Chapter 2
Data Communication Fundamentals

Data: Data is a collection of raw, unorganized facts and details like text, observations, figures, symbols and descriptions of things etc. In other words, data does not carry any specific purpose and has no significance by itself. It is measured in terms of bits and bytes in context of computer storage and processing. Data may be analog or digital, qualitative or quantitative, structured or unstructured etc.

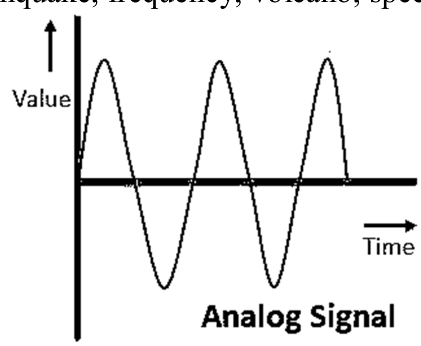
Analog data: Analog data refers to information that is continuous and take continuous values. It has an infinite number of values. Analog data can be represented by continuous waveforms such as sine waves. It varies smoothly over time but susceptible to noise and distortion over long distances. Typical examples of analog data are Human voice in air (sound waves), light, temperature, radio signals etc.

Digital data: Digital data refers to information that has discrete states and takes only discrete values. It has finite number of values. Digital data can be represented using binary numbers i.e. 1 and 0. Digital data are easy to store, process and transmit and less affected by noise. Digital data can easily be compressed and encrypted. Typical examples of digital data are text files, images, videos etc.

Difference between Analog and Digital data

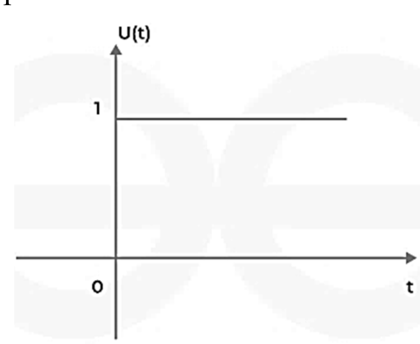
Feature	Analog Data	Digital Data
Nature	Continuous	Discrete
Examples	Sound, temperature	Text, numbers, computer files
Transmission	Analog signals	Digital signals
Noise resistance	Low	High
Storage	Difficult and less reliable	Easy and reliable
Processing	Complex	Simple

Analog signal: Analog signal is a continuous signal in which one time-varying quantity represents another time-based variable. These kind of signals works with physical values and natural phenomena such as earthquake, frequency, volcano, speed of wind, weight, lighting, etc.



There are some basic types of signal

Unit Step signal: The continuous time function u(t) is defined as a positive function for its discrete time counterpart, the unit step is discontinuous at t=0.



$$U(t) = \begin{cases} 1, & for\ t \geq 0 \\ 0, & for\ t < 0 \end{cases}$$

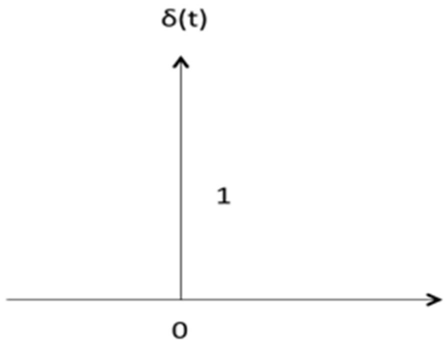
Or

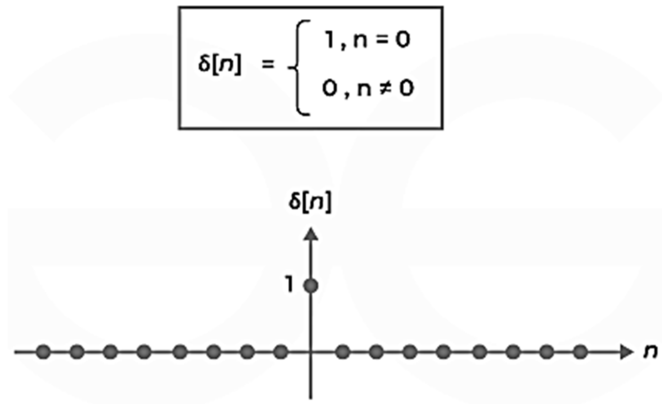
$$U(n) = \begin{cases} 1, & for\ n \geq 0 \\ 0, & for\ n < 0 \end{cases}$$

- It is used as best test signal.
- Area under unit step function is unity.

Unit Impulse Signal or Impulse Signal or Delta Signal
The continuous-time unit impulse function δ(t) is related to the unit step in a manner analogous to the relationship between the discrete-time unit impulse and step functions. Specifically, the continuous-time unit step function is obtained by performing the running integral of the unit impulse function.

$$\delta(t) = \begin{cases} 1, & for\ t = 0 \\ 0, & for\ t \neq 0 \end{cases}$$





$$\int_{-\infty}^{\infty} \delta(t)dt = u(t)$$

$$\delta(t) = \frac{du(t)}{dt}$$

Ramp Signal: Ramp signal is denoted by r(t), and it is defined as

$$r(t) = \begin{cases} t, & for\ t \geq 0 \\ 0, & for\ t < 0 \end{cases}$$ For continuous signal

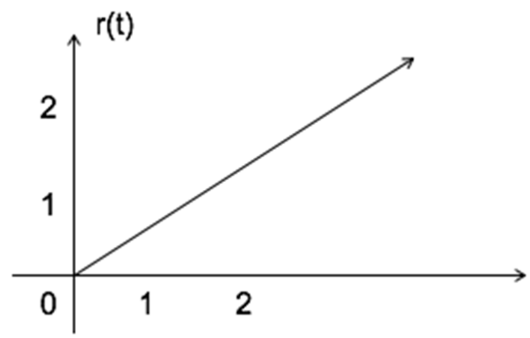
$$r(n) = \begin{cases} n, & for\ n \geq 0 \\ 0, & for\ n < 0 \end{cases}$$ For discrete signal

$$\int u(t)dt = \int 1\ dt = t = r(t)$$

Differentiating both sides with respect to t

$$u(t) = \frac{dr(t)}{dt}$$

Area under unit ramp is unity.



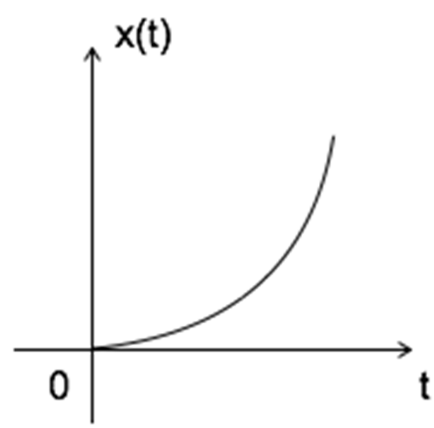
Parabolic Signal: Parabolic signal can be defined as

$$x(t) = \begin{cases} \frac{t^2}{2}, & for\ t \geq 0 \\ 0, & for\ t < 0 \end{cases}$$

$$\iint u(t)dt = \int r(t)dt = \int tdt = \frac{t^2}{2} = parabolicsignal$$

$$\Rightarrow u(t) = \frac{d^2x(t)}{dt^2}$$

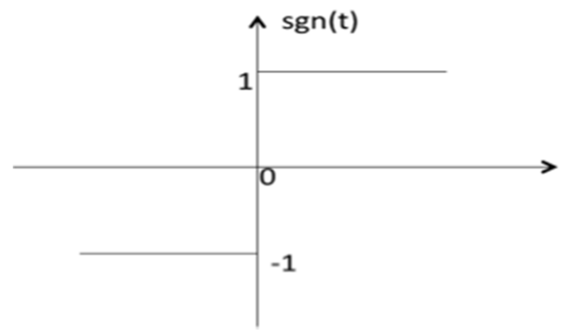
$$\Rightarrow r(t) = \frac{dx(t)}{dt}$$



Signum Function: Signum function is denoted as sgn(t). It is defined as

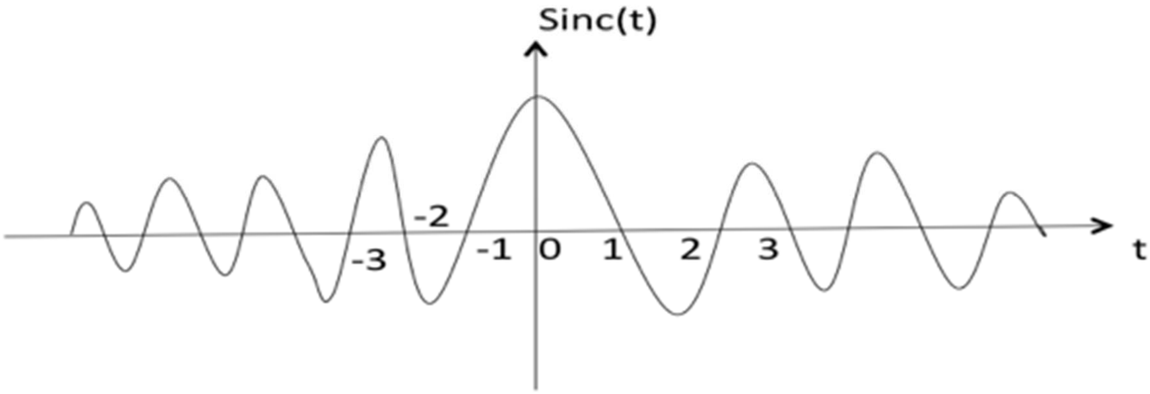
$$sgn(t) = \begin{cases} 1, & for\ t > 0 \\ 0, & for\ t = 0 \\ -1, & for\ t < 0 \end{cases}$$

$$sgn(t) = 2u(t) - 1$$



Sinc Function: It is denoted as sinc(t) and it is defined as

$$Sinc(t) = \frac{\sin \pi t}{\pi t} = 0, \quad for, \quad t = \pm1, \pm2, \pm3, \dots\dots\dots\dots\dots$$
$$= 1, \quad for, \quad t = 0$$

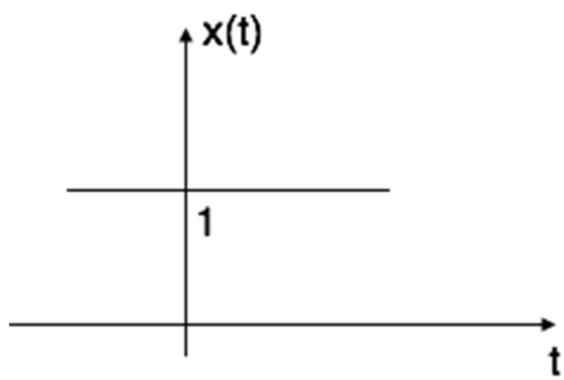


Exponential Signal: Exponential signal is in the form of

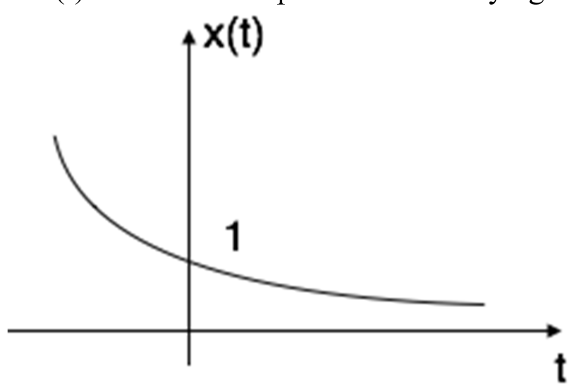
$$x(t) = e^{\alpha t}$$

The shape of exponential can be defined by α.

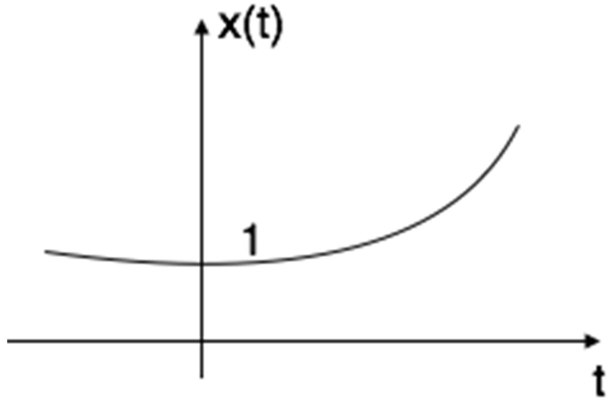
Case i: if $\alpha = 0 \rightarrow x(t) = e^{0} = 1$



Case ii: if $\alpha < 0$ i.e. -ve then $x(t) = e^{-\alpha t}$. The shape is called decaying exponential.

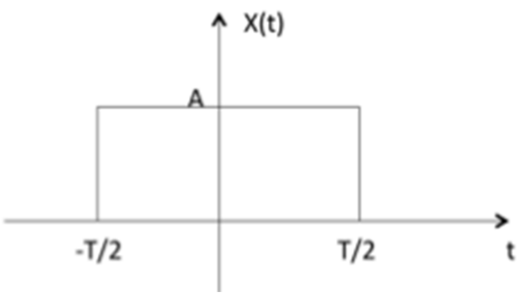


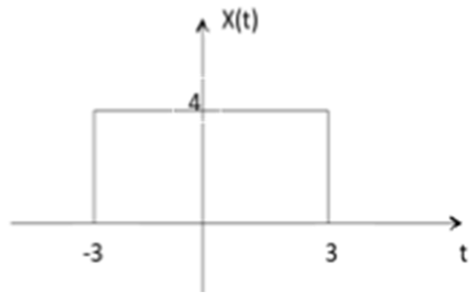
Case iii: if $\alpha > 0$ i.e. +ve then $x(t) = e^{\alpha t}$. The shape is called raising exponential.



