$$

$$\therefore \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{1}{h_{22}}$$

Condition for **symmetrical** network (in terms of h parameter)

- Condition for symmetrical network

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

- In terms of h parameter, we have:

$$\frac{\Delta_h}{h_{22}} = \cancel{-}\frac{1}{h_{22}} \qquad \frac{1}{h_{22}}$$

$$\Delta_h = 1$$

Symmetrical

$\therefore$ For a network to be ~~reciprocal~~ $\Delta_h = 1$

Condition for **symmetrical** network (in terms of **g parameter**)

Condition for **symmetrical** network (in terms of g parameter)

- Condition for symmetrical network in terms of **g parameters** is similar to the condition for symmetrical network in terms of **h parameters**.

So,

symmetrical

$\therefore$ For a network to be ~~reciprocal~~ $\Delta_g = 1$

Here, $g_{11}\, g_{22} - g_{12}\, g_{21} = \Delta_g$

Condition for **symmetrical** network (in summary)

Condition for **symmetrical** network

Parameter	Condition
z	$z_{11} = z_{22}$
y	$y_{11} = y_{22}$
ABCD	$A = D$
A'B'C'D'	$A' = D'$
h	$\Delta_h = 1$
g	$\Delta_g = 1$

Reciprocal

y_{11} y_{12} z y

y_{11} $y_{12} = y_{21}$

$z_{12} = z_{21}$

$z_{11} = z_{22}$

y_{11} y_{12}

y_{21} y_{22}

$y_{12} = y_{21}$

$y_{11} = y_{22}$

Symmetrical

$y_{11} = y_{22}$

Relationship and transformation between sets of parameters

Relationship and transformation between sets of parameters

- Once we know a set of parameters of a network, than based on our needs, we can find the remaining five sets of parameter in terms of the known parameter.
- It is a simple matter to find the relationships of the sets of parameters.

z-parameter in terms of **y-parameter**

- From the definition of y parameter:

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

- From the definition of z parameter:

$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

z-parameter in terms of **y-parameter**

- From the definition of y parameter:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [y][V]$$

$$[V] = [y]^{-1}[I]$$

- From the definition of z parameter:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[V] = [z][I]$$

$$\therefore [z] = [y]^{-1}$$

z-parameter in terms of **y-parameter**

reciprocal
symmetrical

$z = y$

$$[z] = [y]^{-1}$$

$\Delta y = y_{11} y_{22} - y_{12} y_{21}$

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \frac{1}{\Delta_y} \begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}$$

$z_{11} = \frac{y_{22}}{\Delta_y}$	$z_{12} = -\frac{y_{12}}{\Delta_y}$	$z_{21} = -\frac{y_{21}}{\Delta_y}$	$z_{22} = \frac{y_{11}}{\Delta_y}$

$$[y]^{-1} = \frac{1}{\Delta y}\begin{bmatrix} y_{22} & -y_{21} \\ -y_{12} & y_{11} \end{bmatrix}^T$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$$= \frac{1}{\Delta y}\begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}$$

z-parameter in terms of **T-parameter**

- From the definition of T parameter:

$$\begin{cases} V_1 = A\,V_2 - B\,I_2 \\ I_1 = C\,V_2 - D\,I_2 \end{cases}$$

- From the definition of z parameter:

$$\begin{cases} V_1 = z_{11}\,I_1 + z_{12}\,I_2 \\ V_2 = z_{21}\,I_1 + z_{22}\,I_2 \end{cases}$$

- Rewriting the expression of I_1 from the definition of T-parameter, we get:

$$I_1 = C\,V_2 - D\,I_2 \quad \Rightarrow \quad V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2$$

z-parameter in terms of **T-parameter**

$$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2$$

- Putting the value of V_2 in the expression of V_1, we get:

$$V_1 = A V_2 - B I_2 = A \left(\frac{1}{C} I_1 + \frac{D}{C} I_2 \right) - B I_2$$

$$V_1 = \frac{A}{C} I_1 + \frac{A D}{C} I_2 - B I_2 = \underbrace{\frac{A}{C}}_{z_{11}} I_1 + \underbrace{\frac{A D - B C}{C}}_{z_{12}} I_2$$

z-parameter in terms of **T-parameter**

$$V_1 = \frac{A}{C} I_1 + \frac{A\,D - B\,C}{C} I_2 = \frac{A}{C} I_1 + \frac{\Delta_T}{C} I_2$$

Here,

$$A\,D - B\,C = \Delta_T$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad \begin{cases} V_1 = z_{11} I_1 + z_{12} I_2 \\ V_2 = z_{21} I_1 + z_{22} I_2 \end{cases}$$

