WAVE AND OSCILLATION

* **Elastic restoring force:-**
It is the force which have same magnitude of external force but opposite in direction.

Simple Harmonic Motion "SHM":

A particle may be said to execute a simple harmonic motion (S.H.M), if its acceleration is proportional to its displacement from its equilibrium position or any other fixed point in its path and is always directed towards it.

The differential equation of the S.H.M is

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

where, y is the displacement of the particle of mass 'm'.

$\omega = \sqrt{k/m}$ is called angular velocity of object executing SHM.

k is a positive constant, called force constant.

Equation of SHM:

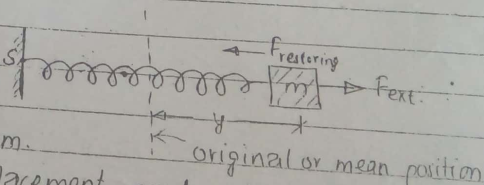

fig: Horizontal oscillation of spring-mass system.

let 'y' is the displacement produced on spring-mass system force (F_{ext}) as shown in figure. Then

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according to Hook's Law, we can write

$f_{ext} \propto y$ [$\because$ within elastic limit]

$\therefore f_{ext} = ky$ ---------- (1)

where,

$F_{restoring} \rightarrow$ Elastic restoring Force

$f_{ext} \rightarrow$ External force.

$y \rightarrow$ displacement

Here,

k is a constant term for a spring called spring constant or force constant. It is defined as the force per unit displacement and its S.I. unit is N/m.

Again,

According to the difinition of elastic restoring force ($F_{restoring}$), we can write,

$$F_{restoring} = -f_{ext}$$

i.e. $F_{restoring} = -ky$ [$\because$ using eq^n (1)]

or, $ma = -ky$

$$\therefore a = \left(\frac{-k}{m}\right) y \text{ -------- (2)}$$

As $\left(\frac{-k}{m}\right)$ is a constant, we get

$\therefore \boxed{a \propto y}$

i.e. motion of the spring-mass system is SHM, and -ve sign indicates the direction of acceleration towards mean position.

Now,

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d^2y}{dt^2}$$

Then eq^n (2) becomes,

$$\frac{d^2y}{dt^2} = \left(\frac{-k}{m}\right) y$$

or, $$\frac{d^2y}{dt^2} + \left(\frac{k}{m}\right) y = 0$$

$$\therefore \frac{d^2y}{dt^2} + \omega^2 y = 0 \text{ -------- (3)}$$

where, $\omega = \sqrt{k/m}$ is a constant term and called angular frequency. It has same dimension of angular velocity.

Hence, Eq^n (3) is the required equation of simple harmonic motion (SHM).

The solution of above eq^n (3) can be expressed as $\boxed{y = A \sin(\omega t + \phi)}$ or $\boxed{y = A \cos(\omega t + \phi)}$

This equation of SHM gives the displacement of a body in any instant 't'.

$A = y_{max}$ = maximum displacement of a body from its mean position in SHM called amplitude of the motion.

$(\omega t + \phi)$: It is the phase of the body in SHM. It depends on different value of 't'.

ϕ : Phase constant or phase angle or phase difference. It depends only the initial value of 't' [i.e. t=0].

Note :-

① Time period (T) : It is the total time required for one complete oscillation.

② Frequency (f) : It is the number of complete oscillation made in one second.

i.e. $\boxed{f = \frac{1}{T}}$

③ Angular frequency (ω) :-

If the total angular displacement 'θ' for one complete oscillation is 2π and time required for one complete oscillation is 'T', then,

Angular frequency (ω) $= \frac{\theta}{T} = \frac{2\pi}{T}$

$\therefore \boxed{\omega = 2\pi f}$

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⇨ Velocity of a body in SHM :

We know,

The displacement of a body in SHM is :

$$y = A \sin(\omega t + \phi) \text{ - - - - - - - - } (1)$$

Also,

$$v = \frac{dy}{dt}$$

or, $v = \frac{d}{dt}[A \sin(\omega t + \phi)]$

or, $v = A \times \omega \cos(\omega t + \phi)$

Squaring both sides, we will get,

$v^2 = A^2 \omega^2 \cos^2(\omega t + \phi)$

or, $v^2 = A^2\omega^2 [1 - \sin^2(\omega t + \phi)]$

or, $v^2 = A^2\omega^2 \left[1 - \frac{y^2}{A^2}\right]$ [Using 1]

or, $v^2 = A^2 \omega^2 \left(\frac{A^2 - y^2}{A^2}\right)$

or, $v^2 = \omega^2 (A^2 - y^2)$

$$\therefore \boxed{v = \pm \omega \sqrt{(A^2 - y^2)}} \text{ - - - - - - - } (2)$$

which is the required expression for the velocity of a body in SHM.

⇒ Acceleration, $\boxed{a = -\omega^2 y}$

⇨ Energy of a body in SHM:

we know,

Kinetic energy (K.E) = $\frac{1}{2} mv^2$

Potential energy (P.E.) = $\frac{1}{2} ky^2$

Total energy E = P.E. + K.E.

$$\text{or, } E = \frac{1}{2} ky^2 + \frac{1}{2} mv^2$$

$$= \frac{1}{2} ky^2 + \frac{1}{2} m\omega^2 (A^2 - y^2) \quad [\because v^2 = \omega^2(A^2 - y^2)]$$

$$= \frac{1}{2} ky^2 + \frac{1}{2} k(A^2 - y^2)$$

$$= \frac{1}{2} ky^2 + \frac{1}{2} kA^2 - \frac{1}{2} ky^2$$

$$\therefore \boxed{E = \frac{1}{2} kA^2}$$

ie. Total energy in SHM remains constant.

Application of SHM:

1. Simple Pendulum and its time Period:-

A simple pendulum is a point mass suspended by an inextensible and weightless thread that oscillates about a point 'O' (mean position).

Here, figure shows the position 'A' of the simple pendulum with linear displacement 'y' from its mean position where 'm' is the mass of the bob and 'l' is called length of pendulum.

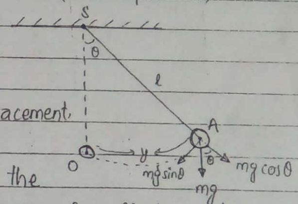


fig: Simple pendulum.

As the weight (mg) of the bob acts vertically downwar[d] we can resolve it into two rectangular components mg sinθ and mg cosθ as in fig. The direction of mg sinθ is forwards mean position and it gives the magnitude of restoring force i.e.

$$F_{res.} = -mg\sin\theta \text{ ------ (1)}$$

As θ is very small, $\sin\theta \approx \theta = y/l$.

Then,

$$F_{res} = -mg(y/l)$$

$$\text{or, } ma = -mg\frac{y}{l}$$

$$\therefore a = \left(-\frac{g}{l}\right) y \text{ -------- (2)}$$

As $(-g/l)$ is a constant term,

$\therefore \boxed{a \propto y}$

i.e. The motion of simple pendulum is simple harmonic.

Also,

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d^2y}{dt^2}$$

Then,

$$\frac{d^2y}{dt^2} = \left(\frac{-g}{l}\right)y$$

or, $$\frac{d^2y}{dt^2} + \left(\frac{g}{l}\right)y = 0$$

$$\therefore \boxed{\frac{d^2y}{dt^2} + \omega^2 y = 0} \quad \text{-------} (3)$$

where, $\omega = \sqrt{g/l}$ is called angular frequency.

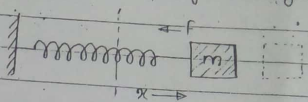
or, $$\frac{2\pi}{T} = \sqrt{g/l}$$

or, $$\boxed{T = 2\pi\sqrt{l/g}}$$

which is the expression for time period of oscillation of simple pendulum.

P.6

⇨ Expression for the time period of Spring Mass System.



Consider a block of mass is oscillating at the end of a mass less spring as shown in fig. Assume that the net force acting on the block is that exerted by the spring which is obtained from the Hook's Law.

i.e. $F = -kx$. ------ (1)

where, x is displacement from equilibrium position and k is force constant.

Then, from Newton's Law,

$F = ma$ -------- (2)

Equating (1) and (2), we get,

$$ma = -kx$$

$$a = \frac{-k}{m}x$$

or, $$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad \text{-------} (3)$$

This equation shows that spring mass system excertes a simple harmonic motion. Comparing equation (3) with differential eq^n of S.H.M, we get,

angular frequency, $\omega = \sqrt{\frac{k}{m}}$

Now, The time period of spring mass is,

$$T = \frac{2\pi}{\omega}$$

$$\therefore \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Angular Harmonic Motion (AHM):

It is that type of motion in which restoring torque ($\tau_{res.}$) is directly proportional to the angular displacement (θ) from mean position.

i.e.

$$\tau_{res.} \propto \theta$$

or, $\tau_{res.} = -C\theta$

or, $I\alpha = -C\theta$

$$\therefore \alpha = \left(\frac{-C}{I}\right)\theta \quad \text{------} \quad (1)$$

As $(-C/I)$ is a constant term,

$$\therefore \boxed{\alpha \propto \theta}$$

i.e. In AHM, angular acceleration (α) is directly proportional to the displacement from mean position and it is directed towards mean position. The -ve sign in above relation (1) indicates the direction of 'α' towards mean position.

Also, $\alpha = \frac{d^2\theta}{dt^2}$, then, eq^n (1) becomes,

$$\frac{d^2\theta}{dt^2} = \left(\frac{-C}{I}\right)\theta$$

or, $\frac{d^2\theta}{dt^2} + \left(\frac{C}{I}\right)\theta = 0$

$$\therefore \boxed{\frac{d^2\theta}{dt^2} + \omega^2\theta = 0} \quad \text{--------} \quad (2)$$

This is the required eq^n of angular harmonic motion.

where,

$\omega = \sqrt{C/I}$ is angular frequency.

i.e. $\omega = \sqrt{C/I}$

or, $\frac{2\pi}{T} = \sqrt{C/I}$

$$\therefore \boxed{T = 2\pi\sqrt{\frac{I}{C}}}$$

which is the required expression for time period of a body in AHM.

Again,

The solution of above equation (2) can be expressed as:-

$\boxed{\theta = \theta_{max} \sin(\omega t + \phi)}$ or $\boxed{\theta = \theta_{max} \cos(\omega t + \phi)}$

Here, θ_{max} is the maximum angular displacement of a body from its mean position in AHM called angular amplitude.

⊛ Angular velocity (Ω):

we know, the angular displacement (θ) of a body in AHM.

$$\theta = \theta_{max} \sin(\omega t + \phi) \text{ ------ (1)}$$

Also,

$$\Omega = \frac{d\theta}{dt} = \frac{d}{dt}[\theta_{max} \sin(\omega t + \phi)]$$

or, $\Omega = \theta_{max}\, \omega \cos(\omega t + \phi)$

Squaring,

or, $\Omega^2 = \theta_{max}^2\, \omega^2 \cos^2(\omega t + \phi)$

or, $\Omega^2 = \theta_{max}^2 \cdot \omega^2 [1 - \sin^2(\omega t + \phi)]$

or, $\Omega^2 = \theta_{max}^2 \cdot \omega^2 \left[1 - \theta^2/\theta_{max}^2\right]$

or, $\Omega^2 = \theta_{max}^2 \cdot \omega^2 \left[\frac{\theta_{max}^2 - \theta^2}{\theta_{max}^2}\right]$

$$\therefore \boxed{\Omega = \pm \omega \sqrt{(\theta_{max}^2 - \theta^2)}} \text{ ---------- (2)}$$

Hence, Eq^n (2) is the required expression for angular velocity.

Case I At the mean position, $\theta = 0$. and above eq^n gives.

$$\Omega = \pm \omega \sqrt{\theta_{max}^2 - 0^2}$$

$$\therefore \boxed{\Omega = \pm \omega\, \theta_{max}}$$

i.e. Angular velocity is maximum at mean position.

Case II

At the extreme position, $\theta = \theta_{max}$ and above eq^n gives.

$$\Omega = \pm \omega \sqrt{\theta_{max}^2 - \theta_{max}^2}$$

$$\therefore \boxed{\Omega_{min} = 0}$$

i.e. Angular velocity is minimum at extreme position.

⊛ Angular Acceleration (α):

we know, angular displacement (θ) of a body in AHM can be expressed as:

$$\theta = \pm \theta_{max} \sin(\omega t + \phi) \text{ -------- (1)}$$

Also, $\alpha = \frac{d\Omega}{dt} = \frac{d^2\theta}{dt^2} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right)$

or, $\alpha = \frac{d}{dt}\left[\frac{d}{dt}\{\theta_{max} \cdot \sin(\omega t+\phi)\}\right]$

or, $\alpha = \frac{d}{dt}\left[\theta_{max} \cdot \omega \cdot \cos(\omega t+\phi)\right]$

or, $\alpha = \theta_{max} \cdot \omega \cdot \frac{d}{dt}\{\cos(\omega t+\phi)\}$

or, $\alpha = -\omega^2 \cdot \theta_{max} \cdot \sin(\omega t+\phi)$

$\therefore \boxed{\alpha = -\omega^2\theta}$ - - - - - - - - (2) [using (1)]

which is the required expression for angular acceleration. The -ve sign in this relation indicates the direction of 'α' towards mean position.

Case I

At the mean position, $\theta = 0$, and above relation gives,

$$\alpha = -\omega^2\theta$$

$$\therefore \boxed{\alpha_{min} = 0}$$

i.e. Angular acceleration is minimum at the mean position.

Case II

At the extreme position, $\theta = \theta_{max}$ and above eq^n gives:

$$\alpha = -\omega^2 \times \theta_{max}$$

$$\therefore \boxed{\alpha_{max} = -\omega^2 \cdot \theta_{max}}$$

i.e.

Angular acceleration is maximum at the extreme position. The -ve sign in above relation indicates the direction of 'α' towards the mean position.

⇒ Difference Bet^n SHM and AHM.

SHM	AHM
(1) $F_{res} \propto y \Rightarrow F_{res} = -ky$ $\Rightarrow a \propto y$	(1) $\tau_{res} \propto \theta \Rightarrow \tau_{res} = -C\theta$. $\Rightarrow \alpha \propto \theta$
(2) $F = ma$	(2) $\tau = I\alpha$
(3) $v = \pm\omega\sqrt{A^2-y^2}$ $a = -\omega^2 y$	(3) $\Omega = \pm\omega\sqrt{\theta_{max}^2 - \theta^2}$ $\alpha = -\omega^2\theta$
(4) $\frac{d^2y}{dt^2} + \omega^2 y = 0$ (Eq^n of SHM)	(4) $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$ (Eq^n of AHM).
(5) $y = A\sin(\omega t+\phi)$	(5) $\theta = \theta_{max} \sin(\omega t+\phi)$
(6) $T = 2\pi\sqrt{m/k}$ for spring mass oscillation.	(6) $T = 2\pi\sqrt{I/C}$

Bar-Pendulum [Physical or Compound Pendulum]:

Physical pendulum is a rigid body fixed at a point and that oscillates in a plane about an axes passing through it. Bar pendulum (a real pendulum) is an example of the physical pendulum.

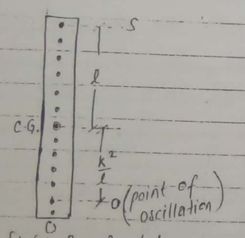


fig (i) Bar-Pendulum at its mean position.

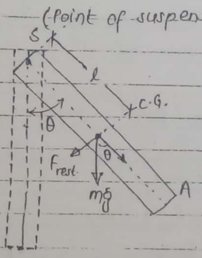


fig (ii): Position 'A' of the bar-pendulum.

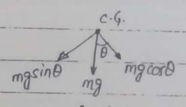


fig (iii).

Bar Pendulum is made of a rectangular uniform metalic body with large no. of holes at equal interval as shown in fig (i). where 'l' is the distance of C.G. from the point of suspension and 'm' is its mass.

fig (ii) shows the position 'A' of the bar pendulum with angular displacement 'θ' from its mean position. The weight (mg) of the pendulum acts vertically downward from C.G. and it can be resolved into two rectangular components $mg\sin\theta$ and $mg\cos\theta$ as in fig (iii). As the $mg\sin\theta$ is directed towards mean position, it gives the magnitude of restoring force.

So,

the restoring torque ($\tau_{res.}$) due to motion of the bar pendulum is:-

$$\tau_{res.} = -mg\sin\theta \cdot l \qquad [\because \tau = F \times r].$$

As 'θ' is very small angle for the SHM of the bar-pendulum, we can take $\sin\theta \approx \theta$ and above relation becomes,

$$\tau_{res.} = -mg \cdot l \cdot \theta.$$

$$\text{or, } I\alpha = -mgl\theta \qquad [\because \text{Torque} = I\alpha].$$

$$\text{or, } \alpha = \left(\frac{-mgl}{I}\right)\theta. \quad \text{-------} (1)$$

where, I = moment of inertia

α = angular acceleration.

As, $\left(\frac{-mgl}{I}\right)$ is a constant term, then.

$\therefore$ Angular acceleration (α) $\propto \theta$.

i.e.

The motion of the bar-pendulum is angular harmonic motion. The -ve sign in above relation indicates the direction of angular acceleration (α) towards mean position.

Again,

$$\text{Angular accl}^n\ (\alpha) = \frac{d\Omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}$$

Then, relation (1) becomes

$$\frac{d^2\theta}{dt^2} = \left(-\frac{mgl}{I}\right)\theta.$$

$$\text{or, } \frac{d^2\theta}{dt^2} + \left(\frac{mgl}{I}\right)\theta = 0.$$

$$\therefore \boxed{\frac{d^2\theta}{dt^2} + \omega^2\theta = 0} \quad \text{---------- (2)}$$

This is the required equation for the oscillation of bar-pendulum.

where, $\omega = \sqrt{\frac{mgl}{I}}$ is called angular frequency.

$$\text{or, } \frac{2\pi}{T} = \sqrt{\frac{mgl}{I}}$$

$$\therefore \boxed{T = 2\pi\sqrt{\frac{I}{mgl}}} \quad \text{-------- (3)}$$

This gives the time period of bar-pendulum in terms of moment of inertia (I)

But,

According to the theorem of parallel axes, the moment of inertia of the bar pendulum is

$$I = ml^2 + I_{c.g.}$$

$$\text{or, } I = ml^2 + mk^2$$

where,

$I_{c.g.}$ is the moment of inertia of the bar-pendulum about an axis passing through c.g. and k is called radius of gyration.

Then,

above relation (3) becomes,

$$T = 2\pi\sqrt{\frac{ml^2 + mk^2}{mgl}}$$

$$\text{or, } T = 2\pi\sqrt{\frac{\cancel{m}(l^2+k^2)}{\cancel{m}gl}}$$

$$\text{or, } T = 2\pi\sqrt{\left(\frac{l^2+k^2}{l}\right)\cdot\frac{1}{g}}$$

$$\text{or, } \boxed{T = 2\pi\sqrt{\frac{(l + k^2/l)}{g}}} \quad \text{------ (4)}$$

or, $\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$ ------- (5)

This gives the time period of bar-pendulum in terms of equivalent length of simple pendulum.
In above relation,

$L = \left(l + \frac{k^2}{l}\right)$ is called equivalent length of simple pendulum. OR

L is the distance between point of suspension and point of oscillation.

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→ Point of oscillation is that point through which the body (bar-pendulum) oscillates.

Note: 1 To prove $K = \sqrt{l_1 l_2}$.

Sol^n

we know, the time period of a physical pendulum (bar-pendulum) can be expressed as:

$$T = 2\pi\sqrt{\frac{(l + k^2/l)}{g}}$$

or, $T^2 = 4\pi^2\left(\frac{l + k^2/l}{g}\right)$

or, $T^2 g = 4\pi^2\left[\frac{l^2 + k^2}{l}\right]$

or, $T^2 g l = 4\pi^2(l^2 + k^2)$

or, $T^2 g l = 4\pi^2 l^2 + 4\pi^2 k^2$

$\therefore$ $\boxed{4\pi^2 l^2 - T^2 g l + 4\pi^2 k^2 = 0}$ -------- (i)

This is the quadratic equation in 'l'. If l_1 and l_2 are the two possible roots of l, then we get.

$l_1 + l_2 = \frac{T^2 g}{4\pi^2}$

or, $L = \frac{T^2 g}{4\pi^2}$

or, $\boxed{g = 4\pi^2 \cdot \frac{L}{T^2}}$

Also,

$l_1 \cdot l_2 = \frac{4\pi^2 k^2}{4\pi^2}$

or, $k^2 = l_1 l_2$

$\therefore$ $\boxed{K = \sqrt{l_1 l_2}}$.

Hence, Proved

Note: $ax^2 + bx + c = 0$.
If α and β are two roots of this equation.
Then,

$\alpha + \beta = -b/a$

and,

$\alpha \cdot \beta = c/a$

Note: 2

Maximum time period of Bar-pendulum:

we know,

Time-period of bar-pendulum is:

$$T = 2\pi\sqrt{\frac{l + k^2/l}{g}} \quad \text{------ (1)}$$

when $l = 0$,

$$T = 2\pi\sqrt{\frac{0 + k^2/0}{g}}$$

or, $$T = 2\pi\sqrt{\frac{0+\infty}{g}}$$

i.e. $\boxed{T_{max} = \infty}$

i.e.

The time period of bar-pendulum is maximum, when $l = 0$, [i.e. when the pendulum is suspended to its C.G.].

Then,

min^n frequency $f_{min} = \frac{1}{T_{max}} = \frac{1}{\infty} = 0$.

Also,

The frequency of pendulum is minimum when $l=0$.

Note: 3

Minimum value of time period of Bar-Pendulum.

we know,

Time period of a bar-pendulum is:

$$T = 2\pi\sqrt{\frac{l + k^2/l}{g}}$$

or, $$T = \frac{2\pi}{\sqrt{g}}\left[l + \frac{k^2}{l}\right]^{1/2}$$

Diff^n. this w.r.t. 'l', we get.

$$\frac{dT}{dl} = \frac{2\pi}{\sqrt{g}} \cdot \frac{1}{2}\left[l + \frac{k^2}{l}\right]^{-\frac{1}{2}} * \left(1 - \frac{k^2}{l^2}\right)$$

Now,

For minimum value of 'T',

$$\frac{dT}{dl} = 0.$$

or, $$\frac{\pi}{\sqrt{g}}\left[l + \frac{k^2}{l}\right]^{-1/2} \cdot \left(1 - \frac{k^2}{l^2}\right) = 0.$$

or, $$1 - \frac{k^2}{l^2} = 0.$$

or, $$k^2 = l^2 \quad \text{------- (1)}$$

$\therefore \boxed{l = \pm k}$ # i.e. The time period of bar-pendulum is minimum, when $l = \pm k$. #

Also, from (1),

$$k^2 = l_1^2$$

or, $$l_1 \cdot l_2 = l_1^2$$

or, $$l_1 = l_2.$$

Note: 4

Center of suspension and center of oscillation are interchangeable in a physical pendulum:

⇒ we know, Time period of a physical pendulum is:

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

i.e. $$T = 2\pi\sqrt{\frac{ml^2 + mk^2}{mgl}} \quad \text{------}(1)$$

or, $$T = 2\pi\sqrt{\frac{l^2 + k^2}{gl}}$$

$$\therefore\ T = 2\pi\sqrt{\frac{\left(l + \frac{k^2}{l}\right)}{g}} \quad \text{-------}(2)$$

Now, let us rotate the physical pendulum and suspend it to its centre of oscillation, then,

$l \longrightarrow \frac{k^2}{l}$ and above relation becomes.

$$T' = 2\pi\sqrt{\frac{\cancel{m}\left(\frac{k^2}{l}\right)^2 + \cancel{m}k^2}{\cancel{m}g\left(\frac{k^2}{l}\right)}}$$

or, $$T' = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{\left(\frac{k^2}{l}\right) + k^2}{\frac{k^2}{l}}}$$

$$\therefore\ T' = 2\pi\sqrt{\frac{\frac{k^2}{l} + l}{g}} \quad \text{-------}(3)$$

From (2) & (3), we get,

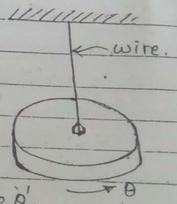
$$T = T'$$

i.e. The center of suspension and centre of oscillation are interchangeable in a physical pendulum.

TORSION PENDULUM

Torsion:

Torsion pendulum consists of a circular disc, suspended by long and thin wire as shown in the figure.



when the disc is rotated, the wire is twisted by an angle 'θ'. In this case, the restoring torque is created on it obeying Hook's law. The disc is set into torsional vibration. The restoring torque is directly proportional to the angular displacement of the wire.

i.e. $\tau_{res} \propto \theta$

or, $\tau_{res} = -C\theta$

where, 'C' is constant called torsion constant.

$$\left\{ C = \frac{\pi \eta r^4}{2l} \right\}$$

The rotational form of Newton's 2nd Law is :-

$$\tau = I\alpha.$$

So,

$$I\alpha = -C\theta$$

or, $\alpha = \left(\frac{-C}{I}\right)\theta$. - - - - - - (1)

Since $(-C/I)$ is constant,

$\therefore$ Angular acceln $(\alpha) \propto \theta$

i.e. motion of torsional pendulum is SHM.

