ⓐ Introduction to Push-down Automata:-

↳ The Context free Languages have a type of automaton that defines them. This automaton is called push-down Automata.

↳ The push down automata can be thought as a ε-NFA with addition of stack. The stack can be read, pushed and popped only at the top just like the stack data structure.

ⓑ. Block Diagram / Structure of PDA:-

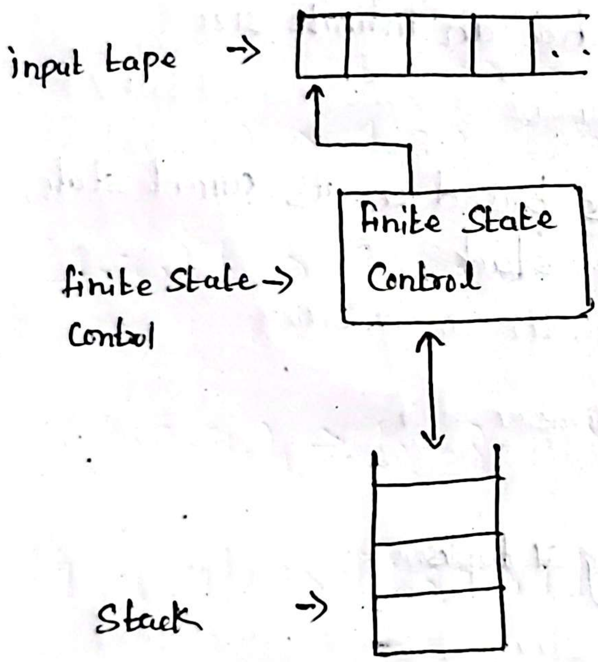


fig: Structure of PDA

↳ Pushdown Automata has three component:-

① Input tape

② finite State Control

③ Stack

* Input tape:- The input tape is divided in many cells. The input head is read-only and may oly move from left to right, one symbol at a time.

* Finite Control:- The finite control has some pointer which points the current symbol which is to be read and generate next state.

* Stack:- The stack is the structure in which we can push and remove the items from one end only. It has an infinite size.

↳ The transition of pushdown automata is based on it's current state, the input symbol and symbol at the top of stack.

* Formal Defination of PDA

Pushdown automata can be formally defined by 7 tuples.

$$P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, f)$$

where,

Q: A finite set of states

Σ: A finite set of input Symbol

Γ: A finite set of stack symbol

δ: Transition function that maps

$Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma \rightarrow Q \times \Gamma^*$ [for DPDA]

$Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma \rightarrow 2^{(Q \times \Gamma^*)}$ [For NDPDA]

q_0 : start state

Z_0 : initial stack symbol

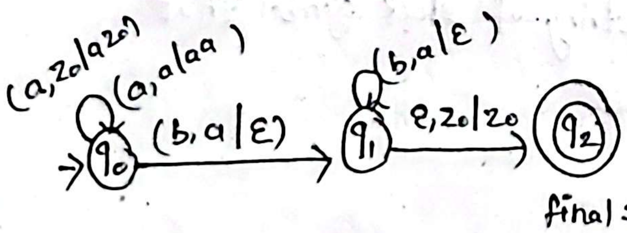
F : set of accepting states of final states i.e $F \subseteq Q$.

Representation of Pushdown Automata :-

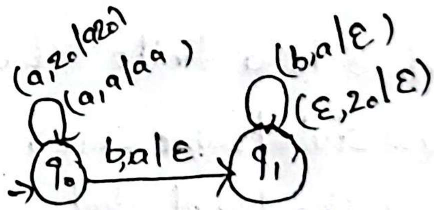
PDA Representation

- State Transition Diagram
  - Acceptance of string by final state
  - Acceptance of string by empty stack
- State Transition function
  - Acceptance of string by final state
  - Acceptance of string by empty stack.

