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Closure Properties of Regular Language:

① The Union of two Regular Language are closed under Regular

⇒ If L_1 and L_2 are two regular language then $L_1 \cup L_2$ is also Regular

Proof:-

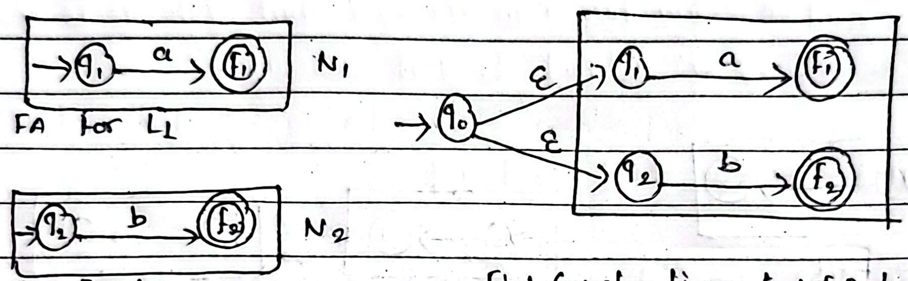


Fig: Construction of NFA to recognize $L_1 \cup L_2$

let $N_1 = \{Q_1, \Sigma_1, S_1, q_1, f_1\}$
let $N_2 = \{Q_2, \Sigma_2, S_2, q_2, f_2\}$

Now, $N = \{Q, \Sigma, S, q_0, f\}$

i.e such that, $L(N) = L_1 \cup L_2$

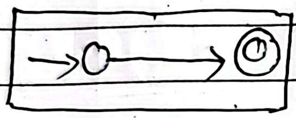
where, $Q_0 = Q_1 \cup Q_2 \cup q_0$
$\Sigma = \Sigma_1 \cup \Sigma_2$
q_0 = initial state
$F = F_1 \cup F_2$

$S(q,a) = S_1(q_0,a)$ for $q \in Q_1$ $\quad S(q,b) = S_1(q,b)$ for $q \in Q_1$
$S(q,b) = S_2(q,b)$ for $q \in Q_2$ $\quad S(q,b) = S_2(q,b)$ for $q \in Q_2$

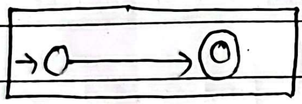
If the current state is q_0 and the input symbol Σ is ε then transition from q_0 to both q_1 ~~an~~ (initial state of Q_1) and q_2 (initial state of Q_2).

② Concatenation of two Regular Language Is Regular.
If L_1 and L_2 are regular language then $L_1 . L_2$ are also regular.

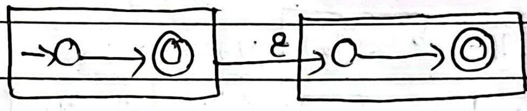
Proof: -



FA for L_1



FA for L_2



FA for $L_1 . L_2$

fig: Construction of NFA to recognize $L_1 . L_2$

$N_1 = \{ Q_1, \Sigma_1, \delta_1, q_1, F_1 \}$

$N_2 = \{ Q_2, \Sigma_2, \delta_2, q_2, f_2 \}$

Now, $N = (Q, \Sigma, \delta, q_0, f \}$

such that $L(N) = L_1 . L_2$

where $Q = Q_1 \cup Q_2$

$\Sigma = \Sigma_1 \cup \Sigma_2$

$q_0 = q_1$

$f = f_2$

$\delta(q, a) = \delta_1(q, a)$ for $q \in Q_1$ and $q \notin f_1$

transition of N_1 for states not in it's final state.

$\delta_2(q, a)$ for $q \in Q_2$ and $q \notin f$

transition of N_2 for states not in it's final state.

3. For $q \in f_1$ (final state of N_1) and $a = \varepsilon$

$\delta(q, a) = \delta_1(q, a) \cup \{q_2\}$

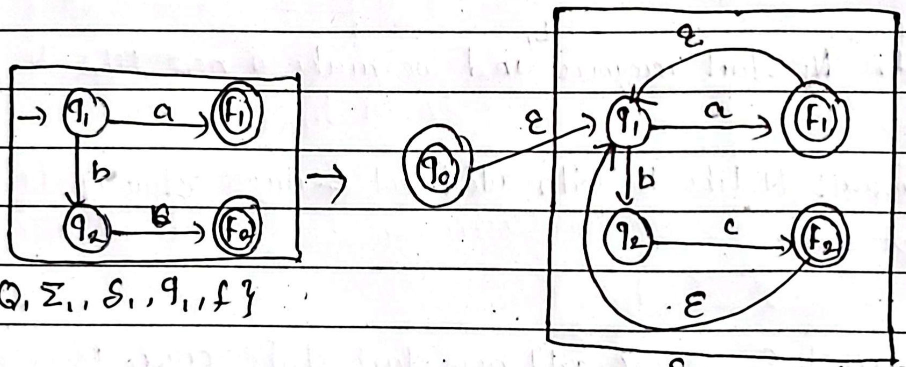
ε-transition connects the final states of N_1 to the initial state of N_2

4. For $\delta(q, a) = \delta_2(q, a)$ for $q \in Q_2$

transition within N_2 remains unchanged.