Also,

$\alpha = \frac{d^2\theta}{dt^2}$, then eqn (1) becomes,

$$\frac{d^2\theta}{dt^2} = \left(\frac{-C}{I}\right)\theta.$$

or, $\frac{d^2\theta}{dt^2} + \left(\frac{C}{I}\right)\theta$

or, $\boxed{\frac{d^2\theta}{dt^2} + \omega^2\theta = 0}$ - - - - - - (2)

This is the eqn of oscillation of torsion pendulum.

where, $\omega = \sqrt{C/I}$ is the angular frequency.

i.e. $\frac{2\pi}{T} = \sqrt{C/I}$

$\therefore$ $\boxed{T = 2\pi\sqrt{\frac{C}{I}}}$ - - - - - - - (3)

This gives the time period of oscillation of torsion pendulum.

Also,

Solution of eqn (2) can be expressed as :-

$$\theta = \theta_{max} \sin(\omega t + \phi)$$

In lab

we can find the modulus of rigidity of given wire with the help of time period of torsional pendulum as below,

we know,

$$C = \frac{\pi r^4 \eta}{2l}$$

where, r = radius of wire.

l = length of wire.

η = modulus of rigidity.

Then, eqn (3) becomes,

$$T = 2\pi\sqrt{\frac{I \times 2l}{\pi r^4 \cdot \eta}}$$

or, $T^2 = \frac{4\pi^2 \times 2l \times I}{\pi r^4 \cdot \eta}$

$\therefore$ $\boxed{\eta = \frac{8\pi I l}{r^4 \cdot T^2}}$

⊛ Free Oscillation :

when a body capable of oscillation is displaced from its equilibrium position and then left free, it begins to oscillate with a definite amplitude and frequency. If the body is not restricted by any kind of friction, the motion continues. Such oscillation is called free oscillation.

Ex: When a bob of simple pendulum is displaced from its mean position and left free, it executes a free oscillation.

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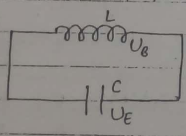
⇒ LC - oscillation :

Undamped Electromagnetic Oscillation:



Let us consider a LC-circuit where the electromagnetic oscillation takes place due to sinusoidal (∼) variation of current with time. In this case, the electromagnetic oscillation takes place according to the priciple of conservation of energy. let 'U' is the total energy in the circuit, then we can write,

$$U = U_E + U_B$$

or, $U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}LI^2$

i.e. rate of change of total energy can be expressed as,

$$\frac{dU}{dt} = \frac{d}{dt}\left[\frac{1}{2C}q^2 + \frac{1}{2}LI^2\right]$$

or, $\frac{dU}{dt} = \frac{1}{2C}\cdot\frac{d(q^2)}{dt} + \frac{1}{2}L\frac{d(I^2)}{dt}$

or, $\frac{dU}{dt} = \frac{1}{2C}\cdot 2q\cdot\frac{dq}{dt} + \frac{1}{2}L\,2I\cdot\frac{dI}{dt}$

or, $\frac{dU}{dt} = \frac{q}{C}\cdot\left(\frac{dq}{dt}\right) + L\cdot I\cdot\left(\frac{dI}{dt}\right)$

or, $\frac{dU}{dt} = \frac{q}{C}\cdot I + L\cdot I\cdot\frac{d^2q}{dt^2}$

$$\therefore \frac{dU}{dt} = I\left[\frac{q}{C} + L\cdot\frac{d^2q}{dt^2}\right] \quad \text{------ (i)}$$

As there is no change of total energy (i.e. U=const) in LC-circuit, i.e. $\frac{dU}{dt} = 0$, above eq^n becomes.

$$I\left[\frac{q}{C} + L\cdot\frac{d^2q}{dt^2}\right] = 0$$

Since $I \neq 0$.

$$\frac{q}{C} + L \cdot \frac{d^2q}{dt^2} = 0.$$

or, $$L \cdot \frac{d^2q}{dt^2} = -\frac{q}{C}$$

or, $$\frac{d^2q}{dt^2} = -\frac{q}{LC}$$

$$\therefore \boxed{\frac{d^2q}{dt^2} + \frac{q}{LC} = 0} \quad \text{-------- (2)}$$

This is required expression for LC-Oscillation.

Now,

Compairing this eq^n (2) with the eq^n of undamped mechanical oscillation i.e. $\frac{d^2y}{dt^2} + \omega^2 y = 0$,

then,

$$\omega^2 = \frac{1}{LC}$$

or, $$\omega = \sqrt{\frac{1}{LC}}$$

or, $$\frac{2\pi}{T} = \frac{1}{\sqrt{LC}}$$

$$\therefore \boxed{T = 2\pi\sqrt{LC}} \text{ and, } \boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$

This gives time period and frequency of undamped electromagnetic oscillation.

Also,

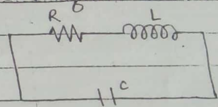
Solution of above eq^n (2) can be expressed as:

$\boxed{q = q_{max} \sin(\omega t + \phi)}$ or, $\boxed{q = q_{max} \cos(\omega t + \phi)}$

Damped Oscillation :

The oscillation whose amplitude goes on decreasing with time is called damped oscillation. Practically, all vibrations/oscillation are damped. A oscillation whose amplitude remains constant with time is called undamped oscillation.

⇒ LCR-Circuit [Damped Electromagnetic Oscillation]



In LCR-circuit, damped electromagnetic oscillation takes place due to the presence of resistance 'R'.

Now, according to the principle of conservation of energy,

$$U = U_E + U_B.$$

or, $$U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}LI^2$$

Forced Oscillation :

When a body is maintained in a state of vibration/oscillation by an external period of force of frequency other than that of natural frequency of the body, the oscillation is called forced oscillation. A body can be forced to oscillate with any frequency depending upon that of the applied periodic force. The oscillation dies out as soon as the applied force is removed.

Resonance :

When a body is maintained in a state of vibration by a periodic force having the same frequency as the natural frequency of the body, the vibration is called resonant vibration. The phenomenon of producing resonant vibration is called resonance.

Conditions for damping :

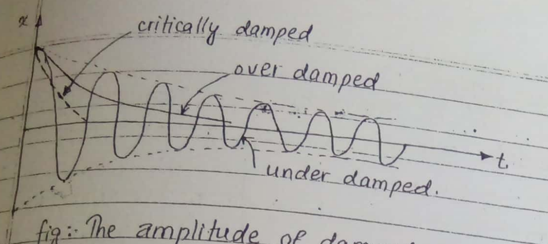


fig:- The amplitude of damped oscillations.

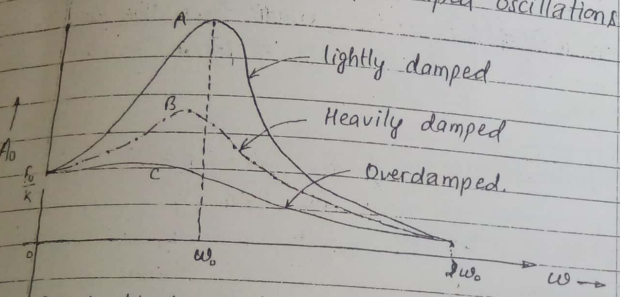


Fig: Amplitude of forced harmonic oscillator as a function of applied frequency ω.

For forced oscillation,

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \sin\omega t.$$

solution of this eq^n is:

$$x = A_0 \sin(\omega t + \phi)$$

where, ϕ is the phase angle bet^n x and external force F_{ext}; and A_0 is amplitude.

then,

$$A_0 = \frac{F_0}{m\sqrt{(\omega^2-\omega_0^2)^2 + \frac{b^2\omega^2}{m^2}}}$$

and,

$$\phi = \tan^{-1}\left[\frac{\omega_0^2-\omega^2}{\omega\left(\frac{b}{m}\right)}\right]$$

—X— —X—

WAVE MOTION

Wave motion in a medium is a form of disturbance that travels from one place to another place due to simple harmonic motion (SHM) of the particles in that medium. There are two types of wave.

1. Mechanical wave :

It requires medium transfer from one place to another place. Ex:- sound wave, wave produced in vibrating string, wave produced in spring wave produced due to motion of gas or liquid molecules, i.e. mechanical wave is either transverse or longitudinal in nature.

2. Non-Mechanical or Electromagnetic wave :

It doesnot requires any medium to transfer from one place to another place. Ex: light wave, x-ray, γ-ray, β-ray, etc. All Electromagnetic waves are transverse in nature.

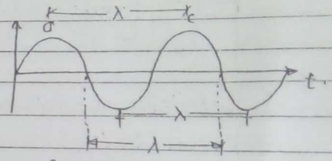
⇒ Terms Used in wave Motion

1. wave length (λ) :-

It is the length of a complete wave.

2. Time period (T) :

It is the total time required for one complete cycle.



3. Frequency (f) :-

It is the number of cycle made in one second. i.e. $f = \frac{1}{T}$

4. Angular frequency (ω) $= \frac{2\pi}{T} = 2\pi f$

5. Speed of wave (v) $= \frac{dx}{dt}$

6. velocity of particle (u) $= \frac{dy}{dt}$

we also have, $\boxed{v = \frac{\lambda}{T} = \lambda f}$

Also,

As the particles are in SHM, the velocity of the particles in a wave is given by:

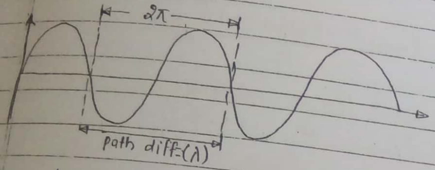
$$u = \pm \omega \sqrt{(A^2 - y^2)}$$

∴ Maximum transverse speed of the particles.

$u_{max} = \omega A$

[u is maximum at mean position i.e $y=0$].

⇒ Relation bet^n Phase difference (ϕ) and path difference (x) :-



we know,

For path difference $= \lambda$, Phase difference $= 2\pi$

i.e. for path diff. $= 1$, Phase diff. $= \frac{2\pi}{\lambda}$

∴ for path diff $= x$, phase difference $= \frac{2\pi}{\lambda} * x$.

∴ $\boxed{\text{Phase difference } (\phi) = \frac{2\pi}{\lambda} * \text{path diff } (x)}$

OR

∴ $\boxed{\text{Path difference } (x) = \frac{\lambda}{2\pi} * \text{phase difference } (\phi)}$

⇒ Equation of Plane Progressive Wave [Travelling Wave] i.e.

Prove $y = y_{max} \sin(\omega t + \phi)$ or, $y = y_{max} (\omega t + kx)$

$y = y_{max} \sin(\pm kx \mp \omega t)$

Let us consider the plane progressive wave pulse along a stretched string with speed 'v'. Initially at $t=0$, the relation between transverse displacement (y) of the particle and displacement (x) of wave for this case is as fig (i), which is given by,

$y = f(x)$ ---------- ①

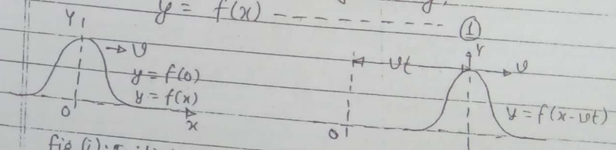


fig (i): Initial position of wave pulse

fig (ii): position of wave pulse after 't'

Fig (ii): shows the position of wave pulse after time 't', the relation of 'y' with displacement (x) and time (t), for this case, it can be expressed as:

$y = f(x - vt)$ ---------- ②

Again,

As the particles in a wave are in SHM, the displacement of the particles at any instant 't', can be expressed as,

$$y = y_{max} \sin(\omega t + \phi) \quad [\because A \text{ or } y_{max} = \text{Amplitude}]$$

$\therefore$ for $t = 0$,

$$y = y_{max} \sin \phi$$

$$\therefore y = y_{max} \sin\left(\frac{2\pi}{\lambda} \cdot x\right) \text{ ------ (3)} \quad \left[\because \phi = \frac{2\pi}{\lambda} \cdot x\right]$$

i.e.

with the help of above eq^n (1), (2), (3),
the transverse displacement 'y' after time 't', can be expressed as,

$$y = y_{max} \cdot \sin \frac{2\pi}{\lambda}(x - vt)$$

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$$\text{or, } y = y_{max} \sin\left[\frac{2\pi}{\lambda} \cdot x - 2\pi \frac{v}{x} \cdot t\right]$$

$$\therefore y = y_{max} \sin\left[\frac{2\pi}{\lambda} x - 2\pi f t\right]$$

$$\boxed{y = y_{max} \sin(kx - \omega t)} \text{ -------- (4)}$$

(along +ve x-axis)

This is the required eq^n of plane progressive wave moving in forward direction.

or

+ve x-axis direction or left to right direction,

where,

$k = \frac{2\pi}{\lambda}$ is called wave number and

$\omega = 2\pi f$ is the angular frequency.

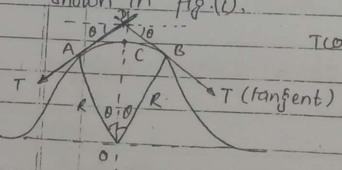
Similary,
the eq^n of plane progressive wave (travelling wave) moving in backward direction or -ve x-direction or right to left direction can be expressed as,

$$\boxed{y = y_{max} \sin(\omega t + kx)}$$ #

⇒ Speed of wave in a medium :-

i.e. $v = f\lambda$ or, $v = \sqrt{T/\mu}$

let us consider a plane progressive transverse wave pulse along a stretched string with speed 'v'. as shown in fig.(i).



fig(i) Speed of wave.

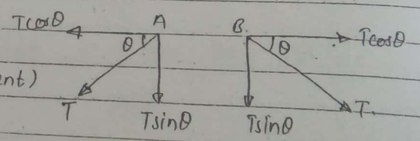


fig (ii).

let 'T' is the tension produced at extreme points A and B of a small element AB with length

δl of the string. We can resolve each 'T' into two rectangular components $T\sin\theta$ and $T\cos\theta$. As the $T\cos\theta$ component of each T has same magnitude and opposite in direction, they cancel each other and sum of vertical components (i.e. $2T\sin\theta$) directed towards center gives the centripetal force.

i.e. $2T\sin\theta = \frac{mv^2}{R}$

or, $2T\frac{\delta l/2}{R} = \frac{mv^2}{R}$ $\left[\because \sin\theta \approx \theta = \frac{\delta l}{2R}\right]$

or, $T \cdot \delta l = mv^2$

or, $v^2 = \frac{T\delta l}{m} = \frac{T}{m/\delta l}$

$\therefore \boxed{v = \sqrt{\frac{T}{\mu}}}$

This is the required expression for the speed of wave in terms of tension 'T' and linear mass density (μ) or mass per unit length.

⇒ Average Power and Intensity of wave :

Let 'dE' is the kinetic energy (K.E.) of a small particle with mass 'dm' and oscillating with velocity 'u' at any instant 't' in a transverse wave along a stretched string. Then we can write.

$$dE = \frac{1}{2} dm\, u^2 \quad \text{-------- (i)}$$

where, $u = \frac{dy}{dt} = \frac{d}{dt}\left[y_{max} \sin(kx - \omega t)\right]$

$= -\omega\, y_{max} \cos(kx - \omega t)$

Then eq^n (i) becomes,

$$dE = \frac{1}{2} dm * \omega^2 y_{max}^2 \cos^2(kx - \omega t)$$

or, $dE = \frac{1}{2} \mu \cdot dl \cdot \omega^2 y_{max}^2 \cos^2(kx - \omega t)$ $\left[\because \mu = \frac{dm}{dl}\right]$

$\therefore$ The rate of change of K.E. of the wave is,

$$\frac{dE}{dt} = \frac{1}{2} \mu \cdot \frac{dl}{dt} \cdot \omega^2 y_{max}^2 \cos^2(kx - \omega t)$$

or, $\frac{dE}{dt} = \frac{1}{2} \mu \cdot v\, \omega^2 \cdot y_{max}^2 \cdot \cos^2(kx - \omega t)$

Here, $v = \frac{dl}{dt}$ is the speed of wave.

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$\therefore$ Average rate of change of K.E. in the wave.

$$\frac{\overline{dE}}{dt} = \frac{1}{4}\mu v \omega^2 y_{max}^2 \quad \text{---- (ii)} \left[\because \overline{\cos^2(kx-\omega t)} = \frac{1}{2}\right]$$

As the potential energy is also associated with the particle in the wave and rate of change of the average value is same as the average value of K.E. given by above eq^n (i), the average ~~power~~ rate of change of total energy [i.e. average power 'P'] is given by the relation,

i.e.

$$\text{Average power }(P) = 2\left(\frac{\overline{dE}}{dt}\right)$$

$$\boxed{\therefore\ P = \frac{1}{2}\mu v \omega^2 y_{max}^2}$$

Again,

As the intensity (I) of wave is defined as the average power per unit area (A), we can write,

$$\text{Intensity }(I) = P/A$$

$$\text{i.e. } I = \frac{\frac{1}{2}\mu v \omega^2 y_{max}^2}{A}$$

$$\text{or, } I = \frac{1}{2}\left(\frac{m}{l A}\right) v \omega^2 y_{max}^2 \quad \left[\because \mu = \frac{m}{l}\right]$$

$$\therefore \boxed{I = \frac{1}{2}\rho_0 v \omega^2 y_{max}^2} \quad \text{Imp}^{\text{n}}.$$

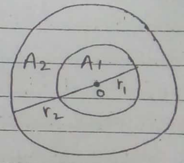
where, ρ_0 is the density of the medium.

Note

1. For spherical wave, $I = \frac{P}{A} = \frac{P}{4\pi r^2}$

2. If P remains constant,

In fig:



$$I_1 = \frac{P}{A_1} = \frac{P}{4\pi r_1^2}$$

and,

$$I_2 = \frac{P}{4\pi r_2^2}$$

Then,

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \implies \boxed{I \propto \frac{1}{r^2}} \quad \text{Imp}^{\text{n}}.$$

⇒ **Standing (a kind of interferance) wave or Stationary wave :**

Standing wave is formed due superposition of two waves having same amplitude, same frequency and wave length with constant phase difference but travelling in opposite direction.

let y_1 and y_2 are the transverse displacement for two waves that produce the standing or stationary wave due to superposition, then we can write,

$y_1 = A\sin(kx - \omega t)$ ------- (i)

$y_2 = A\sin(kx + \omega t)$ ------- (ii)

Now,

According to the principle of superposition, the transverse displacement for resultant wave is:

$y = y_1 + y_2$

or, $y = A\sin(kx-\omega t) + A\sin(kx+\omega t)$

or, $y = A[\sin(kx-\omega t) + \sin(kx+\omega t)]$.

or, $y = 2A.\sin\left\{\frac{(kx-\cancel{\omega t})+(kx+\cancel{\omega t})}{2}\right\}.\cos\left\{\frac{(\cancel{kx}-\omega t)-(\cancel{kx}+\omega t)}{2}\right\}$

or, $y = 2A\sin(kx)\cdot\cos(\omega t)$.

$\therefore \boxed{y = 2A\sin(kx)\cdot\cos(\omega t)}$ ------ (iii)

This is the required equation for standing wave.

The amplitude of standing wave is maximum at the points, where $\sin kx = \pm 1$.

or, $\sin(kx) = \sin(2n+1)\frac{\pi}{2}$.

or, $kx = (2n+1)\frac{\pi}{2}$.

or, $\frac{2\pi}{\lambda}\cdot x = (2n+1)\frac{\pi}{2}$.

$x_n = (2n+1)\frac{\lambda}{4}$.

where $n = 0, 1, 2, \ldots\ldots$

when $n = 0$, $x_0 = \lambda/4$.

when $n = 1$, $x_1 = \frac{3\lambda}{4}$.

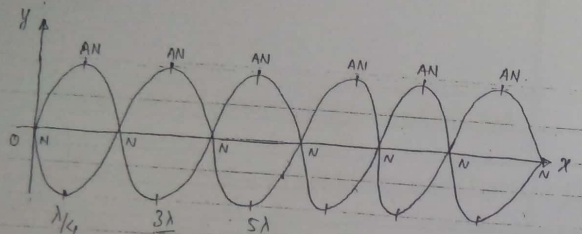
when $n = 2$, $x_2 = \frac{5\lambda}{4}$. and so on....

i.e. The amplitude of standing wave is maximum at the point where $x_n = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots\ldots$ etc.

These points are called antinodes as fig (i) below.

i.e. $\boxed{(x_2 - x_1) = \frac{\lambda}{2}}$ ------- (iv)

$\therefore$ The distance between two consecutive antinodes equal to $\frac{\lambda}{2}$.



Fig(i) : Standing wave

Here,
N → Nodes
AN → Anti-nodes

Similarly,
The amplitude of standing wave is minimum (zero), when $\sin kx = 0$.

or, $\sin kx = \sin n\pi$

or, $kx = n\pi$

or, $\frac{2\pi}{\lambda} \cdot x = n\pi$

$\therefore \boxed{x_n = \frac{n\lambda}{2}}$

when $n=0$, $x_0 = 0$.
when $n=1$, $x_1 = \lambda/2$
when $n=2$, $x_2 = \lambda$
when $n=3$, $x_3 = \frac{3\lambda}{2}$ and so on....

i.e. The amplitude of standing wave is minimum at the point where $x_n = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$..... etc.

These points are called nodes of standing wave.
i.e. $\boxed{(x_2 - x_1) = \lambda/2}$ --------- (v).

From above relation (iv) and (v), it is clear that the distance between any two consecutive nodes or antinodes of standing wave equals to $\lambda/2$.

⇒ Resonance :

When an elastic body is acted by an external force with the frequency equal to any one natural frequency of the body, the body oscillates with high amplitude, this phenomenon is known as resonance.

we know the natural frequency of a vibrating string fixed at two ends is given by the relation,

$$f_n = \frac{n}{2l} \cdot v$$

$$\therefore \boxed{f_n = \frac{n}{2l}\sqrt{\frac{T}{\mu}}} \quad \text{(Natural frequency)}.$$

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when $n=1$, $f_1 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$ $\Rightarrow$ fundamental frequency (1st harmonics)

when $n=2$,

$f_2 = \frac{2}{2l}\sqrt{\frac{T}{\mu}} \Rightarrow 2f_1 \Rightarrow$ 2nd harmonics (1st overtone)

when $n=3$

$f_3 = \frac{3}{2l}\sqrt{\frac{T}{\mu}} = 3f_1 \Rightarrow$ 3rd harmonics (2nd overtone)

and so on.....

i.e.

If the frequency of applied force is equal to any one natural frequency given by above relations, then the string vibrates with high amplitude called resonance.

Similarly,

we know, the natural frequencies of a vibrating string fixed at one end is given by the relation.

$$\boxed{f_n = \frac{2n+1}{4l}\sqrt{\frac{T}{\mu}}}$$

when $n=0$,

$f_0 = \frac{1}{4l}\sqrt{\frac{T}{\mu}}$ $\Rightarrow$ fundamental frequency (1st harmonics)

when $n=1$, $f_1 = 3f_0 \Rightarrow$ 3rd harmonic. (2nd overtone)

when $n=2$,

$f_2 = 5f_0 \Rightarrow$ 5th harmonic (4th overtone)

and so on....

i.e.

If the frequency of applied force is equal to any one natural frequency given by above relations, then the string vibrates with high amplitude called resonance.

—x—x—

ACOUSTICS

⇒ Conditions for a good auditorium :

i) The sound must sufficiently be loud everywhere.

ii) Successive syllable must be clear and distinct i.e. each syllable should die away sufficiently quickly to give place to the next syllable.

iii) There should be no echoes or distortion of the original sound.

iv) There should neither be any focusing of sound nor any zone of silence in any part of the hall.

v) The quality of sound must not change. The relative intensities of the sound components of a complex sound must be maintained.

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⇒ Reverberation and Reverberation time :

The sound produced in a room or Hall suffers multiple reflection from the various walls, ceiling and floor of the Hall, so that in addition to the direct sound, a series of sound waves are heard by a listeners in the Hall. The listeners feels the persistence of sound for some time even when the original sound has cutoff (ceased). This is known as reverberation.

The time taken by a sound to fall to one millionth of its intensity just before the source is cut off is known as reverberation time. It is same as to say "the time to fall the loudness by 60dB.

If 'I_0' is the original value of intensity of sound wave and 'I_t' is the intensity of sound wave after reverberation time 'T', then we get,

$$I_t = 10^{-6} I_0$$

$$\Rightarrow I_t = \frac{I_0}{10^6}$$

i.e. $\boxed{\frac{I_t}{I_0} = 10^{-6}}$ #

⇒ Absorption Coefficient (α):

The absorption coefficient for the material of the surface is defined as the ratio of effective absorbing area (A) to the total surface area (S).

i.e. $\boxed{\alpha = \frac{A}{S}}$

Similarly,

If $A_1, A_2, A_3 \dots$ are the absorption areas ~~coefficient~~ for the surface areas $S_1, S_2, S_3 \dots$ resp, then the average value of absorption coefficient is given by:

$$\alpha_{avg} = \frac{A_1 + A_2 + A_3 + \dots}{S_1 + S_2 + S_3 + \dots}$$

or, $$\alpha_{avg} = \frac{\alpha_1 S_1 + \alpha_2 S_2 + \alpha_3 S_3 + \dots}{S_1 + S_2 + S_3 + \dots}$$

$$\therefore \boxed{\alpha_{avg} = \frac{\sum \alpha_i S_i}{S}}$$ where Total surface area $= S$.

v.v. Imp.

