Chapter-4
Turing Machine

4.1 Introduction to turing Machine, representation of TM, move of a TM, instantaneous description for TM.

* Introduction to turing Machine:-

↳ Turing Machine is an abstract machine developed by Alan Turing in 1936

↳ The turing machine provides the theoretical foundation for modern computers.

↳ The turing machine consists of finite control, which can be in any of finite set of states. There is a tape divided into squares or cells of infinite length.

↳ Each cell can hold any one of finite number of symbol as shown in figure below:-

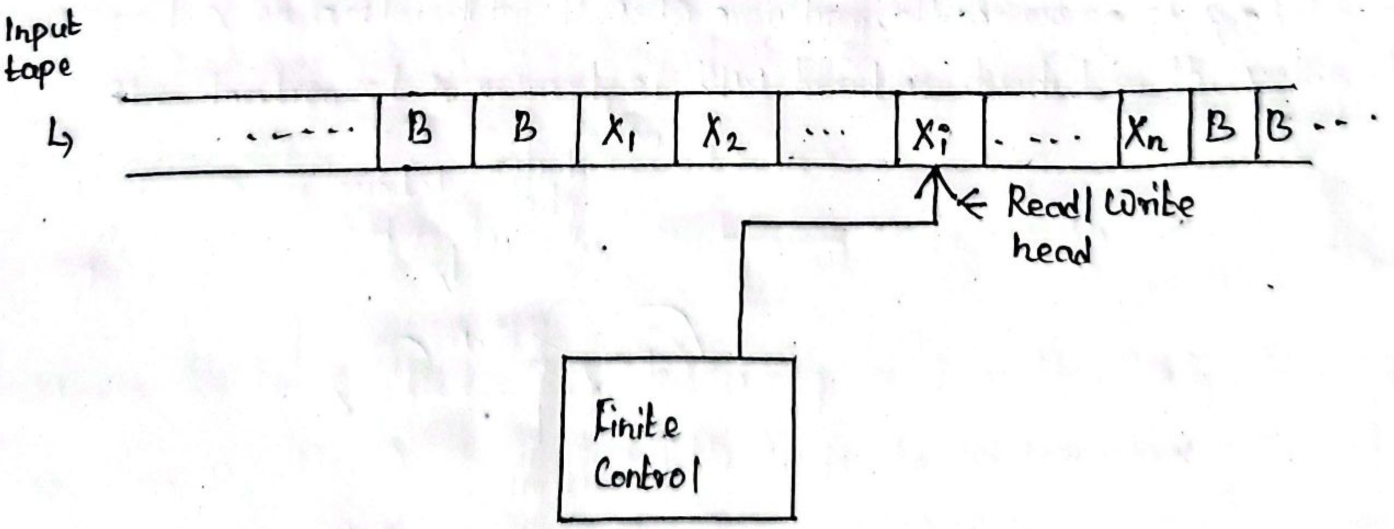


fig: Turing Machine

↳ Initially, the input string is placed on the tape and all other tape cell hold symbol called blank.

↳ The blank Is a tape symbol but not an input symbol, and there may be other tape symbol besides the input symbol.

↳ A move of the turing Machine is a function of the state of turing machine and tape symbol scanned in one move turing machine will:

① change state (or might be same)

② write the tape symbol in the cell scanned. Optionally symbol written may be the same as the symbol currently there.

③ Move the tape head one square right or left or leaving it where it is.

④ formal defination of Turing Machine:-

formally, A Turing Machine is defined by 7-tuples;

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

where,

Q: set of finite states

Σ: finite set of input symbols

Γ: complete set of tape symbols

q_0: start state, $q_0 \in Q$

B: The blank symbol. $B \in \Gamma$ but $B \notin \Sigma$

F: set of final or accepting states $F \subseteq Q$

δ: The transition function defined by,

$Q \times \Gamma \to Q \times \Gamma \times (L, R, S)$ where R, L, S is the direction of head i.e Right/Left/Stationary

Ⓧ Representation of TM

① Transition Diagram:

→ we can use transition systems to represent TM.

→ The states are represented by vertices.

→ Directed edge are used to represent transition of states.

→ start state is represented by arrow head coming from nowhere pointing to the circle and final state is marked a double circle.