Rectangular Signal: Let it be denoted as x(t) and it is defined as

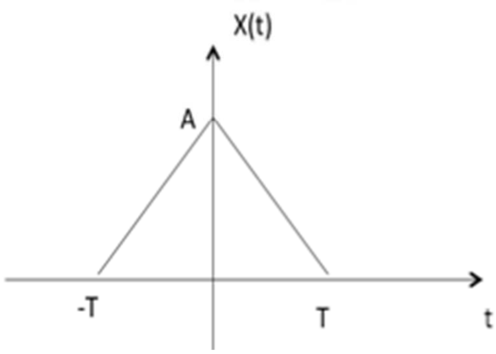
$$x(t) = A\,rect\left[\frac{r}{T}\right] \qquad \text{ex: } 4\,rect\left[\frac{r}{6}\right]$$

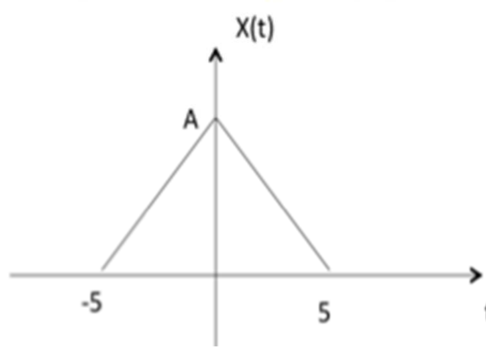




Triangular Signal: Let it be denoted as x(t)

$$x(t) = A\left[1 - \frac{|t|}{T}\right] \qquad \text{ex: } x(t) = A\left[1 - \frac{|t|}{5}\right]$$





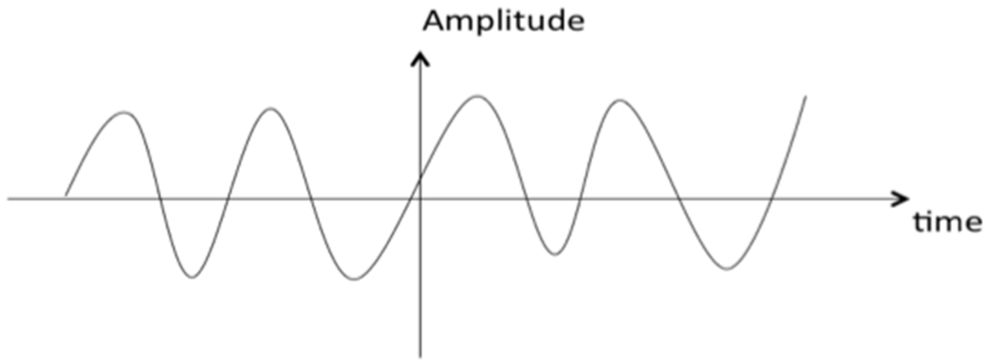
<u>Classification of signals</u>

Signals are classified into the following categories:

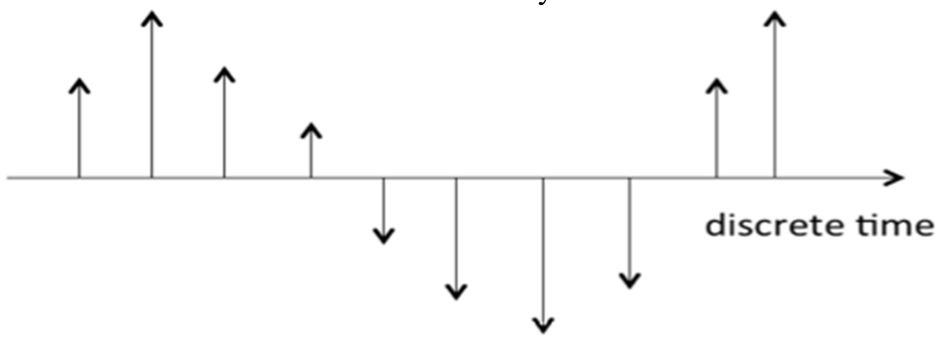
- Continuous Time and Discrete Time Signals
- Deterministic and Non-deterministic Signals
- Even and Odd Signals
- Periodic and Aperiodic Signals
- Energy and Power Signals

Continuous Time and Discrete Time Signals

A signal is said to be continuous when it is defined for all instants of time.

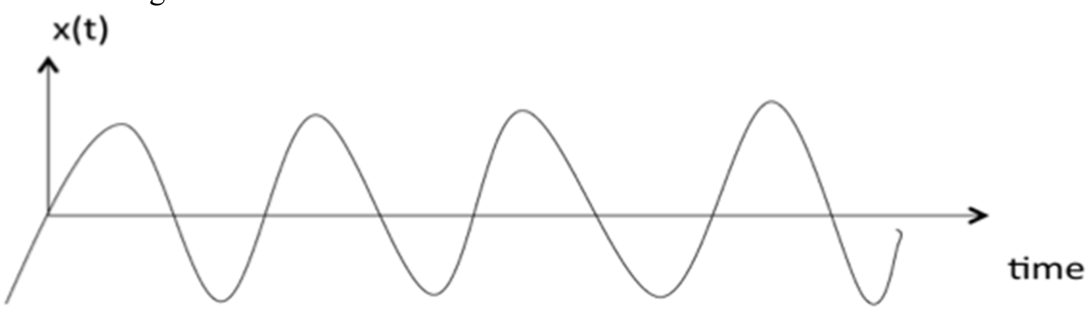


A signal is said to be discrete when it is defined at only discrete instants of time.

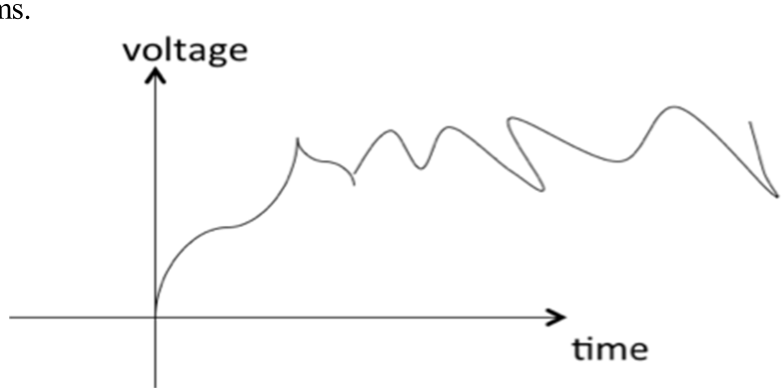


Deterministic and Non-deterministic Signals

A signal is said to be deterministic if there is no uncertainty with respect to its value at any instant of time. Or, signals which can be defined exactly by a mathematical formula are known as deterministic signals.



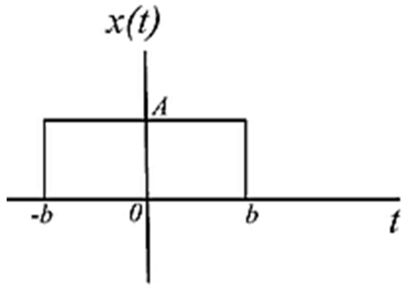
A signal is said to be non-deterministic if there is uncertainty with respect to its value at some instant of time. Non-deterministic signals are random in nature hence they are called random signals. Random signals cannot be described by a mathematical equation. They are modeled in probabilistic terms.

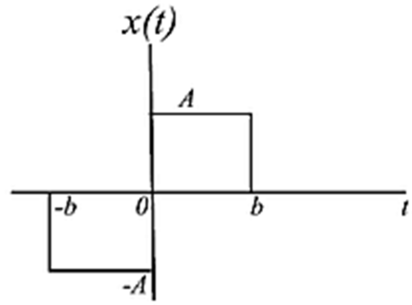


Even and Odd Signals

A signal is said to be even when it satisfies the condition $x(t) = x(-t)$ Or, $x(n) = x(-n)$

A signal is said to be odd when it satisfies the condition $x(t) = -x(-t)$ Or, $x(n) = - x(-n)$





Even signal is symmetric about vertical axis whereas odd signal is anti-symmetric about vertical axis. Any function x(t) can be expressed as the sum of its even function $x_e(t)$ and odd function $x_o(t)$.

$$x(t) = x_e(t) + x_0(t) \qquad \text{.... (1)}$$

Where,

$x_e(t)$ = Even component of the signal x(t)

$x_0(t)$ = Even component of the signal x(t)

Putting t = – t in equation (1), we get,

$$x(-t) = x_e(-t) + x_0(-t) \text{ .... (2)}$$

For the even signal,

$$x_e(t) = x_e(-t) \qquad \text{......(3)}$$

For the odd signal,

$$x_0(-t) = -\, x_0(t) \qquad \text{......(4)}$$

Expression for Even part of x(*t*),

Substituting equation (3) and equation (4) in equation (2)

$$x(-t) = x_e(t) - x_0(t) \qquad \text{......(5)}$$

Adding equation (1) and (5), we get

$$x(t) + x(-t) = 2\, x_e(t)$$
$$x_e(t) = \tfrac{1}{2}[x(t) + x(-t)] \quad \text{....(6)}$$

Expression for odd part of x(*t*),

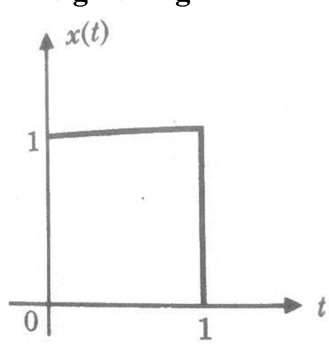
Subtracting equation (5) from equation (1), we get

$$x(t) - x(-t) = 2\, x_0(t)$$
$$x_0(t) = \tfrac{1}{2}[x(t) - x(-t)] \quad \text{....(7)}$$

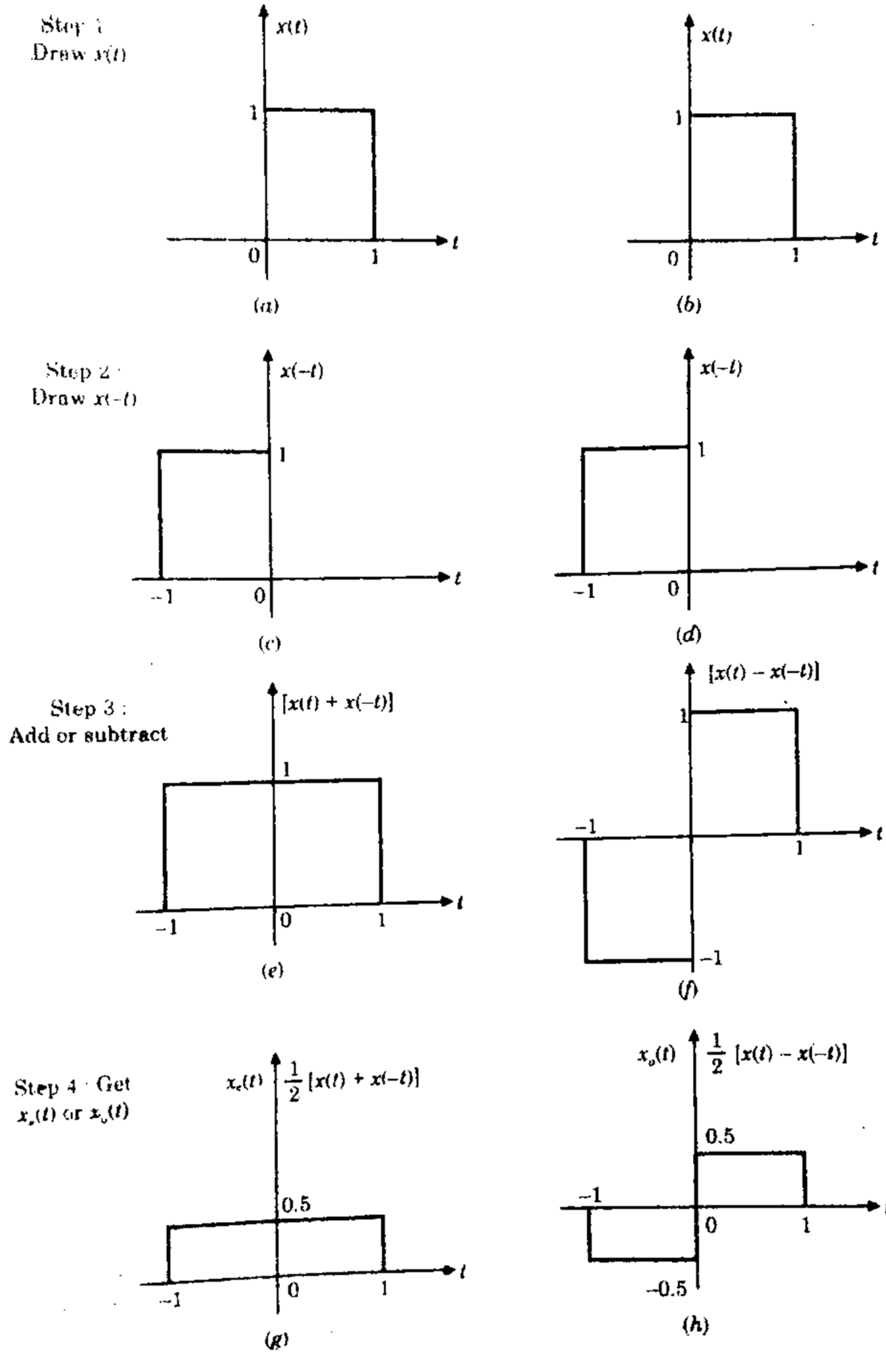
For discrete signal, expressions for even and odd part are as follows

$$x_e(n) = \tfrac{1}{2}[x(n) + x(-n)]$$
$$x_0(n) = \tfrac{1}{2}[x(n) - x(-n)]$$

Q. Draw the even and odd part of the given signal



Solution:



Energy and Power Signals

A signal is said to be energy signal when it has finite energy and zero power.

$$Energy\ (E) = \int_{-\infty}^{\infty} x^2(t)dt \qquad Or, \qquad E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

A signal is said to be power signal when it has finite power and infinite energy.

$$Power\ (P) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)dt \qquad Or, \qquad P = \lim_{N\to\infty} \frac{1}{N} \sum_{n=-N/2}^{N/2} |x(n)|^2$$

NOTE: A signal cannot be both, energy and power simultaneously. Also, a signal be neither energy nor power signal.