Sabine's Relation

It is the relation between volume of hall (V), area of reflecting surface (S), absorption coefficient (α) and reverberation time (T).

let I is the average intensity and δI is the decrease of intensity after small time interval δt. Then it is found that,

$\delta I \propto I$,

$\delta I \propto n$, and

$\delta I \propto \delta t$

Combining these,

$\delta I \propto n I \delta t$

$\delta I = -\alpha n I \delta t$ ------ (1)

Here, -ve sign indicates the decrease of intensity and n is the no. of reflection per second.

Again,

It is found that the distance bet^n two successive reflections [mean free path] is $\frac{4V}{S}$

If v is the velocity of sound wave, then the time interval bet^n two successive reflection is

$= \frac{4V}{Sv}$

No. of reflections per sec. $(n) = \frac{Sv}{4V}$

Then, above relation (1) becomes,

$$\delta I = -\alpha \left(\frac{Sv}{4V}\right) I \delta t$$

or, $$\frac{\delta I}{\delta t} = -\alpha \left(\frac{Sv}{4V}\right) . I$$

For limiting value $\frac{\delta I}{\delta t} = \frac{dI}{dt}$

So, $\frac{dI}{dt} = -\alpha\left(\frac{sv}{4V}\right)I$

or, $\frac{dI}{I} = \left(\frac{-\alpha sv}{4V}\right)dt$

let I_0 is the maximum intensity, when the sound wave is just cut off. and I_t is the decrease of intensity after time 't'. Then, above relation becomes,

$$\int_{I_0}^{I_t} \frac{dI}{I} = -\frac{\alpha sv}{4V}\int_0^t dt.$$

or, $\left[\log I\right]_{I_0}^{I_t} = -\frac{\alpha sv}{4V}.t$

or, $\log I_t - \log I_0 = -\frac{\alpha sv}{4V}.t$

or, $\log\left(\frac{I_t}{I_0}\right) = -\frac{\alpha sv}{4V}.t$

Taking antilog on both sides,

$$\frac{I_t}{I_0} = e^{-\frac{\alpha sv}{4V}.t}$$

But for reverberation time T, $\frac{I_t}{I_0} = 10^{-6}$.

P.30.

or, $10^{-6} = e^{-\frac{\alpha sv}{4V}.t}$

Taking log on both sides we get.

$$\log_e(10^{-6}) = -\frac{\alpha sv}{4V}.T$$

or, $-6 \times \log_e 10 = -\frac{\alpha sv}{4V}.T$

or, $6 \times 2.3026 = \frac{\alpha sv}{4V}.T$

If the velocity of sound $(v) = 350\ ms^{-1}$. Then,

$$\frac{6 \times 2.3026 \times 4V}{\alpha s \times 350} = T$$

$$\therefore \boxed{T = \frac{0.158V}{\alpha s}}$$

This is required expression for sabines relation in MKS system.

Similarly,

If $v = 1120$ ft/sec

Sabines relation becomes.

$$\boxed{T = \frac{0.05V}{\alpha s}} \text{ in FPS system.}$$

⇒ Sound wave :

Sound wave is the longitudinal mechanical wave that travels in solid, liquid or gas. The longitudinal waves which have the frequency (f).

i) If $f < 20\,Hz$; Infrasonic wave.

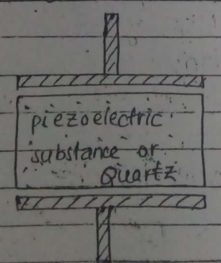
ii) If $20\,Hz \leq f \leq 20\,kHz$ ⇒ Audible range wave.

iii) If $f > 20\,kHz$ ⇒ Ultrasonic wave where supper-sonic wave lies.

⇒ Production of Ultrasonic wave :

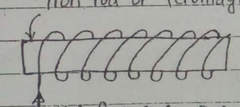
1. Piezo-electric effect or Electrostriction Method :-

If an insulator is placed in an electric field, it gets polarized. On polarizing, its dimension is changed. This phenomenon is



called electrostriction. The substance like quartz called piezoelectric substance, this effect is more pronounced. When the periodically (or, periodic change of electric field) varying voltage is applied to this substance as in fig. the periodic change of length of this substance takes placed and the wave with the frequency greater than $20\,kHz$ is produced which is known as ultrasonic wave.

2. Magnetostriction Method :



A rod of feromagnetic substance is changes its length on magnetization or demagnetization. This is known as magnetostriction method. If an iron rod is placed inside a solenoid carrying high frequency A.C., the length of the rod will changes periodically. This will generate longitudinal waves with the high frequency greater than $20\,kHz$ in the surrounding. This produced waves are called ultrasonic wave.

fig:- Magnetostriction Method.

Uses of Ultrasonic waves or Applications

i) It is used to study the existance of stone inside the land or sea.

ii) It is used to detect the depth of water.

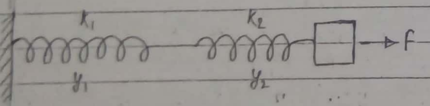
iii) It is used for the treatment of neurological pain.

iv) It is used to determine the speed of sound in water.

v) It is used for different types of scientific researches.

S.H.M.

1. Prove that : $f = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1+k_2)m}}$



let y_1 is the displacement produced on 1st spring from its mean position due to force F, then we get.

$$F = k_1 y_1 \Rightarrow y_1 = \frac{F}{k_1} \quad \text{------ (i)}$$

Similarly,

y_2 is the displacement produced on 2nd spring from its mean position due to force F, then,

$$F = k_2 y_2 \Rightarrow y_2 = \frac{F}{k_2} \quad \text{------ (ii)}$$

If y is the total displacement produced in spring mass system for SHM, then we can write

$$y = y_1 + y_2$$

or, $y = \frac{F}{k_1} + \frac{F}{k_2}$

or, $y = \left(\frac{k_1+k_2}{k_1 k_2}\right) F$

or, $F = y\left(\frac{k_1 k_2}{k_1+k_2}\right)$

or, $ma = y\left(\frac{k_1 k_2}{k_1+k_2}\right)$

or, $m \cdot \omega^2 y = y\left(\frac{k_1 k_2}{k_1+k_2}\right)$

or, $\omega = \sqrt{\frac{k_1 k_2}{(k_1+k_2)m}}$

$$\therefore \boxed{f = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1+k_2)m}}}$$

Hence Proved.

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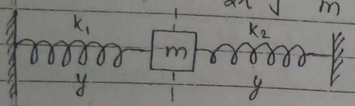
2. Prove that: $f = \frac{1}{2\pi}\sqrt{\frac{(k_1+k_2)}{m}}$.



Here, for 1^{st} spring,

$f_1 = k_1 y$ ------ (i).

For 2^{nd} spring,

$f_2 = k_2 y$ ------ (ii).

∴ Total force f for SHM is:-

$F = f_1 + f_2$

or, $F = k_1 y + k_2 y$.

or, $F = y(k_1+k_2)$

or, $ma = y(k_1+k_2)$

or, $m\omega^2 y = y(k_1+k_2)$

or, $\omega = \sqrt{\frac{(k_1+k_2)}{m}}$

$2\pi f = \sqrt{\frac{(k_1+k_2)}{m}}$

∴ $\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}}$ Hence, Proved.

—— X —— X ——

OPTICAL FIBER

It is made of glass or plastics which is designed to guide the light wave along the axis with the help of total internal reflection from its side walls. It has 3-parts.

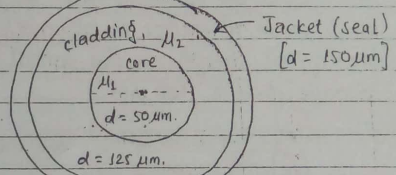


fig:- Optical fibre

1. Core:

It is the inner-most part of the optical fibre. It is made of glass or plastic and its average dia. is 50μm.

2. Cladding:

The core is surrounded by a part in optical fibre called cladding. It is also made of glass or plastics. Its average diameter is 125 μm.

If μ_1 and μ_2 are the refractive indices for core and cladding respectively, then the value of μ_1 is always greater than the value of μ_2

(i.e. $\mu_1 > \mu_2$), for total internal reflection in optical fibre.

3. Jacket :

It is the outermost part of the optical fibre. It is made of plastics or polymer. It gives the mechanical strength to the optical fibre. Its average dia. is 150 μm.

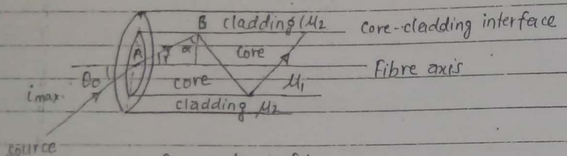


fig: Optical fibre.

$\theta_0 \rightarrow$ Acceptance angle.

let us consider the light wave is incidence at 'A' in an optical fibre as fig.

Now,

Applying snell's law at A.

$$\frac{\mu_1}{\mu_0} = \frac{\sin i}{\sin r}$$

or, $\mu_0 \sin i = \mu_1 \sin r$

or, $\mu_0 \sin i = \mu_1 \sin(90-\alpha)$

or, $\mu_0 \sin i = \mu_1 \cos\alpha$ ---------- (i).

Again,

Applying snell's law at B,

when $i = \theta_0$ & $\alpha = C$ (critical angle).

i.e.

$$\frac{\sin\alpha}{\mu_2} = \frac{\sin 90^\circ}{\mu_1}$$

$$\therefore \boxed{\sin\alpha = \frac{\mu_2}{\mu_1}}$$

$$\therefore \cos\alpha = \sqrt{1 - \frac{\mu_2^2}{\mu_1^2}}$$

Then eq^n (i) becomes,

$$\mu_0 \sin\theta_0 = \mu_1 \sqrt{1 - \frac{\mu_2^2}{\mu_1^2}}$$

or, $\mu_0 \sin\theta_0 = \cancel{\mu_1} \sqrt{\frac{\mu_1^2 - \mu_2^2}{\cancel{\mu_1^2}}}$

or, $\mu_0 \sin\theta_0 = \sqrt{\mu_1^2 - \mu_2^2}$

For air, $\mu_0 = 1$, we get,

$$\boxed{\sin\theta_0 = \sqrt{\mu_1^2 - \mu_2^2}} \Rightarrow \theta_0 = \sin^{-1}\left(\sqrt{\mu_1^2 - \mu_2^2}\right)$$

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Here, θ_0 is the maximum angle of incident at point 'A' of the optical fibre for total internal reflection through it. This angle is called "acceptance angle."

Also, This sine of angle of acceptance (i.e. $\sin\theta_0$) is called "Numerical Aparture (NA)" of optical fibre.

i.e. $\boxed{N.A. = \sin\theta_0 = \sqrt{\mu_1^2 - \mu_2^2}}$

⇒ N.A. measures the light gathering ability of the optical fibre.

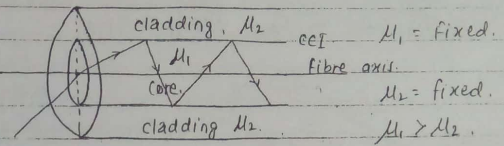
⇒ Modes of Propagation & types of Optical fibre:

According to modes of propagation (i.e. no. of path), Optical fibre is divided into two parts:

i) Monomode (single mode) Optical fibre.
ii) Multimode Optical fibre.

① Monomode Optical fibre:

It is very small and the no. of path of light is single in this optical fibre. The average value of diameter of core is $1\mu m$.



Fig(i) Step-Index monomode Optical fibre.

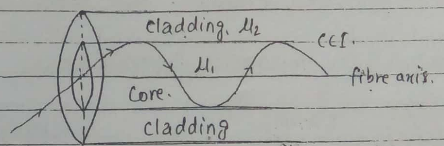


fig (ii) Graded index monomode Optical fibre.

Here,
μ_1 is high near the fibre axis & decreases from the both sides of this axis to the CCI.
μ_2 is fixed & $\therefore \mu_1 > \mu_2$.

② Multimode Optical fibre:

There are large no. of path of light in this optical fibre as below. The average value of diameter of core for this fibre is 100 μm. There are two types of multimode Optical fiber. They are.

1) Step-index multimode Optical fibre.

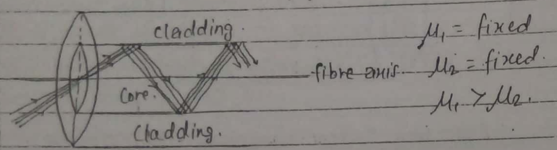


fig. (iii) Step-index multimode Optical fibre.

(ii) Graded-Index multimode Optical fibre.

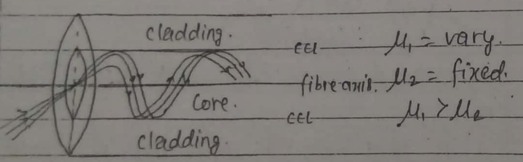


fig (iv) Graded index multimode Optical fibre.

Application of Optical fibre

i) Medical application (Eg. for endoscopy)
ii) Military application (In war)
iii) fibre sensor [Eg. dust particle sensor, temp. sensor)
iv) In Communication
v) In scientific Reasearch.

fibre loss:

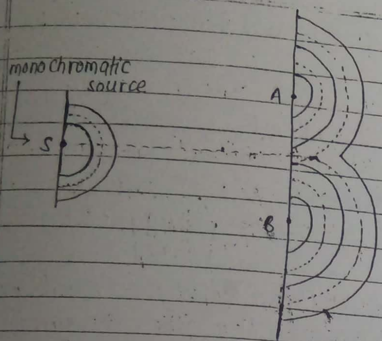
Transmission loss is the ratio of input to output optical power. The ratio is a function of operating wavelength.
The formula for attenuation in decibels per km is:

$$\alpha_{dB} = \frac{1}{L}\left[10 \log_{10}\left(\frac{P_{in}}{P_{out}}\right)\right]$$

where, P_{in} and P_{out} are input and output powers and, L is length of optical fiber in km.

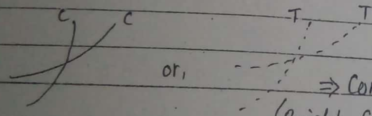
x——x

INTERFERENCE

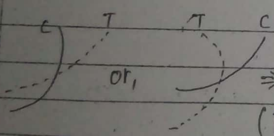


fig(i) Interference of light.

A & B → small slots or virtual coherent sources.



⇒ Constructive Interference (Bright fringes are formed.)



⇒ Destructive Interference (Dark fringes are formed)

When two or more light waves coming from coherent sources meet at a point, then light wave is said to be interference each other i.e. Interference is the phenomenon of superposition of lightwaves coming from coherent sources.

The amplitude of resultant wave is equal to the algebric sum of amplitude of each wave. When the two waves meet at a point are in phase each other, then,

$\boxed{\text{path difference} = n\lambda}$

Similarly, if the two waves are at out of phase, then.

path difference $= (2n+1)\frac{\lambda}{2}$.

⇒ Coherent Sources :-

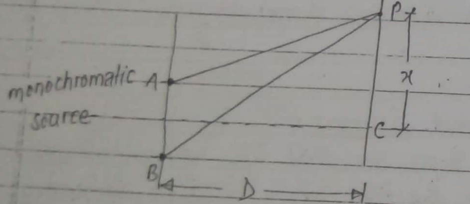
These are that type of sources which produce the light wave of same frequency, same amplitude and same wave length with constant phase difference.

In above fig. two small slits 'A' & 'B' works as the virtual cohrent sources.

⇒ Mathmatical/Analytical treatment of Interference.

let two small slits A & B (virtual coherent sources] are at equal distance from a monochromatic sources. If two waves from A & B meet at a point 'P' on the screen and ϕ is the phase difference between them, then we can write,

$$y_1 = A \sin\omega t \text{ ------ (i)}$$
$$y_2 = A \sin(\omega t + \phi) \text{ ------ (ii)}$$



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Now,
According to the principle of superposition, the displacement of resultant wave is :-

$$y = y_1 + y_2$$
or, $y = A\sin\omega t + A\sin(\omega t + \phi)$
or, $y = A\sin\omega t + A\sin\omega t\cos\phi + A\cos\omega t\sin\phi$
or, $y = A\sin\omega t(1 + \cos\phi) + A\sin\phi\cdot\cos\omega t$.

or, $y = A(1 + \cos\phi)\sin\omega t + A\sin\phi\cdot\cos\omega t$.

let, $A\sin\phi = R\sin\theta$ -------- (iii).
& $A(1+\cos\phi) = R\cos\theta$ -------- (iv).
Then, above relation becomes,
$y = R\cos\theta\cdot\sin\omega t + R\sin\theta\cdot\cos\omega t$.
or, $y = R[\sin\omega t\cdot\cos\theta + \cos\omega t\cdot\sin\theta]$.

$\therefore \boxed{y = R\sin(\omega t + \theta)}$ ------ (v)

This is the required eq^n for resultant wave, where R is its amplitude & θ is the phase constant.

Now,
Squaring (iii) & (iv) and then adding; we get.

$$R^2\sin^2\theta + R^2\cos^2\theta = A^2\sin^2\phi + A^2(1+\cos\phi)^2$$

or, $R^2 = A^2\sin^2\phi + A^2 + 2A^2\cos\phi + A^2\cos^2\phi$

or, $R^2 = A^2(\sin^2\phi + \cos^2\phi) + A^2 + 2A^2\cos^2\phi$
or, $R^2 = 2A^2 + 2A^2\cos^2\phi$
or, $R^2 = 2A^2(1 + \cos^2\phi) = 4A^2\cos^2\phi/2$

$$\therefore R^2 = 4A^2\cos^2\frac{\phi}{2}.$$

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As the intensity of wave varies with the square of the amplitude, we can write,

$$\boxed{I = 4A^2 \cos \phi/_2} \quad \text{------ (vi)}$$

i.e The intensity of resultant wave is maximum (i.e $I_{max} = 4A^2$), when phase difference (ϕ) is.
$0, 2\pi, 4\pi, 6\pi, \ldots\ldots 2n\pi$:

or, path difference $= 0, \lambda, 2\lambda, 3\lambda \ldots\ldots n\lambda$.

$$x = \frac{\lambda}{2\pi} * \phi$$

or, i.e. The intensity of resultant wave is maximum or constructive interference takes place & bright fringes are formed when

$$\boxed{\text{path difference} = n\lambda.}$$

Similarly, the amplitude of resultant wave is minimum or zero, when phase difference (ϕ) is:

$\pi, 3\pi, 5\pi, \ldots\ldots (2n+1)\pi$.

and,
path difference $= \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots\ldots (2n+1)\frac{\lambda}{2}$

i.e. The intensity of resultant wave is minimum or destructive interference takes place & dark fringes are formed, when,

$$\boxed{\text{path difference} = (2n+1)\frac{\lambda}{2}}$$

⇒ Energy or Intensity distribution due to Interference:

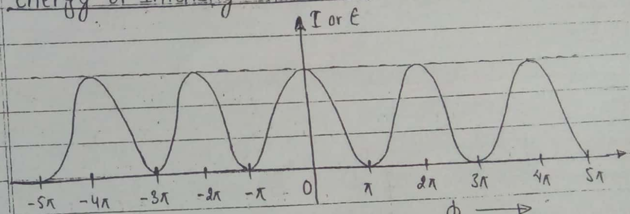

fig: Energy distribution due to interference.

From above figure, it is clear that the intensity or energy is maximum ($4A^2$) at bright points and minimum (zero) at dark points but total energy remains constant i.e. the energy is not created at bright points & not destroyed at dark points. It is just transferred from point of minima to point of maxima.

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⇨ Young's Double Slit Experiment

[To find wave length (λ) or fringe width (β)]

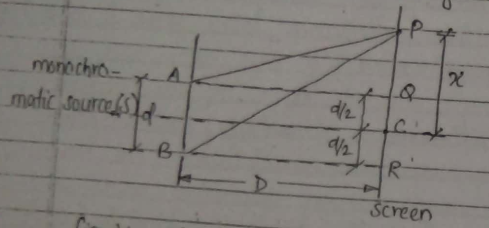

fig. Young's Double slit Experiment.

Let two small slits 'A' and 'B' [virtual coherent source] are at equal distance from a monochromatic source 'S' as in fig. where 'd' is the slit seperation and 'D' is the distance bet slit and screen. If two waves from A and B meet at a point 'P' on the screen with distance 'x' from its centre, then we will get.

$PR = (x + d/2)$ and $PQ = (x - d/2)$

$\therefore BP^2 = BR^2 + PR^2 = D^2 + (x + d/2)^2$ ------ (i)

$\therefore AP^2 = AQ^2 + PQ^2 = D^2 + (x - d/2)^2$ ------ (ii)

Subtracting (ii) from (i),

$$BP^2 - AP^2 = \cancel{D^2} + (x + d/2)^2 - \{\cancel{D^2} + (x - d/2)^2\}$$
$$= x^2 + xd + \frac{d^2}{4} - x^2 + xd - \frac{d^2}{4}$$

$$BP^2 - AP^2 = 2xd.$$

or, $(BP - AP)(BP + AP) = 2xd.$

or $(BP - AP) = \frac{2xd}{(BP + AP)}$

⟹ Path difference $= \frac{2xd}{BP + AP}$

As. $BP \approx AP \approx D$, above relation becomes.

$\therefore$ Path difference $= \frac{2xd}{2D} = \frac{xd}{D}$ ------ (iii)

But, for constructive interference or bright fringes
path difference $= n\lambda$.
Equating with (i).

$$\frac{xd}{D} = n\lambda$$

$$\therefore \boxed{x_n = \frac{n\lambda D}{d}} \text{ ------ (iv).}$$

Hence, x_n is the distance of n^{th} bright fringe from center.

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when $n=0$, $x_0 = 0$

when $n=1$, $x_1 = \frac{\lambda D}{d}$

when $n=2$, $x_2 = \frac{2\lambda D}{d}$

when $n=3$, $x_3 = \frac{3\lambda D}{d}$ and soon....

$$\therefore \boxed{(x_2 - x_1) = \frac{\lambda D}{d}} \quad \text{----------(v)}$$

i.e. the distance between the two successive bright fring is $\frac{\lambda D}{d}$.

Similarly,

for destructive interference,

path difference $= (2n+1)\frac{\lambda}{2}$

Equating this with (iii), we get.

$$\frac{xd}{D} = (2n+1)\frac{\lambda}{2}$$

or, $x = \frac{(2n+1)\lambda D}{2d}$

$$\therefore \boxed{x_n = \frac{(2n+1)\lambda D}{2d}} \quad \text{------(vi)}$$

where x_n is the distance of n^{th} dark fringe from C.

when $n=0$; $x_0 = \frac{\lambda D}{2d}$

when $n=1$; $x_1 = \frac{3\lambda D}{2d}$

when $n=2$, $x_2 = \frac{5\lambda D}{2d}$ and so on....

$$\therefore \boxed{(x_2 - x_1) = \frac{\lambda D}{d}} \quad \text{------(vii)}$$

Hence, from equation's (v) and (vii), it is clear that the distance between any two consecutive bright fringes or dark fringes, is always equal to $\frac{\lambda D}{d}$, which is known as fringe width (β).

i.e. $\boxed{\beta = \frac{\lambda D}{d}}$ Imp. or, $\boxed{\lambda = \frac{\beta d}{D}}$

Thus, we can calculate the value of wavelength (λ) of given light with the help of Young's double slit experiment.

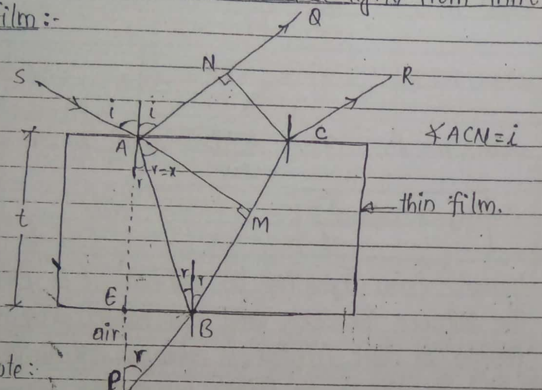
⇒ Condition for Interferences

→ Interference due to reflected light from thin film.

→ Interference due to transmitted light from thin film.

→ Due to wedge shaped thin film of air.

⇒ Interference due to Reflected light from thin film :-



Note :-
Optical path in a medium = distance travelled.
$\mu \rightarrow$ refractive index.

Let, the monochromatic light SA is incident at a point 'A' on the upper surface of a thin film with refractive index 'μ' and thickness 't'. The light wave is partly reflected and partly refracted or transmitted from points A, B, C, D, etc. as in fig. We have to find the optical path difference betⁿ the reflected light from point A & C of the thin film.

From this, let us draw $CN \perp AQ$; $AM \perp BC$ and produced BC in backward direction to meet AE produced at P.

where, ~~[illegible]~~ $\measuredangle APC = r$

$\therefore$ Optical path difference $(x) = (AB+BC)\mu + CR - (AN+CR)$

or, $x = (AB+BC)\mu - AN$ ------- (i).

