Example 1 Design the turing machine accepting language $\{0^n 1^n | n \geq 1\}$

Soln:- → Here the Language consists of any number of 0's followed by same number of 1's.

→ For designing the turing machine, TM should read first symbol 0 at state $q_0$, change it to X and goes to state $q_1$ and move right in the state $q_1$ until 1 is visited on the tape.

→ In state $q_1$, it change 1 to Y and goes to state $q_2$ and moves left over Y's and 0's until it find the symbol X.

→ when it find the symbol X change state to $q_0$ and move right and so on. So, the turing machine for this language can be constructed as

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 |

Let M be the turing machine which is formally defined as

$$M = (Q, \Sigma, \delta, q_0, \Gamma, B, F)$$

where, $Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{0, 1\}$

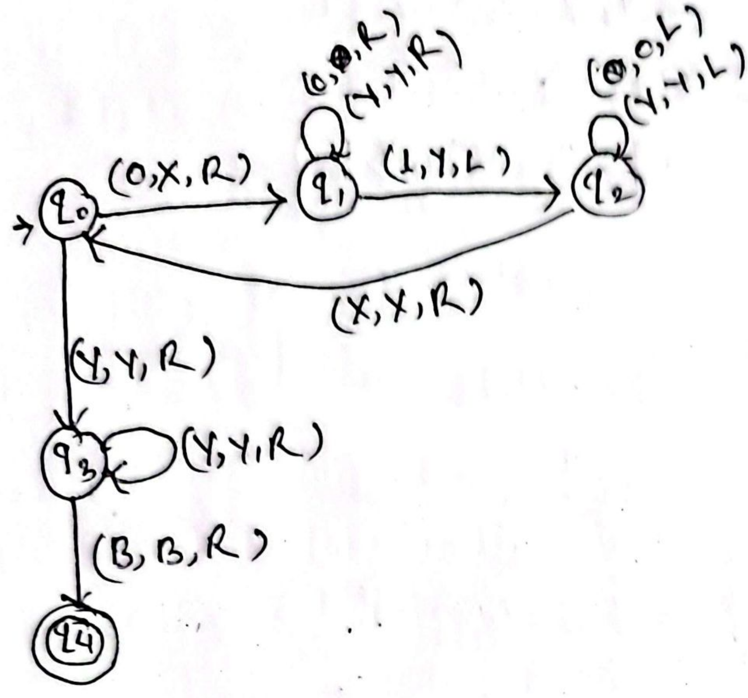
$q_0 = \{q_0\}$

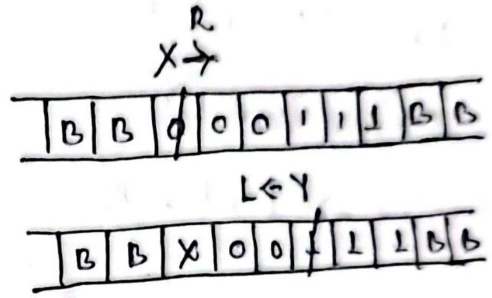
$B = \{B\}$

$\Gamma = \{0, 1, X, Y, B\}$

$F = \{q_4\}$

Transition diagram is given by:



Transition Table is given by:

| Set of States | Tape Symbol 0 | 1 | X | Y | B |
|---|---|---|---|---|---|
| $\rightarrow q_0$ | $X R q_1$ | - | - | $Y R q_3$ | - |
| $q_1$ | $0 R q_1$ | $Y L q_2$ | - | $Y R q_1$ | - |
| $q_2$ | $0 L q_2$ | - | $X R q_0$ | $Y L q_2$ | - |
| $q_3$ | - | - | - | $Y R q_3$ | $B R q_4$ |
| $* q_4$ | - | - | - | - | - |

Transition function ($\delta$) is given by:

1. $\delta(q_0, 0) \rightarrow (q_1, X, R)$
2. $\delta(q_0, Y) \rightarrow (q_3, Y, R)$
3. $\delta(q_1, 0) \rightarrow (q_1, 0, R)$
4. $\delta(q_1, Y) \rightarrow (q_1, Y, R)$
5. $\delta(q_1, 1) \rightarrow (q_2, Y, L)$.
6. $\delta(q_2, 0) \rightarrow (q_2, 0, L)$
7. $\delta(q_2, Y) \rightarrow (q_2, Y, L)$
8. $\delta(q_2, X) \rightarrow (q_0, X, R)$
9. $\delta(q_3, Y) \rightarrow (q_3, Y, R)$
10. $\delta(q_3, B) \rightarrow (q_4, B, R)$

Now let us take $\omega = 0011$.

