Chapter 2 on Instrumentation

Unit 2: **Theory of Measurement**

(2 Hours)

Introduction

- **Measurement** is an act on process of comparison between the unknown physical quantity to its predefined standard in order to find the **value of unknown quantity**.
- The **measurement** is said to be proper if the measured value is **less erroneous**.
- To make error free measurement, the user must know the characteristics and operation of the instrument, basic laws to handle instrument and causes of errors in measurement.
- Thus we have to study about the characteristics of the instruments: **Error and its analysis**, **statistical analysis of error** and **minimization that of error** or **controlling of error**, etc.
- Measurement system characteristics can be divided into two distinct areas:
  - **Static Characteristics**
  - **Dynamic Characteristics**

Static Characteristics

- Some applications involve the measurement of quantities that are either constant or vary slowly with time.
- Under these circumstances, it is possible to define a set of criteria that gives a meaningful description quality of measurement without interfering with dynamic descriptions that involve the use of differential equations.
- These criteria are called **Static Characteristics**.
- Important static characteristics are:
  - Accuracy
  - Precision
  - Sensitivity
  - Resolution
  - Linearity
  - Reproducibility
  - Drift
  - Dead Zone Hystereris

Accuracy

- Accuracy is the closeness with which an instrument reading approaches the true value of the quantity being measured.
- Thus accuracy of a measurement means conformity to truth.
- It is the degree of closeness with which the reading approaches the true value of the quantity to be measured.
- The accuracy can be expressed in following ways:

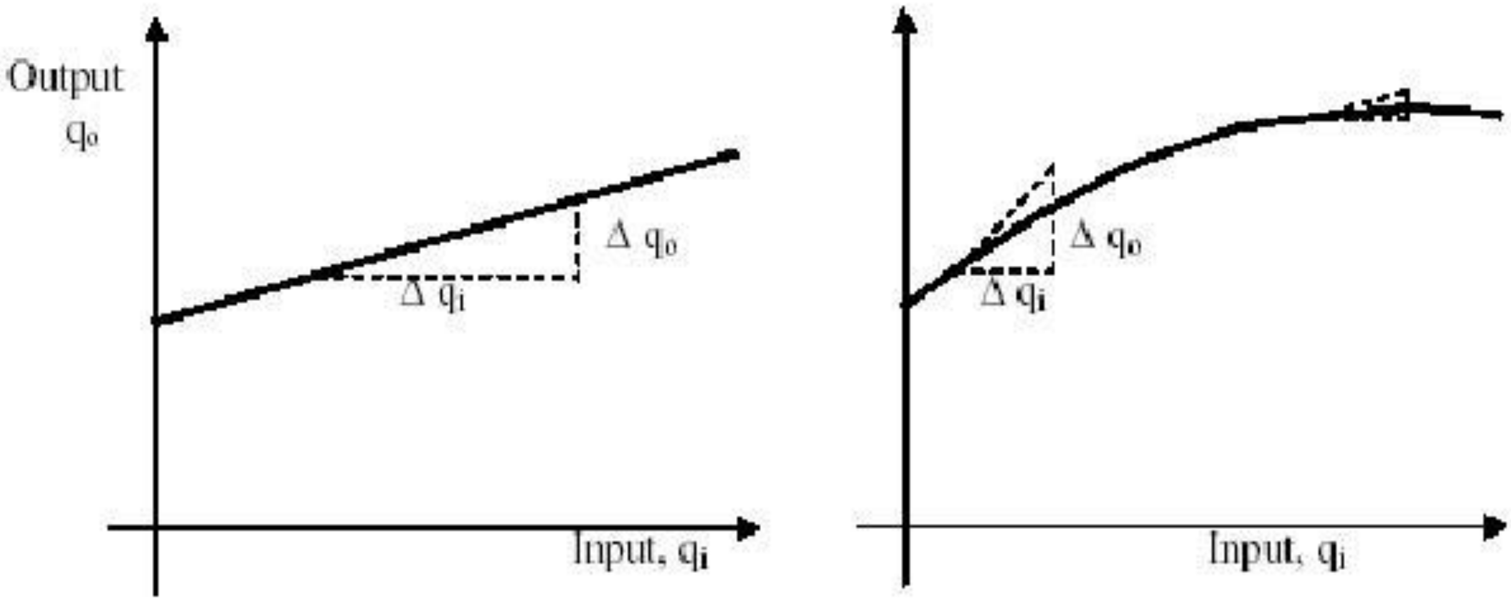
1. **Point Accuracy:**
   - Such accuracy is specified at **only one particular point** of scale.
   - It does not give any information about the accuracy at any other point on the scale.
   - Point accuracy is stated for one or more points in the range.
2. **Accuracy as percentage of scale span:**
   - When an instrument as uniform scale, its accuracy may be expressed in terms of scale range.
   - **Example** – if accuracy of a thermometer with range 500^{o}C with ±0.5% then at 25^{o}C, error will be (500/25*(0.5%)) = 10%, so **misleading**.
3. **Accuracy as percentage of true value:**
   - The best way to conceive the idea of accuracy is to specify it in terms of the true value of the quantity being measured.
   - **Example** – if accuracy is ±0.5% of true value then as reading fet smaller so do that errors, so **more informative**.