Difference between power and energy signal

S.N.	**Energy signal**	Power signal
1.	Total normalized Energy is finite and non-zero.	The normalized average power is finite and non-zero.
2.	$Energy\ (E) = \int_{-\infty}^{\infty} x^2(t)dt$	$Power\ (P) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)dt$
3.	Power of the energy signal is zero	Energy of the power signal is infinite.
4.	Non periodic signals are energy signals	Periodic signals are power signals.
5.	These signals are time limited	These signals can exist over infinite time
6.	Single rectangular pulse is an example of the energy signal	A periodic pulse train is an example of the power signal

Q. Check whether the signal, $x(t) = A\sin t\,,\ for -\infty < t < \infty$, is power or energy signal.
Solution:
As we knows that the signal has time period of 2π
The power of the signal is

$$Power\ (P) = \lim_{T\to\infty} \frac{1}{T} \int_0^T x^2(t)dt = \frac{1}{2\pi}\int_0^{2\pi} A^2 \sin^2 t\, dt = \frac{A^2}{2\pi}\int_0^{2\pi} \frac{1}{2}(1 - cos2t)dt$$

$$P = \frac{A^2}{4\pi}\left[t - \frac{\sin 2t}{2}\right]_0^{2\pi} = \frac{A^2}{4\pi}\left[2\pi - \frac{\sin 4\pi}{2} - 0 + 0\right] = \frac{A^2}{4\pi} * 2\pi = \frac{A^2}{2}$$

The energy of the signal is given as

$$Energy\ (E) = \int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} A^2 \sin^2 t\, dt = A^2 \int_{-\infty}^{\infty} \frac{1}{2}(1 - cos2t)dt$$

$$E = \frac{A^2}{2}\left[t - \frac{\sin 2t}{2}\right]_{-\infty}^{\infty} = \infty$$

Power of the signal is finite and energy of the signal is infinite therefore the signal is power signal.

Q. Check whether the signal, $x(t) = e^{-a|t|},\ for\ a > 0$, is power or energy signal.
Solution: given signal is

$$x(t) = e^{-a|t|}, \qquad for\ a > 0$$

The signal can be expressed as

$$x(t) = \begin{Bmatrix} e^{-at}, & for\ t > 0 \\ e^{at}, & for\ t < 0 \end{Bmatrix}$$

The energy of the signal is given as

$$Energy\ (E) = \int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} (\boldsymbol{e^{-a|t|}})^2 dt$$

$$E = \int_{-\infty}^{0} \boldsymbol{e^{2at}}dt + \int_{0}^{\infty} \boldsymbol{e^{-2at}}dt = \left[\frac{\boldsymbol{e^{2at}}}{2a}\right]_{-\infty}^{0} + \left[\frac{\boldsymbol{e^{-2at}}}{-2a}\right]_0^{\infty}$$

$$E = \left[\frac{1}{2a} - 0\right] + \left[0 + \frac{1}{2a}\right] = \frac{1}{a}$$

The power of the signal is

$$Power\ (P) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)dt = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} (e^{-a|t|})^2 dt$$

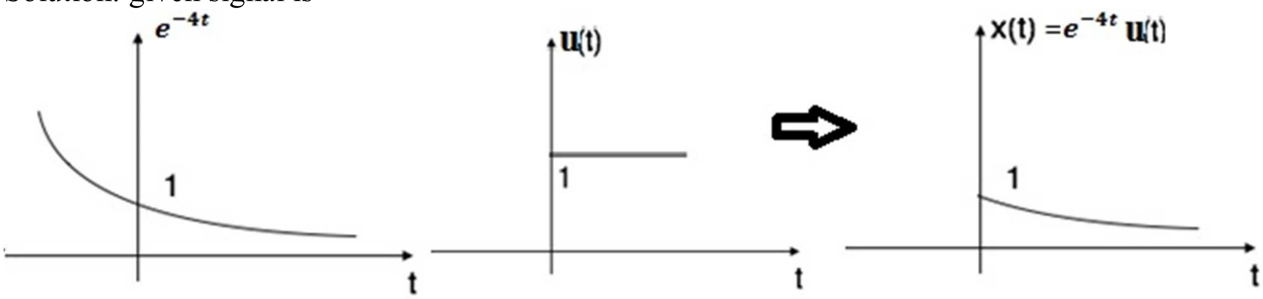
$$P = \lim_{T\to\infty} \frac{1}{T} \left[\int_{-T/2}^{0} e^{2at}dt + \int_{0}^{T/2} e^{-2at}dt \right] = \lim_{T\to\infty} \frac{1}{T} \left[\left[\frac{e^{2at}}{2a}\right]_{-T/2}^{0} + \left[\frac{e^{-2at}}{-2a}\right]_{0}^{T/2} \right]$$

$$P = \lim_{T\to\infty} \frac{1}{T} \left[\frac{1}{2a} - \frac{e^{-aT}}{2a} + \frac{e^{-aT}}{2a} + \frac{1}{2a} \right] = \lim_{T\to\infty} \frac{1}{T} * \frac{1}{a} = \frac{1}{\infty} * \frac{1}{a} = 0$$

Energy of the signal is finite and power is zero, therefore the signal is energy signal.

Q. Check whether the signal, $x(t) = e^{-4t}u(t)$, is power or energy signal.

Solution: given signal is



The energy of the signal is given as

$$Energy\ (E) = \int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} [e^{-4t}u(t)]^2 dt = \int_{0}^{\infty} e^{-8t}dt$$

$$E = \left[\frac{e^{-8t}}{-8}\right]_0^{\infty} = \left[0 - \left(-\frac{1}{8}\right)\right] = \frac{1}{8}$$

The power of the signal is given by

$$P = \lim_{T\to\infty} \frac{E}{T} = \lim_{T\to\infty} \frac{1/8}{T} = 0$$

Energy of the signal is finite therefore the signal is energy signal.

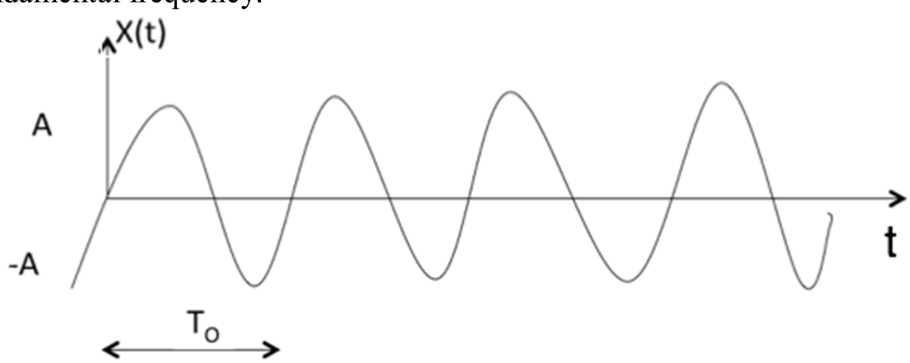
Periodic and Aperiodic Signals

A periodic signal is defined as a signal which has a definite pattern which repeats itself at regular intervals of time. OR, A signal is said to be periodic if x(t) = x(t + T_o) or x(n) = x(n + N).

Where,

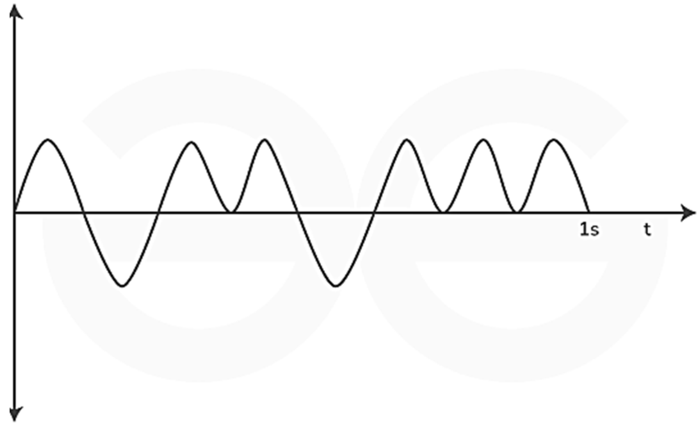
T = fundamental time period,

1/T = f = fundamental frequency.



The above signal will repeat for every time interval T_0 hence it is periodic with period T_0.

A signal which does not repeat at regular intervals of time is known as aperiodic signal. The aperiodic signals are also called the non-periodic signals. OR, A signal is said to be aperiodic if $x(t) \neq x(t + T)$ or $x(n) \neq x(n + N)$.



Notes: periodic signals are power signals and aperiodic signals are energy signals.

Q. shows that the sine wave, $x(t) = A \sin \omega t$, is a periodic signal.
As we know, for periodic signal

$$x(t) = x(t + T)$$

So,

$$x(t + T) = A \sin \omega(t + T) = A \sin(\omega t + \omega T)$$

But,

$$\omega = 2\pi f \quad and \quad T = \frac{1}{f}$$

Therefore,

$$\omega T = 2\pi f T = 2\pi f * \frac{1}{f} = 2\pi$$

$$A \sin(\omega t + \omega T) = A \sin(\omega t + 2\pi) = A[sin\omega t \cos 2\pi + \cos\omega t sin 2\pi]$$

$$A[sin\omega t * 1 + \cos\omega t * 0] = Asin\omega t = x(t)$$

Therefore, the sine wave is periodic signal.

Q. shows that the exponential signal, $x(t) = e^{-at}$, is a non-periodic signal.
As we know, for periodic signal

$$x(t) = x(t + T)$$

So,

$$x(t + T) = e^{-a(t+T)} = e^{-at}e^{-aT}$$

But, T→∞,

$$x(t + T) = e^{-at}e^{-a\infty} = e^{-at}e^{-\infty} = e^{-at} * 0 = 0$$

$$x(t) \neq x(t + T)$$

Therefore, the exponential signal is non-periodic signal.

Periodic Signals Characteristics
Periodic analog signals can be classified as simple or composite. A simple periodic analog signal, a sine wave, cannot be decomposed into simpler signals. A composite periodic analog signal is composed of multiple sine waves.

Most commonly used periodic signal is Sine wave. The main characteristics are as follows:
Peak Amplitude: The peak amplitude of a signal is the absolute value of its highest intensity, proportional to the energy it carries. For electric signals, peak amplitude is normally measured in volts.

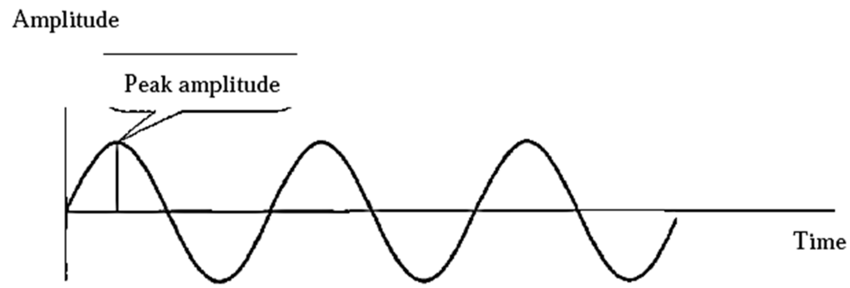


Figure: Signal with high peak amplitude

Period: It is the time required to complete one cycle of signal. It's unit is second.
Frequency: it is defined as the number of oscillation per second. Or it is the number of periods in one second. It's unit is Hertz (Hz)

$$f = \frac{1}{T} \quad and \quad T = \frac{1}{f}$$

Note: Frequency and Period are inverse of each other.

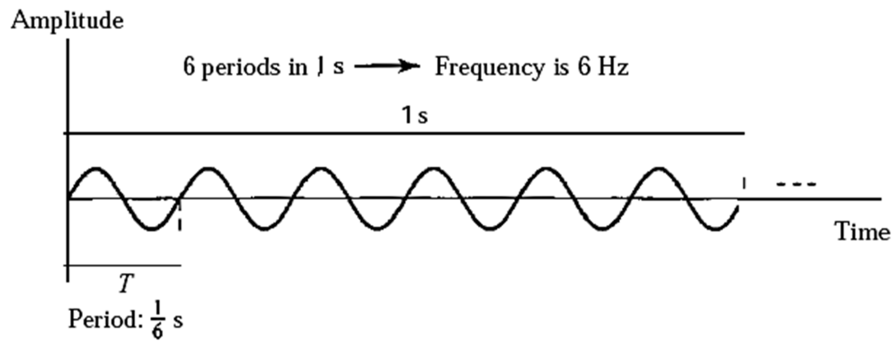


Figure: A signal with a frequency of 6Hz

Phase: Phase describes the position of the waveform relative to time 0.