Again,

$$\mu = \frac{\sin i}{\sin r} = \frac{AN/AC}{CM/AC} = \frac{AN}{CM}$$

$$\therefore \boxed{AN = \mu \cdot CM}$$

Then eqⁿ (i) becomes,

path difference $(x) = (AB+BC)\mu - \mu . CM$

$= (AB+BC - CM)\mu$

$\therefore x = (PB+BC-CM)\mu$ $[\because AB = PB]$

$\Rightarrow \boxed{x = (PC - CM)\mu}$

Optical path difference $(x) = \mu . PM$ ------ (ii)

Again,

From ΔAPM,

$$\cos r = \frac{PM}{AP}$$

or, $PM = AP\cos r$
or, $PM = (AE + EP)\cos r$
or, $PM = 2t\cos r$ $[\because AE = EP = t]$
Then, eq^n (ii) becomes,
optical path difference $(x) = 2\mu t\cos r$ ---- (iii)

This is not the correct path difference for the reflected light from the surface of denser. According to electromagnetic theory, when the light is reflected from the surface of denser medium, a phase change 'π' or equivalent path difference '$\frac{\lambda}{2}$' occurs.

$\therefore$ Correct path diff^n. $= 2\mu t\cos r \pm \frac{\lambda}{2}$ ------ (iv).

Now,
For constructive interference,
Path difference $= n\lambda$ [for n^{th} bright fringe]
Using (iv),
or, $2\mu t\cos r - \frac{\lambda}{2} = n\lambda$

or, $2\mu t\cos r = n\lambda + \frac{\lambda}{2}$

$\therefore$ $\boxed{2\mu t\cos r = (2n+1)\frac{\lambda}{2}}$ $n = 0, 1, 2, 3 \ldots\ldots$

This relation is true for n^{th} bright fringe formed due to reflected light from thin film.
Similarly,
For destructive interference or dark fringes,
path difference $= (2n+1)\lambda/2$.

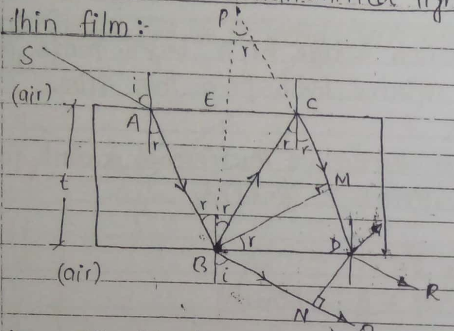
Using (iv),
$2\mu t\cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$.

or, $2\mu t\cos r = (2n+1)\frac{\lambda}{2} + \frac{\lambda}{2}$.

or, $2\mu t\cos r = (2n+1+1)\frac{\lambda}{2}$.

$\therefore$ $\boxed{2\mu t\cos r = (n+1)\lambda}$, $n = 0, 1, 2, 3 \ldots\ldots$
Here, n is an integer and we can replace $(n+1)$ by 'n' such that,
Imp!!
$\therefore$ $\boxed{2\mu t\cos r = n\lambda}$ $n = 1, 2, 3, \ldots\ldots$
This relation is true for n^{th} dark fringe formed due to reflected light from thin film.

⇒ Interference due to transmitted light from thin film :-



Here, $BE = EP = t$

$DR = NQ$

P.44

Let the light wave 'SA' is incident on the surface of a thin film with refractive-index μ and thickness 't'. The light wave is partly reflected or refracted or transmitted through the thin film. We have to find the optical path difference between the transmitted light from B & D.

$\therefore$ Optical path difference $(x) = (BC+CD)\mu + DR - (BM+NQ)$

i.e.

$$x = (BC+CD)\mu - BN \text{ - - - - - - - (i)}$$

Also,

$$\mu = \frac{\sin i}{\sin r} = \frac{BN/BD}{MD/BD} = \frac{BN}{MD}$$

$BN = \mu . MD$

Then, eq^n (i) becomes,

$\therefore$ Optical path diff.^{n} $(x) = (BC+CD)\mu - \mu . MD$

$= (BC + CD - MD)\mu$

$= (PC + CD - MD)\mu$ $[\because BC = PC]$

$= (PD - MD)\mu$

$\therefore x = \mu . PM$ - - - - - - - - - (ii)

Again,

from Δ PMB,

$$\cos r = \frac{PM}{PB}$$

or, $PM = PB . \cos r$

or, $PM = (PE + EB)\cos r$

or, $PM = 2t\cos r$ $[\because PE = EB = t]$

Then, eq^n (ii) becomes,

$x = 2\mu t\cos r$ - - - - - - - - - (iii)

Again,

For constructive interference,

path difference $(x) = n\lambda$

Compairing with (iii),

$\boxed{2\mu t\cos r = n\lambda}$ $n = 0, 1, 2, 3 \ldots\ldots$

This relation is true for n^{th} bright fringe formed d[ue] to transmitted light from thin film.

Similarly,
For destructive interference, we have
path diff. $(x) = (2n+1)\frac{\lambda}{2}$
Using (iii),
i.e. $\boxed{2\mu t\cos r = (2n+1)\frac{\lambda}{2}}$ $n = 0, 1, 2, 3, ----$

This relation is true for n^{th} dark fringe formed due to transmitted light from thin film.

⇒ Interference due to wedge shaped thin film of air :-

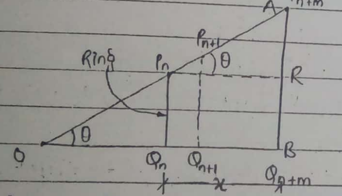


fig:- Wedge shaped thin film.

Let two plane surface OA and OB are inclined at a small angle 'θ' that encloses wedge shaped thin film of air as in fig. When the film is viewed by the monochromatic light with small angle of incidence, the interference of light wave takes place due to the path difference of reflected or transmitted light from thin film and dark and bright fringes are formed.

Let n^{th} bright fringe is formed at P_n due to reflected light from the thin film. Then we will get,

$$2\mu t\cos r = (2n+1)\lambda/2$$

As the thin film is air, $\mu = 1$.
i & r are very small, we can write.
$\cos r \approx \cos 0^{\circ} = 1$.
Then above relation becomes,
$$2t = (2n+1)\frac{\lambda}{2}$$

or. $2 \cdot P_nQ_n = (2n+1)\frac{\lambda}{2}$ ---------(i)

Similarly,
if $(n+1)^{th}$ bright fringe is formed at P_{n+1}, then above relation becomes,
$$2P_{n+1}Q_{n+1} = (2(n+1)+1)\frac{\lambda}{2}$$

i.e. $2P_{n+1}Q_{n+1} = (2n+3)\frac{\lambda}{2}$ --------(ii)

Now, subtracting eqⁿ (i) from (ii), we get,
$$2P_{n+1}Q_{n+1} - 2P_nQ_n = (2n+3)\frac{\lambda}{2} - (2n+1)\frac{\lambda}{2}$$

P.45

or, $2\left[P_{n+1}Q_{n+1} - P_nQ_n\right] = \frac{\lambda}{2}\left[\cancel{2n}+3 - \cancel{2n} - 1\right]$

or, $\boxed{P_{n+1}Q_{n+1} - P_nQ_n = \frac{\lambda}{2}}$

i.e. Thickness of air film is increased by $\frac{\lambda}{2}$, when next bright fringe formed at P_{n+1}.

If 'm' number of bright fringes are formed between P_n and P_{n+m}, then the thickness of air film will be increased by $\frac{m\lambda}{2}$.

$\therefore P_{n+m}Q_{n+m} - P_nQ_m = \frac{m\lambda}{2}$ - - - - - - - (iii)

Now, from above fig, we have,

$\theta \approx \tan\theta = \frac{P_{n+m} \cdot R}{P_n R}$ (as θ is very small)

or, $\theta = \frac{P_{n+m} \cdot Q_{n+m} - P_nQ_n}{Q_n \cdot Q_{n+m}}$

Using (iii),

$\theta = \frac{m\lambda/2}{Q_n \cdot Q_{n+m}} = \frac{m\lambda}{2Q_nQ_{n+m}}$

let $Q_n Q_{n+m} = x$, then.

$\theta = \frac{m\lambda}{2x}$

or, $\frac{x}{m} = \frac{\lambda}{2\theta}$

$\therefore \boxed{\beta = \frac{\lambda}{2\theta}}$

Here, $\beta = x/m$ is called fringe width.

Note:

If the wedge shaped thin film is formed due to material of refractive index 'μ', then above relation becomes.

$$\boxed{\beta = \frac{\lambda}{2\mu\theta}}$$

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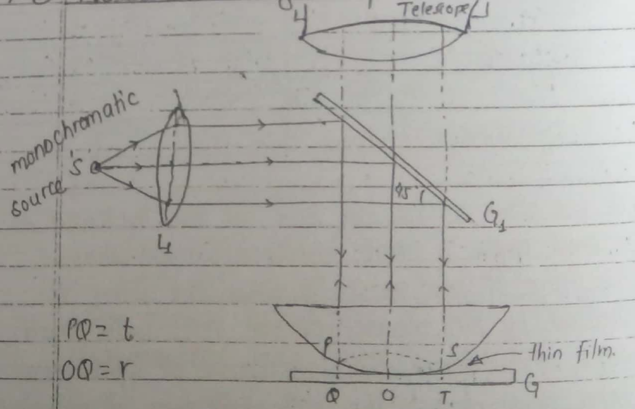
⇒ Newton's Ring Experiment:



PQ = t

OQ = r

P.46

let a plano-convex lens (L) of large radius of curvature (R) is placed on a glass plate'G' that encloses the wedge shaped thin film of air. When the parallel beam of monochromatic light is allowed to fall on the thin film normally with the help of next glass plate G_1, inclined at an angle 45° as in fig. Then, the interference of light takes place due to the path difference betⁿ the reflected or transmitted light from the thin film. Then circular dark & bright fringes are formed called "Newtons Ring".

1. Reflected Case :-

let n^{th} bright fringe is formed at 'P' with distance $OQ = r$ from point of contact 'O' [where $PQ = t$ represents the thickness of air film] then we can write,

path difference $x = n\lambda$ (for bright fringe)

But,

for the reflected light from thin film,

path diff $= 2\mu t \cos r + \lambda/2$.

Then above relation becomes.

$$n\lambda = 2\mu t \cos r + \lambda/2$$

or, $2\mu t \cos r = n\lambda - \lambda/2$.

or, $2\mu t \cos r = (2n-1)\lambda/2$

For thin film of air, $\mu = 1$ and as the light is normally incident on the thin film,

$\cos r \approx \cos 0^\circ = 1$, then above relation becomes.

$$\boxed{2t = (2n-1)\lambda/2} \quad \text{-------(i).}$$

we can find the value of $2t$ with the help of fig.(ii).

where,

$PC^2 = PE^2 + EC^2$

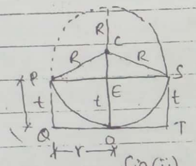


fig.(ii).

or, $PC^2 = PE^2 + (OC - OE)^2$

or, $R^2 = r^2 + (R-t)^2$

or, $R^2 = r^2 + R^2 - 2Rt + t^2$

or, $2Rt = r^2 + t^2$

As the thickness of air film is very small, we can neglect the value of t^2 in above relation, then.

P.47

we get,

$$2Rt = r^2$$

$$\therefore \boxed{2t = \frac{r^2}{R}} \text{ ------- (ii)}$$

To find the thickness of thin film.

Now,

above relation (i) becomes.

$$\frac{r^2}{R} = (2n-1)\frac{\lambda}{2}$$

$$\text{or, } r_n^2 = \frac{(2n-1)\lambda R}{2}$$

$$\therefore \boxed{r_n = \sqrt{\frac{(2n-1)\lambda R}{2}}} \text{ ------ (iii)}$$

Here, r_n is the radius of n^{th} bright ring formed due to reflected light from thin film in Newton's ring experiment.

ie. $\boxed{r_n \propto \sqrt{R}}$ or, $\boxed{r_n \propto \sqrt{\lambda}}$ or, $\boxed{r_n \propto \sqrt{\frac{(2n-1)}{2}}}$

To find λ :

let D_n is the diameter of n^{th} bright ring formed due to reflected light from the thin film, then we can write,

$$\frac{D_n}{2} = \sqrt{\frac{(2n-1)\lambda R}{2}} \quad \text{(using ii)}$$

Squaring,

$$\text{or, } \frac{D_n^2}{4} = \frac{(2n-1)\lambda R}{2}$$

$$\boxed{D_n^2 = 2(2n-1)\lambda R} \text{ ------- (iv)}$$

If m^{th} bright ring is formed due to reflected light from thin film and D_m is its diameter in reflected case, then we will get,

$$\boxed{D_m^2 = 2(2m-1)\lambda R} \text{ -------- (v)}$$

Subtracting eq^n (v) from above relation (iv), we get

$$D_n^2 - D_m^2 = 2(2n-1)\lambda R - 2(2m-1)\lambda R$$

$$= 2\lambda R\{2n - \cancel{1} - 2m + \cancel{1}\}$$

$$\text{or, } D_n^2 - D_m^2 = 4\lambda R(n-m)$$

$$\therefore \boxed{\lambda = \frac{D_n^2 - D_m^2}{4(n-m)R}}$$ Imp^n

Thus, we can calculate the wave length of light used with the help of Newton's Ring experiment.

Similarly,

For destructive interference. (dark rings)

path difference $(x) = (2n+1)\frac{\lambda}{2}$ ------- ⊛

As the light reflected from the thin film, we get,

path difference $(x) = 2\mu t \cos r + \lambda/2$

Using eq^n (*),

$(2n+1)\frac{\lambda}{2} = 2\mu t \cos r + \frac{\lambda}{2}$

or, $2\mu t \cos r = (2n+1)\frac{\lambda}{2} - \frac{\lambda}{2}$

or, $\boxed{2\mu t \cos r = n\lambda}$

For the thin film of air, $\mu = 1$.

As the light is normally incident, $i = r = 0$.

then above relation becomes,

$2t = n\lambda$

Using eq^n (ii),

$\frac{r^2}{R} = n\lambda$

or, $r_n^2 = n\lambda R$

$\therefore \boxed{r_n = \sqrt{n\lambda R}}$ ---------(vi)

Here, r_n is the radius of n^{th} dark ring, which is.

$\boxed{r_n \propto \sqrt{\lambda}}$ or, $\boxed{r_n \propto \sqrt{R}}$ or, $\boxed{r_n \propto \sqrt{n}}$

For $n=0$, the radius of dark ring given by above relation (vi) is zero but the radius of bright ring given by above relation (iii) is not equal to zero (i.e. $r_0 \neq 0$) when $n=0$.

Hence, from these fact, it is clear that the central ring is dark in reflected case.

2. Transmitted Case :-

Newton's rings are also formed due to transmitted light from thin film. i.e. for transmitted n^{th} bright ring is given by the relation,

$2\mu t \cos r = n\lambda$

But,

For air film, $\mu = 1$ and, since light is incident normally, $\cos r \approx \cos 0 = 1$. Then above relation becomes,

$2t = n\lambda$

or, $\frac{r^2}{R} = n\lambda$ (from eq^n ii)

or, $r^2 = n\lambda . R$

$\therefore \boxed{r_n = \sqrt{nR\lambda}}$ ---------(vii)

Here r_n is the radius of n^{th} bright ring formed due to transmitted light from the thin film of air.

Similarly,
for dark ring (destructive interference) formed due to transmitted light from thin film, we can write,

$$2\mu t\cos r = (2n-1)\frac{\lambda}{2}$$

Since $\mu = 1$, $\cos r \simeq \cos 0 = 1$.

$$2t = (2n-1)\lambda/2$$

$$\frac{r^2}{R} = (2n-1)\frac{\lambda}{2}$$

or, $$r^2 = (2n-1)\frac{\lambda R}{2}$$

$$\therefore \boxed{r_n = \sqrt{\frac{(2n-1)\lambda R}{2}}} \text{ ------ (viii)}$$

Here r_n is the radius of dark ring formed due to transmitted light from thin film.

Note :-

⇒ To find diameter, for both cases,

$$\boxed{D_n = 2r_n}$$

⇒ Haidinger fringes :

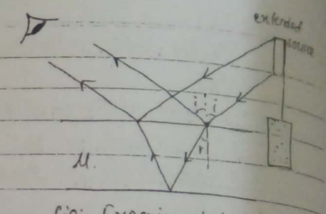
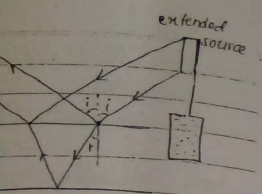


fig:- Experimental setup for formation the. Haidinger fringes

If the thickness of the film is large in thin film experiment, a very small change in 'r' will change the path difference appreciably. Here the ray through a plate appears as parallel beam and generally it is viewed by the eye as in fig.
Fringes are produced in this case due to superposition of rays, which are equally inclined to the normal. These inferring fringes pattern are known as fringes of equal inclination and are different from the Newton's ring. These fringes since were first obtained by Haidinger are called Haidinger fringes.

——x——x——

extended source



Experimental setup for
. Haidinger fringes

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Diffraction

we know, light travels in a straight path. But careful observation shows that the light bends. This phenomenon of bending (spreading) of light around the corners of an obstacles or small slit is called diffraction of light.

⇨ Differences betⁿ Interference & Diffraction of light.

	Interference of light.	Diffraction of light.
1.	It is the phenomenon of superposition of light waves coming from coherent sources.	→ It is the phenomenon of bending of light waves around the corners of an obstacles or small slit.
2.	At least two small slits are used for interference phenomenon.	→ Only one slit is sufficient for difraction phenomena.
3.	The central point is either dark or bright for this case.	→ The central point is always bright with max^m intensity called central maxima.
4.	The point of minima is perfectly dark.	→ The point of minima may not be perfectly dark.
5.	The intensity at points of maxima remains constant as given below.	→ The point of maxima decreases on eithaside of central maxima as given below.

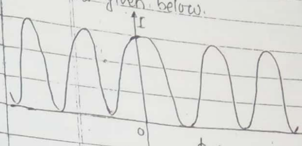

Fig: Intensity distribution in Young's double slit exp.

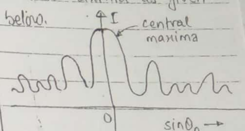

Fig: Intensity distribution due to diffraction of light from a grating.

⇨

	Fresnel's Diffraction	Fraunhofer Difraction
①	In this case, either source or screen or both are at infinete distance from given slit.	① In this case, source & screen are at infinite or appears as at infinite distance from given slit.
②	wave fronts are spherical;	② wave fronts are plane.
③	Mirrors or lens are not necessary to use for the modification of position of source or screen or diffraction pattern.	③ Mirror or lens are necessary to use for the modification of position of source & screen or diffraction pattern.

P.51

P.53

extended source

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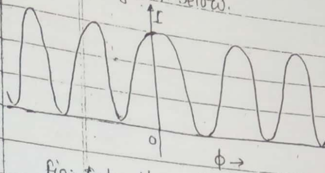
4. The point of minima is perfectly dark.	→ The point of minima may not be perfectly dark.
5. The intensity at points of maxima remains constant as given below.	→ The point of maxima decreases on eithside of central maxima as given below.



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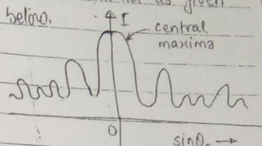

fig: Intensity distribution due to diffraction of light from a grating.

⇒

Fresnel's Diffraction	Fraunhofer Difraction
① In this case, either source or screen or both are at infinite distance from given slit.	① In this case, source & screen are at infinite or appears as at infinite distance from given slit.
② Wave fronts are spherical.	② Wave fronts are plane
③ Mirrors or lens are not necessary to use for the modification of position of source or screen or diffraction pattern.	③ Mirror or lens are necessary to use for the modification of position of source & screen or diffraction pattern.

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⇒ Fraunhofer Diffraction from a single slit :-

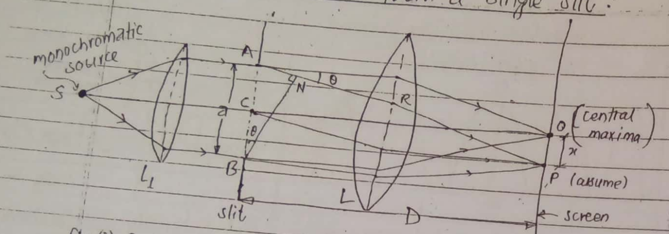


fig (i) Fraunhofer diffraction from a single slit.

P.52

When parallel beam of monochromatic light is allowed to fall on a small slit of width 'a', the light wave suffers diffractions and the diffraction pattern can be observed on the screen with the help of lens L as shown in fig (i).

As the path difference between the light waves from A & B of the given slit in the direction of incident light to the point 'O' is zero, the point 'O' has maximum intensity called central maxima.

let us consider the secondary light wave coming from extreme points A & B in the direction of 'θ' with incident light to the point P. The point 'P' represents point of secondary maxima or minima according to their path difference. for this let us draw RN ⊥ AR and take △ARN, we get,

$$\sin\theta = \frac{AN}{AB}$$

$$AN = AB\sin\theta$$

$$\therefore \boxed{\text{path difference } (AN) = a\sin\theta} \quad \text{------- (i)}$$

let this path difference is equal to 'λ' and the given slit is assumed to be divided into two equal parts AC & CB. Then, the path difference between the light waves coming from extreme point of each half to the point 'P' will be $\lambda/2$ and the point 'P' will give the secondary minima.

Similarly,

if the path difference is 2λ, and the given slit is assumed to be divided into four equal parts. Then the light waves coming from extreme point of each part to the point 'p' will be $\lambda/2$ and will get the point of secondary minima again. i.e. for every point on the upper half, there is corresponding point to the lower half also. and the general relation for the n^{th} secondary minima is given by the path difference $n\lambda$.

i.e. $\boxed{a\sin\theta_n = n\lambda}$ ------- (ii)

where $n = 1, 2, 3, \ldots$

Here, θ_n gives the direction of n^{th} secondary minima.

Similarly,

For n^{th} secondary maxima, the general relation can be expressed as,

$a\sin\theta_n = (2n+1)\lambda/2$ ----------(iii)

where $n = 1, 2, 3, 4, \ldots$

Here, θ_n gives the direction of n^{th} secondary maxima.

⇒ Width of Central Maxima ($2x$):

let point 'P' represents the point of 1^{st} secondary minima, then $n=1$ and above relation (ii) gives

$a\sin\theta = 1.\lambda \Rightarrow \boxed{\sin\theta = \frac{\lambda}{a}}$ ------(iv)

Again if D is the distance between screen and slit and 'x' is the distance of point 'P' from centre 'O', then we get,

$\sin\theta = \frac{x}{D}$ ---------(v) [θ is very small]

From (iv) and (v), we get,

$\frac{x}{D} = \frac{\lambda}{a}$

$\therefore \boxed{x = \frac{\lambda D}{a}}$

Here x is the half width of central maxima.

Then,

$\therefore$ width of central maxima $(2x) = \frac{2\lambda D}{a}$ #

Exceptional Case

If lens is very near to slit, i.e. $D \simeq f$,

where f = focal length of lens 'L'.

Then,

above relation becomes,

width of central maxima $(2x) = \frac{2\lambda f}{a}$.

Note:

The angular width of central maxima (2θ) can be calculated with the help of the relation given by:

$\sin\theta = \frac{\lambda}{a}$ or, $\sin\theta = \frac{x}{D}$

Here, θ represents the half angular width of central maxima.

⇒ Plane Diffraction Grating :

It consists of large no. of transparent parts separated by opaque surface. It is also called transmission gratting.

When parallel beam of monochromatic light is allowed to fall on a grating, the light wave suffers diffraction and diffraction pattern can be observed on the screen with the help of lens L.

(for fig: refer book)

As the path difference between the light waves coming from extreme points of each slit of grating in the direction of incident light to the point O is zero, the point O has maximum intensity called zero maxima.

P.54

Let us consider the light waves coming from extreme point A and C of the slits (of a grating) in the direction θ with the incident light to the point P. The path difference between the light waves is calculated as $(a+b)\sin\theta$. If this path difference equals to $n\lambda$, we will get n^{th} principal maxima.

i.e. $\boxed{(a+b)\sin\theta_n = n\lambda}$ ---------- (i)

$n = 1, 2, 3, \ldots$

Here, θ_n gives the direction of n^{th} principal maxima and, $(a+b)$ is called grating element i.e. a = width of transparent part & b = width of opaque part.

If N is the no. of lines per inch in the grating, then, grating element $(a+b) = \frac{1}{N}$ inch $= \frac{2.54}{N}$ cm.

Let θ_n is increased to $\theta_n + d\theta$ and the path difference between the light from A & C is assumed to be increased by λ/N.

[where N' = total no. of lines in the grating].

Then,

The path difference between the light waves from extreme points of whole grating becomes

$\frac{\lambda}{N} \times N'$

If this grating is assumed to be divided into equal parts, the path difference between the light wave from extreme points of each half will be $\lambda/2$ and we will get first secondary minima after n^{th} principal maxima.

