⊛ Context free Language:-

⇒ In formal Language theory, a Context free language is a language generated by some context free grammar.

⇒ CFL is the language accepted by push down automata.

⊛ Pumping Lemma for Context free Language:-

⇒ Pumping lemma is used to prove a language is not context free.

Statement:-

let 'L' be the Context free Language. let 'n' be the integer constant. Select a string 'm' from 'L' such that $|m| \geq n$. Divide the string 'm' as uvwxy such that ① $|vwx| \leq n$

② $|vx| \geq 1$

③ $uv^i w x^i y$ is in L for $i \geq 0$

Example:- Show that $L = \{a^n b^n c^n \mid n \geq 1\}$ is not a CFL.

Soln:- Let L be the CFL.

Let $n = 3$

$L = \{abc, aabbcc, aaabbbccc, \ldots\}$

Consider $m = aaabbbccc$

i.e $|m| \geq n$, $9 \geq 3$ which is true

Now dividing string m into five parts as $uvwxy$

such that $\underbrace{aa}_{u}\underbrace{a}_{v}\underbrace{b}_{w}\underbrace{b}_{x}\underbrace{bccc}_{y}$

where $u = aa$, $v = a$, $w = b$, $x = b$, $y = bccc$

<u>Checking first Condition</u> :

$|vwx| \leq n$

$\Rightarrow |abb| \leq 3$

$3 \leq 3$ which is true.

<u>Checking second condition</u>:

$|vx| \geq 1$

$|ab| \geq 1$

$2 \geq 1$ which is true

<u>Checking third Condition</u>:

$uv^iwx^iy \in L$ for $i \geq 0$

when $i=0$,

uwy

$\Rightarrow aabbccc \notin L$

$\therefore$ Given Language is not Context free.

(26) Decision algorithm for CFL:

$\Rightarrow$ Different decision algorithms are used for CFLs, first is based on emptiness, followed by finiteness, followed by membership determination.

1. Emptiness

↳ There are algorithm to test emptiness of a CFL i.e $L(G) \neq 0$ or not.

↳ For this, remove useless symbol of Grammar 'G'.

↳ If its start symbol is useless than $L(G) = \phi$ else its not.

2. Finite Theorem

↳ For a given CFG 'G' there exists an algorithm to decide whether $L(G)$ is finite or infinite.

↳ To prove finiteness draw a directed graph whose nodes are variable of 'G' node A to B if there is a production of the form $A \rightarrow \alpha B \beta$

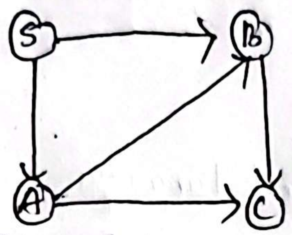
↳ If there is cycle in graph Language of G infinite otherwise finite

For example:

$S \to AB|a$

$A \to BC|b$

$B \to CC|c$



↳ There is no cycle, so the grammar is finite.

③ Membership

↳ For CFG 'G' of any string ω there exists an algorithm to determine whether $\omega \in L(G)$ or not.

↳ If string is accepted by the machine (i.e reached to the final state) it is one of the member of that Language.