Precision (**Precise** – **clearly** or **sharply defined**)

- It is the measure of consistency or reproducibility i.e., given a fixed value of a quantity, precision is a measure of the degree to which successive measurements differ from one another.
- Degree of agreement within a group of measurements.
- The precision is composed of **two** characteristics: **Conformity** and **Number of Significant Figures**.
- **Conformity** is **necessary but not sufficient** for precision because of lack of significant figures obtained.
- **Precision** is a **necessary but not sufficient** condition for accuracy.
- More significant figures in measured value, the grater the precision of measurement.
- Higher degree of conformity of closeness to the true value of the measured value guarantees the accuracy.
- Consider a resistor having true value as 2385692 , which is being measured by an ohmmeter. But the reader can read consistently, a value as 2.4 M due to the nonavailability of proper scale. The error created due to the limitation of the scale reading is a precision error.

Sensitivity

- The sensitivity denotes the smallest change in the measured variable to which the instrument responds.
- It is defined as the ratio of the changes in the output of an instrument to a change in the value of the quantity to be measured.
- Mathematically it is expressed as [**see next slide**]:
- Thus, if the calibration curve is linear, as shown, the sensitivity of the instrument is the slope of the calibration curve.
- If the calibration curve is not linear as shown, then the sensitivity varies with the input.
- **Inverse sensitivity** or **deflection factor** is defined as the reciprocal of sensitivity.
- **Inverse sensitivity** or **deflection factor** = **1/ sensitivity**



$$\text{Sensitivity} = \frac{\text{Infinitesimal change in output}}{\text{Infinitesimal change in input}}$$

$$= \frac{\Delta q_o}{\Delta q_i}$$

Resolution

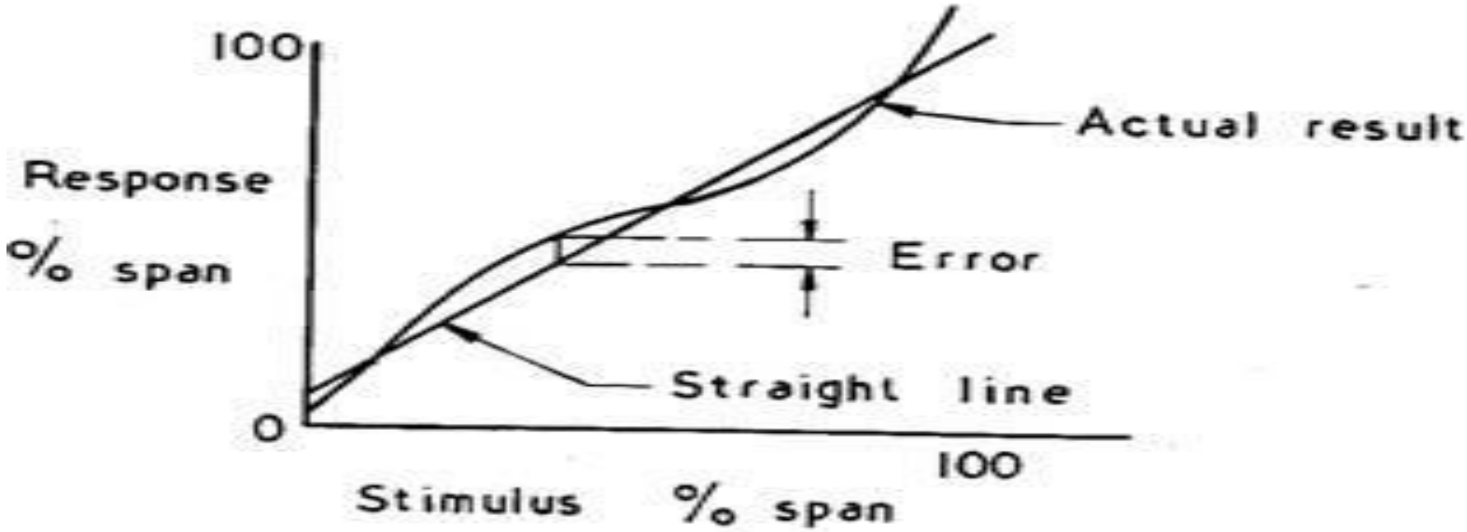
- It is the smallest change in measured value to which the instrument will respond.
- If the input is slowly increased from some arbitrary input value, it will again be found that output does not change at all until a certain increment is exceeded. This **increment** is called **resolution**.
- Thus it is defined as the smallest change in the input which results a detectable output.
- **Analog Instrument** / **Meter** – It is the significant of the smallest division in the scale.
- **Digital Instrument** / **Meter** – It is the significant of the least significant bit (**LSB**).