$q_0 0011 \vdash X q_1 011$

$\vdash X 0 q_1 11$

$\vdash X q_2 0 Y 1$

$\vdash q_2 X 0 Y 1$

$\vdash X q_0 0 Y 1$

$\vdash X X q_1 Y 1$

$\vdash X X Y q_1 1$

$\vdash X X q_2 Y Y$

$\vdash X q_2 X Y Y$

$\vdash X X q_0 Y Y$

$\vdash X X Y q_3 Y$

$\vdash X X Y Y q_3 B$

$\vdash X X Y Y B q_4 B$ Halt & accept

Example 2 Turing Machine for the Language equal number of a's and b's.

Sol$^n$:

Let M be the Turing machine given by $M = (Q, \Sigma, \delta, q_0, \Gamma, B, F)$

Since, the Language contains the string $L = \{abab, aabb, ababbab, \}$

where, $Q = \{q_0, q_1, q_2, q_3, q_4, q_f\}$

$\Sigma = \{a, b\}$

$q_0 = \{q_0\}$

$B = \{B\}$

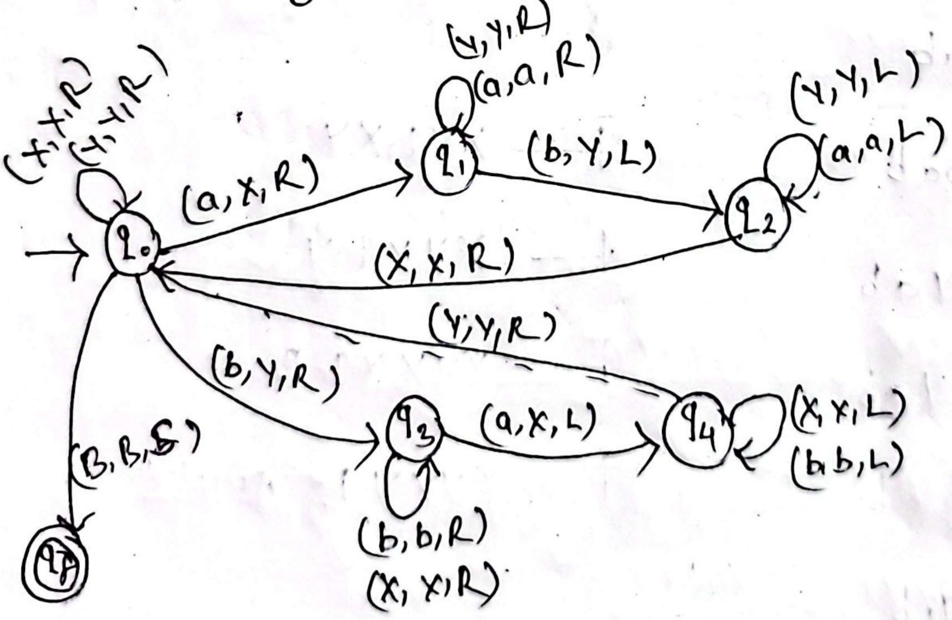
$\Gamma = \{a, b, X, Y, B\}$

$F = \{q_f\}$

| |
|---|
| BaabbB |
| BXa~~Y~~bB |
| BXXYYB |

| |
|---|
| BbbaaB |
| BYbXaB |
| Y~~Y~~XX |

Transition Diagram is given by:



