[5 hrs]

Chapter - 5

Decidability and Computational Complexity

5.1 Church turing thesis

→ Alan Turing defined Turing machines is an attempt to formalize the notion of an 'effective producer' which is usually called as an algorithm these days.

→ Simultaneously mathematicians were working independently on the same problem.

Email Post → Production Systems

Alonzo Church → Lamda Calculus

Noam Chomsky → Unrestricted Grammars

Stephen kleene → Recursive Function Theory

Raymond Smullyn → Formal Systems

→ All of the above formalisms were proved equivalent to one another. This led to

(a) Turing's Thesis (weak form): A Turing machine can compute anything that can be computed by a general-purpose digital computer.

(b) Turing's Thesis (Strong form): A Turing machine can compute anything that can be computed.

The strong form of Turing's Thesis cannot be proved it states a relationship between mathematical concept and "real world."

5.2 Universal turing machine, encoding of turing machine :-

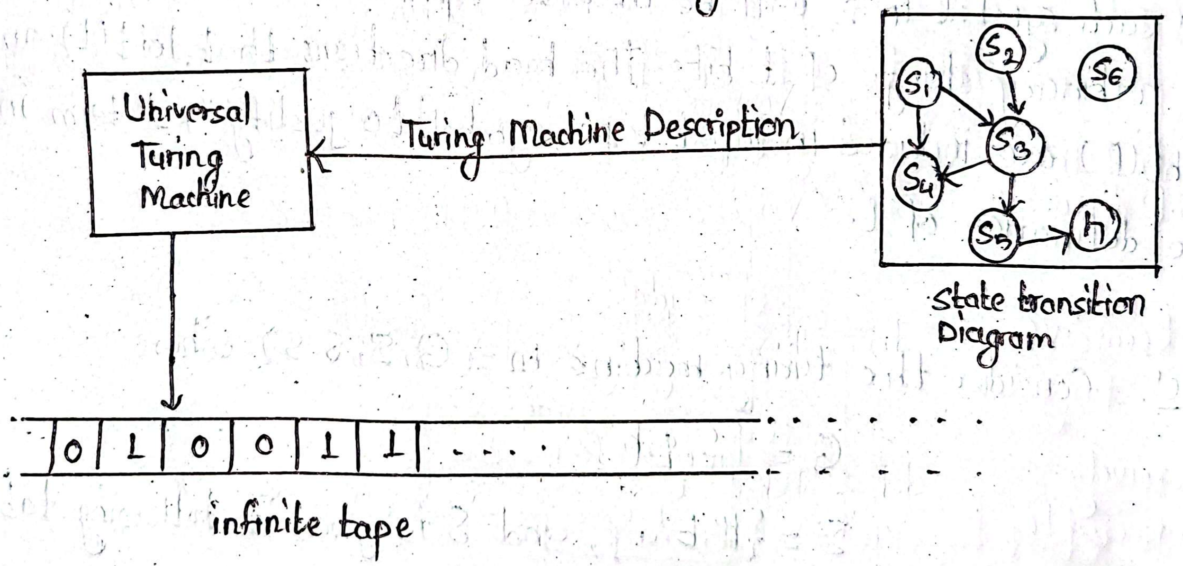
→ A general purpose Turing machine is said to be a Universal Turing machine, if it is powerful enough to simulate the behaviour of any digital computer.

→ A Universal Turing machine can simulate the behaviour of an arbitrary turing machine over any set of input symbols.

→ A Universal Turing machine can accepts two inputs:

1. The input data
2. An algorithm for computation.

→ Turing also showed that it is possible to design a single Turing machine which can simulate the computations of any Turing Machine, given an encoded description of the target TM and it's initial configuration (the input string on it's tape, the initial state and the position of the head. Such a machine is called a Universal Turing Machine.



(*) Working of Universal Turing Machine:-

1. Scan the square or cell on the state area of the tape and read the symbol that Turing machines reads and the initial state of Turing Machine.

2. Move the tape to the program, which contain the finite automation table.

find the row which is headed by the symbol scanned in the previous state.

3. Find the required column.

4. Move the tape to the appropriate square on the data area of the tape, replace the symbol, move the tape in the given direction, read the next symbol, reach the state area and replace the state by the current state and repeat from state 1.

Encoding of Turing Machine:-

Let $Tm = (Q, \Sigma, \delta, s)$ be a turing machine, and k and l be smallest integers such that $2^k \geq |Q|$, and $2^l \geq |\Sigma| + 2$. Then each state in Q will be represented by q followed by a binary string of length k; each symbol in Σ will be likewise represented as letter a followed by a binary strings of l bits. The head directions that left(L) and right(R) are included in input tape symbol to justify "+2" term in the defination of l.