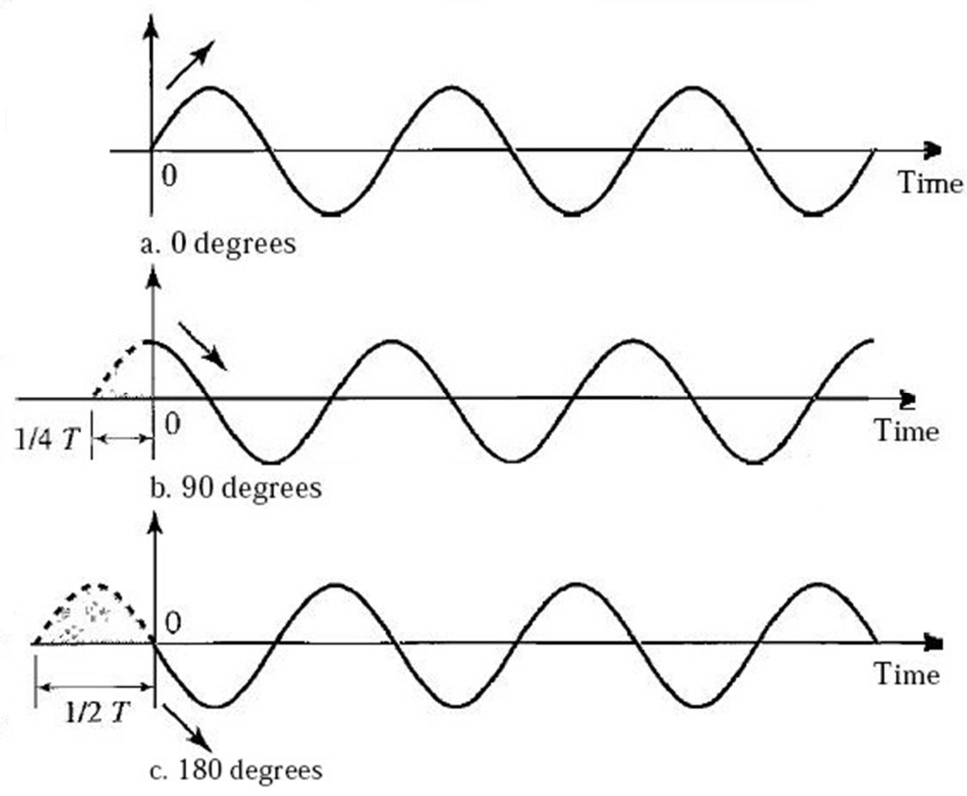
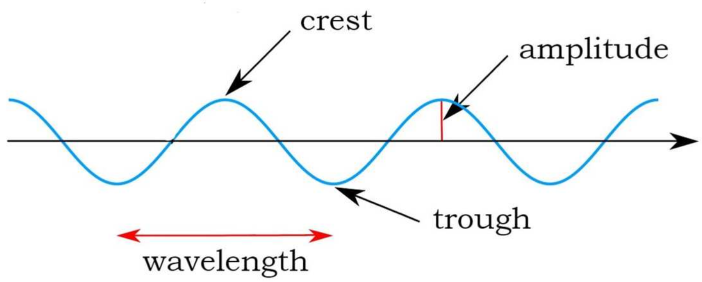


Figure: Three sine wave with same amplitude and frequency but different phase

Wavelength: Wavelength is another characteristic of a signal traveling through a transmission medium. Wavelength binds the period or the frequency of a simple sine wave to the propagation speed of the medium. It is denoted by λ



The frequency of a signal is independent of the medium, the wavelength depends on both the frequency and the medium.

$$Wavelength\ (\lambda) = propagation\ speed * period = C * T = \frac{C}{f}$$

Periodic Signals Characteristics (Time and Frequency Domains)

A sine wave is comprehensively defined by its amplitude, frequency, and phase. We have been showing a sine wave by using what is called a time-domain plot. The time-domain plot shows changes in signal amplitude with respect to time

To show the relationship between amplitude and frequency, we can use what is called a frequency-domain plot. A frequency-domain plot is concerned with only the peak value and the frequency.

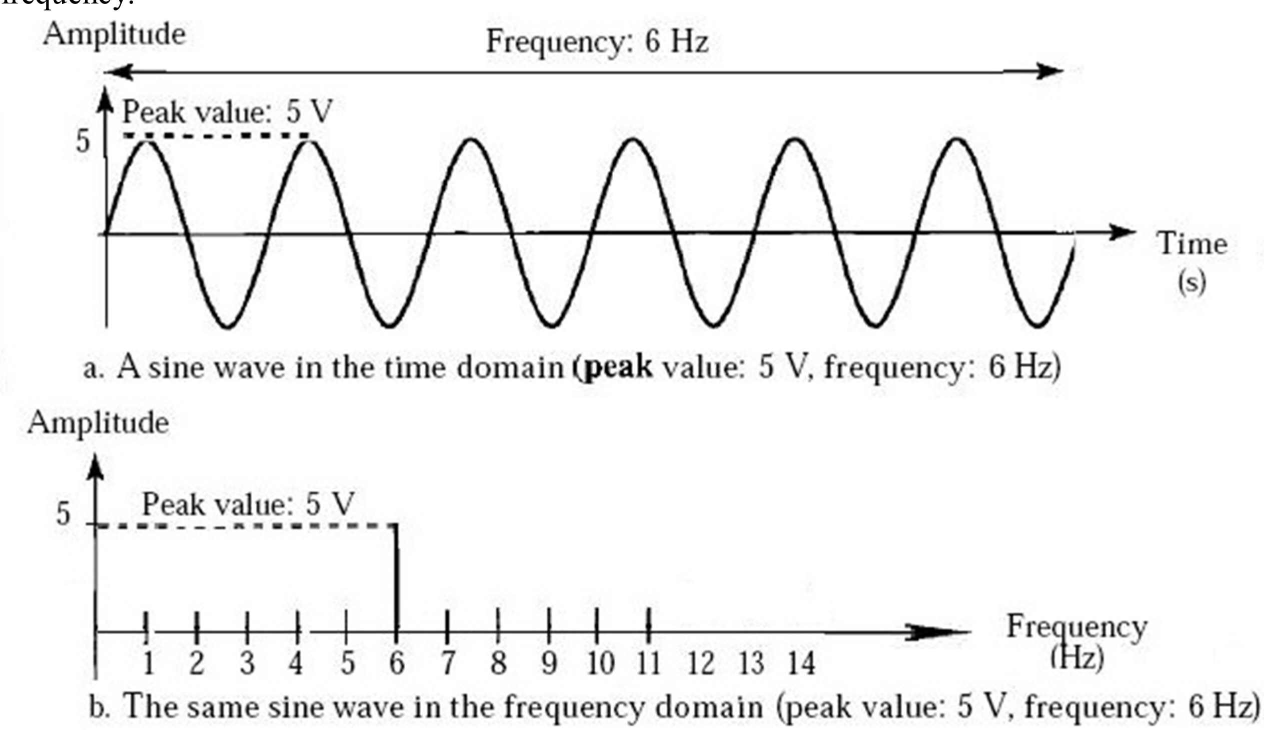


Figure: Sine wave in time and frequency domain

It is obvious that the frequency domain is easy to plot and conveys the information that one can find in a time domain plot. The advantage of the frequency domain is that we can immediately see the values of the frequency and peak amplitude. A complete sine wave is represented by one spike. The position of the spike shows the frequency; its height shows the peak amplitude.

Fourier Series Representation of Periodic Signal

Fourier series represents a periodic waveform in the form of sum of infinite number of sine and cosine terms. It is a representation of the signal in a time domain series form. Fourier series is a tool used to analyze any periodic signal. Spectral analysis of continuous time periodic signal is called continuous time fourier series (CTFS) and Spectral analysis of discrete time periodic signal is called discrete time fourier series (DTFS). Fourier series are applied for file compression (JPG, MPEG etc.), signal processing in communication,cryptography etc.

Types of fourier series

There are three types of fourier series used for the analysis of periodic signals. They are

- Trigonometric or quadrature fourier series
- Polar fourier series
- Exponential fourier series

Trigonometric Fourier Series

A periodic function x(t) may be expressed in the form of trigonometric fourier series comprising the following sine and cosine terms.

$$x(t) = a_0 + a_1 \cos \omega_o t + a_2 \cos 2\omega_o t + \cdots \ldots \ldots \ldots + a_n \cos n\omega_o t + \cdots \ldots \ldots + b_1 \sin \omega_o t + b_2 \sin 2\omega_o t + \cdots \ldots \ldots + b_n \sin n\omega_o t + \cdots \ldots \ldots \ldots$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_o t + b_n \sin n\omega_o t) \qquad (t_0 \le t \le t_0 + T)$$

Here,

$T = \frac{2\pi}{\omega_o}$ $\quad$ *and a_n and b_n are coefficients*

ω_o is called the fundamental frequency and $2\omega_o, 3\omega_o, 4\omega_o$ … … are called the harmonics of ω_o.

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)dt = \frac{1}{T}\int_0^T x(t)dt$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cos n\omega_o t \ dt = \frac{2}{T}\int_0^T x(t)\cos n\omega_o t \ dt$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\sin n\omega_o t \ dt = \frac{2}{T}\int_0^T x(t)\sin n\omega_o t \ dt$$

Symmetric condition

1. If the function x(t) is even, then $b_n = 0$,
2. If the function x(t) is odd, then $a_n = 0 = a_0$,
3. The sum or product of two or more even function is an even function.
4. The sum of two or more odd function is odd function and the product of two odd functions is an even function.

Polar fourier series

It is the modified form of the trigonometric fourier. The trigonometric fourier series is

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_o t + b_n \sin n\omega_o t)$$

Polar fourier series is given expressed as

$$x(t) = D_0 + \sum_{n=1}^{\infty} D_n \cos(n\omega_o t - \emptyset_n)$$

Where,

$$D_0 = a_0, \qquad D_n = \sqrt{{a_n}^2 + {b_n}^2} \qquad and \quad \emptyset_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

Exponential fourier series

The exponential form of fourier series is simpler and more compact and hence this is most widely used in signal analysis.

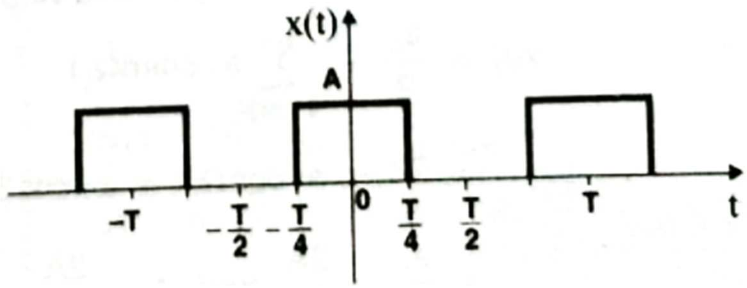
The exponential fourier series is expressed as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{jk\omega_o t}$$

Where,

$$a_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_o t}\, dt$$

Q.1. Determie the fourier series of the given waveform



Solution:

From figure we can say that the waveform is an even symmetric so $b_n = 0$.

The function x(t) is geven as

$$x(t) = A\,; for\ t = 0\ to \frac{T}{4}$$

$$= 0\,; for\ t = \frac{T}{4}\ to \frac{T}{2}$$

$$a_0 = \frac{1}{T}\int_0^T x(t)dt = \frac{2}{T}\int_0^{T/4} A\, dt = \frac{2A}{T}[t]_0^{T/4} = \frac{2A}{T}\left[\frac{T}{4} - 0\right] = \frac{A}{2}$$

$$a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos n\omega_o t\ \ dt = \frac{4}{T}\int_0^{T/4} A\cos n\omega_o t\ \ dt = \frac{4A}{T}\left[\frac{\sin n\omega_o t}{n\omega_o}\right]_0^{T/4}$$

$$a_n = \frac{4A}{T}\left[\frac{\sin n\omega_o T/4}{n\omega_o} - 0\right] = \frac{4A}{T}\left[\frac{\sin n(\frac{n2\pi}{T} * \frac{T}{4})}{\frac{n2\pi}{T}}\right] = \frac{2A}{n\pi}\sin\frac{n\pi}{2}$$

For even values of n,

$$\sin\frac{n\pi}{2} = 0$$

For odd values of n,

$$\sin\frac{n\pi}{2} = \pm 1$$

$$\therefore a_n = 0\ ;\ \ for\ even\ values\ of\ n$$

$$\therefore a_n = \frac{2A}{n\pi}\sin\frac{n\pi}{2}\ ;\quad for\ odd\ values\ of\ n$$

$$\therefore a_1 = \frac{2A}{\pi}\sin\frac{\pi}{2} = \frac{2A}{\pi}$$

$$\therefore a_3 = \frac{2A}{3\pi}\sin\frac{3\pi}{2} = -\frac{2A}{3\pi}$$

Therefor the fourier series of x(t) is

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_o t + b_n \sin n\omega_o t)$$

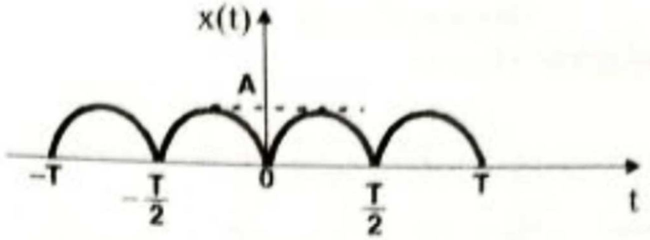
Here, $b_n = 0$,

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_o t$$

$$x(t) = \frac{A}{2} + \frac{2A}{\pi} \cos \omega_o t - \frac{2A}{3\pi} \cos 3\omega_o t + \cdots ....$$

$$\therefore x(t) = \frac{A}{2} + \frac{2A}{\pi} \cos \omega_o t - \frac{\cos 3\omega_o t}{3} + \cdots ....$$

Q.2. Determine the fourier series of the fullwave rectified sine wave



Solution:
From figure we can say that the waveform is an even symmetric so $b_n = 0$.
The function x(t) is geven as

$$x(t) = Asin\omega_o t\,; for\ t = 0\ to \frac{T}{2} \quad and\ \omega_o = \frac{2\pi}{T}$$

The coefficients are

$$a_0 = \frac{1}{T}\int_0^T x(t)dt = \frac{1}{T}\int_{-T/2}^{T/2} Asin\omega_o t\ dt = \frac{2}{T}\int_0^{T/2} Asin\omega_o t\ dt = \frac{2A}{T}\left[-\frac{\cos \omega_o t}{\omega_o}\right]_0^{T/2}$$

$$a_0 = \frac{2A}{T}\left[-\frac{\cos\left(\frac{2\pi}{T} * \frac{T}{2}\right)}{\frac{2\pi}{T}} + \frac{1}{\frac{2\pi}{T}}\right] = \frac{2A}{T} * \frac{T}{2\pi}[-cos\pi + 1] = \frac{2A}{\pi}$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t) \cos n\omega_o t \ \ dt = \frac{4}{T}\int_0^{T/2} Asin\omega_o t \cos n\omega_o t \ \ dt$$

$$a_n = \frac{4A}{2T}\int_0^{T/2} \sin(\omega_o t + n\omega_o t) + \sin(\omega_o t - n\omega_o t) \ \ dt$$

$$a_n = \frac{4A}{2T}\left[-\frac{\cos(1+\mathrm{n})\,\omega_o t}{(1+n)\omega_o} - \frac{\cos(1-\mathrm{n})\,\omega_o t}{(1-n)\omega_o}\right]_0^{T/2}$$

$$a_n = \frac{4A}{2T}\left[-\frac{\cos(1+\mathrm{n})\frac{2\pi}{T} * \frac{T}{2}}{(1+n)\frac{2\pi}{T}} - \frac{\cos(1-\mathrm{n})\frac{2\pi}{T} * \frac{T}{2}}{(1-n)\frac{2\pi}{T}} + \frac{1}{(1+n)\frac{2\pi}{T}} + \frac{1}{(1-n)\frac{2\pi}{T}}\right]$$

$$a_n = -\frac{A\cos(1+\mathrm{n})\pi}{\pi(1+n)} + \frac{A}{\pi(1+n)} - \frac{A\cos(1-\mathrm{n})\pi}{\pi(1-n)} + \frac{A}{\pi(1-n)} \qquad \ldots..(1)$$