Transition Table is given by:

| Set of States(Q) | Tape Symbol | | | | |
|---|---|---|---|---|---|
| | a | b | X | Y | B |
| $\rightarrow q_0$ | $XRq_1$ | $YRq_3$ | $XRq_0$ | $YRq_0$ | $BSq_f$ |
| $q_1$ | $aRq_1$ | $YLq_2$ | - | $YRq_1$ | - |
| $q_2$ | $aLq_2$ | - | $XRq_0$ | $YLq_2$ | - |
| $q_3$ | $XLq_4$ | $bRq_3$ | $XRq_3$ | - | - |
| $q_4$ | - | $bLq_4$ | $XLq_4$ | - | - |
| $*q_f$ | - | - | - | - | - |

Transition function ($\delta$) is given by:-

Let us take $\omega$ = abab

$q_0 abab \vdash X q_1 \underline{b} ab$

$\vdash q_2 \underline{X} Y ab$

$\vdash X q_0 Y ab$

$\vdash XY q_0 ab$

$\vdash XYX q_1 b$

$\vdash XY q_2 XY$

$\vdash X q_2 YXY$

$\vdash XY q_0 XY$

$\vdash XYX q_0 Y$

$\vdash XYXY q_0 B$

$\vdash XYXY q_f$ <u>Accepted</u>

Simple way:-

$$\begin{array}{ccccccc} BababB & \vdash_{(a,X,R)} & BXbabB & \vdash_{(b,Y,L)} & BXYabB & \vdash_{(X,X,R)} & BXYabB \\ \uparrow & & \uparrow & & \uparrow & & \uparrow \\ q_0 & & q_1 & & q_2 & & q_0 \end{array}$$

$$\begin{array}{ccccccc} \vdash_{(Y,Y,R)} & BXYabB & \vdash_{(a,X,R)} & BXYXbB & \vdash_{(b,Y,L)} & BXYXYB \\ & \uparrow & & \uparrow & & \uparrow \\ & q_0 & & q_1 & & q_2 \end{array}$$

$$\begin{array}{ccccccc} \vdash_{(X,X,R)} & BXYXYB & \vdash_{(Y,Y,R)} & BXYXYB & \vdash_{(B,B,S)} & BXYXYB \\ & \uparrow & & \uparrow & & \uparrow \\ & q_0 & & q_0 & & q_f \end{array}$$

Examples:- Turing Machine for the language $a^n b^n c^n \mid n \geq 1$.

Soln:-

Here, the language contains the string as:

$L = \{abc, aabbcc, aaabbbccc, \dots\}$

Let M be the Turing Machine defined as $M = (Q, \Sigma, \delta, q_0, \Gamma, B, F)$