- From T parameter, we have

$$V_1 = \frac{A}{C} I_1 + \frac{\Delta_T}{C} I_2 \quad \text{and} \quad V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2$$

z-parameter in terms of **T-parameter**

- From the definition of T parameter:

$$\begin{cases} V_1 = \dfrac{A}{C} I_1 + \dfrac{\Delta_T}{C} I_2 \\ V_2 = \dfrac{1}{C} I_1 + \dfrac{D}{C} I_2 \end{cases}$$

- From the definition of z parameter:

$$\begin{cases} V_1 = z_{11} I_1 + z_{12} I_2 \\ V_2 = z_{21} I_1 + z_{22} I_2 \end{cases}$$

$z_{11} = \dfrac{A}{C}$	$z_{12} = \dfrac{\Delta_T}{C}$	$z_{21} = \dfrac{1}{C}$	$z_{22} = \dfrac{D}{C}$

z-parameter in terms of **h-parameter**

- From the definition of h parameter:

$$\begin{cases} V_1 = h_{11} I_1 + h_{12} V_2 \\ I_2 = h_{21} I_1 + h_{22} V_2 \end{cases}$$

- From the definition of z parameter:

$$\begin{cases} V_1 = z_{11} I_1 + z_{12} I_2 \\ V_2 = z_{21} I_1 + z_{22} I_2 \end{cases}$$

- Rewriting the expression of I_2 from the definition of h-parameter, we get:

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \Rightarrow \quad V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2$$

z-parameter in terms of **h-parameter**

$$V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2$$

- Putting the value of V_2 in the expression of V_1, we get:

$$V_1 = h_{11} I_1 + h_{12} V_2 = h_{11} I_1 + h_{12} \left(-\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \right)$$

$$V_1 = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

z-parameter in terms of **h-parameter**

$$V_1 = \frac{h_{11}\,h_{22} - h_{12}\,h_{21}}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2 = \boxed{\frac{\Delta_h}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2}$$

Here,

$$h_{11}\,h_{22} - h_{12}\,h_{21} = \Delta_h$$

- From h parameter, we have

$$V_1 = \frac{\Delta_h}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2 \quad \text{and} \quad V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2$$

$\Delta_h \quad \Delta_z \quad \Delta_y \quad \Delta_g \quad \Delta_{T'} \quad \Delta_T$

z-parameter in terms of **h-parameter**

- From the definition of h parameter:

$$V_1 = \frac{\Delta_h}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2$$

- From the definition of z parameter:

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

$z_{11} = \frac{\Delta_h}{h_{22}}$	$z_{12} = \frac{h_{12}}{h_{22}}$	$z_{21} = -\frac{h_{21}}{h_{22}}$	$z_{22} = \frac{1}{h_{22}}$

z	y	T	T'	h	g
z_{11}	$\dfrac{y_{22}}{\Delta_y}$	$\dfrac{A}{C}$	$\dfrac{D'}{C'}$	$\dfrac{\Delta_h}{h_{22}}$	$\dfrac{1}{g_{11}}$
z_{12}	$-\dfrac{y_{12}}{\Delta_y}$	$\dfrac{\Delta_T}{C}$	$\dfrac{1}{C'}$	$\dfrac{h_{12}}{h_{22}}$	$-\dfrac{g_{12}}{g_{11}}$
z_{21}	$-\dfrac{y_{21}}{\Delta_y}$	$\dfrac{1}{C}$	$\dfrac{\Delta_{T'}}{C'}$	$-\dfrac{h_{21}}{h_{22}}$	$\dfrac{g_{21}}{g_{11}}$
z_{22}	$\dfrac{y_{11}}{\Delta_y}$	$\dfrac{D}{C}$	$\dfrac{A'}{C'}$	$\dfrac{1}{h_{22}}$	$\dfrac{\Delta_g}{g_{11}}$

Here, $\Delta_x = x_{11}\, x_{22} - x_{12}\, x_{21}$

y	z	T	T'	h	g
y_{11}	$\dfrac{z_{22}}{\Delta_z}$	$\dfrac{D}{B}$	$\dfrac{A'}{B'}$	$\dfrac{1}{h_{11}}$	$\dfrac{\Delta_g}{g_{22}}$
y_{12}	$-\dfrac{z_{12}}{\Delta_z}$	$-\dfrac{\Delta_T}{B}$	$-\dfrac{1}{B'}$	$-\dfrac{h_{12}}{h_{11}}$	$\dfrac{g_{12}}{g_{22}}$
y_{21}	$-\dfrac{z_{21}}{\Delta_z}$	$-\dfrac{1}{B}$	$-\dfrac{\Delta_{T'}}{B'}$	$\dfrac{h_{21}}{h_{11}}$	$-\dfrac{g_{21}}{g_{22}}$
y_{22}	$\dfrac{z_{11}}{\Delta_z}$	$\dfrac{A}{B}$	$\dfrac{D'}{B'}$	$\dfrac{\Delta_h}{h_{11}}$	$\dfrac{1}{g_{22}}$

