Proof that Language is not Regular Using Pumping Lemma

If A is a Regular Language then A has a pumping length 'p' such that any string 'ω' where $|\omega| \geq p$ may be divided into three parts $\omega = xyz$ such that the following conditions must be true.

(1) $xy^iz \in A$ for every $i \geq 0$

(2) $|y| > 0$

(3) $|xy| \leq p$

To prove that a Language is not Regular using pumping lemma, follow below steps

We prove using contradiction

- Assume that A is Regular
- It has to have a pumping length (say P)
- All strings longer than P can be pumped $|\omega| \geq P$
- Now find a string ω in A such that $|\omega| \geq P$
- Divide ω into xyz
- Show that $xy^iz \notin A$ for some i.
- Then consider all ways that ω cannot be divided into xyz.
- Show that none of these can satisfy all the 3 conditions at the same time.
- ω cannot be pumped == CONTRADICTION

Simple method

Prove that $L = \{a^P \mid P \text{ is a prime number}\}$ is not Regular.

Soln Let 'L' is a Regular Language, 'n' is an integer constant.

Select a string 'w' from L such that $|w| \geq n$.

$L = \{aa, aaa, aaaaa \ldots\}$

Let $n = 3$.

$w = aaa$ such that $|w| \geq n$

$|3| \geq n$

$|3| \geq 3$

Next we have to divide 'w' into three parts as xyz

to satisfy (i) $|xy| \leq n$ (ii) $|y| \geq 1$

(iii) for $i \geq 0$, xy^iz is in the language.

As $w = \underset{x}{a}\underset{y}{a}\underset{z}{a}$.

Checking first condition:

i.e $|xy| \leq n$

$|2| \leq n$ which is true

Checking IInd condition:

i.e $|y| \geq 1$

$|1| \geq 1$ which is true

CamScanner

Checking 3rd condition,

i.e $xy^iz \in L$

For $i = 0$,

$xz = aa \in L$

for $i = 1$

$xyz = aaa \in L$

For $i = 2$

$xy^2z = aaaa \notin L$ which is contradiction

$\therefore$ This language is not Regular.

Q. Prove that language $L = \{a^n b^n \mid n \geq 0\}$ is not Regular.

Soln: Let L is a Regular Language, 'n' is the integer constant.

Select a string 'ω' from L such that $|\omega| \geq n$.

$L = \{\epsilon, ab, aabb \ldots\}$

Let $n = 2$

$\omega = aabb$ ~~aaabbbb~~ such that $|\omega| \geq n$

$4 \geq 2$

Next we have to divide ω into three parts as xyz.

And it has to satisfy three condition (i) $|xy| \leq n$ (ii) $|y| \geq 1$

and (iii) $xy^iz \in L$

Since $\omega = \underset{x}{a}\underset{y}{ab}\underset{z}{b}$

Checking (i) case,

$|xy| \leq 2$

$3 \leq 2$ ~~true~~ which is fals

checking ② condition,

$|y| \geq 1$

$\Rightarrow 2 \geq 1$ true

checking ③ condition,

$xy^iz \in L$,

let $i = 0$

$xz = aab \notin L$ which is contradiction

~~let~~

Thus the language is not Regular.

③ Prove that $L = \{0^{n^2} | n \geq 0\}$ is not Regular.

Solution Let 'L' is the Regular Language, 'n' is a integer constant. ~~let~~ Select a string 'w' from L such that $|w| \geq n$.

$L = \{\varepsilon, 0, 0000, \ldots\}$

Let $n = 2$, Then $w = 0000$ such that $|w| \geq n$

i.e $4 \geq 2$

Next, we have to divide w, into three parts as xyz to satisfy (i) $|xy| \leq n$ (ii) $|y| \geq 1$ (iii) $xy^iz \in L$ for $i \geq 0$

As $w = \underbrace{0}_{x}\underbrace{00}_{y}\underbrace{0}_{z}$,

Checking (i) case

i.e $|xy| \leq n$

$3 \leq 2$ which is false

Checking (ii) condition,

$|y| \geq 1$

$2 \geq 1$ which is true

checking (iii) condition,

$xy^iz \in L$

for $i=0$,

$xz \Rightarrow 00 \notin L$

Thus which is contradiction.

$\therefore$ the Language is not Regular.