From equation (1) is valid for all values of n except n = 1, so we have to calculate a_1 separately.

$$a_1 = \frac{2}{T}\int_{-T/2}^{T/2} x(t) \cos \omega_o t \ \ dt = \frac{4}{T}\int_0^{T/2} Asin\omega_o t \cos \omega_o t \ \ dt$$

$$a_1 = \frac{4A}{2T}\int_0^{T/2} \sin(\omega_o t + \omega_o t) + \sin(\omega_o t - \omega_o t) \; dt = \frac{4A}{2T}\int_0^{T/2} \sin 2\omega_o t + \sin 0 \; dt$$

$$a_1 = \frac{4A}{2T}\left[-\frac{\cos 2\,\omega_o t}{2\omega_o}\right]_0^{T/2} = \frac{A}{2\pi}\left[-\cos 2\frac{2\pi}{T} * \frac{T}{2} + 1\right] = 0$$

Other coefficient are calculated by putting value of n in equatio (1),

$$a_2 = -\frac{A\cos(1+2)\pi}{\pi(1+2)} + \frac{A}{\pi(1+2)} - \frac{A\cos(1-2)\pi}{\pi(1-2)} + \frac{A}{\pi(1-2)} = \frac{A}{3\pi} + \frac{A}{3\pi} - \frac{A}{\pi} - \frac{A}{\pi} = -\frac{4A}{3\pi}$$

$$a_3 = -\frac{A\cos(1+3)\pi}{\pi(1+3)} + \frac{A}{\pi(1+3)} - \frac{A\cos(1-3)\pi}{\pi(1-3)} + \frac{A}{\pi(1-3)} = 0$$

$$a_4 = -\frac{A\cos(1+4)\pi}{\pi(1+4)} + \frac{A}{\pi(1+4)} - \frac{A\cos(1-4)\pi}{\pi(1-4)} + \frac{A}{\pi(1-4)} = \frac{A}{5\pi} + \frac{A}{5\pi} - \frac{A}{3\pi} - \frac{A}{3\pi}$$

$$= -\frac{4A}{15\pi}$$

And for every odd value of n, $a_n = 0$

Therefor the fourier series of x(t) is

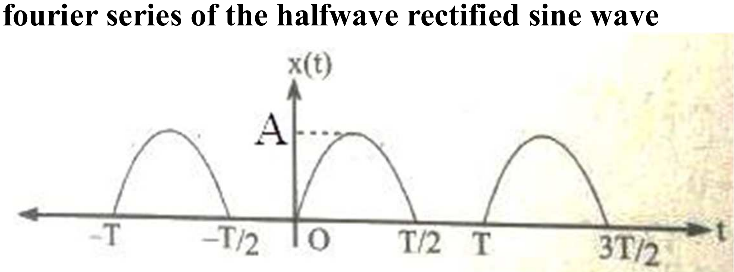
$$x(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_o t + b_n \sin n\omega_o t)$$

Here, $b_n = 0$,

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_o t$$

$$x(t) = \frac{2A}{\pi} - \frac{4A}{\pi}\left[\frac{\cos 2\omega_o t}{3} + \frac{\cos 4\omega_o t}{15} + \cdots \ldots \ldots \ldots \ldots\right]$$

Q.3. Determine the fourier series of the halfwave rectified sine wave



Solution:

From figure we can say that the waveform is neither even nor odd.

The function x(t) is geven as

$$x(t) = Asin\omega_o t\,; for\ t = 0\ to \frac{T}{2} \quad and\ \omega_o = \frac{2\pi}{T}$$

$$= 0\,; \qquad for\ t = \frac{T}{2}\ to\, T$$

The coefficients are

$$a_0 = \frac{1}{T}\int_0^T x(t)dt = \frac{1}{T}\int_0^T Asin\omega_o t\; dt = \frac{1}{T}\left[\int_0^{\frac{T}{2}} Asin\omega_o t\; dt + \int_{\frac{T}{2}}^{T} 0\; dt\right] = \frac{A}{T}\left[-\frac{\cos\omega_o t}{\omega_o}\right]_0^{T/2}$$

$$a_0 = \frac{A}{T}\left[-\frac{\cos\left(\frac{2\pi}{T} * \frac{T}{2}\right)}{\frac{2\pi}{T}} + \frac{1}{\frac{2\pi}{T}}\right] = \frac{A}{T} * \frac{T}{2\pi}[-cos\pi + 1] = \frac{A}{\pi}$$

$$a_n = \frac{2}{T}\int_0^T x(t)\cos n\omega_o t \ \ dt = \frac{2}{T}\left[\int_0^{\frac{T}{2}} Asin\omega_o t\cos n\omega_o t \ \ dt + \int_{\frac{T}{2}}^{T} 0 * \cos n\omega_o t \ dt\right]$$

$$a_n = \frac{2A}{2T}\int_0^{T/2} \sin(\omega_o t + n\omega_o t) + \sin(\omega_o t - n\omega_o t) \ \ dt$$

$$a_n = \frac{A}{T}\left[-\frac{\cos(1+\mathrm{n})\,\omega_o t}{(1+n)\omega_o} - \frac{\cos(1-\mathrm{n})\,\omega_o t}{(1-n)\omega_o}\right]_0^{T/2}$$

$$a_n = \frac{A}{T}\left[-\frac{\cos(1+\mathrm{n})\frac{2\pi}{T} * \frac{T}{2}}{(1+n)\frac{2\pi}{T}} - \frac{\cos(1-\mathrm{n})\frac{2\pi}{T} * \frac{T}{2}}{(1-n)\frac{2\pi}{T}} + \frac{1}{(1+n)\frac{2\pi}{T}} + \frac{1}{(1-n)\frac{2\pi}{T}}\right]$$

$$a_n = \frac{A}{T} * \frac{T}{2\pi}\left[-\frac{\cos(1+\mathrm{n})\,\pi}{(1+n)} - \frac{\cos(1-\mathrm{n})\,\pi}{(1-n)} + \frac{1}{(1+n)} + \frac{1}{(1-n)}\right]$$

$$a_n = \frac{A}{2\pi}\left[-\frac{\cos(1+\mathrm{n})\,\pi}{(1+n)} - \frac{\cos(1-\mathrm{n})\,\pi}{(1-n)} + \frac{1}{(1+n)} + \frac{1}{(1-n)}\right]$$

$$a_n = \frac{A}{2\pi(1+n)(1-n)}[-(1-n)\cos(1+\mathrm{n})\,\pi - (1+n)\cos(1-\mathrm{n})\,\pi + (1-n) + (1+n)]$$

$$a_n = \frac{A}{2\pi(1-n^2)}[(1-n)\{1-\cos(1+\mathrm{n})\,\pi\} + (1+n)\{1-\cos(1-\mathrm{n})\,\pi\}] \quad \ldots..(1)$$

For all odd values of n, $a_n = 0$

The above equation for a_n can be evaluated for all value of n except n = 1, so a_1 can be calculated separately.

$$a_1 = \frac{2}{T}\int_0^T x(t)\cos\omega_o t \ \ dt = \frac{2}{T}\left[\int_0^{\frac{T}{2}} Asin\omega_o t\cos\omega_o t \ \ dt + \int_{\frac{T}{2}}^{T} 0 * \cos\omega_o t \ dt\right]$$

$$a_1 = \frac{2A}{2T}\int_0^{T/2} \sin(\omega_o t + \omega_o t) + \sin(\omega_o t - \omega_o t) \ \ dt = \frac{A}{T}\int_0^{T/2} \sin 2\omega_o t + \sin 0 \ \ dt$$

$$a_1 = \frac{A}{T}\left[-\frac{\cos 2\,\omega_o t}{2\omega_o}\right]_0^{T/2} = \frac{A}{T} * \frac{T}{4\pi}\left[-\cos 2\frac{2\pi}{T} * \frac{T}{2} + 1\right] = 0$$

For all even values of n, a_n can be calculated as

$$a_2 = \frac{A}{2\pi(1-4)}[(1-2)\{1-\cos(1+2)\,\pi\} + (1+2)\{1-\cos(1-2)\,\pi] = -\frac{A}{6\pi}[-2+6]$$

$$= -\frac{2A}{3\pi}$$

$$a_4 = \frac{A}{2\pi(1-16)}[(1-4)\{1-\cos(1+4)\,\pi\} + (1+4)\{1-\cos(1-4)\,\pi]$$

$$= -\frac{A}{30\pi}[-6+10]$$

$$a_4 = -\frac{2A}{15\pi}$$

And so on

$$b_n = \frac{2}{T}\int_0^T x(t)\sin n\omega_o t \ \ dt = \frac{2}{T}\left[\int_0^{\frac{T}{2}} A sin\omega_o t \sin n\omega_o t \ \ dt + \int_{\frac{T}{2}}^{T} 0 * \sin n\omega_o t \ \ dt\right]$$

$$b_n = \frac{2A}{T}\int_0^{\frac{T}{2}} sin\omega_o t \sin n\omega_o t \ \ dt = \frac{2A}{2T}\int_0^{\frac{T}{2}} \cos(\omega_o t - n\omega_o t) - \cos(\omega_o t + n\omega_o t) \ \ dt$$

$$b_n = \frac{A}{T}\left[\frac{\sin(1-\mathrm{n})\,\omega_o t}{(1-n)\omega_o} - \frac{\sin(1+\mathrm{n})\,\omega_o t}{(1+n)\omega_o}\right]_0^{T/2} = \frac{A}{T}\left[\frac{\sin(1-\mathrm{n})\frac{2\pi}{T}*\frac{T}{2}}{(1-n)\frac{2\pi}{T}} - \frac{\sin(1+\mathrm{n})\frac{2\pi}{T}*\frac{T}{2}}{(1+n)\frac{2\pi}{T}}\right]$$

$$b_n = \frac{A}{T} * \frac{T}{2\pi(1-n)(1+n)}[(1+n)\sin(1-\mathrm{n})\pi - (1-n)\sin(1+\mathrm{n})\pi]$$

$$b_n = \frac{A}{2\pi(1-n^2)}[(1+n)\sin(1-\mathrm{n})\pi - (1-n)\sin(1+\mathrm{n})\pi] \qquad \ldots\ldots\ldots.(2)$$

The above equation for b_n can be evaluated for all value of n except n = 1, so a_1 can be calculated separately.

$$b_1 = \frac{2}{T}\int_0^T x(t)\sin \omega_o t \ \ dt = \frac{2}{T}\left[\int_0^{\frac{T}{2}} A sin\omega_o t \sin \omega_o t \ \ dt + \int_{\frac{T}{2}}^{T} 0 * \sin \omega_o t \ \ dt\right]$$

$$= \frac{2A}{T}\int_0^{\frac{T}{2}} sin^2\omega_o t \ \ dt$$

$$b_1 = \frac{2A}{2T}\int_0^{\frac{T}{2}} (1 - cos2\omega_o t)dt = \frac{A}{T}\left[t - \frac{\sin 2\,\omega_o t}{2\omega_o}\right]_0^{T/2} = \frac{A}{T}\left[\frac{T}{2} - \frac{\sin\frac{4\pi}{T}*\frac{T}{2}}{\frac{4\pi}{T}}\right] = \frac{A}{2} - \frac{\sin 2\pi}{\frac{4\pi}{T}} = \frac{A}{2}$$

$b_n = 0$, for all values of n except n = 1.

Therefor the fourier series of x(t) is

$$x(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_o t + b_n \sin n\omega_o t)$$

$$x(t) = \frac{A}{\pi} - \frac{2A}{3\pi}\cos 2\omega_o t - \frac{2A}{15\pi}\cos 4\omega_o t - \cdots \ldots\ldots.. + \frac{A}{2}\sin n\omega_o t$$

Properties of the Fourier Series

The continuous time fourier series (CTFS) representation consists of a number of properties which may be used to devlop conceptualfundamentals and in reducing the complexity of fourier series of signals.

There are many importantant properties of the continuous time fourier series (CTFS) which are described below.

1. Linearity

Let x(t) and y(t) be the two periodic signals with fourier coefficients a_k and b_k respectively.

$$x(t) \overset{FS}{\longleftrightarrow} a_k$$

$$y(t) \overset{FS}{\longleftrightarrow} b_k$$

If C_k be the fourier coefficients of linear combination of x(t) and y(t), then,

$$Z(t) = Ax(t) + By(t) \overset{FS}{\longleftrightarrow} C_k = Aa_k + Bb_k$$

2. Time Shifting: if,

$$x(t) \overset{FS}{\longleftrightarrow} a_k$$

Then,

$$x(t-t_o) \xleftrightarrow{FS} e^{-ik\omega_o t_o} a_k$$

3. **Frequency Shifting:** if

$$x(t) \xleftrightarrow{FS} a_k$$

Then,

$$e^{jM\omega_o t} x(t) \leftrightarrow a_{k-M}$$

4. **Time scaling:** if

$$x(t) \xleftrightarrow{FS} a_k$$

Then,

$$x(\alpha t) \xleftrightarrow{FS} a_k$$

if x(t) is periodic with period T and fundamental frequency ω_0, then x(αt), where α is a positive real number, is periodic with period T/α and fundamental frequency $\alpha\ \omega_0$.