→ The labels are triples of the form (α, β, γ) where $\alpha, \beta \in \Gamma$ and $\gamma \in \{L, R, S\}$

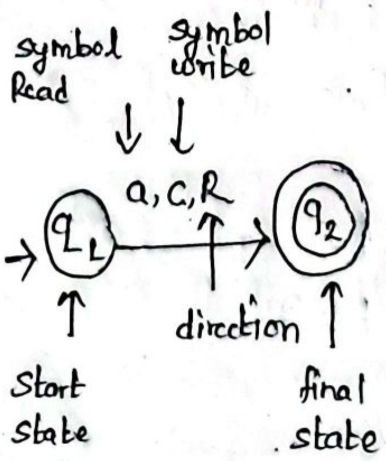
→ when there is a directed edge from q_i to q_j with label (α, β, γ) it means that

$$\delta(q_i, a) = (q_j, \beta, \gamma)$$

where $\alpha, \beta \in \Gamma$ and $\gamma \in \{L, R, S\}$

The triple (α, β, γ) indicates that the symbol under R/W head is α and α is replaced by β as a resulting of processing of symbol α and the direction of movement of R/W head is decided by the value of γ.

eg.



(2) Transition Table :-

→ We will define 'δ' in terms of a table called transition table.

→ If $\delta(q,a) = (\gamma, \alpha, \beta)$ we write $\alpha\beta\gamma$ under a-column and q-row.

→ So, if we get $\alpha\beta\gamma$ in the table, it means that α is written in the current cell, β gives the movement of head (L or R) and γ denotes the new state into which the Turing Machine enters.

eg:

Set of States	Tape Symbol	
	a	b
$\rightarrow q_1$	cRq_2	–
$* q_2$	–	cLq_2

next state

symbol written

head movement

(3) Instantaneous Description :-

↳ ID of a TM is snapshot of TM to describe the current situation of the TM.

↳ Let the initial ID of a TM is $x_1 x_2 \ldots x_{i-1}\, q\, x_i\, x_{i+1} \ldots x_n$

So,

$$x_1 x_2 \ldots x_{i-1}\, q\, x_i\, x_{i+1} \ldots x_n \;\vdash^{\delta(q,x_i)=(P,y,L)}\; x_1 x_2 \ldots x_{i-2}\, P\, x_{i-1}\, y\, x_{i+1} \ldots x_n$$

$$x_1 x_2 \ldots x_{i-1}\, q\, x_i\, x_{i+1} \ldots x_n \;\vdash^{\delta(q,x_i)=(P,y,R)}\; x_1 x_2 \ldots x_{i-1}\, y\, P\, x_{i+1} \ldots x_n$$

(*) Moves of a Turing Machine:-

→ Suppose $\delta(q_1, x_3) = (q_2, y, R)$ and the input string to be processed is $x_1, x_2, \dots x_n$ and the current symbol under Read/write is x_3. Therefore, ID before processing the input symbol x_3 is

$$x_1 x_2 q_1 x_3 x_4 \dots x_n$$

→ After processing the input symbol x_3, the resulting ID will be

$$x_1 x_2 y q_2 x_4 x_5 \dots x_n$$

because according to $\delta(q_1, x_3) = (q_2, y, R)$ the symbol 'x_3' is replaced by 'y' and there is a right move. Therefore, the current symbol under R/W head is 'x_4'. The change of ID can be represented as

$$x_1 x_2 q_1 x_3 x_4 \dots x_n \vdash x_1 x_2 y q_2 x_4 x_5 \dots x_n$$

Similarly, if there is a transition $\delta(q_1, x_3) = (q_2, y, L)$ on place of $\delta(q_1, x_3) = (q_2, y, R)$ then the change of ID is represented as

$$\cancel{x_1 x_2 q_1 x_3 x_4}$$

$$x_1 x_2 q_1 x_3 x_4 \dots x_n \vdash x_1 q_2 x_2 y x_4 \dots x_n$$

If we denote an ID by I_J for some J and another ID by I_K and the machine is able to reach from ID I_J to I_K in some moves then it is represented by

$$I_J \vdash^{*} I_K$$

Language acceptability by Turing Machine / Language of Turing Machine:

Suppose the Turing Machine $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$. A string w in Σ^* is said to be accepted by M if

$$q_0 w \vdash^* \alpha_1 p \alpha_2 \text{ for some } p \in F \text{ and } \alpha_1 \text{ and } \alpha_2 \in \Gamma^*$$

M doesnot accept w if the machine M either halts in a non-accepting state or doesnot halt.

So, a turing machine is said to accept a Language when it starts with the initial state and input string and processing that string it reaches to the final state i.e

$$\underset{\substack{\uparrow\\ \text{current}\\ \text{state}}}{q_0}\ (\underset{\substack{\uparrow\\ \text{input}\\ \text{string}}}{w}) \vdash^* \underset{\substack{\uparrow\\ \text{final}\\ \text{state}}}{q_f}$$

Role of Turing Machine [4.2 Computing with Turing Machine]:-

① **As a Language Recognizer**:- Turing Machine can be used for accepting a Language like finite Automata & PDA.

② **As a computer of function**:- Turing Machine can represents a particular function.

③ **As an enumerator of strings of a Language**:- In this role, turing machine outputs the strings of a language one at a time in some systematic order.