where, $Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$

$\Sigma = \{a, b, c\}$

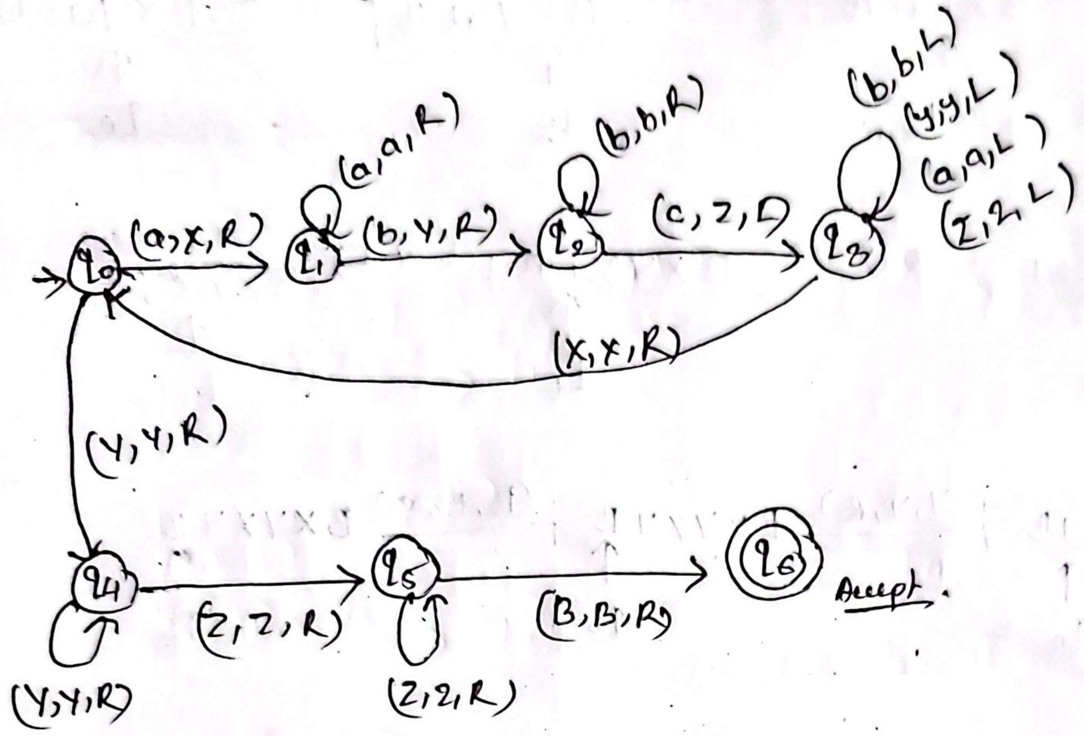
$q_0 = \{q_0\}$

$\Gamma = \{a, b, c, X, Y, Z, B\}$

$B = \{B\}$

$F = \{q_6\}$

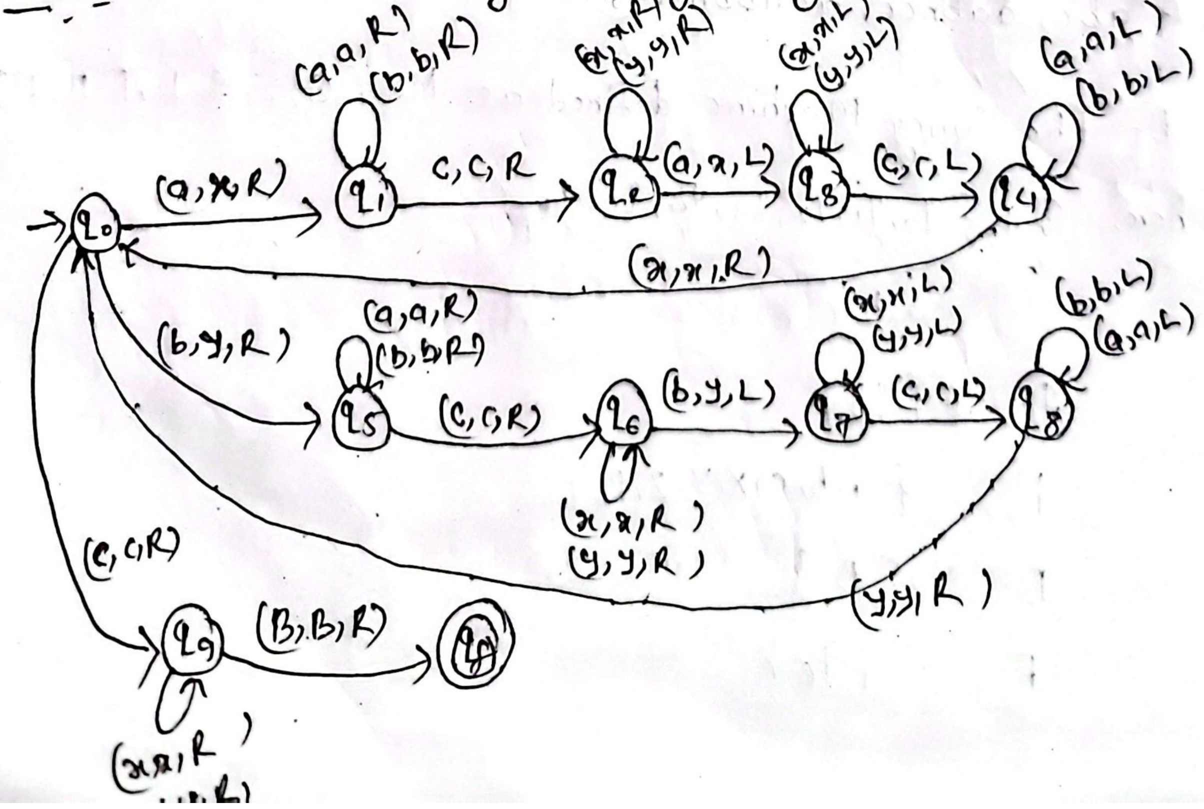
Transition Diagram is given by:




B a b c B
B X b c B
B X Y c B
B X Y Z B

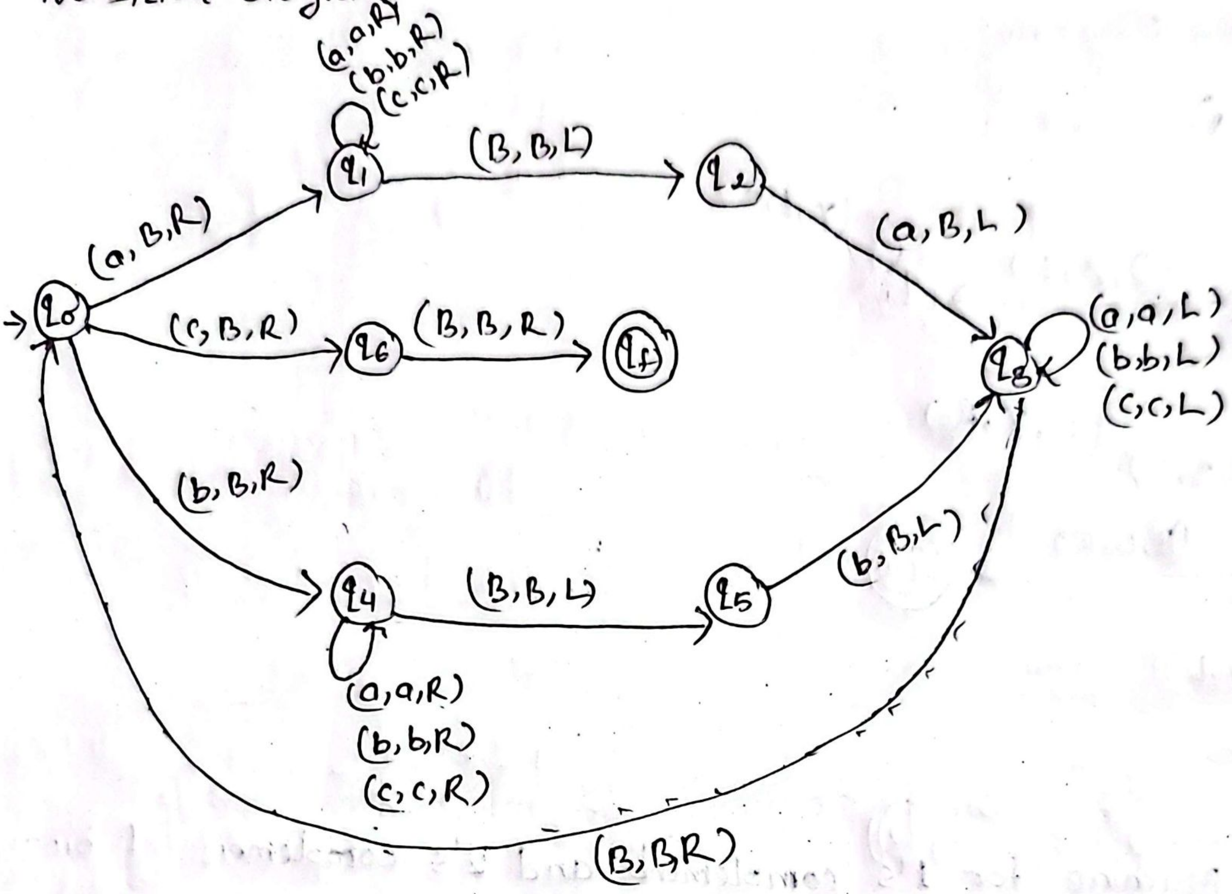
**Example 4:** Design Turing Machine for the language $WCW \mid W \in \{a,b\}^*$