Linearity

- It means constant sensitivity throughout whole measurement range.
- It is defined as the ability to produce the input characteristics symmetrical and this can be expressed as:

$$y = m\,x + c$$

- When the input-output points of the instrument are plotted on the calibration curve and resulting curve may not be linear.
- This would be only if the output is proportional to input.
- Linearity is the measure of maximum deviation of these points from the straight line.
- The departure from the straight line relationship is non-linearity, but it is expressed as linearity of the instrument.
- This departure from the straight line could be due to non-linear elements in the measuring system or the elastic after effects of the mechanical system.



- Linearity is expressed in many different ways:
  - **Independent Linearity**: It is the maximum deviation from the straight line so placed as to minimize the maximum deviation.
  - **Zero based linearity**: It is the maximum deviation from the straight line joining the origin and so placed as to minimize the maximum deviation.
  - **Terminal based linearity**: It is the maximum deviation from the straight line joining both the end points of the curve.

Dynamic Characteristics

- Many measurements are concerned with rapidly varying quantities and therefore, for such cases we must examine the dynamic relations which exits between output and input.
- Normally done with the help of differential equations.
- Performance criteria based upon dynamic relations constitute the **Dynamic Characteristics**.
- It is used for study of behavior of the system between the time that output value changes and the time the value has settled down to its steady state value.
- Important dynamic characteristics are: **Speed of Response**, **Response Time**, **Measuring Lag**, **Frequency Response**, **Bandwidth**, **SNR**, **Damping Factor**, **Rise Time**, **Fall Time**, **Settling Time**, **Fidelity**, **Dynamic Error**, etc.

Speed of Response

- It indicates how fast the input given to the measurement system , an instrument brings the output.
- It is defined as the rapidity with which a measurement system responds to changes in the measured quantity.

Response Time

- It is defined as the time required by the system to settle to its final steady state position after the application of the input.

Measuring Lag

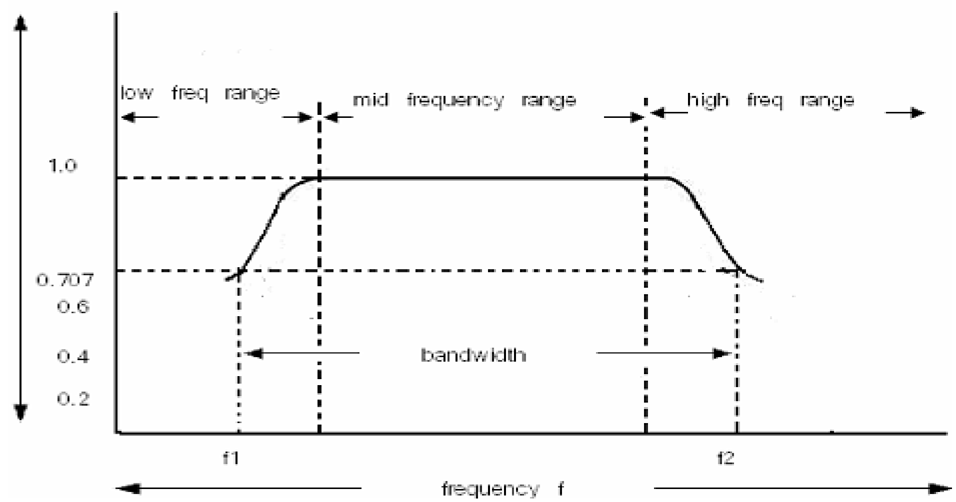
- It is the retardation or delay in the response of a measurement system to changes in the measured quantity.
- The measuring lags are of two types:
  - **Retardation Type**: In this case the response of the measurement system begins immediately after the change in measured quantity has occurred.
  - **Time Delay Lag**: In this case the response of the measurement system begins after a dead time after the application of the input.
  - **Fidelity**: It is defined as the degree to which a measurement system indicates changes in the measurand quantity without dynamic error.