Similarly,

If the path difference between the extreme points A & C. is $\frac{2\lambda}{N'}$, $\frac{3\lambda}{N'}$, $\frac{4\lambda}{N'}$, ... etc.

then we will get 2^{nd}, 3^{rd}, 4^{th}, ... etc. secondary maxima. after n^{th} principal maxima as fig(ii).

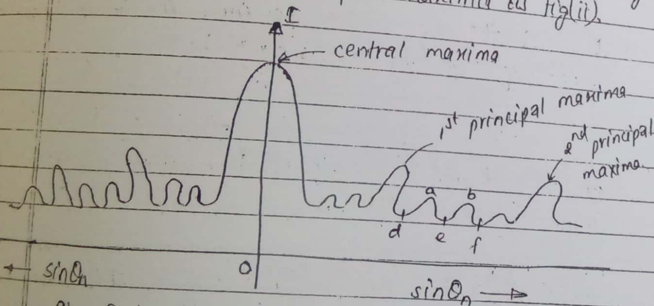


fig: Intensity distribution due to diffraction from a grating.

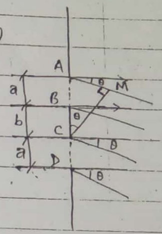
Here, a, b → Secondary maxima.

d, e, f → secondary minima

Note: For Double slit Experiment :

Path difference (AM) = AC $\sin\theta$

= $(a+b)\sin\theta$



But, for principal maxima or primary or interference maxima,

Path difference = $n\lambda$

so,

$\therefore \boxed{(a+b)\sin\theta = n\lambda}$ for n^{th} principal maxima.

Similarly, for principal minima,

$$\boxed{(a+b)\sin\theta_n = (2n+1)\frac{\lambda}{2}}$$

—x—x—

POLARIZATION

⇒ Polarization of light :-

When the direction of vibration of light wave is converted into one plane, then the light wave is said to be polarized. This phenomenon explains the transverse nature of light wave.

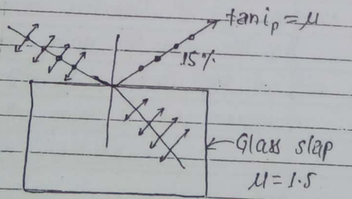
↕↕ ↕ ↕ ⇒ unpolarized or ordinary light wave.

↕↕↕↕ ⇒ polarized light wave with the directⁿ vibration parallel to the plane of the paper.

• • • • ⇒ polarized light with the direction of vibration perpendicular to the plane of the paper.

P.56

⇒ Brewsters law [Polarization of Reflection] :-



When the ordinary light is incident on a glass plate with angle of

fig:- Polarization by reflection.

polarization (i_p), a part of its perpendicular component is reflected from the glass plate & its remaining component with all parallel component is refracted from the glass plate as shown in fig.

Brewster's law states that : "the tangent of angle of polarization is numerically equal to refractive index (μ) of the glass plate."

i.e. $\tan i_p = \mu$

$\therefore \frac{\sin i_p}{\cos i_p} = \mu$ ------ (i).

Again,

From snell's law, we have,

$\frac{\sin i_p}{\sin r} = \mu$ ------ (ii).

From eqⁿ (i) & (ii), we get,

$\cos i_p = \sin r$

or, $\sin\left(\frac{\pi}{2} - i_p\right) = \sin r$

or, $\pi/2 - i_p = r$

$\therefore \boxed{i_p + r = \pi/2}$

i.e. sum of angle of polarization "i_p" & angle of refraction is $\pi/2$.

Malus Law :-

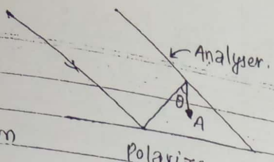


fig (i).

When the light wave from a polariser is incident on the next polariser (analyser) with angle of polarization, the intensity of light wave coming from the analyser depends on the value of angle between polarizer and analyser.

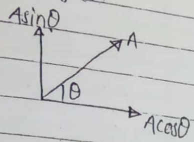


fig: (ii)

"Malus law" states that 'Intensity of transmitted light from the analyser is directly proportional to the square of the cosine of the angle betn polarizer and analyser."

If θ is the angle between polariser and analyser, 'A' is the amplitude of incident light to the analyser. then according to the Malus Law,

$$I \propto \cos^2\theta$$

$$\therefore I = I_0 \cos^2\theta$$

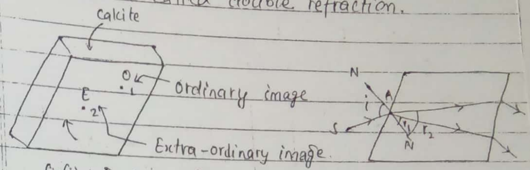
This is the required expression for the Malus law. where,

I = intensity of transmitted light from the analyser.

I_0 = maxm intensity of light incident to the analyer.

Double Refraction :

When the light wave is passed through the double refracting uniaxial crystal like (calcite or quartz), we will get two refracted rays. This phenomenon is called double refraction.



fig(i) : Double refraction.

we have,

$$\mu_E = \frac{\sin i}{\sin r_2} \neq \text{constant.}$$

$$\mu_0 = \frac{\sin i}{\sin r_1} \neq \text{costant.}$$

where μ_0 and μ_E are the refractive index for ordinary rays and extra-ordinary rays respectively.

Note: For calcite $\mu_0 > \mu_E \Rightarrow$ -ve crystal

For quartz $\mu_E > \mu_0 \Rightarrow$ +ve crystal.

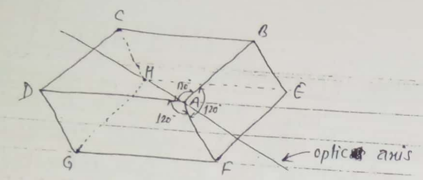


fig:- Rhombohedron shape of calcite.

Here, A & H ⇒ blunt corners.

⊛ Optic axis :

Any line passing through the blunt corners A & B is called optic axis. The line parallel to the optic axis also gives the direction of optic axis.

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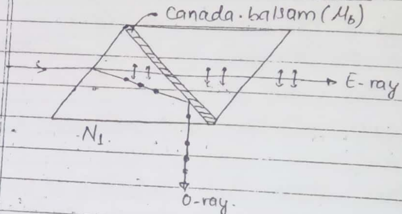
→ When the light is incident parallel to the direction of optic axis it doesnot split, i.e. The ordinary rays and extra-ordinary rays moves with same velocity in same direction.

→ When the light is incident perpendicular to the direction of optic axis, the ordinary ray and extra-ordinary ray move in same direction with different velocities.

Nicol Prism :-

It is made of calcite crystal with length three times of its breadth and it is used to produce polarised light as well as to detect (analyse) it.

1. Nicol Prism as a Polarizer :-



$\mu_o = 1.65$

$\mu_b = 1.55$

$\mu_E = 1.48$

Fig(i) Nicol prism as a polarizer.

When the light wave is incident to the Nicol prism, the light splits into ordinary ray and extra-ordinary ray. As the refractive index of calcite crystal for O-ray is 1.65 and for E-ray is 1.48, ~~and~~ so, the refractive index for canada balsam used in Nicol prism is 1.55, the ordinary ~~and~~ ray is eliminated, total internal reflection and only E-ray is passed through the Nicol prism as in fig (i).

Hence, Nicol prism N_1 is used as a polariser.

2. Nicol Prism as an Analyser :-

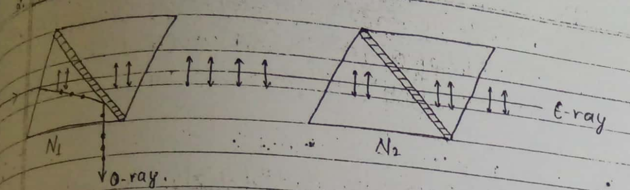


Fig (i): [N_1 & N_2 are parallel to each other]

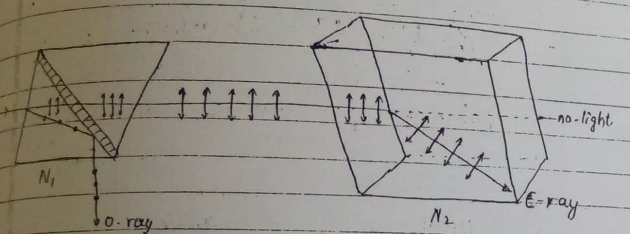


Fig (ii) Nicol Prism : N_1 and N_2 are perpendicular to each other.

When Nicol prism N_1 and N_2 are parallel to each other, we will get the light transmitted from N_2 as shown in fig(i). If the Nicol prism N_1 rotates, the intensity of light coming from N_2 gradually decreases and it becomes zero, when N_1 is perpendicular to N_2 as in fig(ii). From this experiment, it is clear that the light wave coming from N_1 has only one direction or it is polarised which is detected by N_2. So, Nicol prism N_2 works as an Analyser.

Quarter Wave Plate :

It is made up of double refracting uniaxial crystal like calcite or quartz whose reflecting face is cut parallel to the direction of optical axis. If 't' is the thickness of quarter wave plate and μ_o & μ_E are the refractive indices for ordinary and extra-ordinary rays respectively, then path difference betn them is found that:

$(\mu_o - \mu_E)t \rightarrow$ for -ve plate (calcite)

and,

$(\mu_E - \mu_o)t \rightarrow$ for +ve plate (quartz)

But,

for quarter wave plate, path diff $= \lambda/4$

$\therefore$ $\boxed{(\mu_o - \mu_E)t = \frac{\lambda}{4}}$ for -ve crystal.

$\boxed{(\mu_E - \mu_o)t = \lambda/4}$ for +ve crystal.

If the plane polarized light whose plane of vibration is inclined at an angle of 45° to the optic axis, is incident on a quarter wave plate the emergent light is circularly polarised.

➾ Half Wave Plate :-

It is made up of double refracting uniaxial crystal like calcite or quartz whose reflecting face is cut parallel to the direction of an optic axis. If 't' is the thickness of half wave plate and, μ_0 and μ_E are the refractive indices for ordinary and extra-ordinary rays respectively, then path difference betn them is found to be :

$(\mu_0 - \mu_E)t$ for -ve plate (calcite)

and,

$(\mu_E - \mu_0)t$ for +ve plate (quartz)

But for half wave plate,

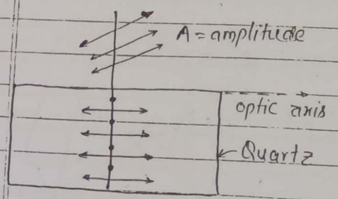
path difference $= \lambda/2$

$\therefore \boxed{(\mu_0 - \mu_E)t = \lambda/2}$ → for -ve crystal

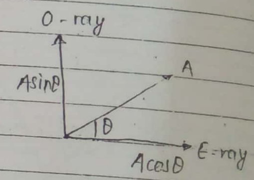
$\therefore \boxed{(\mu_E - \mu_0)t = \frac{\lambda}{2}}$ → for +ve crystal.

If the plane-polarized light whose plane of vibration is inclined at an angle 45° to the optic axis, is incident on a half wave plate, the emergent light is plane polarised.

➾ Plane, Circularly and Elliptically polarized light :



fig(i) N_1



fig(ii).

When the light wave coming from Nicol prism N_1 is incident on a quartz plate in the direction perpendicular to the direction of optic axis. as in fig(i). where A is the amplitude of incident light and θ is the angle made by the direction of vibration of light with optic axis. The light wave splits into two different rays [O-ray : Ir. E-ray] that move in same direction with different velocities. We can resolve the amplitude (A) of incident light into two

rectangular components $A\sin\theta$ [amplitude of O-ray] and $A\cos\theta$ [amplitude of E-ray] as in fig (ii).
If ϕ is the phase difference between O-ray and E-ray, their displacements can be expressed as:

$$x = A\sin\theta \cdot \sin\omega t \quad \text{------(i) [O-ray]}$$

$$y = A\cos\theta \cdot \sin(\omega t + \phi) \quad \text{----(ii) [E-ray]}$$

let,

$A\sin\theta = a$. and $A\cos\theta = b$.

Then, eq^n (i) becomes,

$$x = a\sin\omega t$$

$$\Rightarrow \sin\omega t = x/a \quad \text{--------(iii)}$$

i.e. $\cos\omega t = \sqrt{1-\sin^2\omega t} = \sqrt{1 - x^2/a^2}$ ---(iv).

similarly, Eq^n (ii) can be expressed as:

$$y = b\sin(\omega t + \phi)$$

or, $y = b\,[\sin\omega t\cos\phi + \cos\omega t\sin\phi]$

or, $y = b\left[\frac{x}{a}\cos\phi + \sqrt{1-\frac{x^2}{a^2}}\cdot\sin\phi\right]$ Using (iii) & (iv).

or, $\frac{y}{b} = \frac{x}{a}\cos\phi + \sqrt{1-\frac{x^2}{a^2}}\,\sin\phi$

or, $\frac{y}{b} - \frac{x}{a}\cos\phi = \sqrt{1-\frac{x^2}{a^2}}\,\sin\phi$

Squaring both sides, we get

$$\left[\frac{y}{b} - \frac{x}{a}\cos\phi\right]^2 = \left(1-\frac{x^2}{a^2}\right)\sin^2\phi$$

or, $\frac{y^2}{b^2} - 2\frac{y}{b}\frac{x}{a}\cos\phi + \frac{x^2}{a^2}\cos^2\phi = \sin^2\phi - \frac{x^2}{a^2}\sin^2\phi$

or, $\frac{y^2}{b^2} - \frac{2xy}{ab}\cos\phi + \frac{x^2}{a^2}[\cos^2\phi + \sin^2\phi] = \sin^2\phi$

or, $\boxed{\frac{y^2}{b^2} - \frac{2xy}{ab}\cos\phi + \frac{x^2}{a^2} = \sin^2\phi}$ -------(v)

Case I

If $\phi = 0$, then eq^n (v) becomes,

$$\frac{y^2}{b^2} - \frac{2xy}{ab} + \frac{x^2}{a^2} = 0$$

or, $\left(\frac{y}{b} - \frac{x}{a}\right)^2 = 0$

$\frac{y}{b} - \frac{x}{a} = 0$

$\therefore \frac{y}{b} = \frac{x}{a}$

$\boxed{y = \frac{b}{a}x}$ -----(vi).

which is eq^n of straight line.

i.e. The emergent light from quartz plate is plane polarized if the phase difference (ϕ) bet^n

O-ray and E-ray is zero.

Case II

when $\phi = \pi/2$, and $a=b$, then eqⁿ (v) becomes,

$$\frac{y^2}{a^2} - \frac{2xy}{a^2}\cos\frac{\pi}{2} + \frac{x^2}{a^2} = \sin^2\frac{\pi}{2}.$$

or, $$\frac{y^2}{a^2} + \frac{x^2}{a^2} = 1.$$

or, $$\boxed{x^2+y^2 = a^2} \quad \text{------- (vii)}.$$

which is the eqⁿ of circle.

i.e. The emergent light from quartz is circularly polarized when phase difference betⁿ O-ray and E-ray is $\pi/2$ and amplitude of both of rays are same i.e. $a=b$.

Case III

when $\phi = \pi/2$ and $a \neq b$. then eqⁿ (v) becomes:

$$\frac{y^2}{b^2} - \frac{2xy}{ab}\cdot\cos\frac{\pi}{2} + \frac{x^2}{a^2} = \sin^2\frac{\pi}{2}.$$

or, $$\boxed{\frac{y^2}{b^2} + \frac{x^2}{a^2} = 1} \quad \text{------ (viii)}$$

which is eqⁿ of ellipse.

i.e. The emergent light from quartz is elliptically polarized when $\phi = \pi/2$ and $a \neq b$.

Optical Activity & Specific Rotation :-

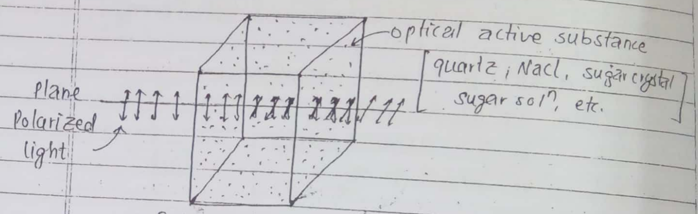


fig (i) Optical Activity.

When the plane polarized light is passed through the optical active substance like quartz, NaCl, sugar crystal, sugar solⁿ, etc, the direction of vibration of the polarized light rotates as in fig(i). This phenomenon is known as optical activity. The substance that rotates the direction of vibration of plane polarized light is called optically active substance.

let θ = Rotation produced in plane polarized light by optically active substance.

l = length of tube having optical active substance. (gm/cc).
C = density of concentration of the optical active substance.
Then we will get.

$$\theta \propto l \text{ ------- (i)}.$$

and $\theta \propto C$ ------ (ii).

i.e. from above relation (i) & (ii), we get.

$$\theta \propto lC$$

$\therefore \boxed{\theta = SlC}$ [In MKS system)

where S is a constant called specific rotation with constant temperature (°C) & wave length(λ).

But, In C.G.S. system.

The specific rotation produced by a decimeter (10cm) long column of the liquid containing 1 gm of the optical active substance in one cc of the solution.

Then:.

$$S = \frac{10\theta}{lC}$$

$\therefore \boxed{10\theta = SlC}$ Imp (Numerical).

#.

—— X —— X ——

LASER

LASER stands for "Light Amplification by Stimulated Emission of Radiation".

It is coherent, monochromatic and unidirectional beam of light that can travel long distance without loss of its intensity.

An atom can emit radiation by two different methods.
1. Spontaneous emission.
2. Stimulated emission.

⇒ Induced Absorption.

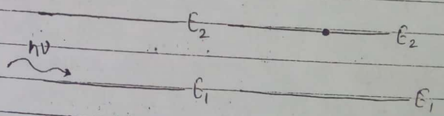


fig: Induced absorption

⇒ Spontaneous emission of radiation:

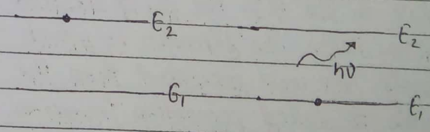


fig:- spontaneous emission.

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⇒ Stimulated Emission of radiation :-

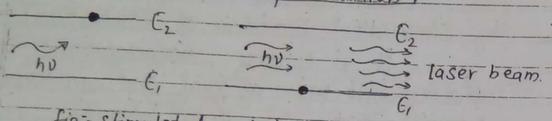

fig:- stimulated emission

Hence, LASER is produced due to the stimulated emission of radiation.

⇒ Population Inversion :-

we know that population is maximum in the ground state and decrease exponentially as one goes to higher energy states. Therefore, whenever an electromagnetic wave is incident, there is net absorption of the radiation. For lasing action, it is necessary that stimulated emission predominate over spontaneous emission. This happens only if no. of electrons in ground state is less than the no. of electrons in higher state. Hence, The situation in which the upper levels are more populated than the lower levels is called population inversion.
This occurs when higher levels are at metastable state in which the electron remains longer than usual.

⇒ Pumping :

The state of population inversion canot be achieved thermolly. Generally, population inversion is achieved by exciting the medium with suitable form of energy. Such a phenomenon of population inversion occurs with a process called pumping.
Examples of pumping are :

i) Optical pumping (ii) chemical pumping.
iii) Electric pumping (iv) Direct conversion.
v) Inelastic atom-atom collision, etc.

Optical Pumping :-

A light source is used to supply luminous energy in optical pumping. Usually this energy comes in the form of light flashes. This method was used in rubby laser first by T.H. Maimam and widely used in solid state lasers. In such case the lasing material is placed inside a helical xenon flash lamp.

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He-Ne Laser

The first gas laser to be operated successfully was the He-Ne laser. A Javan in 1960, operated the first continuous wave laser using the mixture of these gases in a discharge tube.

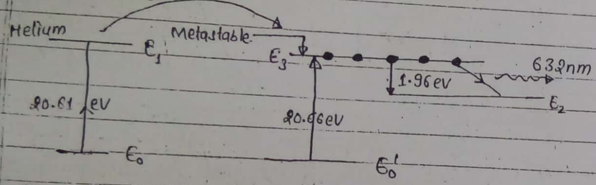
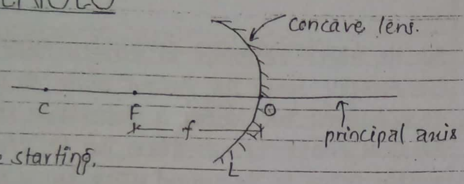


fig:- Energy levels in the He-Ne laser.

The He-Ne laser consists of a long and narrow discharge tube of diameter about 2-8 mm and length 10-100cm. The lasing material is the mixture of the gases with a concentration of about 15% helium and 85% Neon. The mixture works as lasing material because of same compatible properties of the two gases. The electrodes in the discharge tube are connected to a high voltage source of few KV d.c. So an electric discharge takes place within the gas. With this high voltage some of the He atoms are raised to a metastable state at $E_1 = 20.61eV$ above the ground state as shown in fig. It so happens that Ne has a metastable state at nearly the same energy, $E_3 = 20.66eV$. The He atom donot quickly return to the ground state by spontaneous emission. Rather it transfers the energy to Ne-atoms during collision. With such collision the energy of excited He atoms will be transfered and it drops to ground state. However, getting the excess energy Ne atom is excited to the state E_3. The small difference of 0.05 eV is supplied the kinetic energy of the atoms. In this way the higher state E_3 of Neon, becomes the metastable state, than E_2. Therefore the population for inversion is achieved for Ne-atoms. Hence the lasing action takes place by stimulated emission between the E_3 and E_2 states of Neon. The laser light emitted is of about 632.8 nm.

x x

LENSES



Before starting,
Here,

L is concave lens.
O is optical center of lens.
F is the principal focus.
C is the center of curvature.
f is the focal length which is the distance between O and F.

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Refraction Through a lens:-

The most important, simple optical device is the thin lens. A lens is a portion of a transparent refracted medium. A thin lens is usually round and its two faces are each a portion of a sphere. Although cylinder surfaces are also possible, we will concentrate on spherical ones. Such lens is buit up from a series of small angled prism whose refracting angle changes continuously.

let us consider a thin lens of material of

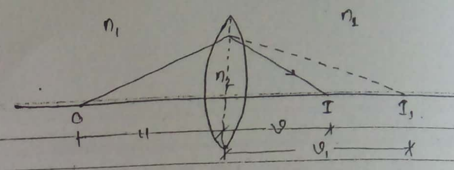


fig:- Refraction through a thin lens.

refractive index n_2 seperates a medium of refractive index n_1 on its two sides. The radius of curvature of the co-axial refracting surfaces is R_1 and R_2. Consider a point 'O' be kept on the principle axis at a distance 'u' from the 1st refracting surface as shown in fig.

When light from the object O falls on the 1st refracting surface of the lens the image I_1 is formed at a distance v_1 from 1st refracting surface. So,

$$\frac{n_2}{v_1} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \quad \text{-----(i)}$$

This image I_1 acts as virtual object in medium of refractive index n_2 for the 2nd refracting surface and forms the final image I at a distance v from the second refracting surface in medium of refractive index n_1. Again

$$\frac{n_1}{v} - \frac{n_2}{v_1} = \frac{n_1 - n_2}{R_2} \quad \text{-----(ii)}$$

Adding (i) & (ii), we get.

$$\frac{n_1}{v} - \frac{n_1}{u} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or, $\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

or, when the lens is in air,

$$\frac{1}{v} - \frac{1}{u} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{ -----(iii)}$$

where n is the refractive index of the material of the lens with respect to air.

Eq^n (ii) holds true for paraxial rays for which the angles made by the rays are very small, for thin lenses.

But we know that, $\frac{1}{v} - \frac{1}{u} = f$ (focal length)

$$\Rightarrow \frac{1}{v} - \frac{1}{u} = f$$

Then eq^n (iii) becomes,

$$f = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore \boxed{f = +(n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}$$

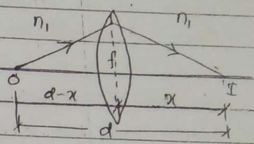
This is called lens maker formula.

⇒ **Least Possible distance between an object and its Real Image**

Consider a thin convex lens of focal length 'f' as shown in fig.

Then,

image distance $(v) = x$



object distance $(u) = -(d-x)$

we know,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or, $\frac{1}{x} - \frac{1}{-(d-x)} = \frac{1}{f}$

or, $\frac{d-x+x}{x(d-x)} = \frac{1}{f}$

or, $fd = dx - x^2$

or, $x^2 - dx + fd = 0$.

This is quadratic in x. so, its solution is :-

$$x = \frac{-(-d) \pm \sqrt{(-d)^2 - 4fd}}{2 \times 1}$$

or, $x = \frac{d \pm \sqrt{d^2 - 4fd}}{2}$

For Image to be real, the distance x should be real

This condition is satisfied if

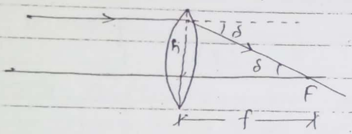
$$d^2 - 4fd \geq 0$$

or, $d^2 \geq 4fd$

or, $d \geq 4f$

Thus, d should be greater than 4f or at least should be equal to 4f for the image to be real. Therefore, the minimum distance between real object and real image is 4f.