**Soln.:-** Transition Diagram is given by:-

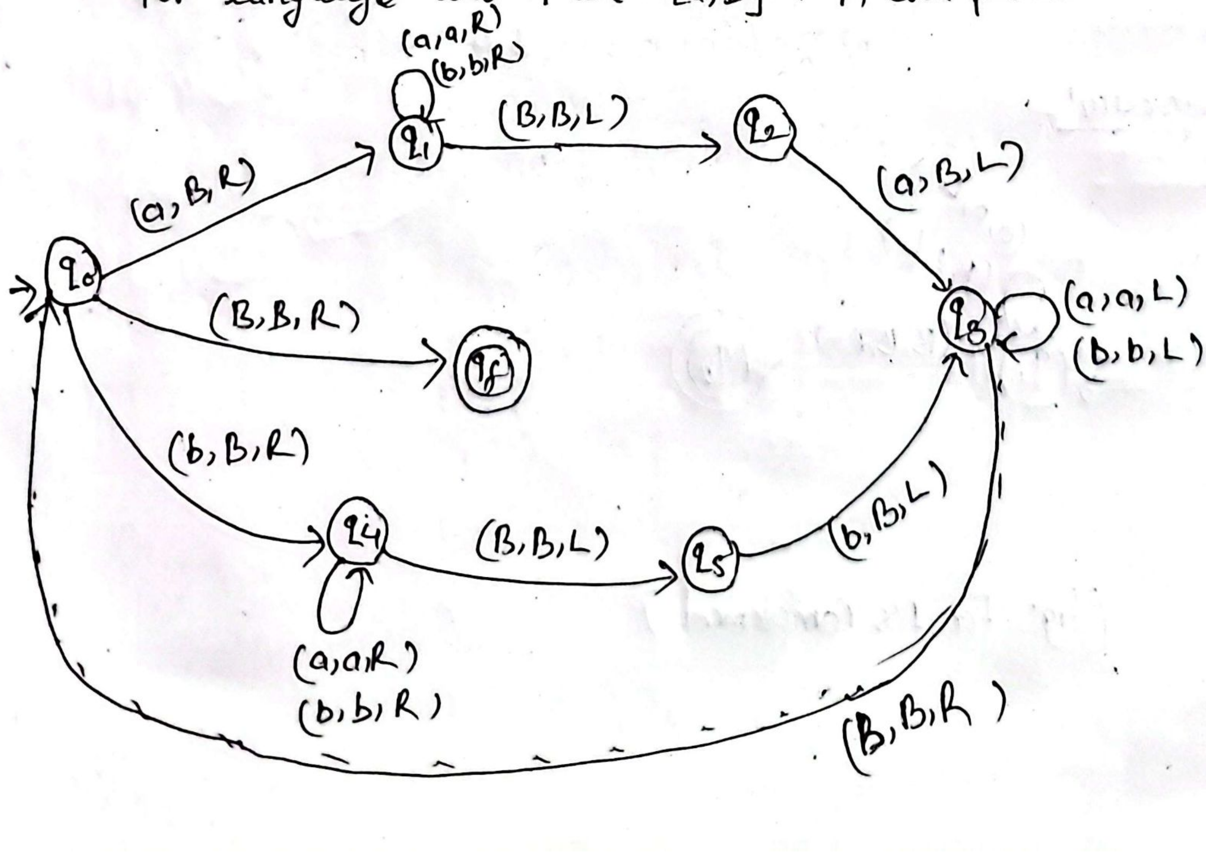


for language $wcw^R \mid w \in \{a,b\}^*$. // odd palindrome.