Frequency Response

- When the analysis of the measurement system is done by plotting the gain (output/input) with respect to the frequency then the resulting response is called **Frequency Response**.
- It is of two types: Gain vs Frequency (**Magnitude or Amplitude Response**) and Phase Angle vs Frequency (**Phase Response**).
- The summation of the both – **Bode Plot.**

Bandwidth

- Range of frequency over which an instrument is designed to operate the output receiving the input signal or quantity with a constant gain.
- Calculated by magnitude response and unit is **Hz**.
- The quantity of the measurement system taken in the interval between those frequencies where the power gain of the system has dropped to one – half of its maximum value or voltage gain dropped by a factor of 0.707 (1/√2).



Errors in Measurement and Their Statistical Analysis

Error in Measurement

- Error of Measurement can be defined as **difference** between the **actual value** of a quantity and the **value obtained** by a measurement.
- Repeating the measurement will improve (reduce) the random error (caused by the accuracy limit of the measuring instrument) but not the systemic error (caused by incorrect calibration of the measuring instrument).

1. Absolute Error

- Can be defined as the absolute of difference between the **expected value** (or **T.V.**) of the variable and the **measured value** (**M.V.**) of the variable.

e = absolute error

Y_n = expected value

X_n = measured value

$$e = Y_n - X_n$$

2. Relative Error (Percent of Error)

$$Percent\ error = \frac{absolute\ error}{expected\ error}\ x\ 100\%$$

From equation 1.1 where $Absolute\ error, e = Y_n - X_n$, so

$$Percent\ error = \frac{e}{Y_n}\ x\ 100\%$$

or

$$Percent\ error = \frac{Y_n - X_n}{Y_n}\ x\ 100\%$$

TYPE OF ERROR

- Errors are generally categorized under the following **three major types**:

1. Gross Error

- This class of errors is generally the fault of the person using the instruments such as incorrect reading of instruments, incorrect recording of experimental data or a incorrect use of instruments.
- As long as human beings are involved, some gross errors will definitely commit.
- Although complete eliminating of gross errors is probably impossible, one should try to avoid them.
- The following actions may be necessary to reduce the effects of gross errors.
  - Great care should be taken in reading and recording the data.
  - Two or more readings should be taken by different experimenters.

2. Systematic Error

- All the error due to the shortcoming of an instrument such as less accuracy in the scale calibration, defective parts and effects of the environment on the equipment or the users.
- Systematic errors can be divided into four categories:

i) Instrumental Error

- These errors arise due to main reasons:
  - Due to inherit shortcoming in the instruments (may be caused by the construction, calibration or operation of mechanical structure in the instruments).
  - Due to misuse of the instruments. For example, these may be caused by failure to adjust zero of the instruments.
  - Due to loading effect of the instruments.
  - These errors can be eliminated or at least reduced by using the following methods:
    - The procedure of measurements must be carefully planned.
    - Correction factors should be applied after detection of these errors.
    - Re-calibration the instrument carefully.
    - Use the instrument intelligently.

ii) Observational Errors

- Due to the types on instrument display, whether it is analog or digital.
- Due to parallax (eye should be directly in line with the measurement point).
- ***Note: These errors can be eliminated completely by using digital display instruments.***

iii) Environmental Errors

- Due to conditions external to the measuring device such as the area, surrounding the instrument.
- These conditions may be caused by the changes in pressure, humidity, dust, vibration or external magnetic or electrostatic fields.
- These errors can be eliminated or reduced by using corrective measure such as:
  - Keep the condition as constant as possible.
  - Use instrument/equipment which is immune to these effects.
  - Employ technique which eliminates these disturbances.

iv) Simplification Errors

- Due to simplification of a formula: For example: $\mathbf{A = B + C + D^2}$.
- If **D** is too small, then the formula is simplified to: $\mathbf{A = B + C}$.
- There will be a different result between the first and the second equation.
- In high accuracy requirements, a formula should not be simplified to avoid these types of errors.

3. Random Error

- In some experiments, the results shows variation from one to another, even after all systematical and gross errors have been accounted for.
- The cases of these errors are not recognized, therefore the elimination or reduction of these errors are not possible.
- When these types of errors are occurred, the best result can be determined by statistical analysis.

Statistical Analysis of Experimental Data or Error in Measurement

- It is important to define some pertinent terms before discussing some important methods of statistical analysis of experimental data.