⇒ Deviation by a lens:



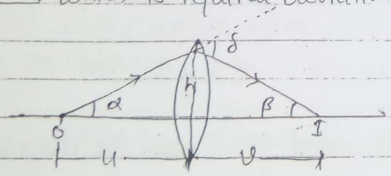
from fig:

$$\tan\delta = \frac{h}{f}$$

Since δ is very small, $\tan\delta \approx \delta$.

$\therefore \boxed{\delta = \frac{h}{f}}$ —— ① which is required deviation.

Similarly,



For this fig: we have,

$$\delta = \alpha + \beta \quad \text{------ (i)}$$

But,

$$\tan\alpha = \frac{h}{-u}$$

since α is very small, $\tan\alpha \approx \alpha$.

$$\therefore \alpha = \frac{h}{-u} \quad \text{----- (ii)}$$

and, $\tan\beta = \frac{h}{v}$ $(\because \tan\beta \approx \beta)$

$$\text{or, } \beta = \frac{h}{v} \quad \text{----- (iii)}$$

From (i), (ii) & (iii), we get,

$$\delta = \frac{h}{-u} + \frac{h}{v}$$

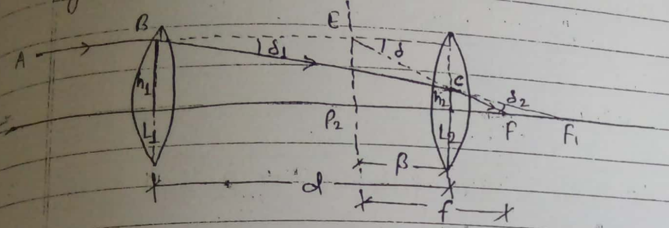
$$\text{or, } \delta = h\left(\frac{1}{v} - \frac{1}{u}\right)$$

or, $\boxed{\delta = \frac{h}{f}}$ —— ②.

Hence,

From ① and ②, it is clear that deviation produced by a lens is independent of the object position (u).

Combination of Two thin lenses Separated by a finite distance :



Consider two thin lenses L_1 and L_2 having focal lengths f_1 and f_2 are placed coaxially as shown in fig. let 'd' be the separation of two lenses. Then, for the monochromatic ray AB parallel to principal axis, it follows the path as shown in fig.

If δ_1 is the deviation of lens L_1, then.

$$\delta_1 = \frac{h_1}{f_1} \quad \text{------(i)}.$$

If δ_2 is the deviation of lens L_2, then.

$$\delta_2 = \frac{h_2}{f_2} \quad \text{------(ii)}.$$

Now, if we produce the 1st incident ray AB and final emergent ray CF, they will intersect at a point E. Thefore if a thin lens is placed at the same ~~direction~~ ... it will produce the same

If 'f' is the focal length of equivalent lens. and δ be the deviation produced, then.

$$\delta = \frac{h_1}{f} \quad \text{--------(iii)}$$

From geometry, we can write,

$$\delta = \delta_1 + \delta_2$$

or, $$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2} \quad \text{-------(iv)}$$

Now, from similar triangle BL_1F_1 and CL_2F_1.

$$\frac{BL_1}{L_1F_1} = \frac{CL_2}{L_2F_1}$$

or, $$\frac{h_1}{f_1} = \frac{h_2}{f_1 - d}$$

or, $$h_2 = \frac{h_1(f_1 - d)}{f_1} \quad \text{-------(v)}.$$

Using (v) in (iv), we get.

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1(f_1 - d)}{f_1 f_2}$$

or, $$\frac{1}{f} = \frac{1}{f_1} + \frac{(f_1 - d)}{f_1 f_2}.$$

or, $$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}} \quad \text{----(vi)}$$

which provides the equivalent focal length of thin lenses separated by a distance d.

Determination of Principal points

For this we proceed from eqⁿ (vi).

Let β be the distance between the imaginary equivalent lens and the second lens L_2.

From similar triangles EP_2F and CL_2F,

$$\frac{P_2F}{L_2F} = \frac{EP_2}{CL_2}$$

or, $$\frac{f}{f-\beta} = \frac{h_1}{h_2}$$

Since distance β is in the left side of L_2, following the sign convention, we can rewrite,

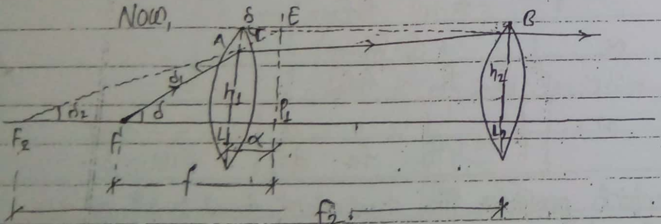
$$\frac{f}{f+\beta} = \frac{h_1}{h_2} = \frac{h_1 f_1}{h_1(f_1-d)}$$

or, $$\frac{f}{f+\beta} = \frac{f_1}{(f_1-d)}$$

or, $$ff_1 - fd = ff_1 + f_1\beta$$

or, $$\boxed{\beta = -\frac{fd}{f_1}} \quad \text{------ (1)}$$

Now,



From similar triangles EP_1F and AL_1F.

$$\frac{EP_1}{P_1F} = \frac{AL_1}{L_1F}$$

or, $$\frac{h_2}{f} = \frac{h_1}{f-\alpha} \quad \text{------ (vii)}$$

also, From similar triangles, BL_2F_2 and AL_1F_2.

$$\frac{BL_2}{L_2F_2} = \frac{AL_1}{L_1F_2}$$

or, $$\frac{h_2}{f_2} = \frac{h_1}{f_2-d} \quad \text{------ (viii)}$$

Solving (vii) & (viii), we get,

$$\frac{h_1 f_2}{(f_2-d)} = \frac{h_1 f}{(f-\alpha)}$$

or, $$ff_2 - \alpha f_2 = ff_2 - fd$$

or, $$\boxed{\alpha = \frac{fd}{f_2}} \quad \text{------ (2)}$$

Hence, Equation (1) and (2) denotes the position of principal points.

In case if the two lenses are in contact, i.e. $d=0$, the equivalent focal length is given by,

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}$$

Power of a lens :-

Power of a lens is its magnifying or diverging capacity. It is defined as the reciprocal of focal length expressed in meter.

i.e. Power (P) = $\frac{1}{\text{focal length (f)}}$

The unit of lens power is diopter (D).

→ The power of a converging lens is positive, whereas that for a diverging lens is negative.

Cardinal Points

There are six different points which are considered as reference points to measure various distances in the refraction through a thick lens and in a system of co-axial lenses. These six points of reference are called cardinal points of an optical system. They are:

a) Two principal focal points
b) Two principal points
c) Two nodal points.

Chromatic Aberration

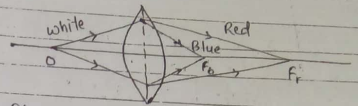


fig: Chromatic aberration produced by a lens.

A lens is composed of a large number of prism of varying refracting anglas placed one after another. The refracting angle of the prism goes on decreasing at a uniform rate from its centre to outwards in convex lens. If a ray, consisting of various colours is allowed to fall on a lens, it is dispersed into its constituent colours. The deviation for different colours is different. Therefore the dispersed ray focous at different points in the axis and there will be coloured fringes in the image. This phenomenon of formation of different images is known as chromatic aberration.

Consider a chromatic object O in the principal axis of a lens as shown in figure. The ray after refracting through the lens disperse into the constituent colours let F_r be focus for red and F_b be that for blue colour with focal length f_r and f_b respectively. $(f_r - f_b)$ is a measure of longitudinal chromatic aberration.

Since the focal length depends upon the colour or wavelength of light, the images of different colours are magnified to different extents. This defect is called lateral chromatic aberration. The combined effect causes a blurred image. This defect in colour produced by a lens is known as chromatic aberration.

Now, The focal length of a lens is given by,

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or, $\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{f(n-1)}$ ------ (i)

So,

$$\frac{1}{f_b} = (n_b-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or, $\frac{1}{f_b} = \frac{n_b-1}{f(n-1)}$ using (i) ---- (ii)

Similarly,

$\frac{1}{f_r} = \frac{n_r-1}{f(n-1)}$ ------- (iii)

where f and n are focal length and refractive index of mean colour.

From (ii) & (iii), we can write,

$$\frac{1}{f_b} - \frac{1}{f_r} = \frac{n_b - n_r}{(n-1)f}$$

or, $\frac{f_r - f_b}{f_r f_b} = \frac{\omega}{f}$ ------- (iv)

where ω is the dispersive power of the material of the lens.

Now,

$f_r f_b = f^2$ and $(f_r - f_b)$ = longitudinal chromatic aberration.

So, from (iv)

$\boxed{f_r - f_b = \omega f}$ ------- (v)

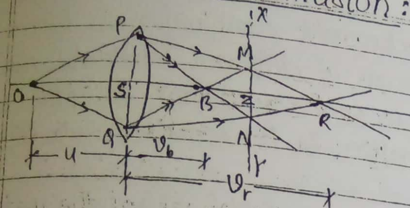
Therefore,

longitudinal chromatic aberration = dispersive power * mean focal length.

Note:

Single lens cannot form an image free from chromatic aberration.

Circle of least Confusion:



let $MN = d$

$PQ = D$

Consider a lens of focal length 'f' as shown in fig. Let a white light coming from O is refracted through the lens due to which blue and red image will form at B and R resp. as shown in fig. If XY screen is placed as shown, the image of least chromatic aberration is observed in the screen. Then,

u = object distance

v_b = image distance for blue

v_r = image distance for red.

$MN = d$ be the diameter of circle of least confusion.

we have, $\frac{1}{v_b} - \frac{1}{u} = \frac{1}{f_b}$ ------(i)

and,

$\frac{1}{v_r} - \frac{1}{u} = \frac{1}{f_r}$ ------(ii)

Subtracting (ii) from (i), we get.

$$\frac{1}{v_b} - \frac{1}{v_r} = \frac{1}{f_b} - \frac{1}{f_r}$$

or, $\frac{v_r - v_b}{v_b \cdot v_r} = \frac{f_r - f_b}{f_r f_b}$

but $v_r v_b \simeq v^2$ and $f_r f_b \simeq f^2$.

or, $\frac{v_r - v_b}{v^2} = \frac{f_r - f_b}{f^2}$

or, $v_r - v_b = \frac{v^2 \omega f}{f^2} = \frac{\omega v^2}{f}$ ------(iii)

From geometry, of similar triangles PQR & MNR.

$\frac{SR}{PQ} = \frac{ZR}{MN}$ ------(iv)

Also, from similar triangles PBQ and MNB.

$\frac{SB}{PQ} = \frac{ZB}{MN}$ ------(v).

Adding (iv) and (v), we get.

$\frac{SR + SB}{PQ} = \frac{ZR + ZB}{MN}$

or, $\frac{v_r + v_b}{D} = \frac{v_r - v_b}{d}$

where, D is the diameter of the lens.

Since, $v_r + v_b = 2v$ and $v_r - v_b = \frac{\omega v^2}{f}$

$\frac{2v}{D} = \frac{\omega v^2}{fd}$

or, $d = \frac{D\omega . v}{2f}$

For a parallel beam of the incident light $v = f$.

$\therefore \boxed{d = \frac{1}{2} . D . \omega}$

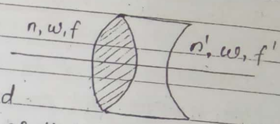
Hence, when a parallel beam of light is incident on the lens, diameter of circle of least confusion depends on the diameter of lens (or lens aperture) and the dispersive power of the material of the lens but is independent of the focal length of the lens.

Achromatism

The phenomenon of elimination of the defect chromatic aberration is known as achromatism. The lenses used for achromatism, in general have different refractive indices. ∴ Normally, one lens is converging and other is diverging and they are cemented together. Such type of combination is called as achromatic doublet or colour corrected lens.

Achromatism of two lenses in Contact:

The achromatic combination is made by placing two lenses of different materials and suitable focal lengths in contact, so that the focal length of the combination is the same for both the colours.



Consider the two lenses have the mean focal lengths f and f' and mean refractive index of their materials as n and n' respectively. Let f_b, f_r and f_b', f_r' are the corresponding colour focal lengths of the lenses.

We know, the lens maker's formula for a lens is:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Differentiating,

$$-\frac{df}{f^2} = dn\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or, $$-\frac{df}{f^2} = \frac{dn}{(n-1)f} \quad \text{-----------} \quad (i)$$

where dn is the change in refractive index $(n_b - n_r)$.

Hence,

Dispersive power $(\omega) = \frac{dn}{n-1} = \frac{n_b - n_r}{n-1}$

$$\therefore \; -\frac{df}{f^2} = \frac{\omega}{f} \quad \text{-------- (ii)}$$

when two lenses of focal lengths f and f' are placed in contact to form an achromatism, then the focal length F of the combination is given by,

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f'}$$

Upon differenting,

$$-\frac{dF}{F^2} = -\frac{df}{f^2} - \frac{df'}{f'^2}$$

To bring different colours into focus at a point;
dF, the change in focal length of the combination should be zero. So,

$$\frac{df}{f^2} + \frac{df'}{f'^2} = 0$$

$$\therefore \; \boxed{\frac{\omega}{f} + \frac{\omega'}{f'} = 0}$$

which gives condition of achromatism.
where,
ω and ω' are the dispersive powers of the materials of the two lenses and are positive quantity.

Achromatism of two lenses separated by a finite distance :

we know, the equivalent focal length for the combination of two ~~length~~ lenses of the focal lengths f_1 and f_2 is :

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

where, x is the distance of separation of two co-axial lenses.
differentiating (i), we get,

$$-\frac{df}{f^2} = -\frac{df_1}{f_1^2} + -\frac{df_2}{f_2^2} - x\left(\frac{df_1}{f_1^2 \cdot f_2} - \frac{df_2}{f_1 \cdot f_2^2}\right)$$

But for achromatism,

$$df = 0.$$

also, $-\frac{df_1}{f_1} = -\frac{df_2}{f_2} = \omega$ for lenses of same materials.

where, ω is the dispersive power of the lens material.
Then,

above eq^n reduces to,

$$\frac{\omega}{f_1} + \frac{\omega}{f_2} - x\left(\frac{\omega}{f_1 f_2} + \frac{\omega}{f_1 f_2}\right) = 0$$

or,

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{2x}{f_1 f_2}$$

$$\therefore \; \boxed{x = \frac{f_1 + f_2}{2}}$$

#

Photons and Matter Waves.

de-Broglie's Relation
[Wave-particle duality].

According to the de-Broglile, the small particles like electron also exihibit the wave nature. This wave is called matter wave or de-Broglie's wave.

Now,

According to quantum theory of radiation, energy of photon is,

$$E = h\nu$$

i.e. $E = \frac{hc}{\lambda} \longrightarrow (i)$.

Where,

ν = frequency of radiation

c = velocity of radiation (light)

$= 3 \times 10^{8}$ m/s.

h = planck's constant $= 6.628 \times 10^{-34}$ Js

λ = wave length of the radiation.

Again,

From Einstein's mass energy relation,

$$E = mc^{2} \longrightarrow (ii)$$

From above eqns (i) & (ii), we can write.

$$mc^2 = \frac{hc}{\lambda}$$

$$\therefore \boxed{\lambda = \frac{h}{mc}} \longrightarrow \text{(iii)}$$

Let, small particles (like electron) of mass 'm' is moving with velocity 'v', then above relation for this particles can be expressed as,

$$\lambda = \frac{h}{mv}$$

$$\therefore \boxed{\lambda = \frac{h}{p}} \longrightarrow \text{(iv)}$$

This relation is known as de-Broglie's relation, where $p = mv$ is the linear momentum of the electron and λ is the wave length of the wave associated with it. [i.e. matter wave]

Note :-

(i) $E = h\nu = \frac{h}{2\pi} \cdot 2\pi\nu$

$$\therefore \boxed{E = \hbar\omega}$$

$\hbar$ = planck's constant per cycle = $\frac{h}{2\pi}$

ω = angular frequency

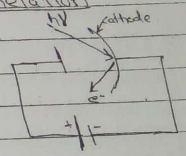
(ii) From de-Broglie's relation,

$$\lambda = \frac{h}{p}$$

$$p = \frac{h}{\lambda} = \frac{h}{2\pi} \cdot \frac{2\pi}{\lambda}$$

$$\therefore \boxed{p = \hbar k}$$

(iii) Einstein's photoelectric relation



$$h\nu = \phi + (K.E)_e$$

or, $h\nu = h\nu_{min} + \frac{1}{2}mv^2$

or, $\frac{hc}{\lambda} = \frac{hc}{\lambda_{max}} + \frac{1}{2}mv^2$

where ϕ = work function.

Heisenberg's Uncertainity Principle:-

Heisenberg's uncertainity principle states that "it is impossible to determine precisely and simultaneously two complementary variables to arbitrary accuracy." Such pairs of variables are canonically conjugate variables. Position q and momentum p, energy E and time t, angular momentum τ and angular displacement θ, etc, are canonically conjugate variables. The Heisenberg's uncertainity principle for position and momentum is:

$$\Delta q \Delta p \geq \frac{\hbar}{2} ; \quad \hbar = \frac{h}{2\pi}$$

This uncertainity principle usually describes one or more of the following statements.

i) It is impossible to predict states in which position and momentum are simultaneously well located.

ii) It is impossible to measure position and momentum simultaneously.

iii) It is impossible to measure position without disturbing momentum and vice versa.

Schrodinger Wave Equation:-

It is the fundamental mathematical expression that describes the wave nature of small particle like electron. i.e it is a form of de-Broglie's relation.

We know,

The equation of a plane progressive wave can be expressed as

$$y = A \sin(\omega t - kx) \longrightarrow (i)$$

In exponential form, it can be expressed as,

$$y = Ae^{-i(\omega t - kx)}$$

Differentiating eq^n (i) with respect to x, we get,

$$\frac{dy}{dx} = \frac{d}{dx}[A \sin(\omega t - kx)]$$

$$\therefore \frac{dy}{dx} = -Ak \cos(\omega t - kx)$$

Again, diff. this rel^n w.r.t 'x', we get

$$\frac{d^2y}{dx^2} = -k^2 A \sin(\omega t - kx)$$

or, $\frac{d^2y}{dx^2} = -k^2 y$ [Using eq^n (i)]

or, $\frac{d^2y}{dx^2} = -\frac{4\pi^2}{\lambda^2} \cdot y$ $\left[\because k = \frac{2\pi}{\lambda}\right]$

or, $\frac{d^2y}{dx^2} + \frac{4\pi^2}{\lambda^2} y = 0$

$$\therefore \frac{d^2y}{dx^2} + \frac{4\pi^2 p^2}{h^2} y = 0 \longrightarrow (ii)$$

$[\because \lambda = h/p]$

If E is the total energy and V is the potential energy of the small particle like electron, than its kinetic energy can be expressed as,

$$K.E = E - V \qquad [\because E = K.E + P.E]$$

or, $\frac{1}{2} mv^2 = E - V$

or, $\frac{p^2}{2m} = [E - V] \qquad [\because p = mv]$

$\therefore p^2 = 2m[E - V]$

Then, above eq^n (ii) becomes,

$$\therefore \frac{d^2y}{dx^2} + \frac{4\pi^2 \times 2m[E-V]}{h^2} y = 0$$

$$\therefore \frac{d^2y}{dx^2} + \frac{8\pi^2 m[E-V]}{h^2} y = 0$$

For general relation, let us write ψ (sai) for y and above relation becomes,

$$\therefore \boxed{\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m[E-V]\psi}{h^2} = 0} \longrightarrow (iii)$$

This is the required expression for schrodinger time independent [one dimensional].

~~In 3-dimensional,~~ $\cancel{\nabla^2\psi + \frac{2mE}{\hbar^2}\psi}$

For free particle [free electron], the potential energy (V) is assumed to be zero (as K.E is maximum] and above relation (iii) becomes,

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 mE}{h^2}\psi = 0$$

$$\therefore \boxed{\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0} \qquad \left[\because \hbar = \frac{h}{2\pi}\right]$$

This is the required expression of schrodinger time independent equation for free particle.

In 3D,

$$\nabla^2\psi + \frac{2mE}{\hbar^2}\psi = 0$$

where, $\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2} \Rightarrow$ laplacian operator

P.81

We know,

$$\psi = Ae^{-i(\omega t - kx)}$$

Diff this relation w.r.t time 't', we get,

$$\frac{d\psi}{dt} = -i\omega \cdot Ae^{-i(\omega t - kx)}$$

or, $\frac{d\psi}{dt} = -i\omega\psi$

or, $\frac{d\psi}{dt} = -i\left(\frac{E}{\hbar}\right)\psi \quad [\because E = \hbar\omega]$

or, $\frac{d\psi}{dt} = -i^2 \frac{E\psi}{i\hbar}$

or, $\frac{d\psi}{dt} = \frac{E\psi}{i\hbar} \quad [\because i^2 = -1]$

$\therefore E\psi = i\hbar \frac{d\psi}{dt} \longrightarrow$ (iv)

Then above eq^n (iii) becomes,

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left[i\hbar\frac{d\psi}{dt} - V\psi\right] = 0$$

$$\therefore \boxed{\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar}\left[i\hbar\frac{d\psi}{dt} - V\psi\right] = 0} \quad \left[\because \hbar = \frac{h}{2\pi}\right]$$

This is the required expression for Schrodinger time dependent equation.

For free particle, V=0 and above relation becomes,

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar}\left[i\hbar\frac{d\psi}{dt}\right] = 0}$$

Physical significance of wave function(ψ)

(i) ψ is the solution of the equation,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left[i\hbar\frac{d\psi}{dt} - V\psi\right] = 0$$

It helps to describe the complete description of the behaviour of small particle like electron and the wave associated with it.

(ii) It helps to find the probability of the particle at any instant 't'.

(iii) It satisfies the equation of continuity i.e $\nabla \cdot J = -\frac{\partial \rho}{\partial t}$

(iv) It satisfies the eigen value eq^n of type,

$$\underset{\text{operator}}{\hat{A}}\psi = \underset{\text{eigen value}}{a}\psi$$

(v) The wave function is said to be normalised if $\int \psi \cdot \psi^* \, dx = 1$

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(vi) The wave function is said to be orthogonal if $\int \psi \cdot \psi^* \, dx = 0$

Application of Schrodinger Wave Equation:-

1. Infinite Potential well:- [A particle in a Box]

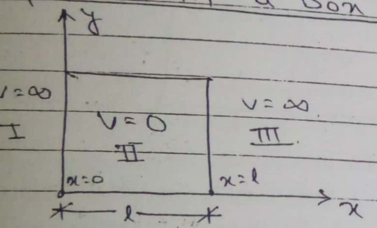


Fig. (i)

Let us consider a small body like electron is free to move inside a box of side 'l' as fig. (i) i.e. the potential energy for the particle at different regions can be expressed as,

$V = \infty$ when $x \leq 0$

$V = 0$ when $0 < x < l$

$V = \infty$ when $x \geq l$

As the particle or electron doesnot exists outside the box, the value of ψ vanishes in region I and II and the particle is free to move inside the region II, where $V = 0$ and schrodinger time independent equation becomes,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}[E - 0]\psi = 0 \quad [\because V = 0, \text{ for free particle}]$$

or, $$\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 mE}{h^2}\psi = 0 \quad \left[\because \hbar = \frac{h}{2\pi}\right]$$

$$\therefore \boxed{\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0} \longrightarrow (i)$$

Here, $k^2 = \frac{8\pi^2 mE}{h^2}$ and the solution of this equation can be expressed as,

$$\psi = A \sin kx + B \cos kx \longrightarrow (ii)$$

Here, A & B are the constant terms and we can find the value of 'k' by using two boundary conditions given by.

(a) $\psi = 0$, when $x = 0$, for all values of 't'

(b) $\psi = 0$ when $x = l$ for all values of 't'

From (a),

$$0 = A \sin 0 + B \cos 0$$

$$\therefore B = 0$$

$\therefore$ Eq^n (ii) becomes,

$$\psi = A \sin kx \longrightarrow \text{(iii)}$$

From (6),

$$0 = A \sin kl . \quad [\because x = l \;\&\; \psi = 0]$$

As, $A \neq 0$, $\sin kl = 0$

$$\therefore \sin kl = \sin n\pi$$

$$kl = n\pi$$

$$\therefore k = \frac{n\pi}{l}$$

So, eq^n (iii) becomes,

$$\psi_n = A \sin\left(\frac{n\pi}{l}\right) x \longrightarrow \text{(iv)}$$

Simillarly,

E [Total energy] = K.E + P.E.

$$= \frac{1}{2} mv^2 + 0$$

$$= \frac{1}{2} \frac{p^2}{m} \quad [\because p = mv]$$

$$\text{or}, E = \frac{1}{2} \frac{(\hbar k)^2}{m} \quad [\because p = \hbar k]$$

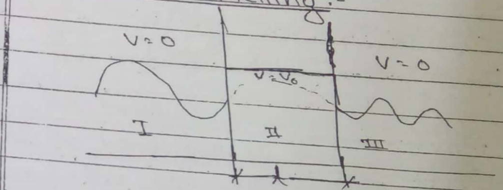
$$= \frac{1}{2m} \hbar^2 . k^2$$

$$= \frac{1}{2m} \frac{h^2}{4\pi^2} . \frac{n^2\pi^2}{l}$$

$$\therefore \boxed{E_n = \frac{n^2 h^2}{8ml^2}} \; ; n = 1, 2, 3 \ldots$$

This gives the energy of free electron at different levels i.e. for different value of n, there is discrete energy level of the electron or the energy levels of the electron is quantized [discrete]

2. Barrier Tunnelling :-



Let us consider a potential barrier of height V_0 [potential energy] and small width l. When an electron with energy 'E' less than V_0 is incident on the barrier as in figure, then there is certain probability of the electron to transmit through the barrier in additional to the probability of the reflection from the barrier. This phenomenon is known as tunnelling effect.