Example:- Consider the turing machine $M = (Q, \Sigma, \delta, s)$ where

$$Q = \{s, q, h\}$$

$\Sigma = \{\#, b, a\}$, and δ is given in following table

states	symbol	δ
s	a	(q, #)
s	#	(h, #)
s	b	(s, R)
q	a	(s, a)
q	#	(s, R)
q	b	(q, R)

Since, there are three states, these symbols in Σ, we have $k=2$ and $l=3$. These are the smallest integers such that $2^k \geq 3$ and $2^l \geq 3+2$. The states and symbols are represented as follows:

State/ Symbol	Representation
s	100
q	101
h	111
#	0000
b	0001
L	0010
R	0011
a	0100

Thus representation of baa#a is

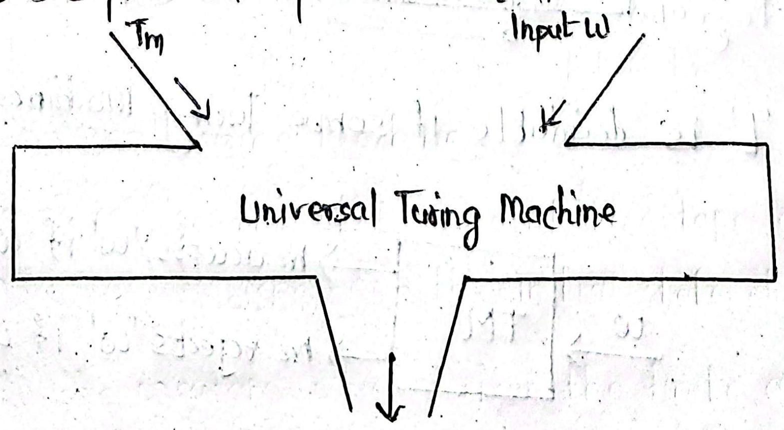
"baa#a" = 0001 0100 0100 0000 0100

Representing "Tm" of the turing machine Tm is the following strings:

"Tm" = (100, 0100, 101, 0000), (100, 0000, 111, 0000), (100, 0001, 100, 001

(101, 0100, 100, 0011), (101, 0000, 100, 0011), (101, 0001, 101, 011

In the last let us see pictorial representation of universal turing machine



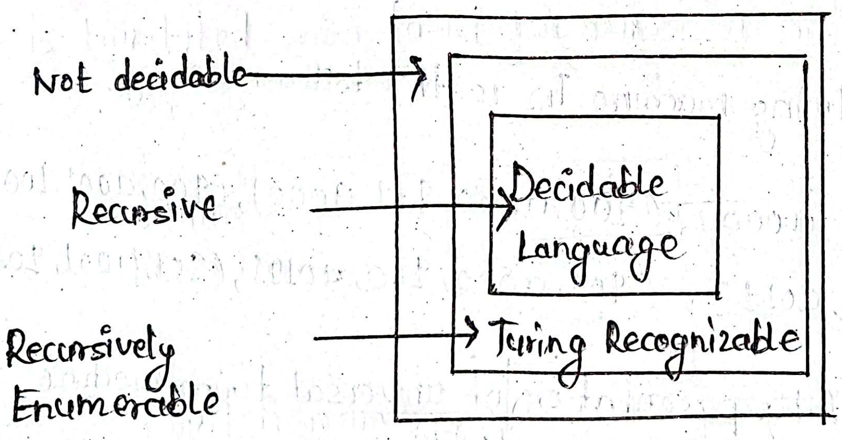
Result of calculation by Tm on input w.

5.3 Undecidable problem about turing machines, halting problems and its implications.

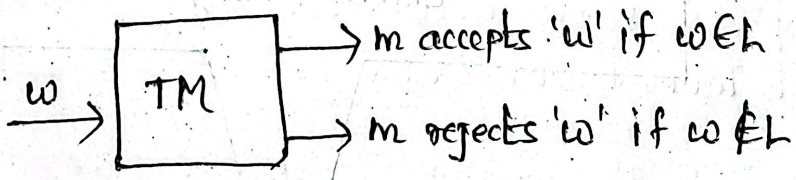
(a) Decidable Language / Recursive Language:-

→ As we know that reccursive language are those language which are accepted by at least one turing machine and these sets of recursive language are subclass of reccursively enumerable language.

→ A problem whose Language is reccursive is said to be decidable, otherwise problem is undecidable i.e a problem is undecidable if there exists no algorithm that takes an input at instances of the problem and determine whether the answer to instances is 'yes' or 'no'.



→ A Language 'L' is decidable if some Turing Machines decides it



→ Decidable Language is also knoown as decidable Language or computable language or solvable language.

→ Example:- Regular language and Context free Language.

→ Recursive Enumerable language also knoown as Turing Recognizable or Partially decidable or semi-decidable.