Push down automata

Example: Design the pushdown automata for the language $L=\{a^n b^n | n>0\}$

Solution:

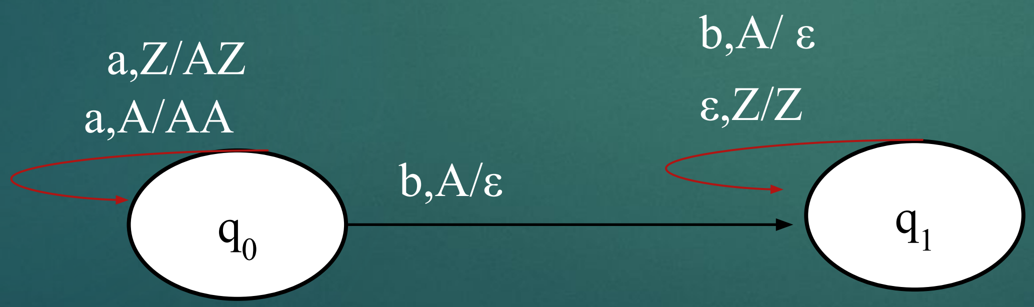
Where ,

$Q=\{q_0 , q_1\}$

$\sum=\{a,b\}$

$\Gamma=\{A,Z\}$

State diagram:



Push down automata

δ:

$\delta(q_0,a,Z)=\{q_0,AZ\}$

$\delta(q_0,a,A)=\{q_0,AA\}$

$\delta(q_0,b,A)=\{q_1,\varepsilon\}$

$\delta(q_1,b,A)=\{q_1, \varepsilon\}$

Let us see how this automata works for aaabbb.

Row	**State**	**Input**	**δ**	**Stack**	**State after move**
1	q_0	aaabbb		Z	q_0
2	q_0	aaabbb	δ(q0,a,Z)={q0,AZ}	AZ	q_0
3	q_0	aaabbb	δ(q0,a,Z)={q0,AA}	AAZ	q_0
4	q_0	aaabbb	δ(q0,a,Z)={q0,AAA}	AAAZ	q_0
5	q_0	aaabbb	δ(q0,a,Z)={q0, ε}	AAZ	q_1
6	q_1	aaabbb	δ(q_1,a,Z)={q0, ε}	AZ	q_1
7	q_1	aaabbb	δ(q1,a,Z)={q0, ε}	Z	
8	q_1	ε	δ(q1,a,Z)={q0, ε}	ε	q_1

Push down automata

Practice:

Design a PDA that accepts $L=\{a^n b^n|n>=1\}$

Solution:

The language contain the strings L={ab,aabb,aaabbb,,,,}

$\delta(q_0,a,z_0)=(q_0,az_0)$

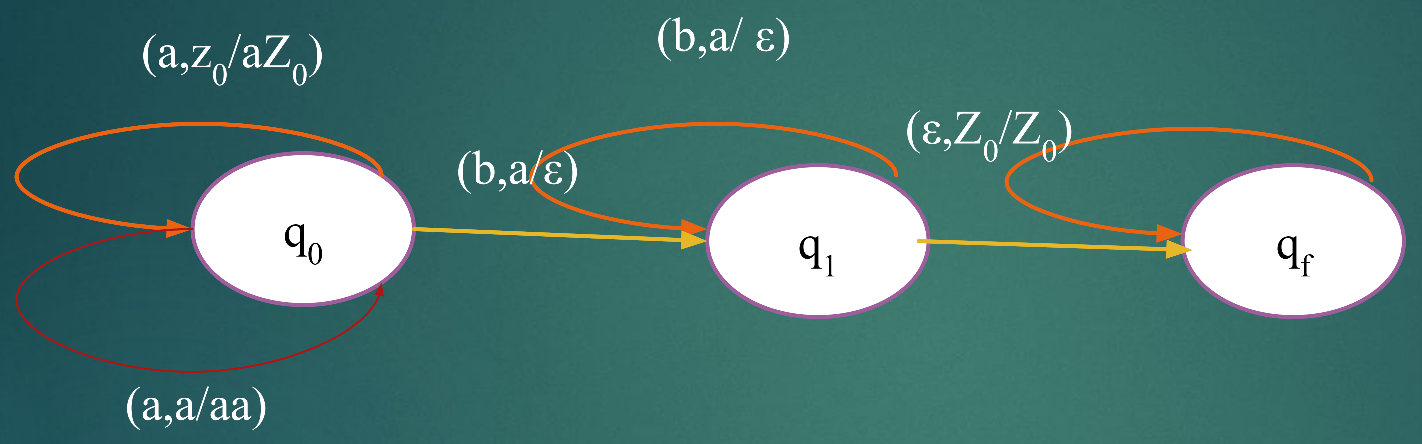
$\delta(q_0,a,a)=(q_0,aa)$

$\delta(q_0,b,a)=(q_1,\varepsilon)$

$\delta(q_0,b,a)=(q_0, \varepsilon)$

$\delta(q_0, \varepsilon,z_0)=(q_0,z_0)$ or$(q_f, \varepsilon)=(q_0,z_0)$

Push down automata



Push down automata

- Let us consider a string aabb.

$\delta(q_0,aabb,z_0)$

$\vdash\delta(q_0,abb,az_0)$

$\vdash\delta(q_0,bb,aaz_0)$

$\vdash\delta(q_1,b,az_0)$

$\vdash\delta(q_0,\varepsilon,z_0)$

$\vdash\delta(q_f)$

Thus the string is accepted by push down automata.

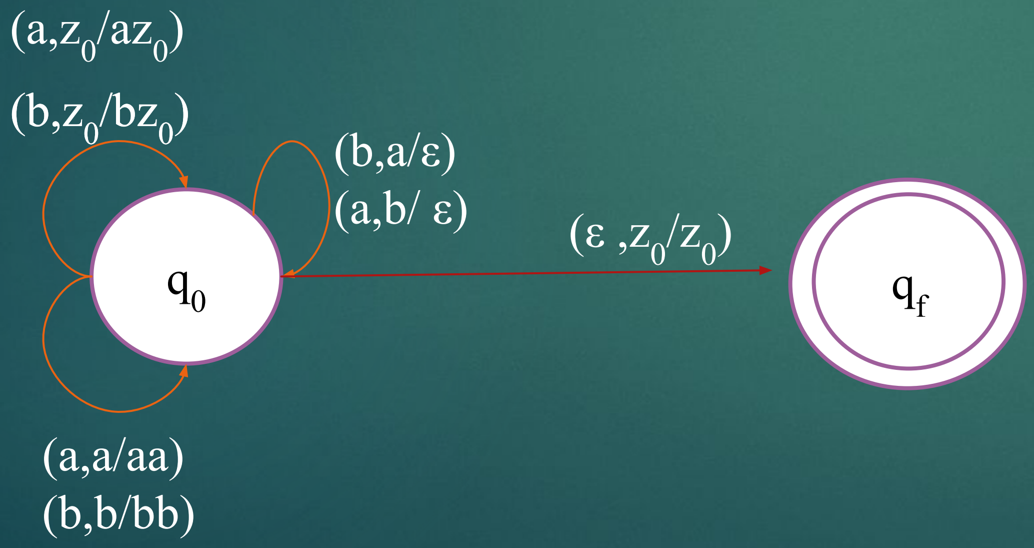
Push down automata

Example: construct PDA that accepts $L=\{w/n_a(w)=n_b(w)\}$.

Solution:

The language contains the string L={abab,baba,,,)