Here, $\Delta_x = x_{11}\, x_{22} - x_{12}\, x_{21}$

T	z	y	T'	h	g
A	$\frac{z_{11}}{z_{21}}$	$-\frac{y_{22}}{y_{21}}$	$\frac{D'}{\Delta_{T'}}$	$-\frac{\Delta_h}{h_{21}}$	$\frac{1}{g_{21}}$
B	$\frac{\Delta_z}{z_{21}}$	$-\frac{1}{y_{21}}$	$\frac{B'}{\Delta_{T'}}$	$-\frac{h_{11}}{h_{21}}$	$\frac{g_{22}}{g_{21}}$
C	$\frac{1}{z_{21}}$	$-\frac{\Delta_y}{y_{21}}$	$\frac{C'}{\Delta_{T'}}$	$-\frac{h_{22}}{h_{21}}$	$\frac{g_{11}}{g_{21}}$
D	$\frac{z_{22}}{z_{21}}$	$-\frac{y_{11}}{y_{21}}$	$\frac{A'}{\Delta_{T'}}$	$-\frac{1}{h_{21}}$	$\frac{\Delta_g}{g_{21}}$

Here, $\Delta_x = x_{11}\,x_{22} - x_{12}\,x_{21}$

$$
\begin{array}{cccccc}
T' & z & y & T & h & g \\
A' & \dfrac{z_{22}}{z_{12}} & -\dfrac{y_{11}}{y_{12}} & \dfrac{D}{\Delta_T} & \dfrac{1}{h_{12}} & -\dfrac{\Delta_g}{g_{12}} \\[2ex]
B' & \dfrac{\Delta_z}{z_{12}} & -\dfrac{1}{y_{12}} & \dfrac{B}{\Delta_T} & \dfrac{h_{11}}{h_{12}} & -\dfrac{g_{22}}{g_{12}} \\[2ex]
C' & \dfrac{1}{z_{12}} & -\dfrac{\Delta_y}{y_{12}} & \dfrac{C}{\Delta_T} & \dfrac{h_{22}}{h_{12}} & -\dfrac{g_{11}}{g_{12}} \\[2ex]
D' & \dfrac{z_{11}}{z_{12}} & -\dfrac{y_{22}}{y_{12}} & \dfrac{A}{\Delta_T} & \dfrac{\Delta_h}{h_{12}} & -\dfrac{1}{g_{12}}
\end{array}
$$