Transition Diagram:

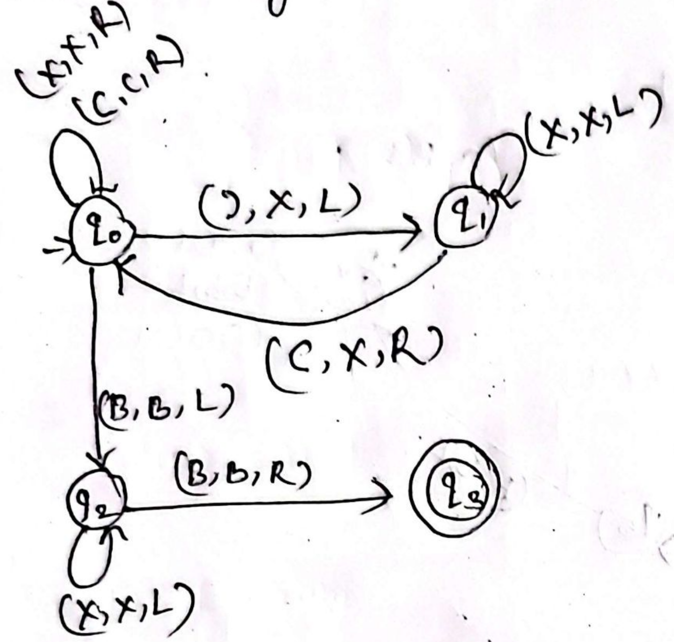



For language $ww^R \mid w \in \{a,b\}^*$. // even palindrome.



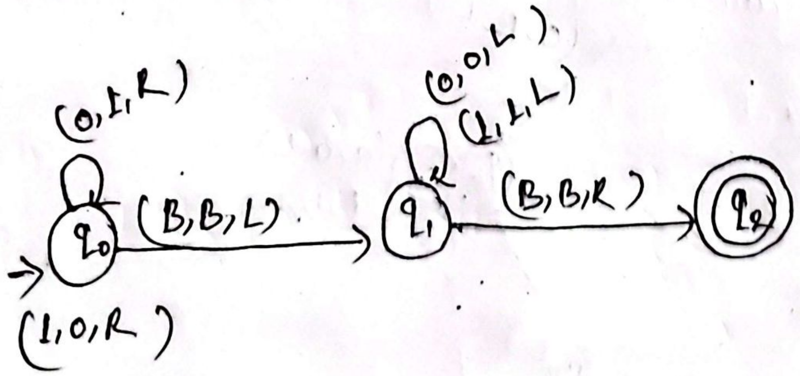
## Turing Machine for balanced parenthasis:-

### Transition Diagram:-



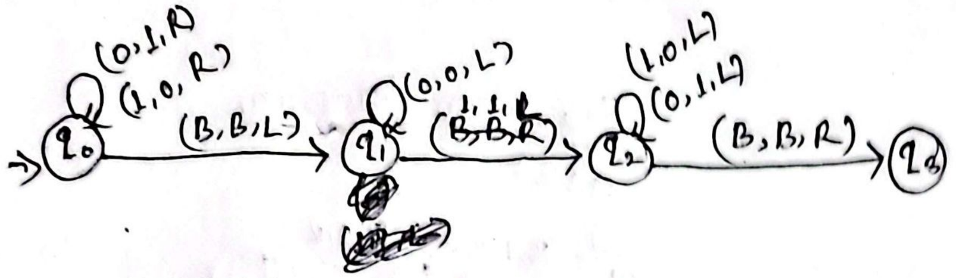

## Turing Machine for 1's complement and 2's complement of binary number. [Turing Machine as a Transducer].

### Transition Diagram:-




(fig:- for 1's Complement)

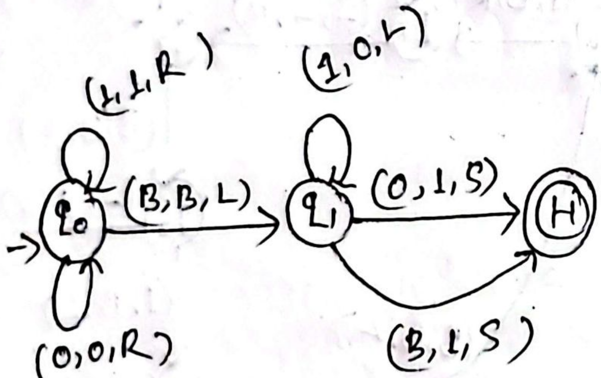
Transition Diagram:-



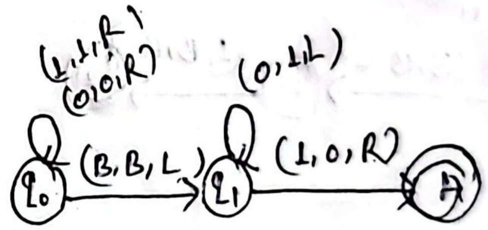

(fig:- for 2's complement)

Turing Machine that increment any binary strings by 1 and decrement any binary string by 1.