The state is diagram is given by



Push down automata

Let us consider the acceptance of the string w=baab

$\delta(q_0,baab,z_0)$

$\vdash\delta(q_0,aab,bz_0)$

$\vdash\delta(q_0,ab,z_0)$

$\vdash\delta(q_0,b,az_0)$

$\vdash\delta(q_0, \varepsilon,z_0)$

$\vdash\delta(q_f)$

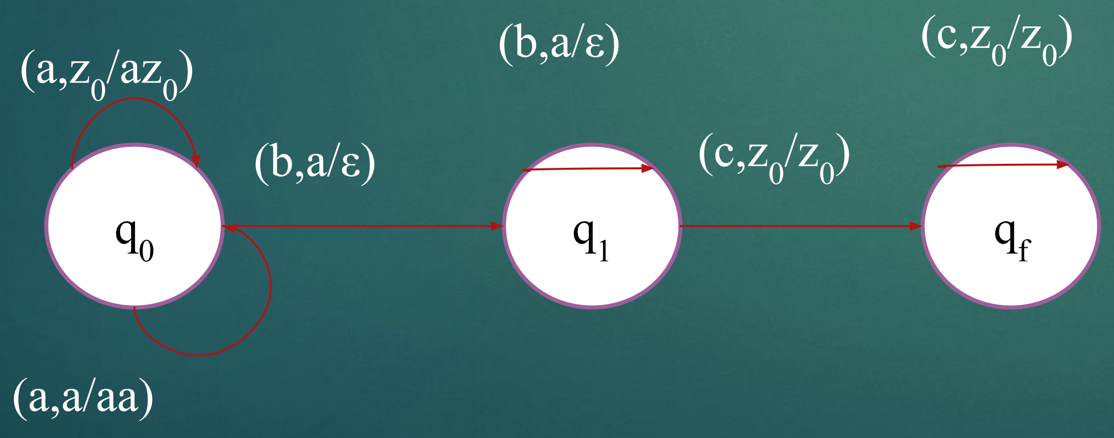
Push down automata

Example: Design a PDA for language $a^n b^n c^m$ |n,m=>1.

Solution:

The language will contain the string L={abc,aabbc,,,}

The state diagram is given as:



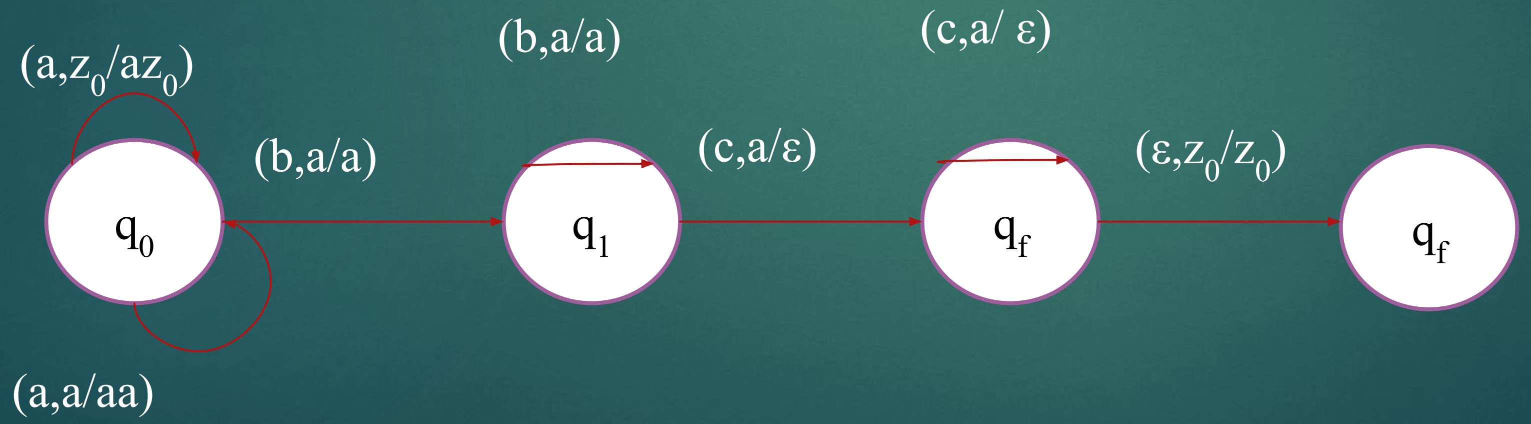
Push down automata

Example: Design a PDA for language $a^n b^m c^n \mid n,m=>1$.

Solution:

The language will contain the string L={abc,aabcc,,,}

The state diagram is given as:



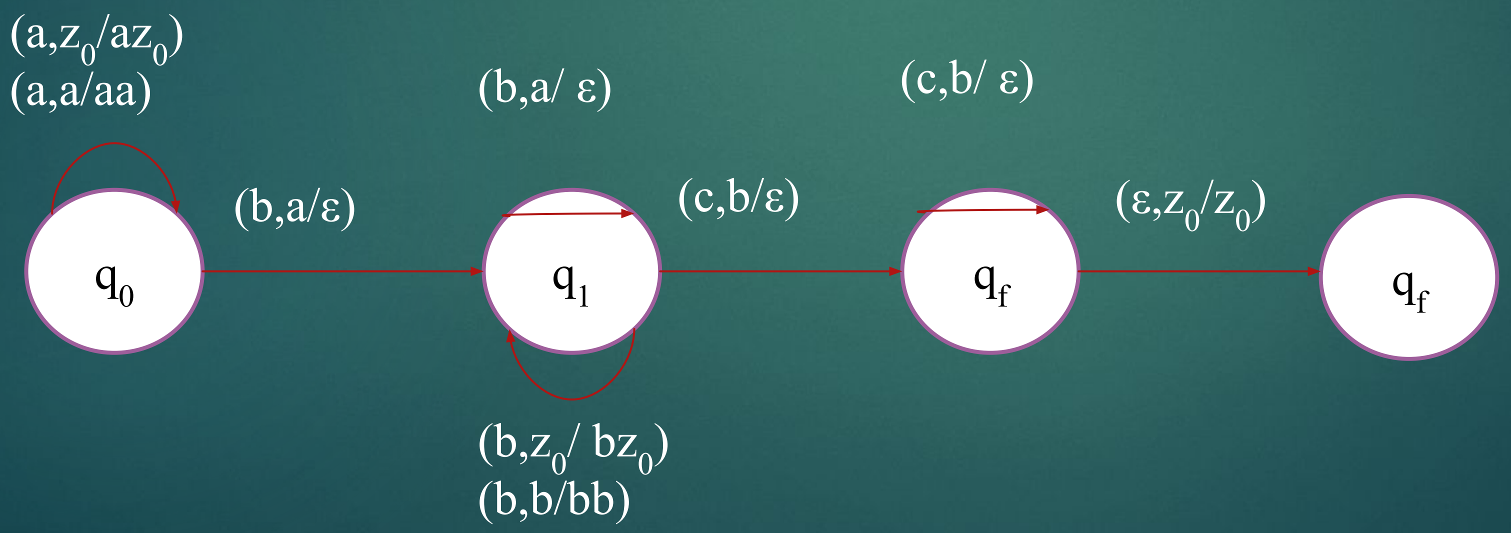
Push down automata

Example: Design a PDA for language $a^n b^{n+m} c^m$ |n,m=>1.

Solution:

The language can be simplified as $a^n.b^n.b^m c^m$

The state diagram is given as:

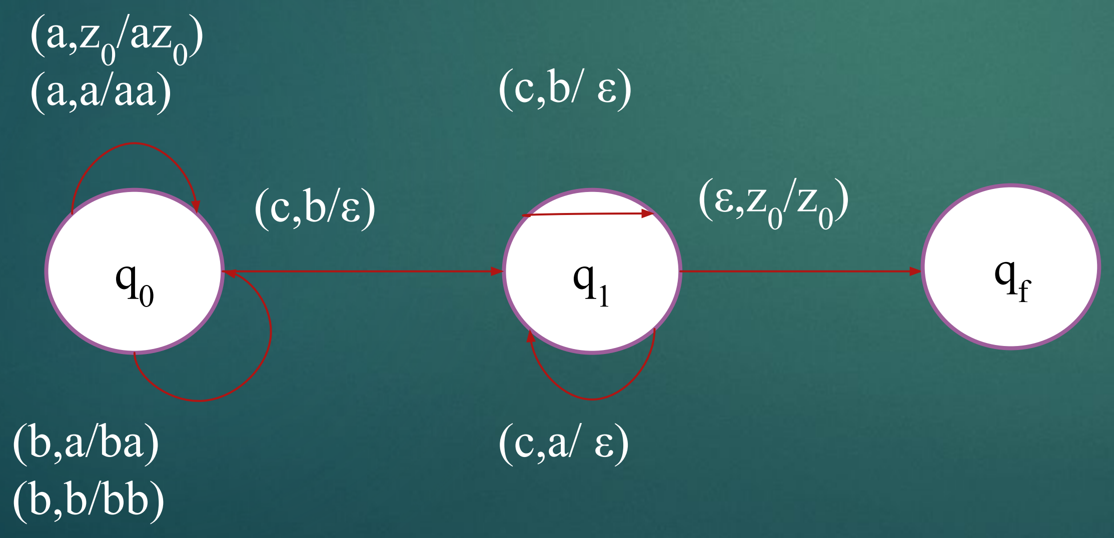


Push down automata

Example: Design a PDA for language $a^n b^m c^{n+m}$ |n,m=>1.