Now,

Applying the schrodinger eq^n to region I,

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$$\frac{\partial^2 \psi_I}{\partial x^2} + \frac{8\pi^2 m}{h^2}[E-0]\psi_I = 0$$

$$\frac{\partial^2 \psi_I}{\partial x^2} + \alpha^2 \psi_I = 0 \longrightarrow (i)$$

where $\alpha = \sqrt{\frac{8\pi^2 mE}{h^2}}$

The solution of this eq^n can be expressed as,

$$\psi_I = Ae^{i\alpha x} + Be^{-i\alpha x} \longrightarrow (ii)$$

where, 'A' is the amplitude of incident wave at the boundary of region I & II and 'B' is the amplitude of reflected wave from this boundary.

Similarly,

Applying the schrodinger eq^n to region II (where $V = V_0$), we get,

$$\frac{\partial^2 \psi_{II}}{\partial x^2} + \frac{8\pi^2 m}{h^2}[E-V_0]\psi_{II} = 0$$

or, $$\frac{\partial^2 \psi_{II}}{\partial x^2} - \frac{8\pi^2 m}{h^2}[V_0-E]\psi_{II} = 0 \quad [\because V_0 > E]$$

or, $$\boxed{\frac{\partial^2 \psi_{II}}{\partial x^2} - \beta^2 \psi_{II} = 0} \longrightarrow (iii)$$

where, $\beta = \frac{8\pi^2 m}{h^2}[V_0 - E]$

The solution of this eq^n can be expressed as,

$$\psi_{II} = Ce^{\beta x} + De^{-\beta x} \longrightarrow (iv)$$

For III region,

$$\frac{\partial^2 \psi_{III}}{\partial x^2} + \frac{8\pi^2 m}{h^2}[E-0]\psi_{III} = 0 \quad [\because V=0]$$

or, $$\frac{\partial^2 \psi_{III}}{\partial x^2} + \frac{8\pi^2 mE}{h^2}\psi_{III}$$

$$\therefore \boxed{\frac{\partial^2 \psi_{III}}{\partial x^2} + \alpha^2 \psi_{III} = 0} \longrightarrow (v)$$

This sol^n of this eq^n can be expressed as,

$$\psi_{III} = Fe^{i\alpha x} + Ge^{-\alpha x} \longrightarrow (vi)$$

As, there is no reflection on wave from region III, $G=0$ & above sol^n becomes,

$$\psi_{III} = Fe^{i\alpha x} \longrightarrow (vii)$$

We can find constants A, B, C, D & F by using boundary conditions.

$\psi_I(x) = \psi_2(x) = \psi_3(x)$ and,

$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} = \frac{\partial \psi_3}{\partial x}$ for values of $x=0$ & $x=l$.

Now, the transmission coefficient is defined by,

$$T = \frac{|F|^2}{|A|^2} = T_0 e^{-2\beta l}$$

And, the reflection coefficient is

$$R = \frac{|B|^2}{|A|^2} = 1 - T.$$

After calculating we get,

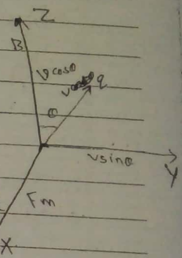
$$\boxed{T = \frac{16E}{V_0}\left(1 - \frac{E}{V_0}\right) e^{-2\beta l}}$$

where, $\beta = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$

— x —— x —

Magnetic Fields:-

⇒ **Force experienced by a moving charge inside the magnetic field:-**



Let us consider a charge 'q' is moving in Y-Z plane as in figure. If 'B' is the strength of magnetic field, where the charge 'q' is moving, then the magnitude of magnetic force experienced by the moving charge will be,

$F_m \propto q \longrightarrow$ (i)

and $F_m \propto v \sin\theta \longrightarrow$ (ii)

From above eq^n (i) and (ii), we can write,

$F \propto q v \sin\theta.$

$\therefore \boxed{F_M = Bqv\sin\theta.}$

where, B = magnetic field intensity

v = velocity of moving charge

Note:-

If $\theta = 0$, $F_M = 0$

If $v = 0$, $F_M = 0$

If $\theta = 90°$, $F_M = Bqv$ [Max^m value]

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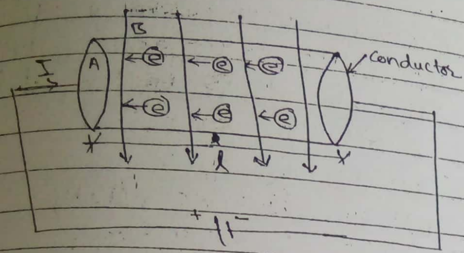
Force experienced by current carrying conductor inside the magnetic field:-



Let us consider a conductor of length 'l' and cross-sectional area 'A'. If the charge (free electrons) are moving inside the conductor in the direction perpendicular to the direction of magnetic field B' as in figure, then the force experienced by a electron in the conductor is given by,

$F_1 = BeV_d \sin 90^\circ$ $[\because \theta = 90^\circ]$

i.e. $F_1 = BeV_d \longrightarrow (i)$

Here, V_d is the drift velocity of electron.

Current density $(J) = neV_d$

$\Rightarrow V_d = \frac{J}{ne}$

Then, eqⁿ (i) becomes,

$F_1 = Be \cdot \frac{J}{ne}$

$\therefore F_1 = \frac{BJ}{n} \longrightarrow (ii)$

Here, n = no. of electrons per unit volume.

Now,

Total force on the free electron in the wire is given by,

$F = F_1 \times A l \times n$

or, $F = \frac{BJ}{n} \times A l \times n$

or, $F = \frac{B}{n} \times \frac{I}{A} \times A l \times n$ $\left[\because J = \frac{I}{A}\right]$

$\therefore \boxed{F = BIl} \longrightarrow (iii)$

If θ is the angle made by moving charge with the direction of magnetic field then above relation becomes,

$\boxed{F = BIl \sin\theta} \longrightarrow (iv)$

This is required equation for magnetic force on current carrying conductor.

In vector form above relation can be written as,

$$\boxed{\vec{F} = I(\vec{l} \times \vec{B})}$$

Biot-Savart Law:-

It states that "the magnitude of magnetic field 'dB' due to the current element 'Idl' at any point 'P' with distance 'r' from current element of length 'dl' of a conductor carrying current 'I' is.

(i) directly proportional to current I.
i.e. $dB \propto I$

(ii) directly proportional to length 'dl'
i.e. $dB \propto dl$.

(iii) directly proportional to sine of angle 'θ' between 'dl' and 'r'.
i.e. $dB \propto \sin\theta$.

(iv) inversely proportional to square of dist. 'r'
i.e. $dB \propto \frac{1}{r^2}$

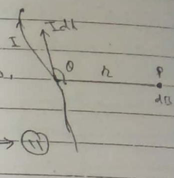
Combining all above equations, we get.

$$dB \propto \frac{I dl \sin\theta}{r^2}$$

$$\therefore dB = \frac{k I dl \sin\theta}{r^2} \longrightarrow (i)$$

In CGS system, $k = 1$, so,

$$dB = \frac{I dl \sin\theta}{r^2}$$



In SI system, $k = \frac{\mu_0}{4\pi}$, so,

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin\theta}{r^2} \longrightarrow (ii)$$

This is the required expression for the Biot savart Law in MKS system.

Where, $\frac{\mu_0}{4\pi} = 10^{-7}$.

$\therefore \mu_0 = 4\pi \times 10^{-7}$ wb/A-m which is known as permeability of free space.

In vector form,

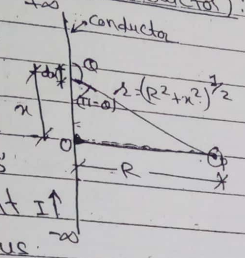
$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I dl \sin\theta \, \hat{r}}{r^2}$$

i.e. $$\boxed{\vec{dB} = \frac{\mu_0}{4\pi} \frac{I(\vec{dl} \times \vec{r})}{r^3}} \quad \left[\because \hat{r} = \frac{\vec{r}}{r}\right]$$

Application of Biot-Savart Law:-

① For infinite length of wire (Conductor):-

Figure shows the long straight wire with current 'I'. We have to find the magnetic field induction 'B' at point 'P' due to current I↑ of the wire. For this, let us take a small element 'dx' with distance 'x' from 'O'. Then the magnitude of 'B' at 'P' with distance r from the current element Idx is given by



$$dB = \frac{\mu_0}{4\pi}\frac{Idx \sin\theta}{r^2}$$

$$\text{or, } dB = \frac{\mu_0}{4\pi}\frac{Idx}{(R^2+x^2)^{2\times 1/2}}\sin(\pi-\theta)$$

$$= \frac{\mu_0}{4\pi}\frac{Idx}{(R^2+x^2)}\cdot\frac{R}{(R^2+x^2)^{1/2}} \quad \left[\because \sin(\pi-\theta) = \frac{R}{r}\right]$$

$$\therefore dB = \frac{\mu_0}{4\pi}\frac{IdxR}{(R^2+x^2)^{3/2}}$$

The total magnetic field 'B' at 'P' due to the current through the long wire is

$$B = \frac{\mu_0 IR}{4\pi}\int_{-\infty}^{\infty}\frac{dx}{(R^2+x^2)^{3/2}}$$

$$\text{or, } B = \frac{2\times\mu_0 IR}{4\pi}\int_0^{\infty}\frac{dx}{(R^2+x^2)^{3/2}}$$

$$\text{or, } B = \frac{\mu_0 IR}{2\pi}\int_0^{\infty}\frac{dx}{(R^2+x^2)^{3/2}}$$

Let $x = R\tan\theta \Rightarrow dx = R\sec^2\theta\, d\theta$

when, $x = 0$, $\theta = 0$

$x = \infty$, $\theta = \pi/2$

Then, above relation becomes,

$$B = \frac{\mu_0 IR}{2\pi}\int_0^{\pi/2}\frac{R\sec^2\theta\, d\theta}{(R^2 + R^2\tan^2\theta)^{3/2}}$$

$$= \frac{\mu_0 I\cancel{R^2}}{2\pi}\int_0^{\pi/2}\frac{\cancel{\sec^2\theta}\, d\theta}{R^{\cancel{3}}\sec^{\cancel{3}}\theta}$$

$$= \frac{\mu_0 I}{2\pi R}\int_0^{\pi/2}\cos\theta\, d\theta$$

$$= \frac{\mu_0 I}{2\pi R}\left[\sin\theta\right]_0^{\pi/2}$$

or, $B = \frac{\mu_0 I}{2\pi R}[\sin \pi/2 - \sin 0]$

or, $B = \frac{\mu_0 I}{2\pi R} \times 1$

∴ $\boxed{B = \frac{\mu_0 I}{2\pi R}}$

This is the required expression for magnitude of magnetic field induction due to current through long straight wire.

Note:-

For finite length wire

$$B = \frac{\mu_0 I}{4\pi}\int_{-l/2}^{+l/2} \frac{dl \sin\theta}{r^2}$$

$$= \frac{2.\mu_0 I}{4\pi}\int_{0}^{l/2} \frac{dl \sin\theta}{r^2}$$

∴ $\boxed{B = \frac{\mu_0 I}{4\pi R}[\sin\phi_1 - \sin\phi_2]}$

② Magnetic field at a point due to circular current coil.

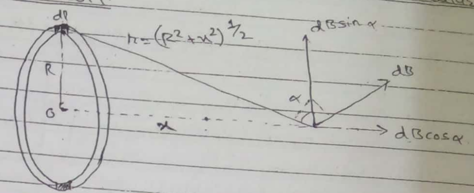


Fig:- Magnetic field due to circular coil

Let us consider a circular current coil with current I and radius R. We have to find the magnitude of magnetic field B' at the point 'P' due to this coil. Let us take a small element 'dl' with distance 'r' from point 'P'. According to Biot-savart law, we will get,

$$dB = \frac{\mu_0}{4\pi}\frac{I dl \sin\theta}{r^2}$$

As the small element 'dl' is perpendicular to plane of paper,

$\sin\theta = \sin \pi/2 = 1$. Then above rel^n becomes

$$dB = \frac{\mu_0}{4\pi}\frac{I dl}{(R^2+x^2)} \longrightarrow (1)$$

As the similar element 'dl' is present at the next end coil and the perpendicular component of ~~magnetic~~ magnetic field at P [i.e. $dB\sin\alpha$] cancel eachother and only $dB\cos\alpha$ component is effective. So, the total magnetic field 'B' at 'P' due to current through conductor is,

$$B = \int dB\cos\alpha.$$

$$\therefore B = \int_0^{2\pi R} \frac{\mu_0}{4\pi}\frac{I\,dl}{(R^2+x^2)}\cos\alpha.$$

$$= \frac{\mu_0 I}{4\pi}\int_0^{2\pi R}\frac{dl}{(R^2+x^2)}\cdot\frac{R}{(R^2+x^2)^{1/2}} \quad \left[\because \cos\alpha = \frac{R}{r}\right]$$

$$= \frac{\mu_0 I}{4\pi}\cdot\frac{R}{(R^2+x^2)^{3/2}}\int_0^{2\pi R} dl$$

$$= \frac{\mu_0 I}{2\cancel{4\pi}}\cdot\frac{R}{(R^2+x^2)^{3/2}} \times 2\pi R$$

$$\therefore \boxed{B = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}} \longrightarrow \text{(ii)}$$

This gives the magnetic field induction at P due to circular current coil of radius R.

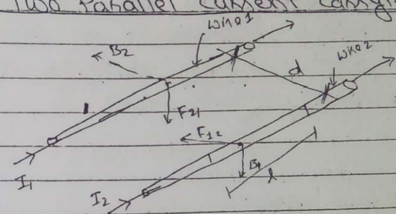
Special Case:- :-

For centre of coil, $x=0$ and above relⁿ becomes,

$$B = \frac{\mu_0 I R^2}{2R^3}$$

$$\therefore \boxed{B = \frac{\mu_0 I}{2R}} \longrightarrow \text{(iii)}$$

Two Parallel current carrying conductors:-



Let us consider wire 1 and wire 2 separate by d with current I_1 & I_2 respectively. The wire 1 carrying I_1 produces B_1 and given by,

$$B_1 = \frac{\mu_0 I_1}{2\pi d} \longrightarrow \text{(i)}$$

& magnetic force is given by,

$$F_{21} = B_1 l I_2 = I_2 l\left(\frac{\mu_0 I_1}{2\pi d}\right) = \frac{\mu_0 I_1 I_2 l}{2\pi d} \longrightarrow$$

Simillarly,

$$B_2 = \frac{\mu_0 I_2}{2\pi d} \longrightarrow (iii)$$

And

$$F_{12} = I_1 l B_2 = \frac{\mu_0 I_1 I_2 l}{2\pi d} \longrightarrow (iv)$$

Hence,

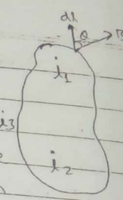
From above relation (ii) and (iv), it is clear that the forces that the two wires exert on eachother are equal in magnitude and opposite in direction i.e "parallel current attracts and antiparallel current repell".

Ampere's Law :-

It states that the line integral of magnetic induction (B) around a closed loop for vaccum is equal to μ_0 layers the current enclosed by the loop.

i.e. $\oint_C \vec{B} \cdot \vec{dl} = \mu_0 i \longrightarrow (i)$

Here, $i = i_1 - i_2$ (out of page) is the current enclosed by the loop as in figure.



Simillarly, if θ is the angle made by magnetic induction with small element of length dl then we will get,

$$\oint \vec{B} \cdot \vec{dl} = \oint B\, dl \cos\theta \longrightarrow (ii)$$

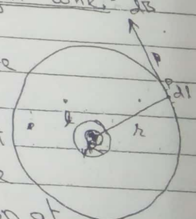
∴ From above eq^n (i) & (ii), we can write,

$$\boxed{\oint B\, dl \cos\theta = \mu_0 i}$$

Application of Ampere's Law:-

(1) An infinitely long straight wire:- (Outside the wire)

Let us consider a infinite length of a straight wire with radius 'R' and current 'i'. We have to find the magnetic field induction at a point 'P' with distance r from the wire. For this let us draw an



amperian loop passing through 'P' as in figure.

Then according to Ampere's law, we can write,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 i \quad [i = \text{current enclosed by the loop}]$$

or, $\oint B\,dl\cos\theta = \mu_0 i$

As angle between B and dl is zero, i.e. $\cos\theta = \cos 0° = 1$

$\therefore \oint_c B.dl = \mu_0 i$

or, $B\oint_c dl = \mu_0 i$

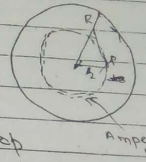
As $\oint_c dl = 2\pi r$, so we write

or, $B \times 2\pi r = \mu_0 i$

$$\therefore \boxed{B = \frac{\mu_0 i}{2\pi r}}$$

This gives the magnetic field induction at a point outside of the given wire.

② **At a point inside the long straight wire:-**



Amperian loop

Figure: c/s of wire

To find the magnetic field induction 'B' at a point inside the given wire. Let us draw an amperian loop of radius 'r' as in figure. If 'i'' is the current enclosed by this loop, then according to Ampere's law we can write,

$$\oint_c \vec{B}\cdot\vec{dl} = \mu_0 i'$$

or, $\oint B\,dl\cos\theta = \mu_0 i'$

As, $\cos\theta = \cos 0° = 1$, then,

$B\oint dl = \mu_0 i'$

or, $B \times 2\pi r = \mu_0 i'$ $\quad [\because \oint dl = 2\pi r]$

$$\therefore B = \frac{\mu_0 i'}{2\pi r} \longrightarrow (i)$$

Here, πR^2 area contains current 'i'.

So, πr^2 " " " $i' = \frac{i}{\pi R^2} \times \pi r^2$

$$\therefore i' = \frac{i r^2}{R^2}$$

Then equation (i) becomes,

$$B = \frac{\mu_0}{2\pi r} \times \frac{ir^2}{R^2}$$

$$\therefore \boxed{B = \frac{\mu_0 i r}{2\pi R^2}}$$

This gives the magnetic field inside the wire at a distance 'r' from its centre.

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⇒ Faraday's Law of Electromagnetic Induction :-

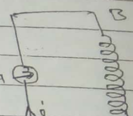
According to Faraday Laws of electromagnetic induction,

(i) Whenever there is change in the magnetic lines of force or magnetic flux and induced emf is generated which lasts as the change in the magnetic flux continues.

(ii) The magnitude of induced emf is directly proportional to the rate of change of magnetic flux

i.e. $(Emf)_{induced} \propto \frac{d\phi_B}{dt}$

$$\therefore E = -\frac{d\phi_B}{dt}$$

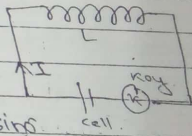


If there is N no. of turns in the coil, we can write,

$$\boxed{E = -N\left(\frac{d\phi_B}{dt}\right)}$$

Here, -ve sign indicates the direction of induced emf or back emf which ~~appears~~ opposes the change in magnitude

⇒ Self induction and self inductance(L):-



The phenomenon in which the induced emf is produced as a result of change in the current passing through the coil is known as self inductance(L) in series with battery and key (K). If ϕ is the magnetic flux and I is the current in the coil, then we will get,

$\phi \propto I$.

i.e. $\phi = LI \longrightarrow$ (i)

Here, L is called coefficient of induction or self inductance

$\therefore L = \phi / I$.

If $I = 1$, then $L = \phi$ i.e. self inductance of a coil is defined as the flux linked across the coil, when the current passing through it is unity.

Also,

$$E = \frac{-d\phi}{dt}$$

$$= \frac{-d}{dt}(LI)$$

$$\therefore \boxed{E = -L\frac{dI}{dt}} \longrightarrow \text{(ii)}$$

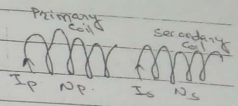
Induced emf,

if $\frac{dI}{dt} = 1$; $E = L$ [numerical value]

So, self inductance is numerically equal to induced emf when $\frac{dI}{dt} = 1$.

~~RL Circuit~~

⇒ Mutual Induction and Mutual Inductance



The phenomenon of production of an emf in the circuit when the current changes in another circuit is called mutual inductance. The mutual inductance, M of two coils is defined by the equations,

$$M = -\frac{\mathcal{E}}{dI_p/dt}$$

and,

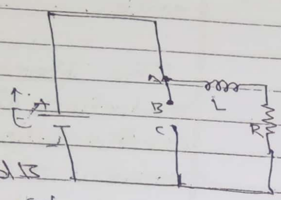
$$\boxed{M = \frac{N_s \phi_s}{I_p}}$$

where, N_s, ϕ_s and I_p are number of turns of coil in secondary, flux linked with secondary and current flowing in primary coil.

RL circuit

① Rise of current:-

Let us consider a RL circuit with cell of emf (E) as in figure. When the point A and B is connected we will get,



$$E = IR + L\frac{dI}{dt} \qquad [\because E = V_R + V_L]$$

or, $E - IR = L\frac{dI}{dt}$

or, $\frac{dI}{E - IR} = \frac{1}{L}dt$

or, $\frac{dI}{R(E/R - I)} = \frac{1}{L}dt$

or, $\frac{dI}{\left(\frac{E}{R} - I\right)} = \frac{R}{L}dt$

Integrating both sides, we get,

$$\int_0^I \frac{dI}{\left(\frac{E}{R} - I\right)} = \frac{R}{L}\int_0^t dt$$

or, $-\log\left[\frac{E}{R} - I\right] = \frac{Rt}{L} + A \longrightarrow$ (i)

Here, A is the constant of integration, for $t = 0$, $I = 0$ and above relation gives,

$$-\log\left(\frac{E}{R}\right) = A$$

substuting this value of 'A' in eq^n (i), we get,

$$-\log_e\left[E/R - I\right] = \frac{Rt}{R} - \log(E/R)$$

or, $\log_e\left[\frac{E}{R} - I\right] - \log\left[\frac{E}{R}\right] = -\frac{Rt}{L}$

or, $\log_e\left[\frac{E/R - I}{E/R}\right] = -\frac{Rt}{L}$

Taking antilog both sides, we get,

$$\frac{\left(\frac{E}{R} - I\right)}{E/R} = e^{-\frac{Rt}{L}}$$

or, $\frac{E}{R} - I = \frac{E}{R} \cdot e^{-Rt/L}$

or, $I = \frac{E}{R} - \frac{E}{R} \cdot e^{-et/L}$

or, $I = \frac{E}{R}\left[1 - e^{-Rt/L}\right]$

$\therefore I = I_0\left[1 - e^{-\frac{Rt}{L}}\right] \longrightarrow$ (ii)

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This gives the rise current in RL circuit for time 't' where $I_0 = E/R$ is the maximum current in the circuit.

Also,

$\tau = L/R$ is called inductive time constant and above relation in terms of τ becomes,

$$I = I_0\left[1 - e^{-t/\tau}\right]$$

i.e. for $t = \tau$, $I = I_0[1 - e^{-1}] = 0.63\, I_0$

$\therefore I = 0.63\, I_0$

Therefore, the inductive time constant is defined as the time in which current increases to 0.63 (63%) times of its maximum value.

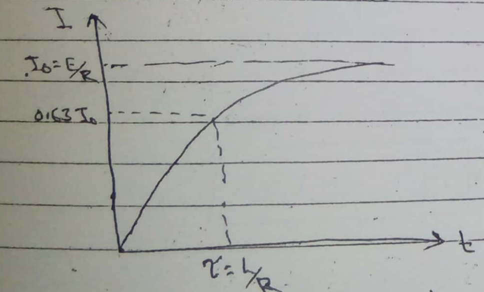


Fig: Rise of current.

(ii) **Decay of current:-**

When the point A and B is disconnected and the point B is connected to C, we will get,

$$IR + L\frac{dI}{dt} = 0 \quad [\because E = 0 \text{ for open circuit}]$$

or, $L \cdot \frac{dI}{dt} = -IR$

or, $L \cdot \frac{dI}{I} = -R\,dt$

$\therefore \frac{dI}{I} = -\frac{R}{L}\,dt$

Integrating both sides, we get,

$$\int \frac{dI}{I} = \int -\frac{R}{L}\,dt$$

or, $\log I = \frac{-Rt}{L} + B$

Here, B is the constant of integration. When $t = 0$, $I = I_0$ and $B = \log I_0$

Then above relation becomes,

$$\log I = -\frac{R}{L}t + \log I_0$$

or, $\log\left(\frac{I}{I_0}\right) = \frac{-Rt}{L}$

Taking antilog both sides, we get.

$$\frac{I}{I_0} = e^{-\frac{Rt}{L}}$$

$$\therefore I = I_0 e^{-Rt/L}$$

This is the required expression for decay of current in RL circuit.

for $t = \tau$, $I = I_0 e^{-t/L/R} = I_0 e^{-1}$ $[\because \tau = L/R]$

$$\therefore I = 0.37\, I_0$$

i.e. Inductive time constant during decay of current is defined as the time in which the current is 0.37 of its maximum value.