Arithmetic Mean

- When a set of readings of an instrument is taken, the individual readings will vary somewhat from each other, and the experimenter is usually concerned with the mean of all the readings.
- If each reading is denoted by $\boldsymbol{x_i}$ and there are $\boldsymbol{n}$ *readings,* the arithmetic mean is given by

$$x_m = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Deviation

The deviation, d, for each reading is given by

$$d_i = x_i - x_m \quad .$$

We may note that the average of the deviations of all readings is zero since

$$d_i = \frac{1}{n}\sum_{i=1}^{n} d_i = \frac{1}{n}\sum_{i=1}^{n} (x_i - x_m)$$

$$= x_m - \frac{1}{n}(nx_m)$$

$$= 0 \quad .$$

The average of the absolute value of the deviations is given by

$$|\bar{d}_i| = \frac{1}{n}\sum_{i=1}^{n} |d_i|$$

$$= \frac{1}{n}\sum_{i=1}^{n} [x_i - x_m] \quad .$$

Note that the quantity is not necessarily zero.

Standard Deviation

- It is also called **root mean-square deviation**.
- It is defined as:

$$\sigma = \left[\frac{1}{n} \sum_{i=1}^{n} (x_i - x_m)^2 \right]^{1/2}$$

Variance

- The square of standard deviation is called variance.
- This is sometimes called the population or biased standard deviation because it strictly applies only when a large number of samples is taken to describe the population.

Geometrical Mean

- It is appropriate to use a geometrical mean when studying phenomena which grow in proportion to their size.
- This would apply to certain biological processes and growth rate in financial resources. The geometrical mean is defined by

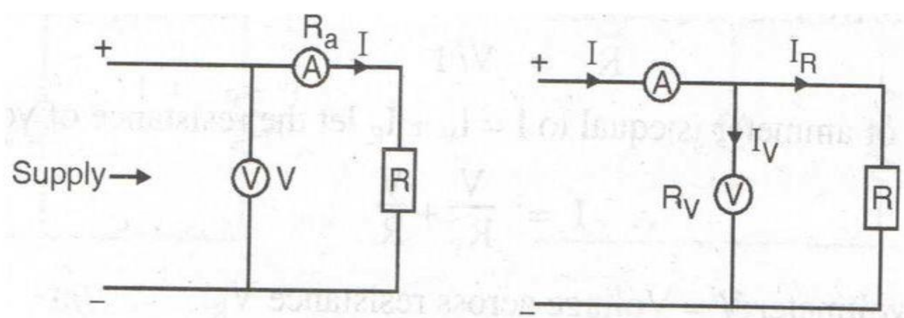
$$x_g = [x_1 \cdot x_2 \cdot x_3 \ldots x_n]^{\frac{1}{n}}$$

Measurement of Resistance (Low, Medium and High)

Measurement of Low Resistance

- This is **Ammeter – Voltmeter Method**.
- This method (**a** and **b**) is very common as voltmeter and ammeter is available in all labs.
- There are two methods of connecting voltmeter and ammeter for measurement of resistances as shown in **Fig (a)** and **(b)**.
- In both the cases the measured value of the unknown resistances is equal to the reading of voltmeter divided by reading of ammeter.
- Let the reading of voltmeter is **V** and ammeter **I**, hence measured value of the resistances **= R_m =V/I**.

Measurement of Medium Resistance



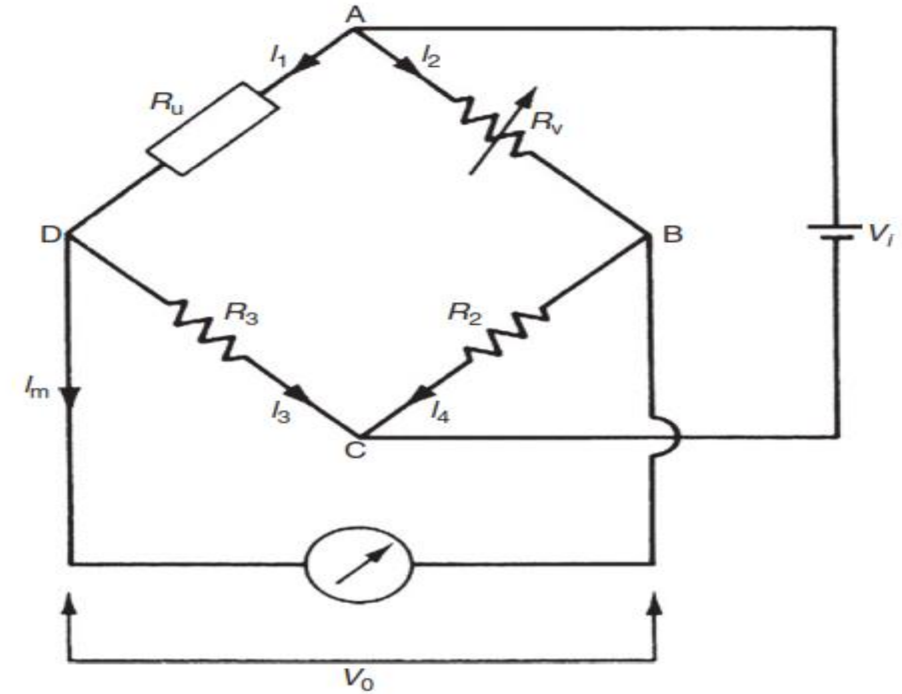
(a) (b)