Transition Diagram:-




(fig:- For increment)

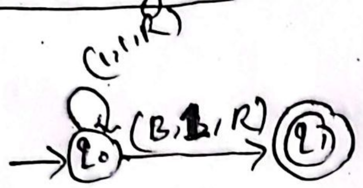



(fig:- For decrement)

# Turing Machine as a function Calculator

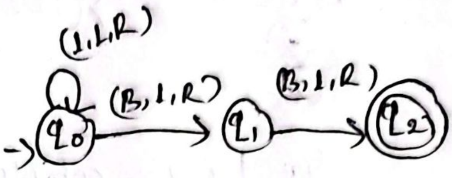
**Example:-** $f(n) = n+1$

Transition Diagram



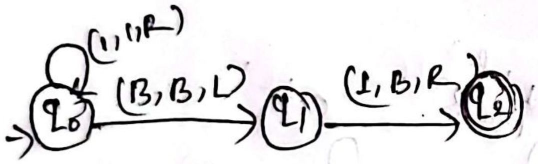

$f(n) = n+2$

Transition Diagram



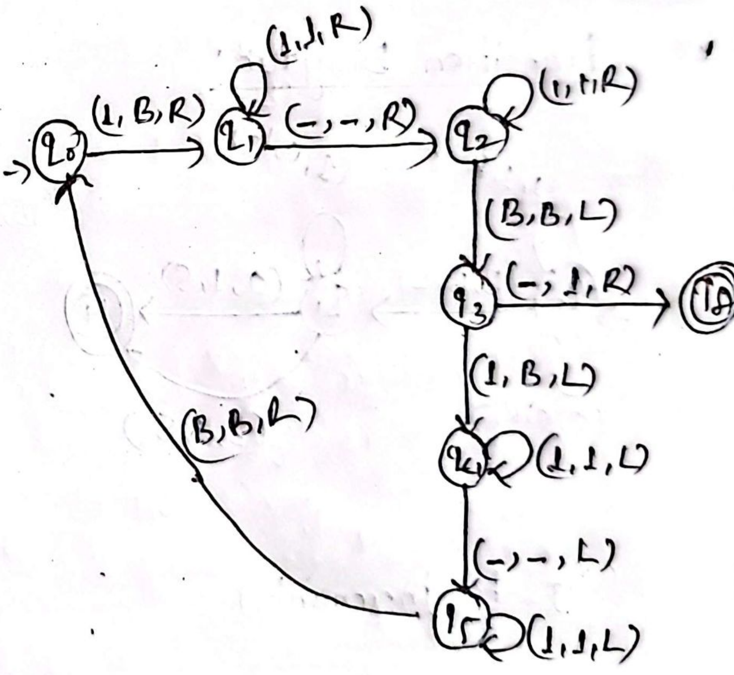

$f(n) = n-1$

Transition Diagram



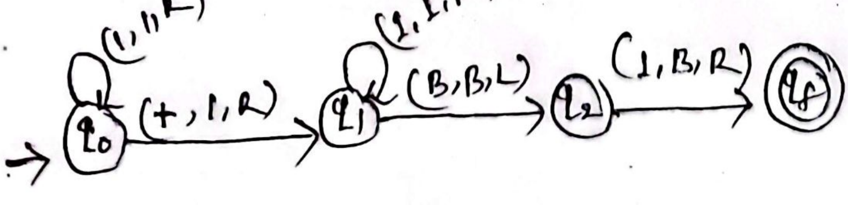

$f(n) = n-m$

Transition Diagram




$f(n) = n+m$

Transition Diagram



# Turing Machine as a Copier Machine