5. **Time Reversal:** if

$$x(t) \xleftrightarrow{FS} a_k$$

Then,

$$x(-t) \xleftrightarrow{FS} a_{-k}$$

6. **Multiplication:** if

$$x(t) \xleftrightarrow{FS} a_k$$
$$y(t) \xleftrightarrow{FS} b_k$$

Then,

$$z(t) = x(t)y(t) \xleftrightarrow{FS} h_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$$

7. **Differential:** if

$$x(t) \xleftrightarrow{FS} a_k$$

Then,

$$\frac{dx(t)}{dx} \xleftrightarrow{FS} jk\omega_o a_k$$

8. **Integration:** if

$$x(t) \xleftrightarrow{FS} a_k$$

Then,

$$\int_{\tau=-\infty}^{t} x(\tau)\, d\tau \xleftrightarrow{FS} \frac{1}{jk\omega_o}\ a_k$$

9. **Conjugation:** if

$$x(t) \xleftrightarrow{FS} a_k$$

Then,

$$x^*(t) \xleftrightarrow{FS} a^*{}_{-k}$$

10. **Convolution:** if

$$x(t) \xleftrightarrow{FS} a_k$$
$$y(t) \xleftrightarrow{FS} b_k$$

Then,

$$z(t) = x(t) \otimes y(t) \xleftrightarrow{FS} T a_k b_k$$

11. **Parseval's relation:** if

$$x(t) \xleftrightarrow{FS} a_k$$

Then,

$$P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt = \sum_{k=-\infty}^{\infty} |a_k|^2$$

Where, P is average power

<u>Fourier Transform of Aperiodic Signal</u>

The main drawback of Fourier series is, it is only applicable to periodic signals. There are some naturally produced signals such as nonperiodic or aperiodic, which we cannot represent using Fourier series. To overcome this shortcoming, Fourier developed a mathematical model to transform signals between time (or spatial) domain to frequency domain & vice versa, which is called 'Fourier transform'.

Fourier transform has many applications in physics and engineering such as analysis of LTI systems, RADAR, astronomy, signal processing etc.
Fourier Transform can be expressed as

$$X(j\omega) = F[x(t)] = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \qquad (1)$$

Inverse Fourier Transform can be expressed as

$$x(t) = F^{-1}[X(j\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega \qquad (2)$$

X(jω) and x(t) are called the Fourier Transform Pair where X(jω) is called Fourier Transform and x(t) is Inverse Fourier Transform. Symbolically, this may be expressed as

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

Conditions for Existence of Fourier Transform

Any function x(t) can be represented by using Fourier transform only when the function satisfies Dirichlet's conditions. i.e.

- The function x(t) has finite number of maxima and minima.
- There must be finite number of discontinuities in the signal x(t), in the given interval of time.
- It must be absolutely integrable in the given interval of time i.e.

$$\int_{-\infty}^{\infty} |x(t)|\ dt < \infty$$

Properties of Fourier Transform

The Fourier Transform has several important properties that are useful in signal analysis and processing:

1. **Linearity:** If

$$x(t) \xleftrightarrow{FT} X(j\omega)$$
$$\boldsymbol{and}, y(t) \xleftrightarrow{FT} Y(j\omega)$$

Then

$$z(t) = a\,x(t) + by(t)\,FT \xleftrightarrow{FT} Z(j\omega) = a\,X(j\omega) + b\,Y(j\omega)$$

Thus, The FT of linear combination of the signals is equal to linear combination of their Fourier transforms.

2. **Time shifting:** If

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

Then,

$$y(t) = x(t - t_o) \xleftrightarrow{FT} Y(j\omega) = e^{-j\omega t_o} . X(j\omega)$$

A shift of '$\boldsymbol{t_0}$' in time domain is equivalent to introducing a phase shift of '$-\,\boldsymbol{\omega t_0}$' in frequency domain. But amplitude remains same.

3. **Frequency Shifting:** If

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

Then,

$$e^{j\omega_o t} . x(t) \xleftrightarrow{FT} X[j(\omega - \omega_o)]$$

Shifting the frequency by $\boldsymbol{\omega_0}$ in frequency domain is equivalent to multiplying the time domain signal by $e^{j\omega_o t}$.

4. **Time scaling:** if

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

Then,

$$x(\alpha t) \xleftrightarrow{FT} \frac{1}{|\alpha|} X\left(\frac{j\omega}{\alpha}\right)$$

5. **Time reversal:** if

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

Then,

$$x(-t) \xleftrightarrow{FT} X(-j\omega)$$

6. **Differentiation:** if

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

Then,

$$\frac{dx(t)}{dx} \xleftrightarrow{FT} j\omega\, X(j\omega)$$

Differentiation in time domain corresponds to multiplying by $\boldsymbol{j\omega}$ in frequency domain.

7. **Time Integration:** if

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

Then,

$$\int_{\tau=-\infty}^{t} x(\tau)\ d\tau \xleftrightarrow{FT} \frac{1}{j\omega} X(j\omega)$$

8. Conjugation: if

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

Then,

$$x^*(t) \xleftrightarrow{FT} X^*(-j\omega)$$

9. Time convolution: if

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

$$\textbf{\textit{and}}, \quad y(t) \xleftrightarrow{FT} Y(j\omega)$$

Then,

$$x(t) \otimes y(t) \xleftrightarrow{FT} X(j\omega)\, Y(j\omega)$$

A convolution operation in time domain is transformed to multiplication in frequency domain.

10. Frequency convolution: if

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

$$\textbf{\textit{and}}, \quad y(t) \xleftrightarrow{FT} Y(j\omega)$$

Then,

$$2\pi\, x(t)y(t) \xleftrightarrow{FT} X(j\omega) \otimes Y(j\omega)$$

11. Parseval's theorem: if

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

Then,

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2\, d\,\omega$$

Q.1. Calculate Fourier Transform for the signal

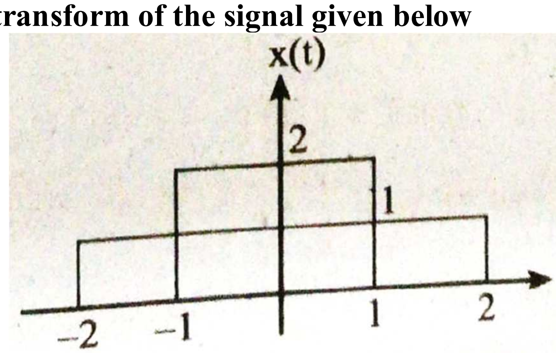
$$\boldsymbol{x(t) = e^{-at}u(t), \qquad a > 0}$$

Solution:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} e^{-at}u(t)\, e^{-j\omega t}dt = \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}dt = \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$X(j\omega) = \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^{\infty} = \frac{e^{-(a+j\omega)\infty} - e^0}{-(a+j\omega)} = \frac{0-1}{-(a+j\omega)} = \frac{1}{(a+j\omega)}$$

Q.2. Determine fourier transform of the signal given below



Solution:

$$\begin{aligned} x(t) &= 1\,, \qquad for -2 \le t \le -1 \\ &= 2\,, \qquad for -1 \le t \le 1 \\ &= 1\,, \qquad for \quad 1 \le t \le 2 \end{aligned}$$

$$X(j\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt=\int_{-2}^{-1}x(t)e^{-j\omega t}dt+\int_{-1}^{1}x(t)e^{-j\omega t}dt+\int_{1}^{2}x(t)e^{-j\omega t}dt$$
$$=\int_{-2}^{-1}e^{-j\omega t}dt+\int_{-1}^{1}2e^{-j\omega t}dt+\int_{1}^{2}e^{-j\omega t}dt=\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-2}^{-1}+2\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-1}^{1}+\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{1}^{2}$$
$$=\frac{1}{-j\omega}\left[\left(e^{j\omega}-e^{2j\omega}\right)+2\left(e^{-j\omega}-e^{j\omega}\right)+\left(e^{-2j\omega}-e^{-j\omega}\right)\right]$$

Digital Signals

In addition to being represented by an analog signal, information can also be represented by a digital signal. For example, a I can be encoded as a positive voltage and a 0 as zero voltage. A digital signal can have more than two levels. In this case, we can send more than 1 bit for each level.

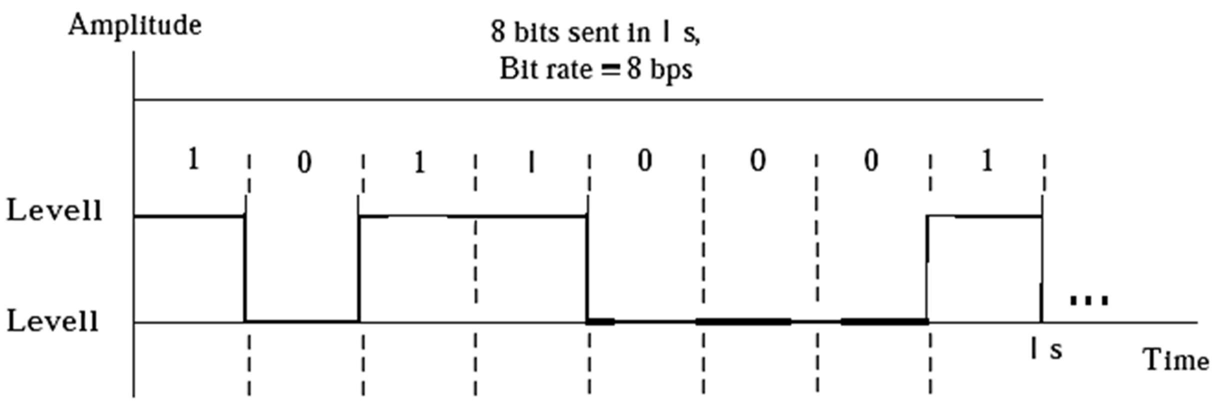


Figure: A Digital signal with two levels

- **If a signal has *L* levels, each level needs *log₂L* bits.**

Example: A digital signal has eight levels. How many bits are needed per level?

We calculate the number of bits from the formula

Number of bits per level = $\log_2 8 = 3$

Each signal level is represented by 3 bits.

Digital Signal Characteristics

Most digital signals are non-periodic, and thus period and frequency are not appropriate characteristics.

Bit Rate: The bit rate is the number of bits sent in 1second and expressed in bits per second (bps).

Example: Assume we need to download text documents at the rate of 100 pages per minute. What is the required bit rate of the channel?

Solution:

A page is an average of 24 lines with 80 characters in each line. If we assume that one character requires 8 bits, the bit rate is

100 x 24 x 80 x 8 =1,636,000 bps =1.636 Mbps

Bit Length: The bit length is the distance one bit occupies on the transmission medium.

$$Bit\ Length = Propagation\ speed * bit\ duration$$

Analog and Digital Transmission

Analog transmission sends analog signals regardless of their content, which can represent either analog or digital data. As the signal travels, it weakens (attenuates), so amplifiers are used to boost it. However, amplifiers also amplify noise, leading to increased distortion over long distances. While analog data like voice can tolerate some distortion, digital data is more sensitive, and repeated amplification can cause errors.

Digital transmission assumes binary data and is limited by attenuation and noise. To extend the distance, repeaters are used to reconstruct and retransmit the original digital signal, eliminating accumulated noise. When analog signals carry digital data, repeaters can also be used to recover the digital content and send a clean analog signal, preventing noise buildup.

The digital signal can be transmitted by using two approaches i.e. Baseband transmission and Broadband transmission.

Baseband Transmission

Baseband transmission means sending a digital signal over a channel without changing the digital signal to an analog signal. The following figure shows baseband transmission.

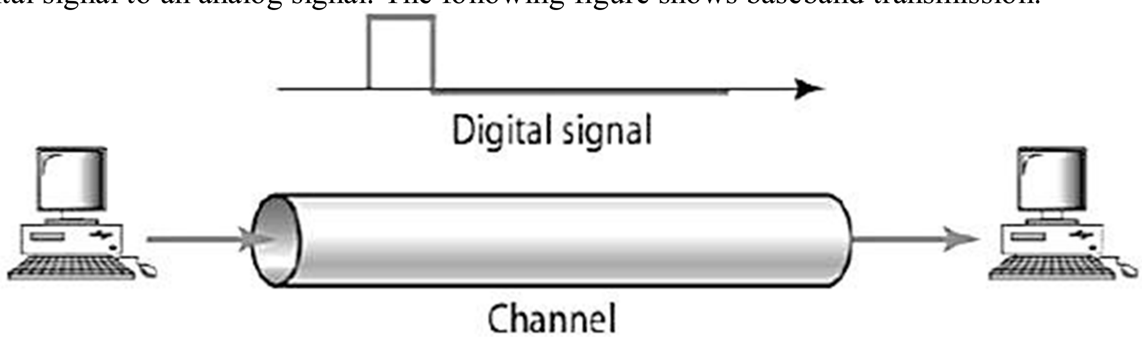


Figure: Baseband transmission

Baseband transmission requires a low-pass channel, a channel with a bandwidth that starts from zero. This is the case if we have a dedicated medium with a bandwidth constituting only one channel. For example, the entire bandwidth of a cable connecting two computers is one single channel. As another example, we may connect several computers to a bus, but not allow more than two stations to communicate at a time.

Broadband Transmission (Using Modulation)

Broadband transmission or modulation means changing the digital signal to an analog signal for transmission. Modulation allows us to use a band pass channel – a channel with a bandwidth that does not start from zero. This type of channel is more available than a low-pass channel. The following figure shows a band pass channel.