③ kleene star of two Regular Language is regular.

Proof:



$N_1 = \{Q_1, \Sigma_1, \delta_1, q_1, f\}$

$N = \{Q, \Sigma, \delta, q_0, f\}$

let $N_1 = \{Q_1, \Sigma_1, \delta_1, q_1, f\}$ recognize L.

Now $N = \{Q, \Sigma, \delta, q_0, f\}$ such that

$L(N) = L^*$

Here, $Q = Q_1 \cup q_0$ { states of N includes all states of N_1 plus a new state q_0 which becomes the new start state.

$\Sigma = \Sigma_1$

$q_0 = q_0$

$F = F_1 \cup q_0$ { final states includes the original final state of N_1 and the new start state q_0 }.

δ:

$\delta(q, a) = \delta_1(q, a)$ for $q \in Q_1$ and $q_1 \notin F$ (Transition for N_1)

$\delta(q, a) = \delta_1(q, a) \cup \{q_1\}$ for $q \in F_1$ and $a = \epsilon$

[Add ϵ-transition from the initial states of N_1 back to it's initial state q_1, enabling repeated cycles of L.]

$\delta(q_0, a) = \delta_1(q_0, a) \cup \{q_1\}$ for $a = \epsilon$

[Add an ϵ-transition from the new start state q_0 to the original start state q_1. allowing the automata to start Processing L.

$\delta(q, a) = \phi$ for $q = q_0$ and $a \neq \epsilon$.

→ Here NFA N_1 start recognize L_1 and we make a new NFA N For L^*

→ We construct N Like N_1 with additional ϵ-arrow returning to start state from accept state.

→ N also accept ϵ, so we, add new start state which is also final state and has ϵ-arrow to old input state.

4. Regular Language are closed under complementation.

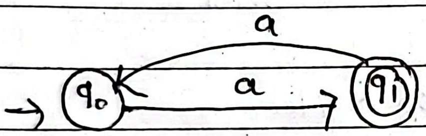
If L is regular then L' is also Regular.

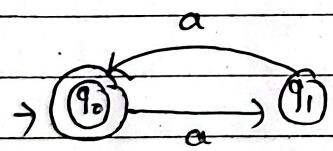
Proof:

Let $M = \{Q, \Sigma, \delta, q_0, f\}$ be a finite automata then the computing Language $\Sigma^* - L(M)$ is accepted by finite automata.

$$\bar{M} = \{Q, \Sigma, \delta, q_0, Q - f\}$$

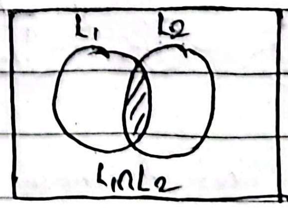
That is $\bar{M}$ is identical to M except that final and non-final states are interchanged.





5. Intersection: [Regular Language are closed under Intersection]
If L_1 and L_2 are regular than $L_1 \cap L_2$ is also Regular

Proof:



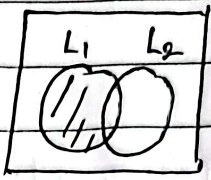
By demorgan's Law
$L_1 \cap L_2 = \overline{\bar{L}_1 \cup \bar{L}_2}$

Here, L_1 and L_2 are regular. The complement of regular Language is regular. So $\bar{L}_1$ and $\bar{L}_2$ also regular. Again, we already prove that union of two regular language are regular. So $\bar{L}_1 \cup \bar{L}_2$ also Regular. So, $L_1 \cap L_2 = \overline{\bar{L}_1 \cup \bar{L}_2}$ is also regular

6. If L_1 and L_2 are regular then $L_1 - L_2$ is also Regular.

Proof:-

$$L_1 - L_2 = L_1 \cap \bar{L}_2$$



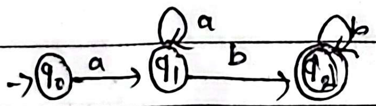
Since, complement of L_2 is regular as L_2 is regular. Intersection of L_1 and $\bar{L}_2$ is also regular. Hence the difference of two regular language is Regular.

7. Regular Language are closed under Reverse or Transpose!

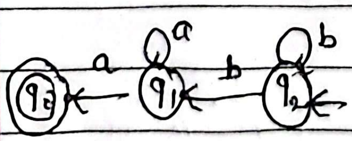
If L is regular then L^R / L^r is also Regular.

Proof:

Let $L = \{a^n b^m \mid n, m \geq 1\}$



$L^R = \{b^m a^n \mid n, m \geq 1\}$



So finite Automata for L^R is obtained by making start to final and final to start state and flip the direction of arrow ~~form~~ of FA of L.

Since we get FA for L^R Hence L^R is also Regular.