Fig. 3.3.1

- From Fig (b), $\mathbf{R_m = V/I}$.
- The reading of ammeter is equal to $\mathbf{I = I_V + I_R}$
- Let the resistance of voltmeter by '$\mathbf{R_v}$' then $\mathbf{I = V/R_v + V/R}$
- Reading of voltmeter $\mathbf{V}$ = Voltage across resistance $\mathbf{V_R}$.
- $\mathbf{R_m = V/(V/R+V/R_v) = 1/(1/R+1/R_v) = R/(1+R/R_v)}$
- $\mathbf{R = V/I_R = V/(I - I_V) = V/(I - V/R_v) = V/[I(1 - V/R_v)]}$
  $\mathbf{= R_m/(1-R_m/R_v)}$
- From this expression the **true value** of resistance $\mathbf{R=R_m}$ only, when resistance of voltmeter is '**∞**'(This is the ideal case).
- But practically $\mathbf{R_v}$ is in medium or high resistance class.
- Hence this method is used for measurement of low resistance as in this case $\mathbf{R_m/R_v}$ will be approximately equal to zero.
- $\mathbf{R = R_m/(1-R_m/R_v) = R_m(1-R_m/R_v)^{-1} = R_m(1+R_m/R_v)}$ if $\mathbf{R_m \ll R_v}$.

Measurement of Medium Resistance: Wheatstone Bridge Method

- **Bridge circuits** are used very commonly as a variable conversion element in measurement systems and produce an output in the form of a voltage level that changes as the measured physical quantity changes.
- They provide an accurate method of measuring resistance, inductance, and capacitance values and enable the detection of very small changes in these quantities about a nominal value.
- They are of immense importance in measurement system technology because so many transducers measuring physical quantities have an output that is expressed as a change in resistance, inductance, or capacitance.
- Normally, excitation of the bridge is by a **d.c.** voltage for **resistance measurement** and by an **a.c. voltage** for **inductance** or **capacitance** measurement.
- Both **null** and **deflection types** of bridges exist, and, in a like manner to instruments in general, null types are employed mainly for calibration purposes and deflection types are used within closed loop automatic control schemes.



- The four arms of the bridge consist of the unknown resistance $\mathbf{R_u}$, two equal value resistors $\mathbf{R_2}$ and $\mathbf{R_3}$, and variable resistor $\mathbf{R_v}$ (usually a decade resistance box).
- A d.c. voltage $\mathbf{V_i}$ is applied across the points **AC**, and resistance $\mathbf{R_v}$ is varied until the voltage measured across points **BD** is zero.
- This null point is usually measured with a high sensitivity galvanometer.

To analyze the Whetstone bridge, define the current flowing in each arm to be $I_1 \ldots I_4$ as shown in Figure 9.1. Normally, if a high impedance voltage-measuring instrument is used, current I_m drawn by the measuring instrument will be very small and can be approximated to zero. If this assumption is made, then, for $I_m = 0$: $I_1 = I_3$ and $I_2 = I_4$.

Looking at path ADC, we have voltage V_i applied across resistance $R_u + R_3$ and by Ohm's law:

$$I_1 = \frac{V_i}{R_u + R_3}.$$

Similarly, for path ABC,

$$I_2 = \frac{V_i}{R_v + R_2}.$$

Now we can calculate the voltage drop across AD and AB:

$$V_{AD} = I_1 R_v = \frac{V_i R_u}{R_u + R_3} \quad ; \quad V_{AB} = I_2 R_v = \frac{V_i R_v}{R_v + R_2}.$$

By the principle of superposition, $V_o = V_{BD} = V_{BA} + V_{AD} = -V_{AB} + V_{AD}$.