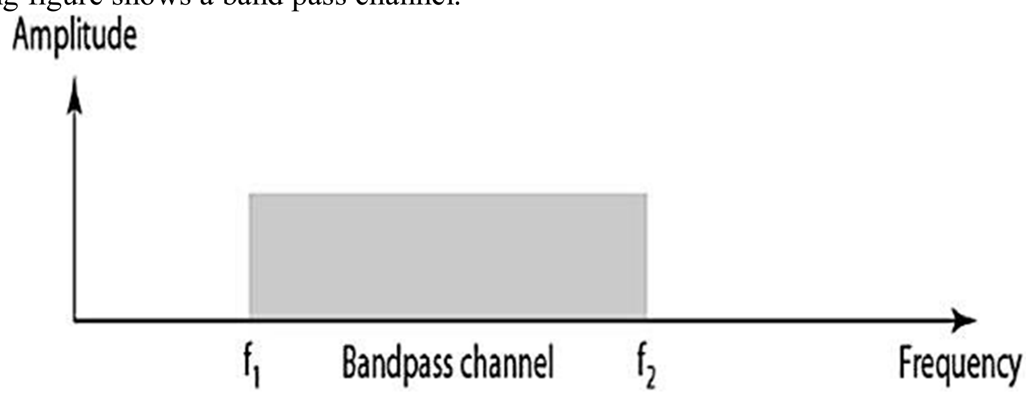


Figure: Bandwidth of a band pass channel

Transmission Modes

The transmission mode decides how data is transmitted between two computers. The binary data in the form of 1s and 0s can be sent in two different modes: Parallel and Serial.

Parallel Transmission:

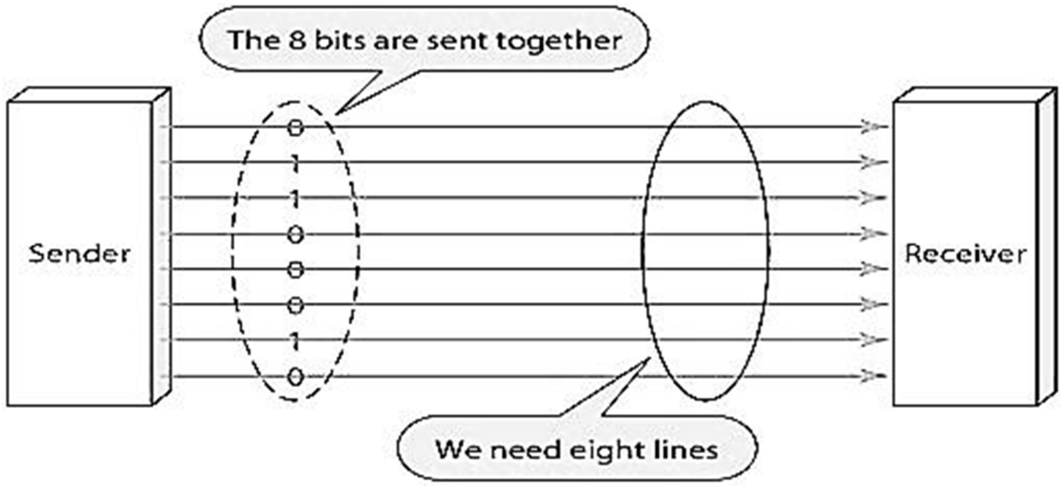


Figure: Parallel Transmission of data

The binary bits are organized into groups of fixed length. Both sender and receiver are connected in parallel with the equal number of data lines. Both computers distinguish between high order and low order data lines. The sender sends all the bits at once on all lines. Because the data lines are equal to the number of bits in a group or data frame, a complete group of bits (data frame) is sent in one go.

Advantage of Parallel transmission is high speed and disadvantage is the cost of wires, as it is equal to the number of bits sent in parallel.

Serial Transmission

In serial transmission, bits are sent one after another in a queue manner. Serial transmission requires only one communication channel rather than n to transmit data between two communicating devices.

The advantage of serial over parallel transmission is that with only one communication channel, serial transmission reduces the cost of transmission over parallel by roughly a factor of n. Serial transmission occurs in one of three ways: asynchronous, synchronous, and isochronous.

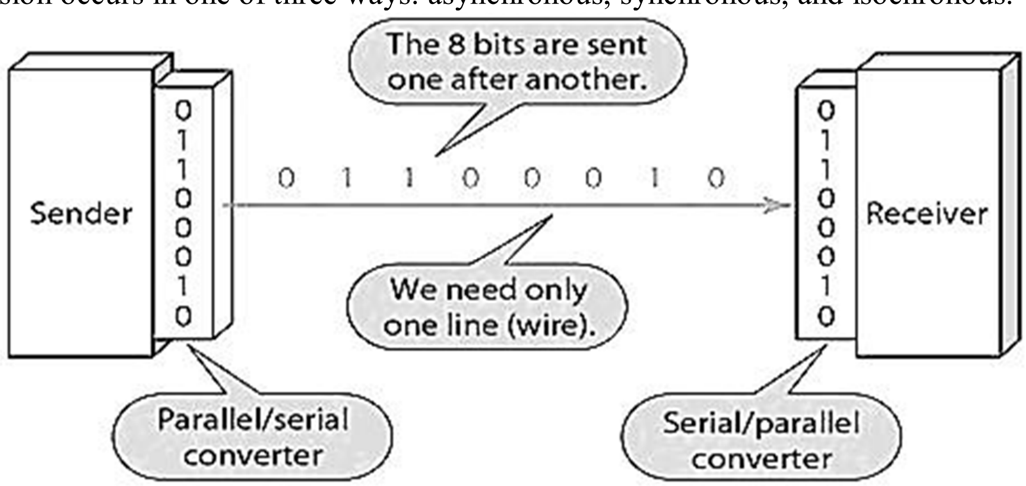


Figure: Serial transmission of data

Asynchronous Serial Transmission

- The receiving device does not need to be synchronized with the transmitting device.
- Transmitting device send data units when it is ready to send data.
- Each data unit must contain one start bit (0) at the beginning, 8 bit information at the middle and one or more stop bits (1s) at the end of data unit. There may be a gap between each byte or data unit.
- The addition of stop and start bits and the insertion of gaps into the bit stream make asynchronous transmission slower than forms of transmission that can operate without the addition of control information. But it is cheap and effective, two advantages that make it an attractive choice for situations such as low-speed communication.

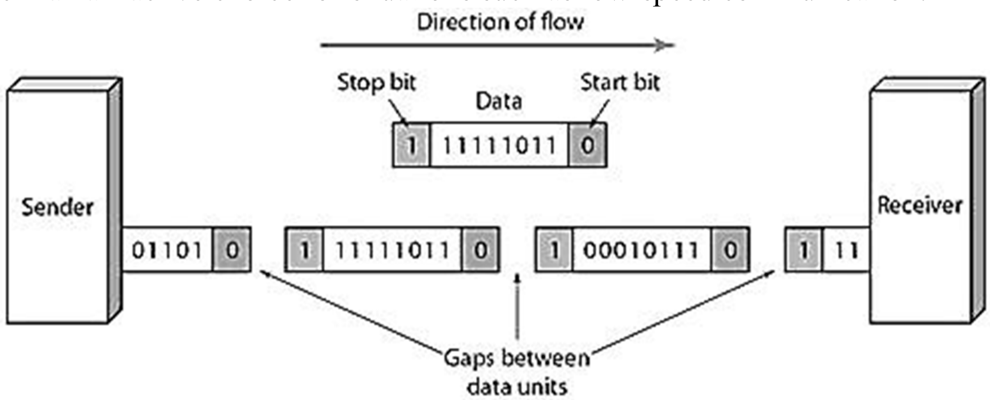


Figure – Asynchronous serial data transmission

Synchronous Serial Transmission

- Data is transmitted or received based on a clock signal i. e. synchronously.
- The bit stream is combined into longer "frames," which may contain multiple bytes.
- In synchronous transmission, we send bits one after another without start or stop bits or gaps. It is the responsibility of the receiver to group the bits.
- The accuracy of the received information is completely dependent on the ability of the receiving device to keep an accurate count of the bits as they come in.
- Synchronous transmission is faster than asynchronous transmission.
- For this reason, it is more useful for high-speed applications such as the transmission of data from one computer to another.
- There may be uneven gaps between frames.

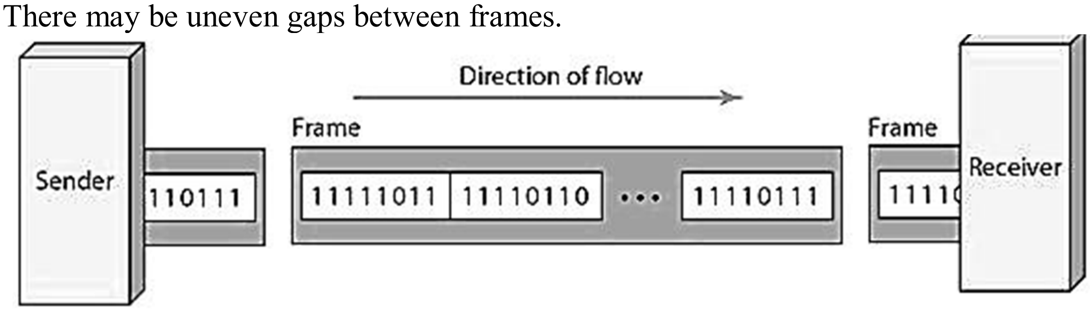


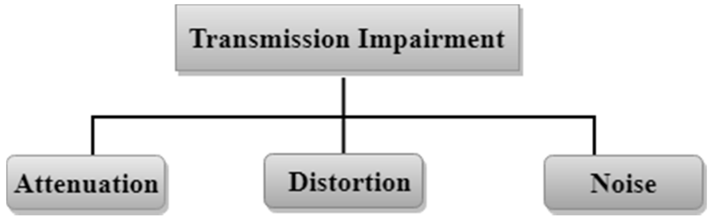
Figure: Synchronous serial transmission format

Isochronous

In real-time audio and video, in which uneven delays between frames are not acceptable, synchronous transmission fails. For example, TV images are broadcast at the rate of 30 images per second; they must be viewed at the same rate. If each image is sent by using one or more frames, there should be no delays between frames. For this type of application, synchronization between characters is not enough; the entire stream of bits must be synchronized. The isochronous transmission guarantees that the data arrive at a fixed rate.

Transmission Impairment

Signals travel through transmission media, which are not perfect. The imperfection causes signal impairment. This means that the signal at the beginning of the medium is not the same as the signal at the end of the medium. Three causes of impairment are attenuation, distortion, and noise.



Attenuation: It means loss of energy. The strength of signal decreases with increasing distance which causes loss of energy in overcoming resistance of medium. This is also known as attenuated signal. Amplifiers are used to amplify the attenuated signal which gives the original signal back and compensate for this loss.

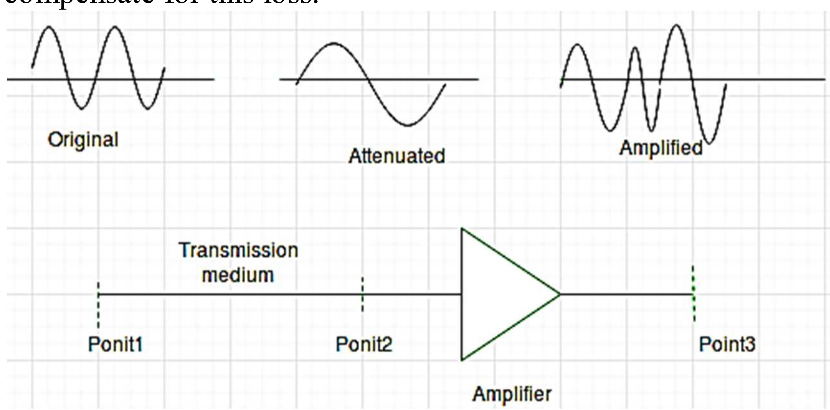


Figure: Attenuation

Attenuation is measured in **decibels (dB)**. It measures the relative strengths of two signals or one signal at two different points.

Note that the decibel is negative if a signal is attenuated and positive if a signal is amplified.

$$Attenuation\ (dB) = 10\log\left(\frac{P_2}{P_1}\right)$$

Where, P_1 is the power at sending end

P_2 is the power at receiving end

Some where the decibel is also define in terms of voltage instead of power. In this case because power is proportional to the square of the voltage the formula is

$$Attenuation\ (dB) = 20\log\left(\frac{V_2}{V_1}\right)$$

Where, V_1 is the power at sending end

V_2 is the power at receiving end

Q.1. A signal travels through a transmission medium and its power is reduced to one-half. Calculate the attenuation?

Solution:

Given,

$P_2 = 0.5\ P_1$

$$Attenuation\ (dB) = 10\log\left(\frac{P_2}{P_1}\right) = 10\log\left(\frac{0.5P_1}{P_1}\right) = 10 * (-0.3) = -3dB$$

A loss of 3 dB (-3 dB) is equivalent to losing one-half the power.

Q.2. A signal travels through an amplifier, and its power is increased 10 times. Then calculate attenuation?

Solution:

Given,

$P_2 = 10 P_1$

$$Attenuation\ (dB) = 10\log\left(\frac{P_2}{P_1}\right) = 10\log\left(\frac{10P_1}{P_1}\right) = 10 * 1 = 10dB$$

Q.3. If the signal at the beginning of a cable with -0.3 dB/km has a power of 2 mW, what is the power of the signal at 5 km?

Solution:

The loss in the cable in decibels = 5 x (-0.3) = -1.5 dB. We can calculate the power as

$$Attenuation\ (dB) = 10\log\left(\frac{P_2}{P_1}\right) = -1.5$$

$$\left(\frac{P_2}{P_1}\right) = 10^{-0.15} = 0.71$$

$$P_2 = 0.71P_1 = 0.71 * 2mW = 1.4mW$$

Distortion: It means changes in the form or shape of the signal. This is generally seen in composite signals made up with different frequencies. Each frequency component has its own propagation speed travelling through a medium. And that's why it delays in arriving at the final destination every component arrive at different time which leads to distortion. Therefore, they have different phases at receiver end from what they had at senders end.