Let us take $L = \{a^n b^n \mid n \geq 1\}$



Acceptance by final state



Acceptance by empty stack

State transition function

$\delta(q_0, a, z_0) \rightarrow (q_0, az_0)$ (push)	$\delta(q_0, a, z_0) \rightarrow (q_0 A z_0)$
$\delta(q_0, a, a) \rightarrow (q_0, aa)$ (push)	$\delta(q_0, a, a) \rightarrow (q_0, aa)$
$\delta(q_0, b, a) \rightarrow (q_1, \epsilon)$ (pop)	$\delta(q_0, b, a) \rightarrow (q_1, \epsilon)$
$\delta(q_1, b, a) \rightarrow (q_1, \epsilon)$ (pop)	$\delta(q_1, b, a) \rightarrow (q_1, \epsilon)$
$\delta(q_1, \epsilon, z_0) \rightarrow (q_2, z_0)$ (skip)	$\delta(q_1, \epsilon, z_0) \rightarrow (q_1, \epsilon)$

⊛ Transition function of PDA' -

Transition function δ of PDA takes as argument a triplet i.e $\delta(q, a, X)$ where,

① q is a state in Q.

② a is either input symbol in Σ or $a = \epsilon$, the empty string.

③ X is the stack symbol that is the member of Γ

The output of δ is a finite set of Pairs (P, α) where

P is the new state/same state, α is the string of stack symbol that replaces X at the top of stack.

if $\alpha = \epsilon$, then stack is poped

If $\alpha = X$, then stack is unchanged

If $\alpha = YZ$ then X is replaced by Z and Y is pusped into the stack.

⊛ Transition Diagram of PDA:-

↳ We can use transition diagram to represent PDA, where

ⓐ The node correspond to the state of PDA.

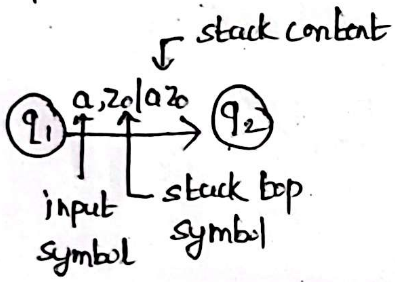
ⓑ An arrow labelled start indicates start state and doubly circled states are accepting / final state.

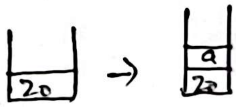
ⓒ The arc correspond to transitions of the PDA as:

↳ An arc labelled $a, X | \alpha$ from state q to p means that $\delta(q, a, X) \rightarrow (p, \alpha)$

⊛ Operation Performed on Stack:-

① Push Operation



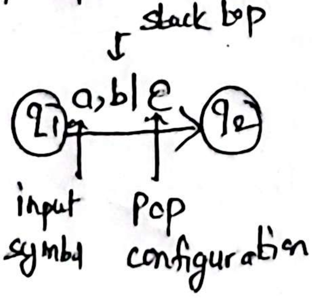


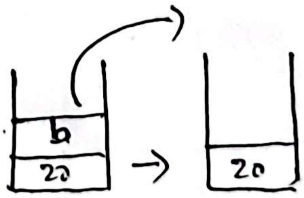
Push

transition Diagram:

$$\delta(q_1, a, z_0) \rightarrow (q_2, a z_0)$$

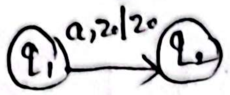
② Pop Operation

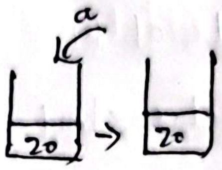




Pop

③ skip: operation





skip

$\delta(q_1, a, z_0) \rightarrow (q_2, z_0)$

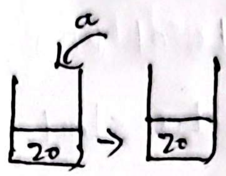
Note:- $\delta(q_1, a, z_0) \rightarrow (q_2, az_0)$ - push

$\delta(q_1, a, b) \rightarrow (q_2, \epsilon)$ - pop

$\delta(q_1, a, z_0) \rightarrow (q_2, z_0)$ - skip

③ skip: operation

$$q_1 \xrightarrow{a, z_0 | z_0} q_2$$



$\delta(q_1, a, z_0) \rightarrow (q_2, z_0)$

skip

Note:- $\delta(q_1, a, z_0) \rightarrow (q_2, az_0)$ - Push

$\delta(q_1, a, b) \rightarrow (q_2, \epsilon)$ - Pop

$\delta(q_1, a, z_0) \rightarrow (q_2, z_0)$ - skip