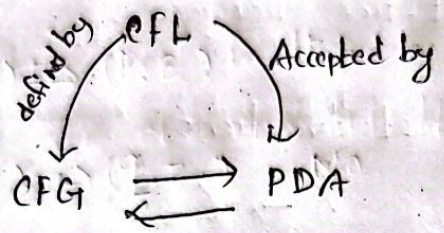
$E \rightarrow E+P \mid P$

$P \rightarrow P*Q \mid Q$

$Q \rightarrow id$

(A) Equivalence of CFG and PDA:



Rule 1: for each variable A

$\delta(q, \varepsilon, A) = (q, \beta)$ where $A \rightarrow \beta$ is a production of Grammar

Rule 2: for each terminal 'a'

$\delta(q, a, a) = (q, \varepsilon)$

Example: Construct the equivalent PDA For the given grammar

$S \rightarrow 0S1 | 00 | 11$

Soln:-

Here, variable is 'S'.

$\delta(q, \varepsilon, S) \rightarrow (q, 0S1)$ ①

$\delta(q, \varepsilon, S) \rightarrow (q, 00)$ ②

$\delta(q, \varepsilon, S) \rightarrow (q, 11)$ ③

∴ Using Rule for Variable

$\delta(q, 0, 0) \rightarrow (q, \varepsilon)$ ④

$\delta(q, 1, 1) \rightarrow (q, \varepsilon)$ ⑤

∴ Using Rule For terminal

Let us take $\omega = 0111$

$\delta(q, 0111, S)$ using ①

$\delta(q, 0111, 0S1)$ using ①

$\delta(q, 111, S1)$ using ④

$\delta(q, 11, 11)$ using ⑤

$\delta(q, 1, 1)$ using ⑤

$\delta(q, \varepsilon, \varepsilon)$ using ⑥

Accept

⑧ Conversion of PDA to CFG

Example:- Let $M = (\{P, q\}, \{0, 1\}, \{x, z\}, \delta, q, z)$

① $\delta(q, 1, z) \to (q, xz)$ Push

② $\delta(q, 1, x) \to (q, xx)$ Push

③ $\delta(q, \varepsilon, x) \to (q, \varepsilon)$ Pop

④ $\delta(q, 0, x) \to (P, x)$ No

⑤ $\delta(P, 1, x) \to (P, \varepsilon)$ Pop

⑥ $\delta(P, 0, z) \to (q, z)$ No

Solution:-

Let S be the start symbol for our grammar.

$S \to [q, z, q]$

$S \to [q, z, P]$

[Production for start Symbol]

① $\delta(q, 1, z) \to (q, xz)$ [Push Operation]

$[q, z, q] \to 1[q, x, q]\ [q, z, q]$

$[q, z, q] \to 1[q, x, P]\ [P, z, q]$

$[q, z, P] \to 1[q, x, q]\ [q, z, P]$

$[q, z, P] \to 1[q, x, P]\ [P, z, P]$

② $\delta(q, 1, x) \rightarrow (q, xx)$ [Push Operation]

$[q, x, q] \rightarrow 1[q, x, q] \quad [q, x, q]$

$[q, x, q] \rightarrow 1[q, x, p] \quad [p, x, q]$

$[q, x, p] \rightarrow 1[q, x, q] \quad [q, x, p]$

$[q, x, p] \rightarrow 1[q, x, p] \quad [p, x, p]$

③ $\delta(q, \varepsilon, x) \rightarrow (q, \varepsilon)$ [Pop operation]

$[q, x, q] \rightarrow \varepsilon$

④ $\delta(q, 0, x) \rightarrow (p, x)$ [No operation]

$[q, x, q] \rightarrow 0[p, x, q]$

$[q, x, p] \rightarrow 0[p, x, p]$

⑤ $\delta(p, 1, x) \rightarrow (p, \varepsilon)$ [Pop Operation]

$(p, x, p) \rightarrow 1$

⑥ $\delta(p, 0, z) \rightarrow (q, z)$

$[p, z, q] \rightarrow 0[q, z, q]$

$[p, z, p] \rightarrow 0[q, z, p]$

Original value	New value
[q, 2, q]	A
[q, 2, P]	B
[P, 2, q]	C
[P, 2, P]	D
[q, x, q]	E
[q, x, P]	F
[P, x, q]	G
[P, x, P]	H

$S \to A$

$S \to B$

$A \to 1EA$

$A \to 1FC$

$B \to 1EB$

$B \to 1FD$

$E \to 1EE$

$E \to 1FG$

$F \to 1EF$

$F \to 1FH$

$E \to \varepsilon$

$E \to 0G$

$F \to 0H$

$H \to 1$

$C \to 0A$

$D \to 0B$