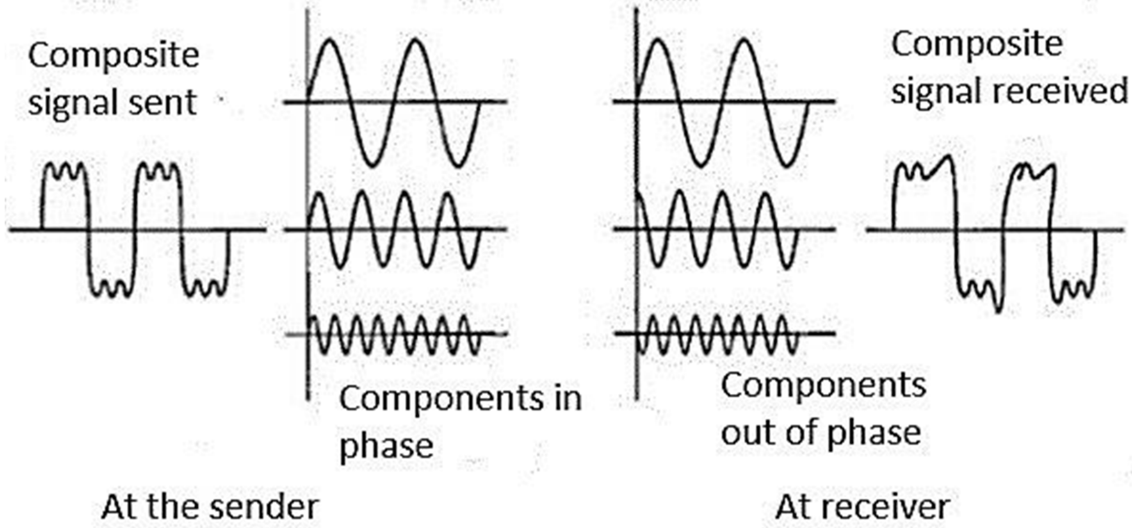


Figure: Distortion

Distortion can be categorized as:

- Linear Distortion
- Non-linear Distortion

Linear Distortion:

The term "linear distortion" means any change to the signal that does not change the shape of the individual sine wave components of the signal. Linear distortion may be amplitude distortion or phase distortion.

Amplitude Distortion: When different frequency components of the input signal are amplified or attenuated by different amounts, the output signal consists of distortion, known as amplitude distortion.

Phase Distortion: If the phase of the output signal is different from the phase of the input signal, then such distortion is known as phase distortion. Phase distortion leads to delay in the transmission of the signal. Hence, it is also known as delay distortion.

Non-linear distortion

When the sine wave components of the signal are changed by nonlinear elements, then the distortion is called nonlinear distortion.

Noise: The random or unwanted signal that mixes up with the original signal is called noise. There are several types of noise such as induced noise, crosstalk noise, thermal noise, impulse noise etc. which may corrupt the signal.

Induced noise: It comes from sources such as motors and appliances. These devices act as sending antenna and transmission medium act as receiving antenna.

Thermal noise: The random movement of electrons in wire which creates an extra signal known as thermal noise. This noise is also known as Johnson noise or white noise.

The noise power is calculated as

$$P_n = KTB$$

Where, P_n = power of the noise

K = Boltzmann's constant = $1.38*10^{23}$ j/k

T = Absolute temperature

B = Bandwidth over which noise is measured

The corresponding equivalent rms noise voltage is given as

$$E_n = \sqrt{4KTRB}$$

Q.1. Fin the rms noise voltage at the input of amplifier at an ambient temperature of 17°C if we are using resistor of 8KΩ and amplifier works in frequency range of 12 – 1.5 MHz.

Solution: given

T = 17°C = 17+273K =290K

R = 8KΩ = $8*10^3$Ω

Bandwidth (B) = (15.5 – 12) MHz = $3.5*10^6$Hz

The RMS noise voltage is

$$E_n = \sqrt{4KTRB} = \sqrt{4 * 1.38 * 10^{-23} * 290 * 8 * 10^3 * 3.5 * 10^6} = 2.11 * 10^{-5}$$

Q.2. Calculate the thermal noise power density in watts/Hz at a temperature of 17°C. The Boltzmann's constant is $1.38*10^{23}$ J/K.

Solution: given

T = 17°C = 17+273K =290K

K = Boltzmann's constant = $1.38*10^{23}$ j/k

The noise power is given by

$$P_n = KTB$$

The noise power density is calculated as

$$\frac{P_n}{B} = KT = 1.38 * 10^{-23} * 290 = 4 * 10^{-21} W/Hz$$

Crosstalk noise: It comes when one wire affects the other wire.

Impulse noise: It is a signal with high energy that comes from lightning or power lines.

Atmospheric noise or static noise: it is produced by lightening discharge in thunder storm and other natural electrical disturbances.

Industrial noise: it is produced by industrial source such electrical motors, automobiles etc.

Signal-to-Noise Ratio (SNR): SNR is actually the ratio of signal to noise. A high SNR means the signal is less corrupted by noise; a low SNR means the signal is more corrupted by noise. Because SNR is the ratio of two powers, it is given as

$$SNR = \frac{Average\ signal\ power}{Average\ noise\ power}$$

SNR is often described in decibel units. Mathematically, SNR in dB is given as

$$SNR_{dB} = 10 \log SNR$$

Q. The power of a signal is 10 mW and the power of the noise is 1 µW; what are the values of SNR and SNR_{dB}?

Solution:

$$SNR = \frac{Average\ signal\ power}{Average\ noise\ power} = \frac{10mW}{1\ \mu W} = 10000$$

$$SNR_{dB} = 10 \log SNR = 10 \log 10000 = 40$$

Data Rate Limits

A very important consideration in data communications is how fast we can send data, in bits per second. Over a channel. Data rate depends on three factors:

1. The bandwidth available
2. The level of the signals we use
3. The quality of the channel (the level of noise)

Two theoretical formulas were developed to calculate the data rate:

- By Nyquist for a noiseless channel
- By Shannon for a noisy channel

Noiseless Channel: Nyquist Bit Rate

For a noiseless channel, the Nyquist bit rate formula defines the theoretical maximum bit rate

$$Bit\ Rate = 2 * bandwidth * \log_2 L$$

Where,

Bandwidth is the bandwidth of the channel,
L is the number of signal levels used to represent data, and
Bit Rate is the bit rate in bits per second.

According to the formula, we might think that, given a specific bandwidth, we can have any bit rate we want by increasing the number of signa1leve1s. Although the idea is theoretically correct, practically there is a limit. When we increase the number of signal1eve1s, we impose a burden on the receiver. If the number of levels in a signal is just 2, the receiver can easily distinguish between a 0 and a 1. If the level of a signal is 64, the receiver must be very sophisticated to distinguish between 64 different levels. In other words, increasing the levels of a signal reduces the reliability of the system.

Q.1. Consider a noiseless channel with a bandwidth of 3000 Hz transmitting a signal with two signal levels. What is the maximum bit rate?

Solution:

The maximum bit rate can be calculated as

$$Bit\ Rate = 2 * bandwidth * \log_2 L = 2 * 3000 * \log_2 2 = 6000\ bps$$

Q.2. Consider the same noiseless channel transmitting a signal with four signal levels (for each level, we send 2 bits). What is the maximum bit rate?

Solution:
The maximum bit rate can be calculated as

$$Bit\ Rate = 2 * bandwidth * \log_2 L = 2 * 3000 * \log_2 4 = 12000\ bps$$

Q.3. We need to send 265 kbps over a noiseless channel with a bandwidth of 20 kHz. How many signal levels do we need?

Solution:
The maximum bit rate can be calculated as

$$Bit\ Rate = 2 * bandwidth * \log_2 L$$
$$265000 = 2 * 20000 * \log_2 L$$
$$\log_2 L = 6.625 \qquad ,L = 2^{6.625} = 98.7\ Levels$$

Since this result is not a power of 2, we need to either increase the number of levels or reduce the bit rate. If we have 128 levels, the bit rate is 280 kbps. If we have 64 levels, the bit rate is 240 kbps.

Noisy Channel: Shannon Capacity

In reality, we cannot have a noiseless channel; the channel is always noisy. In 1944, Claude Shannon introduced a formula, called the Shannon capacity, to determine the theoretical highest data rate for a noisy channel:

$$Capacity = bandwidth * \log_2 (1 + SNR)$$
$$C = B \log_2 (1 + SNR)$$

Where,

Bandwidth is the bandwidth of the channel,
SNR is the signal-to noise ratio, and
Capacity is the capacity of the channel in bits per second.

Note that in the Shannon formula there is no indication of the signal level, which means that no matter how many levels we have, we cannot achieve a data rate higher than the capacity of the channel. In other words, the formula defines a characteristic of the channel, not the method of transmission.

Q.1. Consider an extremely noisy channel in which the value of the signal-to-noise ratio is almost zero. In other words, the noise is so strong that the signal is faint. What is the channel capacity?

Solution:
For this channel the capacity C is calculated as

$$C = B \log_2 (1 + SNR) = B \log_2 (1 + 0) = B * 0 = 0$$

This means that the capacity of this channel is zero regardless of the bandwidth. In other words, we cannot receive any data through this channel.

Q.2. We can calculate the theoretical highest bit rate of a regular telephone line. A telephone line normally has a bandwidth of 3000 Hz (300 to 3300 Hz) assigned for data communications. The signal-to-noise ratio is usually 3162.

Solution:
For this channel the capacity is calculated as

$$C = B \log_2 (1 + SNR) = 3000 * \log_2 (1 + 3162)$$
$$C = 3000 * \log_2 3163 = 3000 * 11.62 = 34860\ bps$$

This means that the highest bit rate for a telephone line is 34.860 kbps. If we want to send data faster than this, we can either increase the bandwidth of the line or improve the signal-to-noise ratio.

Q.3. The signal-to-noise ratio is often given in decibels. Assume that SNR_{dB} = 36 and the channel bandwidth is 2 MHz. Calculate channel capacity.

Solution:

The theoretical channel capacity can be calculated as

$$SNR_{dB} = 10 \log_{10}(SNR)$$

$$SNR = 10^{SNR_{dB}/10} = 10^{36/10} = 3981$$

$$C = B \log_2 (1 + SNR) = 2 * 10^6 * \log_2(3982) = 24\ Mbps$$

Q.4. We have a channel with a 1-MHz bandwidth. The SNR for this channel is 63. What are the appropriate bit rate and signal level?

Solution:

First, we use the Shannon formula to find the upper limit.

$$C = B \log_2 (1 + SNR) = 10^6 * \log_2 (1 + 63) = 10^6 * \log_2 64 = 6Mbps$$

The Shannon formula gives us 6 Mbps, the upper limit. For better performance we choose something lower, 4 Mbps, for example. Then we use the Nyquist formula to find the number of signal levels.

$$Bit\ Rate = 2 * bandwidth * \log_2 L$$

$$4\ Mbps = 2 * 1 * 10^6 * \log_2 L$$

$$L = 4$$

The Shannon capacity gives us the upper limit; the Nyquist formula tells us how many signal levels we need.

Performance of the Network

Network performance refers to the quality and speed of a network's transmission of data between devices. It is typically measured by factors such as bandwidth, latency, jitter and throughput.

Network performance is important because it determines how well devices can communicate with each other and accesses the resources they need, such as the internet or shared files. Poor network performance can lead to slow response times, reduced productivity, and other problems.

Bandwidth

Bandwidth can be used in two different contexts with two different measuring values: bandwidth in hertz and bandwidth in bits per second.

Bandwidth in Hertz: It is the range of frequencies contained in a composite signal or range of frequencies a channel can pass. For example, we can say the bandwidth of a subscriber telephone line is 4 KHz.

Bandwidth in Bits per Seconds: Bandwidth can also refer to the number of bits per second that a channel, a link, or even a network can transmit. For example, one can say the bandwidth of a Fast Ethernet Network is a maximum of 100 Mbps. This means that this network can send 100 Mbps.

Throughput: It is a measure of how fast we can actually send data through a network. Bandwidth in bits per second and throughput are different. A link may have a bandwidth of B bps, but we can only send T bps through this link with T always less than B. For example, we may have a link with a bandwidth of 1 Mbps, but the devices connected to the end of the link may handle only 200 kbps. This means that we cannot send more than 200 kbps through this link.

Q. A network with bandwidth of 10 Mbps can pass only an average of 12,000 frames per minute with each frame carrying an average of 10,000 bits. What is the throughput of this network?

Solution:

The throughput is calculated as

$$Throughput = \frac{12000 * 10000}{60} = 2000000 = 2\ Mbps$$

Latency (Delay): The latency or delay defines how long it takes for an entire message to completely arrive at the destination from the time the first bit is sent out from the source. We can say that latency is made of four components: propagation time, transmission time, queuing time and processing delay.

$$Latency\ = propagation\ time\ + transmission\ time\ + queuing\ time\ + \ processing\ delay$$

Propagation Time: Propagation time measures the time required for a bit to travel from the source to the destination. The propagation time is calculated by dividing the distance by the propagation speed.

$$Propagation\ time = \frac{Distance}{Propagation\ Speed}$$

Transmission time: It is the time between the first bit leaving the sender and the last bit arriving at the receiver. The first bit leaves earlier and arrives earlier; the last bit leaves later and arrives later. The time required for transmission of a message depends on the size of the message and the bandwidth of the channel.

$$Transmission\ time = \frac{Message\ Size}{Bandwidth}$$

Queuing Time: It is the time needed for each intermediate or end device to hold the message before it can be processed. The queuing time is not a fixed factor; it changes with the load imposed on the network. When there is heavy traffic on the network, the queuing time increases. An intermediate device, such as a router, queues the arrived messages and processes them one by one.

Jitter: Another performance issue that is related to delay is jitter. Jitter is a problem if different packets of data encounter different delays and the application using the data at the receiver site is time-sensitive (audio and video data, for example).

Bit Error Rate (BER): It is defined as the rate at which error occurs in transmission system. This can be directly translated into the number of errors that occur in string of a stated number of bits. The BER can be calculated as

$$BER = \frac{Number\ of\ errors}{Total\ number\ of\ bits\ send}$$