$$V_o = -\frac{V_i R_v}{R_v + R_2} + \frac{V_i R_u}{R_u + R_3}. \tag{9.1}$$

At the null point $V_o = 0$, so

$$\frac{R_u}{R_u + R_3} = \frac{R_v}{R_v + R_2}.$$

Inverting both sides,

$$\frac{R_u + R_3}{R_u} = \frac{R_v + R_2}{R_v},$$

that is,

$$\frac{R_3}{R_u} = \frac{R_2}{R_v}$$

or

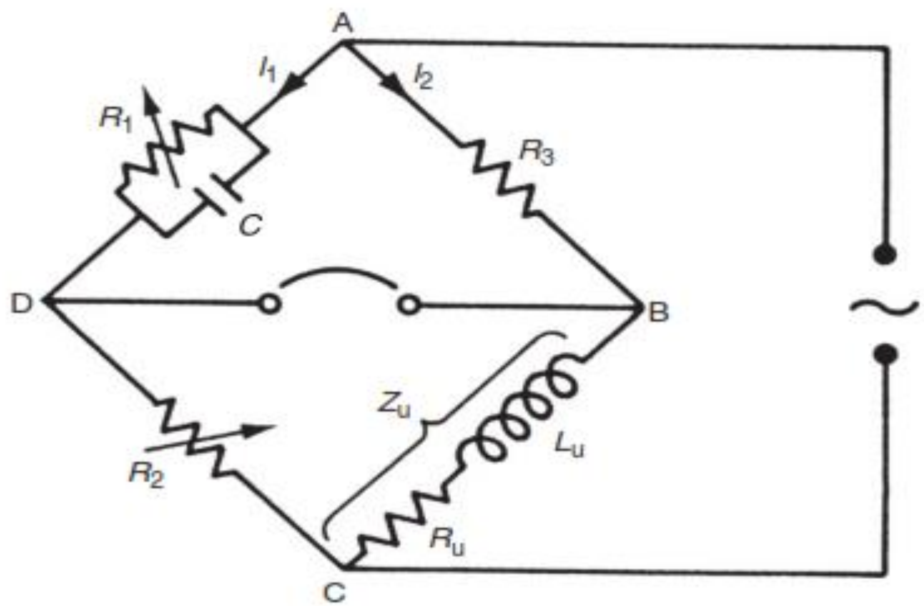
$$R_u = \frac{R_3 R_v}{R_2}. \tag{9.2}$$

Thus, if $R_2 = R_3$, then $R_u = R_v$. As R_v is an accurately known value because it is derived from a variable decade resistance box, this means that R_u is also accurately known.

Measurement of Inductance : AC Bridge – Maxwell Bridge

- Bridges with a.c. excitation are used to measure unknown impedances (capacitances and inductances).
- Both **null** and **deflection types** exist.
- As for d.c. bridges, null types are more accurate but also more tedious to use.
- Therefore, null types are normally reserved for use in calibration duties and any other application where very high measurement accuracy is required.
- Otherwise, in all other general applications, deflection types are preferred.

- A **Maxwell bridge** is shown in Figure.
- The requirement for a variable inductance box is avoided by introducing instead a second variable resistance.
- The circuit requires one standard fixed-value capacitor, two variable resistance boxes, and one standard fixed-value resistor, all of which are components that are readily available and inexpensive.
- Referring to Figure, we have at the null output point:



$$I_1 Z_{AD} = I_2 Z_{AB} \quad ; \quad I_1 Z_{DC} = I_2 Z_{BC}$$

Thus,

$$\frac{Z_{BC}}{Z_{AB}} = \frac{Z_{DC}}{Z_{AD}}.$$

or

$$Z_{BC} = \frac{Z_{DC} Z_{AB}}{Z_{AD}} \tag{9.12}$$

The quantities in Equation (9.12) have the following values:

$$\frac{1}{Z_{AD}} = \frac{1}{R_1} + j\omega C \quad \text{or} \quad Z_{AD} = \frac{R_1}{1 + j\omega C R_1}$$

$$Z_{AB} = R_3 \quad ; \quad Z_{BC} = R_u + j\omega L_u \quad ; \quad Z_{DC} = R_2$$

Substituting the values into Equation (9.12),

$$R_u + j\omega L_u = \frac{R_2 R_3 (1 + j\omega C R_1)}{R_1}.$$

Taking real and imaginary parts:

$$R_u = \frac{R_2 R_3}{R_1} \quad ; \quad L_u = R_2 R_3 C. \tag{9.13}$$

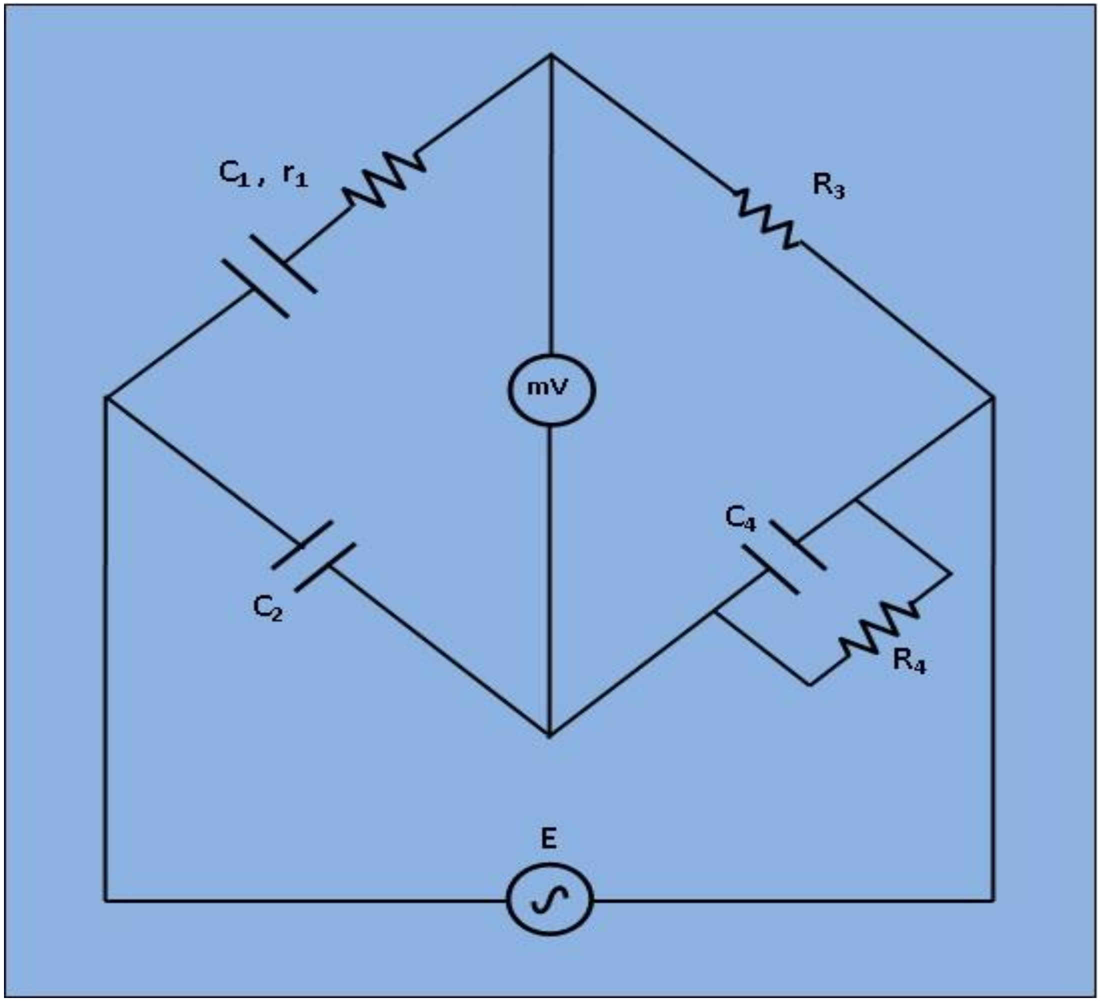
This expression [Equation (9.13)] can be used to calculate the quality factor (Q value) of the coil:

$$Q = \frac{\omega L_u}{R_u} = \frac{\omega R_2 R_3 C R_1}{R_2 R_3} = \omega C R_1.$$

If a constant frequency ω is used, $Q \approx R_1$.

Thus, the Maxwell bridge can be used to measure the Q value of a coil directly using this relationship.

Measurement of Capacitance: AC Bridge – Schering Bridge



Let

- $\mathbf{C_1}$ = capacitor whose capacitance is to be measured.
- $\mathbf{r_1}$ = a series resistance representing the loss in the capacitor $\mathbf{C_1}$.
- $\mathbf{C_2}$ = a standard capacitor.
- $\mathbf{R_3}$ = a non inductive resistance.
- $\mathbf{C_4}$ = a variable capacitor.
- $\mathbf{R_4}$ = a variable non inductive resistance.

At balance: $\mathbf{Z_1 Z_4 = Z_2 Z_4}$

$$\mathbf{(r_1+1/j\omega C_1)\cdot(R_4/(j\omega C_4R_4+1)) = R_3/j\omega C_2} ......(1)$$

$$\mathbf{r_1R_4 - jR_4/\omega C_1 = -jR_3/\omega C_2 + R_3R_4C_4/C_2} ......(2)$$

Or Equating the real and imaginary terms in equa. (2), we obtain

$$\mathbf{r_1 = R_3\cdot C_4/C_2} ......(3)$$

$$\mathbf{C_1 = R_4\cdot C_2/R_3} ......(4)$$