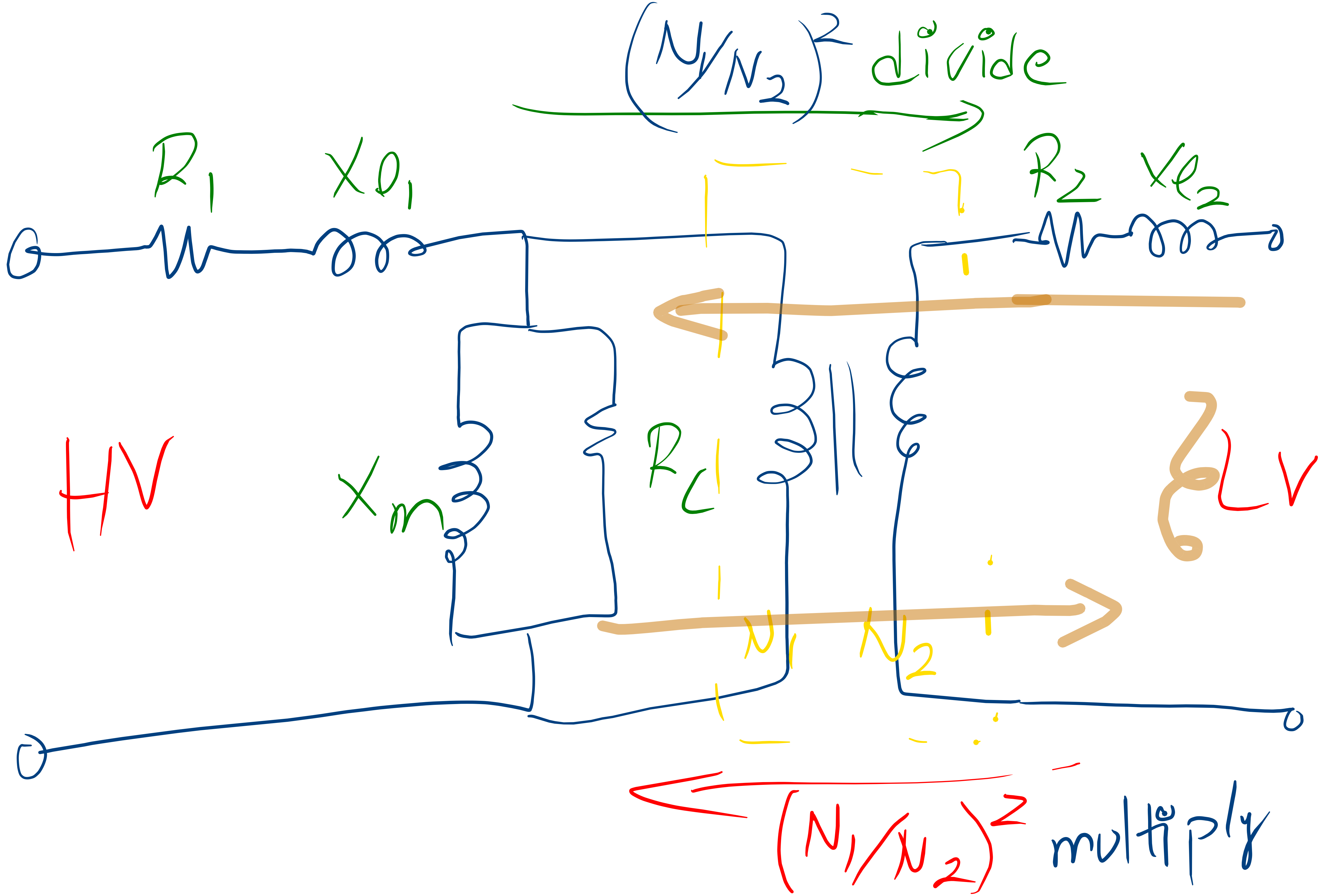
Here, $\Delta_x = x_{11}\, x_{22} - x_{12}\, x_{21}$

h	z	y	T	T'	g
h_{11}	$\frac{\Delta_z}{z_{22}}$	$\frac{1}{y_{11}}$	$\frac{B}{D}$	$\frac{B'}{A'}$	$\frac{g_{22}}{\Delta_g}$
h_{12}	$\frac{z_{12}}{z_{22}}$	$-\frac{y_{12}}{y_{11}}$	$\frac{\Delta_T}{D}$	$\frac{1}{A'}$	$-\frac{g_{12}}{\Delta_g}$
h_{21}	$-\frac{z_{21}}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$-\frac{1}{D}$	$-\frac{\Delta_{T'}}{A'}$	$-\frac{g_{21}}{\Delta_g}$
h_{22}	$\frac{1}{z_{22}}$	$\frac{\Delta_y}{y_{11}}$	$\frac{C}{D}$	$\frac{C'}{A'}$	$\frac{g_{11}}{\Delta_g}$

Here, $\Delta_x = x_{11}\, x_{22} - x_{12}\, x_{21}$

g	z	y	T	T'	h
g_{11}	$\frac{1}{z_{11}}$	$\frac{\Delta_y}{y_{22}}$	$\frac{C}{A}$	$\frac{C'}{D'}$	$\frac{h_{22}}{\Delta_h}$
g_{12}	$-\frac{z_{12}}{z_{11}}$	$\frac{y_{12}}{y_{22}}$	$-\frac{\Delta_T}{A}$	$-\frac{1}{D'}$	$-\frac{h_{12}}{\Delta_h}$
g_{21}	$\frac{z_{21}}{z_{11}}$	$-\frac{y_{21}}{y_{22}}$	$\frac{1}{A}$	$\frac{\Delta_{T'}}{D'}$	$-\frac{h_{21}}{\Delta_h}$
g_{22}	$\frac{\Delta_z}{z_{11}}$	$\frac{1}{y_{22}}$	$\frac{B}{A}$	$\frac{B'}{D'}$	$\frac{h_{11}}{\Delta_h}$

Here, $\Delta_x = x_{11}\, x_{22} - x_{12}\, x_{21}$


$(N_1/N_2)^2$ divide
R_1
X_{e1}
R_2
X_{e2}
HV
X_m
R_c
N_1
N_2
LV
$(N_1/N_2)^2$ multiply