Solution:

The state diagram is given as:



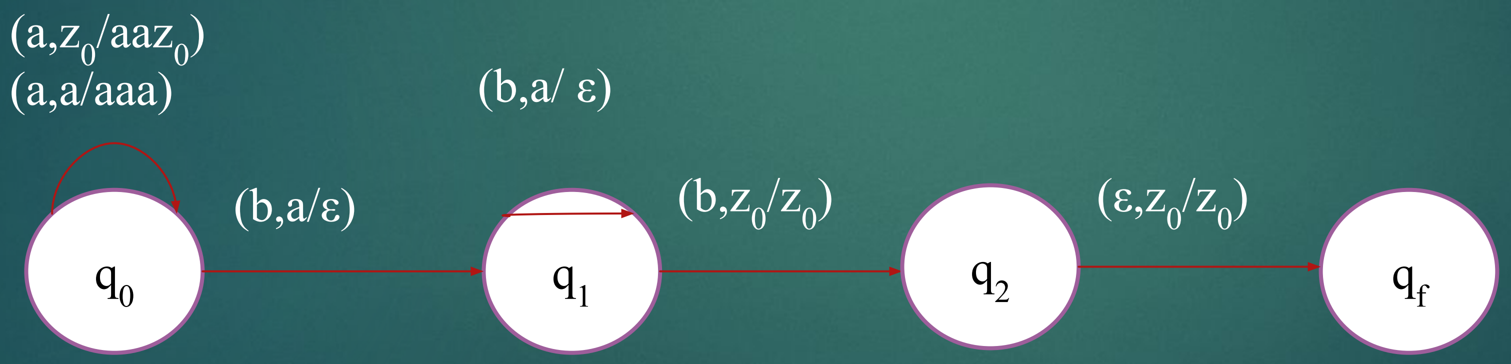
Push down automata

Example: Design a PDA for language $a^n b^{2n+1}$ |n,=>1.

Solution:

We use logic here for single ‘a’ push two a’s into the stack.

The state diagram is given as:



Push down automata

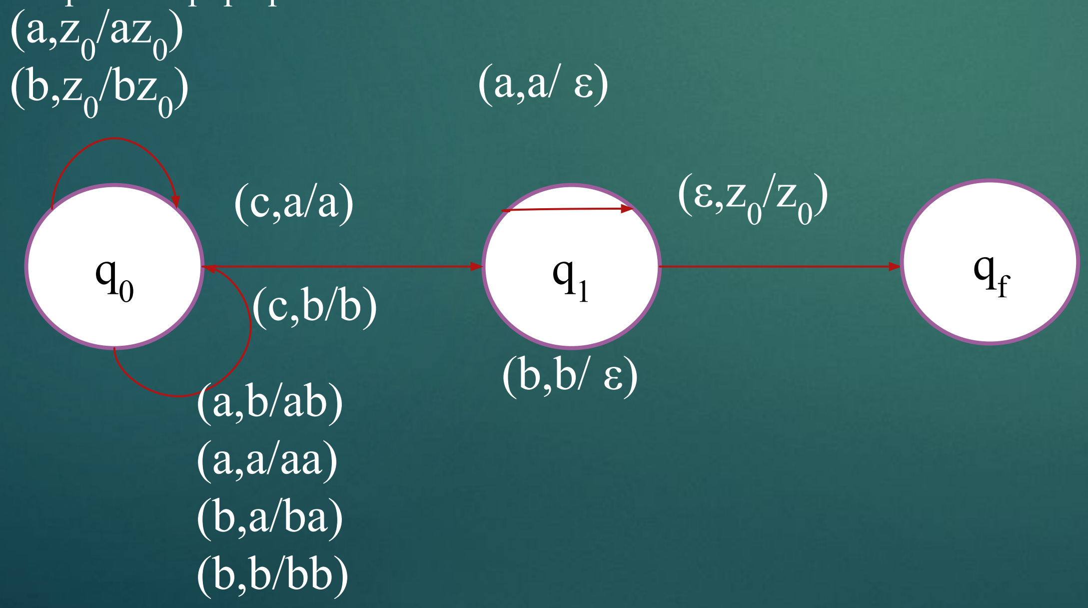
Example: Design a PDA for language $wcw^R|w\text{Є}(a+b)^{+}$ and ww^R

Solution:

For $L=wcw^R$

Logic: In this we perform push operation until c reached.

After c perform pop operation.



Push down automata

Example: Design a PDA for language $ww^R \mid w \in (a+b)^{*}$.

Solution:

For $L=ww^R$

Here DPDA is not possible so NPDA is used.

If top of stack and input symbol are same then assuming im in center or

If top of stack and input symbol differ then assume we are not in center

After c perform pop operation.