Example 1 Design the turing machine accepting language $\{0^n 1^n \mid n \geq 1\}$

Soln:- → Here the language consists of any number of 0's followed by same number of 1's.

→ For designing the turing machine, TM should read first symbol 0 at state q_0, change it to X and goes to state q_1 and move right in the state q_1 until 1 is visited on the tape.

→ In state q_1, it change 1 to Y and goes to state q_2 and moves left over Y's and 0's until it find the symbol X.

→ when it find the symbol X change state to q_0 and move right and so on. So, the turing machine for this language can be constructed as

0	0	0	1	1	1

Let M be the turing machine which is formally defined as

$$M = (Q, \Sigma, \delta, q_0, \Gamma, B, F)$$

where, $Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{0, 1\}$

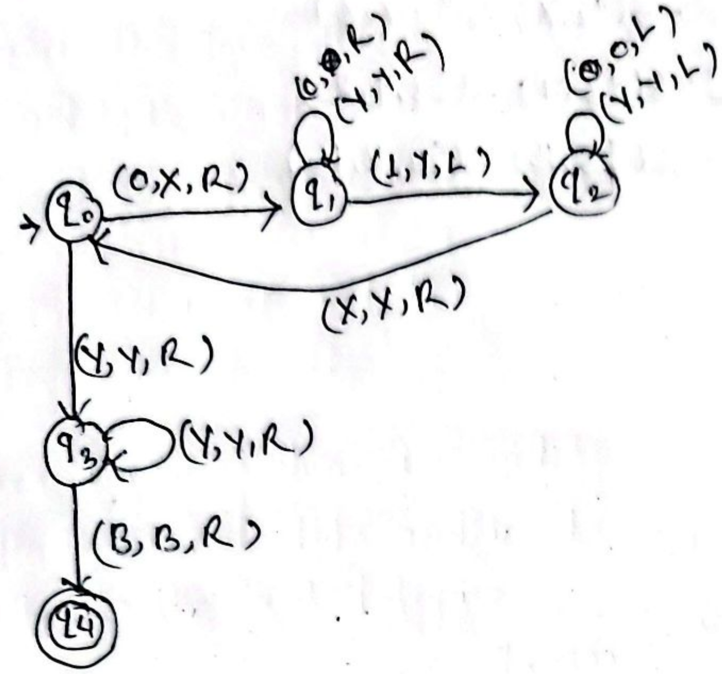
$q_0 = \{q_0\}$

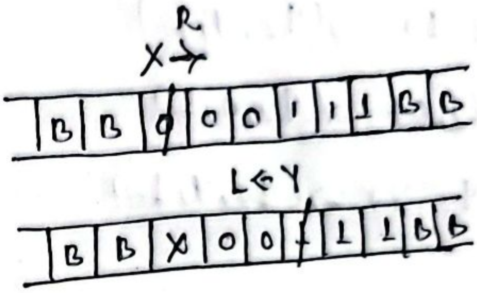
$B = \{B\}$

$\Gamma = \{0, 1, X, Y, B\}$

$F = \{q_4\}$

Transition diagram is given by:





Transition Table is given by:

Set of States	Tape Symbol 0	1	X	Y	B
$\rightarrow q_0$	XRq_1	-	-	YRq_3	-
q_1	$0Rq_1$	YLq_2	-	YRq_1	-
q_2	$0Lq_2$	-	XRq_0	YLq_2	-
q_3	-	-	-	YRq_3	BRq_4
$* q_4$	-	-	-	-	-

Transition function (δ) is given by:

1. $\delta(q_0,0) \rightarrow (q_1,X,R)$
2. $\delta(q_0,Y) \rightarrow (q_3,Y,R)$
3. $\delta(q_1,0) \rightarrow (q_1,0,R)$
4. $\delta(q_1,Y) \rightarrow (q_1,Y,R)$
5. $\delta(q_1,1) \rightarrow (q_2,Y,L)$.
6. $\delta(q_2,0) \rightarrow (q_2,0,L)$
7. $\delta(q_2,Y) \rightarrow (q_2,Y,L)$
8. $\delta(q_2,X) \rightarrow (q_0,X,R)$
9. $\delta(q_3,Y) \rightarrow (q_3,Y,R)$
10. $\delta(q_3,B) \rightarrow (q_4,B,R)$

Now let us take $\omega = 0011$.

$q_0 0011 \vdash X q_1 011$

$\vdash X0q_1 11$

$\vdash Xq_2 0Y1$

$\vdash q_2 X0Y1$

$\vdash Xq_0 0Y1$

$\vdash XXq_1 Y1$

$\vdash XXYq_1 1$

$\vdash XX\cancel{q}q_2 YY$

$\vdash Xq_2 XYY$

$\vdash XXq_0 YY$

$\vdash XXYq_3 Y$

$\vdash XXYYq_3 B$

$\vdash XXYYBq_4 B$ Halt & accept