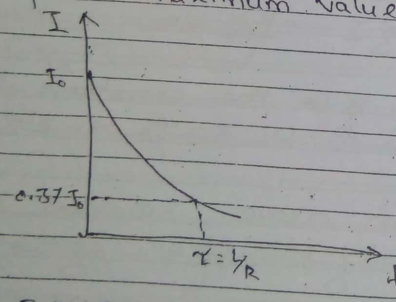


Fig:- Decay of current.

Energy stored in Magnetic Field:-

We know:-

Rate of energy supplied by battery $= EI$

" " " appeared as Joule's law of heat $= I^2R$

" " " stored due to $L = LI\frac{dI}{dt}$

From conservation of energy

$$EI = I^2R + LI\frac{dI}{dt}$$

We know,

Rate of magnetic energy stored $= LI.\frac{dI}{dt}$

or, $\frac{dU_B}{dt} = LI.\frac{dI}{dt}$; $[\because U_B = \text{magnetic energy}]$

Total energy stored in inductance is,

$$U_B = \int_0^t \left(\frac{dU_B}{dt}\right)dt = \int_0^t LI\left(\frac{dI}{dt}\right)dt$$

$$= \int_0^I LI.dI$$

$$\therefore \boxed{U_B = \frac{1}{2}LI^2}$$

Induced Magnetic Field:-

According to Faraday's laws of electromagnetic induction,

$$\oint_c E\cdot dl = -\frac{d\phi_B}{dt} \longrightarrow (i)$$

In similar manner,

$$\oint_c \vec{B}\cdot\vec{dl} = \mu_0 \epsilon_0 \frac{\partial \phi_E}{\partial t} \longrightarrow (ii)$$

Also, According to Ampere's law,

$$\oint_c B\cdot dl = \mu_0 i \longrightarrow (iii)$$

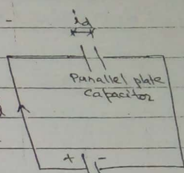
With the help of above relations (ii) & (iii), it is concluded that,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 i + \mu_0 \underbrace{\boxed{\epsilon_0 \frac{\partial \phi_E}{\partial t}}}_{i_d = \text{displacement current}} \longrightarrow (iv)$$

This relation is known as Ampere's modified law or Ampere-Maxwell relation.

i = conduction current

Displacement current:-



The idea of displacement current allows us to retain the notation of continuity of current through a capacitor. However, it is not a transfer of charge. The additional term $\epsilon_0 \frac{\partial \phi_E}{\partial t}$ in above relation (iv) is called displacement current.

i.e.

$$i_d = \epsilon_0 \frac{\partial \phi_E}{\partial t}$$

$$= \epsilon_0 \frac{\partial [E\cdot A]}{\partial t} \qquad [\because \phi_E = E\cdot A]$$

$$\therefore i_d = \epsilon_0 A \frac{\partial E}{\partial t}$$

Also for parallel plate capacitor, capacitance $(C) = \frac{\epsilon_0 A}{d}$

$$\Rightarrow \epsilon_0 A = Cd$$

So, above relation becomes,

$i_d = \epsilon_0 d \frac{\partial E}{\partial t}$

or, $i_d = C \frac{\partial}{\partial t}(E \cdot d)$

$\therefore \boxed{i_d = C \frac{\partial V}{\partial t}} \longrightarrow (i)$

This gives the displacement current between the parallel plate capacitor.

Again,

For parallel plate capacitor,

$E = \frac{q}{\epsilon_0 A}$

i.e, $q = \epsilon_0 E A$

$\therefore$ Rate of flow of charge is,

$\frac{\partial q}{\partial t} = \epsilon_0 A \frac{\partial E}{\partial t}$

i.e. $i = C d \frac{\partial E}{\partial t}$ [i = conduction current]

or, $i = C \frac{\partial}{\partial t}(E \cdot d)$

or, $i = C \frac{\partial V}{\partial t} \longrightarrow (ii)$

From (i) & (ii),

$\boxed{i = i_d}$

V.V. Imp

Hall Effect:-

Whenever a magnetic field is applied to the current carrying conductor, an electric field is set up in a direction perpendicular to both magnetic field and current. This effect is called Hall effect and developed field is called Hall field.

Consider a slab of material subjected to an external magnetic field $\vec{B}$ acting along Z-direction & electric field $\vec{E}$ acting along X-direction so that current flows along X-direction.

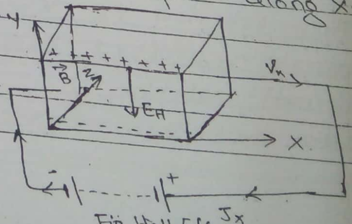


Fig Hall Effect.

~~Electromagnetic Idea~~

Under the influence of magnetic field, electron experiences force acting downward along y-axis so that lower surface collects the positive charge. This develops an electric field called Hall field (E_H). Ultimately, the force due to the Hall field cancels the Lorentz force and steady state is achieved.

The Lorentz force experienced by an electron of charge '-e' moving with velocity $\vec{v}$ in an electromagnetic field $\vec{E}$ and $\vec{B}$ is

$$\vec{F} = (-e)[\vec{E} + \vec{v} \times \vec{B}]$$

At steady state,

$$F_y = 0$$

$$\text{or, } -e[E_H - v_x B] = 0$$

$$\therefore E_H = v_x B \longrightarrow (i)$$

From Ohms law,

$$I_x = n(-e)v_x \longrightarrow (ii)$$

Now,

Hall coefficient (R_H) is defined by.

$$R_H = \frac{E_H}{J_x B} = \frac{v_x B}{n(-e)B v_x} = -\frac{1}{ne}$$

$$\therefore \boxed{R_H = -\frac{1}{ne} \ (SI)} \longrightarrow (iii)$$

$$\therefore \boxed{R_H = -\frac{1}{nec} \ (C.G.S)} \longrightarrow (iv)$$

These relations point out that Hall coefficient R_H gives sign and concentration of carriers.

→ The Hall mobility;

$$\mu = \frac{v_x}{E_x} \longrightarrow (v)$$

$$\text{or, } E_H = \mu E_x B. \longrightarrow (vi)$$

Thus,

$$R_H = \frac{E_H}{J_x B} = \frac{\mu E_x B}{\sigma E_x B} = \frac{\mu}{\sigma}$$

where, σ = electric ~~res~~ conductivity

$$\text{i.e. } \mu = \sigma R_H \ (SI) \longrightarrow (vii)$$

$$\mu = \sigma R_H c \ (C.G.S)$$

→ Hall angle, θ_H is defined by relⁿ.

$$\tan\theta_H = \frac{E_H}{E_x} = \frac{BJ_x}{neE_x} = \frac{nev_x B}{neE_x} = \frac{v_x B}{E_x}$$

$$\text{Also, } \tan\theta_H = \mu B. \longrightarrow (viii)$$

Application of Hall effect:-

(i) Determinations of (a) nature of charge carriers (b) concⁿ of carriers (c) mobilities of holes & electrons (d) power flow in EM waves.

(ii) Measurement of magnetic flux density.

(Refer Book.)

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P.100

Electromagnetic Waves

Note:-

* Gauss divergence theorem:-

$$\oint_V (\vec{\nabla}\cdot\vec{A})\,dV = \oint_S \vec{A}\cdot\vec{ds}$$

* Stoke's theorem:-

$$\oint_S (\vec{\nabla}\times\vec{A})\,ds = \oint_C \vec{A}\cdot\vec{dl}$$

Maxwell's Equations:-

(i) Gauss law in electrostatics:

Integral form: $\oint_S \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$

Differential form: $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$

(ii) Gauss law in Magnetism.

$\oint_S \vec{B}\cdot\vec{ds} = 0$ $\quad$ $\vec{\nabla}\cdot\vec{B} = 0$

(iii) Faraday's law of electromagnetic induction.

$\oint \vec{E}\cdot\vec{dl} = -\frac{\partial \phi_B}{\partial t}$ $\quad$ $\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}$

(iv) Ampere's modified law.

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 i + \mu_0\epsilon_0 \frac{d\phi_E}{dt} \qquad \vec{\nabla}\times\vec{B} = \mu_0 J + \mu_0\epsilon_0 \frac{\partial E}{\partial t}$$

⇒ Conversion of integral form of Maxwell's equation to its differential form:

i) According to Gauss law in electrostatics,

$$\oint_S \vec{E}\cdot\vec{ds} = q/\epsilon_0$$

or, $\oint_S \vec{E}\cdot\vec{ds} = \frac{1}{\epsilon_0}\oint_V \rho\, dV$ $\left[\because \rho = dq/dV \Rightarrow q = \oint_V \rho\, dV\right]$

Applying Gauss divergence theorem to L.H.S. we get,

$$\oint_V (\vec{\nabla}\cdot\vec{E})\, dV = \frac{1}{\epsilon_0}\oint_V \rho\, dV$$

$\therefore \boxed{\nabla\cdot E = \rho/\epsilon_0}$ proved.

This is the differential form of maxwell's 1st equation.

(ii) According to Gauss law in magnetism,

$$\oint_S \vec{B}\cdot\vec{ds} = 0$$

After applying Gauss divergence theorem to LHS, we get,

$$\oint_V (\vec{\nabla}\cdot\vec{B})\, dV = 0$$

$\therefore \boxed{\vec{\nabla}\cdot\vec{B} = 0}$

is reqd diff. form of Maxwell's 2nd eqn.

(iii) According to Faraday's law of electrostatics,

$$\oint_C \vec{E}\cdot\vec{dl} = -\frac{\partial \phi_B}{\partial t}$$

or, $\oint_C \vec{E}\cdot\vec{dl} = -\frac{\partial}{\partial t}\oint_S \vec{B}\cdot\vec{ds}$ $\left[\because B = \frac{\partial \phi_B}{\partial t}\right]$

By stokes theorem, we get,

$$\oint_C (\vec{\nabla}\times\vec{E})\cdot\vec{ds} = -\frac{\partial}{\partial t}\oint \vec{B}\cdot\vec{ds}$$

$\therefore \boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}}$

is reqd diff. form of maxwell's 3rd equation.

(iv) According to Amperes modified law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 i + \mu_0\epsilon_0 \frac{\partial \phi_E}{\partial t}$$

or, $\oint \vec{B}\cdot\vec{dl} = \mu_0 \oint \vec{J}\cdot\vec{ds} + \mu_0\epsilon_0 \oint \vec{E}\cdot\vec{ds}$

$$\left[\because J = di/ds \text{ and } i = \oint_s \vec{J}\cdot\vec{ds}\right]$$

Applying stokes theorem, we get,

$$\oint (\vec{\nabla}\times\vec{B})\vec{ds} = \mu_0 \oint \vec{J}\cdot\vec{ds} + \mu_0\epsilon_0 \frac{\partial}{\partial t}\oint \vec{E}\cdot\vec{ds}$$

$$\therefore \boxed{\vec{\nabla}\times\vec{B} = \mu_0 J + \mu_0\epsilon_0 \frac{\partial E}{\partial t}}$$

is the diff. form of maxwells 4th equation.

Electromagnetic wave equations for free space.

We know, the maxwell's equation for free space can be expressed as,

(i) $\vec{\nabla}\cdot\vec{E} = 0$ $\quad \left[\because \rho = 0, \text{ so, } \rho/\epsilon_0 = 0\right]$

(ii) $\vec{\nabla}\cdot\vec{B} = 0$

(iii) $\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}$

(iv) $\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0 \frac{\partial E}{\partial t}$

Taking curl of eq^n (iii), we get,

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\frac{\partial}{\partial t}(\vec{B}\times\vec{\nabla})$$

or, $\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \vec{E}(\vec{\nabla}\cdot\vec{\nabla}) = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$

or, $0 - \vec{\nabla}^2\vec{E} = -\frac{\partial}{\partial t}\left[\mu_0\epsilon_0 \frac{\partial E}{\partial t}\right]$

using eq^n (i) & (iv),

$$\therefore \nabla^2 E = \mu_0\epsilon_0 \frac{\partial^2 E}{\partial t^2} \longrightarrow (v)$$

This is the required electric wave eq^n for free space similarly, taking curl in eq^n (iv), we get,

$$\vec{\nabla}\times\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E})$$

$$\text{or, } \vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \vec{B}(\vec{\nabla}\cdot\vec{\nabla}) = \mu_0\epsilon_0\frac{\partial}{\partial t}[\vec{\nabla}\times\vec{E}]$$

$$\text{or, } 0 - \vec{\nabla}^2\vec{B} = \mu_0\epsilon_0\frac{\partial}{\partial t}\left[-\frac{\partial B}{\partial t}\right]$$

Using eq^n (ii) & (iv)

$$\text{or, } -\nabla^2 B = -\mu_0\epsilon_0\frac{\partial^2 B}{\partial t^2}$$

$$\therefore \nabla^2 B - \mu_0\epsilon_0\frac{\partial^2 B}{\partial t^2} = 0 \longrightarrow \text{(vi)}$$

This is the required magnetic wave eq^n for free space.

But, the standard form for the electromagnetic wave for free space is,

$$\nabla^2\psi - \frac{1}{C^2}\cdot\frac{\partial^2\psi}{\partial t^2} = 0 \longrightarrow \text{(vii)}$$

Comparing (v) & (vi) with (vii), we get,

$$\frac{1}{C^2} = \mu_0\epsilon_0 \Rightarrow C = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

where C is the velocity of light for free space i.e. The electromagnetic wave propagates with velocity of light in free space.

EM wave equations in conducting medium:-

Considering maxwell's third equation in conducting medium, i.e.

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

taking curl both sides, we get,

$$\nabla\times(\nabla\times\vec{E}) = -\left(\nabla\times\frac{\partial\vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}[\nabla\times\vec{B}]$$

Using 4th Maxwell's eq^n, we get,

$$\nabla\cdot(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = -\frac{\partial}{\partial t}\left[\mu\vec{J} + \mu\epsilon\frac{\partial\vec{E}}{\partial t}\right]$$

For constant charge density of medium,

$$\nabla(\nabla\cdot\vec{E}) = 0, \text{ so,}$$

$$\nabla^2\vec{E} = \epsilon\mu\frac{\partial^2\vec{E}}{\partial t^2} + \mu\sigma\frac{\partial\vec{E}}{\partial t} \longrightarrow \text{(i)}$$

$$[\because \vec{J} = \sigma\vec{E}]$$

Ratio of electric field & magnetic field in plane electromagnetic wave along x-axis.

[Or, Prove, $\frac{E_m}{B_m} = C = \frac{E}{B}$]

We know,

The electric field and magnetic field for the EM wave can be expressed as,

$E = E_m \sin(kx - \omega t) \longrightarrow (i)$

& $B = B_m \sin(kx - \omega t) \longrightarrow (ii)$

As the wave is moving along x-axis only, we will get,

$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \longrightarrow (iii)$

Diff. eq^n (i) w. r. to x, we get.

$\frac{\partial E}{\partial x} = k . E_m \cos(kx - \omega t)$

lly, diff eq^n (ii), w.r. to 't', we get,

$\frac{\partial B}{\partial t} = -\omega B_m \cos(kx - \omega t)$

Then,

Eq^n (iii) becomes,

$k E_m \cancel{\cos(kx-\omega t)} = +\omega B_m \cancel{\cos(kx-\omega t)}$

$\therefore \frac{E_m}{B_m} = \frac{\omega}{k}$

or, $\frac{E_m}{B_m} = \frac{2\pi f}{2\pi/\lambda} = \lambda f$

$\therefore \frac{E_m}{B_m} = C$; velocity of EM wave $\longrightarrow (iv)$

Also, dividing eq^n (i) by (ii), we get,

$\boxed{\frac{E}{B} = \frac{E_m}{B_m} = C}$ proved.

V.V. Imp

Poynting vector (S):-

The rate of change of energy per unit area in a plane EM wave is called poynting vector (S). It is denoted by S.

i.e. $S = \frac{1}{A}\frac{dU}{dt} \longrightarrow (i)$

Here,

$dU = (u_E + u_B) A\, dx$

[$\because u = \frac{U}{\text{Volume}} = \frac{U}{A\,dx}$]

or, $dU = \left[\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2\right] A.dx$

$= \left[\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}\frac{E^2}{C^2}\right] A.dx \quad \left[\because \frac{E}{B} = C\right]$

$= \left[\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}\mu_0\epsilon_0 E^2\right] A.dx \quad \left[\mu_0\epsilon_0 = \frac{1}{C^2}\right]$

$= \left[\cancel{2}.\frac{1}{\cancel{2}}.\epsilon_0 E^2\right] A.dx$

Then, equation (i) becomes,

$S = \frac{1}{A}.\epsilon_0 E^2. A\frac{dx}{dt}$

$= \epsilon_0 E^2 C$

$= \epsilon_0 E.EC$

$= \epsilon_0 E.BC^2$

$= \epsilon_0 E.B.\frac{1}{\mu_0\epsilon_0}$

$\therefore \boxed{S = \frac{1}{\mu_0}(E.B)}$

This gives the magnitude of poynting vector.

Now, in vector form, we have,

$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$

But, $E = E_m \sin(Kx-\omega t)$ & $B = B_m \sin(Kx-\omega t)$

so,

$S = \frac{1}{\mu_0} E_m B_m \sin^2(Kx-\omega t)$

For average value of poynting vector,

$\bar{S} = \frac{1}{\mu_0} E_m B_m \left[\cancel{\sin^2(Kx-\omega t)}\right]_{avg}$ or $\frac{1}{2}$

$\boxed{\bar{S} = \frac{1}{2\mu_0} E_m B_m}$

This gives the average value of poynting vector.

Equation of Continuity :-

The rate of flow of electric charge is called electric current. The rate of loss of charge density is equivalent to divergence of current density. This is the statement of equation of

Continuity.
i.e.

$$\boxed{\vec{\nabla}\cdot\vec{J} = -\frac{\delta \rho}{\partial t}}$$

Proof:-

From definition,

$I = -\frac{dq}{dt}$ [for loss of current]

or, $\oint_s \vec{J}\cdot d\vec{s} = -\frac{d}{dt}\oint_v \rho dv$ $\left[\because \rho = \frac{dq}{dv}\right]$

Using Gauss divergence theorem,

$$\oint_s (\vec{\nabla}\cdot\vec{J})\, dv = -\oint_v \frac{\partial \rho}{\partial t} dv$$

$$\therefore \boxed{\nabla\cdot J = -\frac{\partial \rho}{\partial t}}$$

proved.

For steady state current, ρ = constant.

$$\therefore \boxed{\nabla\cdot J = 0}$$

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P.106

Direct Current

⇒ Ohm's Law:

It states that, "the potential difference across any two points on a conductor is directly proportional to steady current through it, provided that its physical conditions remain unchanged."

$V \propto I$

$V = RI$

where, R is resistance of conductor.

Also,

$$I = \frac{V}{R} = \frac{E\cdot l}{\rho l/A} = \frac{EA}{\rho}$$

or, $\frac{I}{A} = \frac{1}{\rho}E$

$\therefore \boxed{J = \sigma E}$

where, $\frac{I}{A} = J$ and $\sigma = \frac{1}{\rho}$

In vector form,

$\vec{J} = \sigma\vec{E}$

Where, J = current density and, σ = conductivity.

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Resistance, Resistivity and Conductivity:-

An electrical resistance in conducting medium is due to collision of mobile electrons with free electrons.

If an electron of mass 'm' is applied in an electric field of magnitude E, the electron will experience an acceleration given by Newton's law;

$$a = \frac{F}{m} = \frac{eE}{m} \longrightarrow (i)$$

The nature of the oscillations experienced by conduction electrons is such that after a typical collision, each electron will completely lose its drift velocity. Each electron will then start off fresh after every encounter, moving off in random direction. In the average time τ between the collisions, the average electron will acquire a drift speed of $v_d = a\tau$.

Moreover, if we measure the drift speeds of all the electrons at any instant, we will find that their average drift speed is also '$a\tau$'.

Thus at any instant, the electron will have drift speed.

$$v_d = a\tau \longrightarrow (ii)$$

Then eq^n (i) gives,

$$v_d = a\tau = \frac{eE\tau}{m} \longrightarrow (iii)$$

since,

$$\vec{J} = ne\vec{v_d} \longrightarrow (iv)$$

where, n = no. of carriers per unit volume

Combining equation (iii) with magnitude form of eq^n (iv) leads us to:

$$v_d = \frac{J}{ne} = \frac{eE\tau}{m}$$

$$\text{or, } E = \left(\frac{m}{e^2 n\tau}\right) J. \longrightarrow (v)$$

Comparing eq^n (v) with $E = \rho J$, we get,

$$\boxed{\rho = \frac{m}{e^2 n\tau} = \frac{mv}{e^2 n\lambda}}$$

This is the required expression for resistivity of metals in terms of mean free path(λ).

The conductivity of metals is,

$$\sigma = \frac{1}{\rho} = \frac{e^2 n \lambda}{mv}$$

i.e. conductivity of metal is directly proportional to mean free path and inversely proportional to average velocity of electrons.

The conductivity of semiconductor is,

$$\sigma_s = e(n\mu_e + p\mu_n)$$

where n & p are concentration of electrons and holes, μ_e and μ_n are mobilities of electrons and holes respectively.

$$\mu_e = \frac{e\tau_e}{m_e}$$

$$\mu_n = \frac{p\tau_n}{m_n}$$

The resistivity of metal increases with temperature. It is due to an increase in collision rate of carriers.

In case of semiconductor, $conc^n$ of carriers is small but increases with increase of temperature. This causes decrease in resistivity with increasing temperature.

Semiconductors:-

The metals whose electrical properties lies between conductor and insulator are known as semi-conductor.

According to Band theory of solid, the semiconductors are partially filled conduction band, partially filled valence band and ~~gap~~ band gap is in the order of 1 ev (electron volts).

There are two types of semiconductors.

(i) Intrinsic semiconductor:-

The semiconductor in its pure ~~present~~ form is called intrinsic semiconductor. In intrinsic semiconductor, number of electrons is equal to the number of holes. Examples:-

Germanium → band gap = 0.7 ev

Silicon → band gap = 1.1 ev.

(ii) Extrinsic semiconductor:-

When some impurities elements i.e. pentavalent phosphorous, antimony and trivalent aluminium are added to pure semiconductor then resulting semiconductor is called extrinsic semiconductor.

These are two types of extrinsic semiconductor:-

(a) n-type semiconductor:- When pentavalent impurities like antimony, phosphorous are dropped with the pure semiconductor, the four electron of impurity forms covalent bond with semiconductor but one electron still remaining. In this way by the addition of pentavalent impurities increases the concentration of electron and forms n-type semiconductor.

(b) p-type semiconductor:- When the trivalent impurities like boron is dropped into the pure semiconductor, it decreases the concentration of electron and forms the P-type semiconductor. (i.e. there is the formation of ~~strong~~ large number of holes).

Superconductors:-

Kamerlingh onnes first observed that the resistivity of certain metals and alloys drop abruptly to zero when they are cooled to sufficiently low temperature. This phenomenon is called superconductivity and the substance exhibiting this phenomenon is called superconductors. For instance, the resistivity of pure mercury drops to zero, when it is cooled to temperature of about 4.2 K.

The temperature at which the electrical resistivity of substance becomes zero is called critical temperature or threshold temperature. Below the critical temperature substance is in superconducting state and above the critical temperature substance is in normal state.

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P.110

Electrostatics.

⇒ Electric charge:-

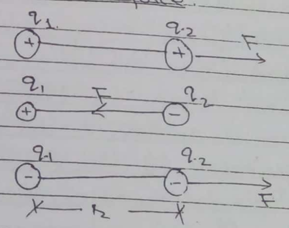
Charge:- It is the intrinsic property of the fundamental particle.

Conservation of charge:- Total charges of the system always remain constant.

Quantization of charge:-

$$q = Ne \;; \quad N = 1, 2, 3 \dots$$

⇒ Electric force:-



Coulomb's Law:-

$$F \propto q_1 q_2 \longrightarrow (i)$$

$$F \propto \frac{1}{r^2} \longrightarrow (ii)$$

combining (i) & (ii), we get.

$$F = K\frac{q_1 q_2}{r^2}$$

Where K is a constant depends on (i) units (ii) medium.

In SI and vaccum,

$$K = \frac{1}{4\pi\epsilon_0} \approx 9\times10^9\ m/F.$$

ϵ_0 = permitivity of free space
= 8.84×10^{-12} F/m.

In SI and other medium,

$$K = \frac{1}{4\pi\epsilon}$$

ϵ = permittivity of the medium.

Relative permittivity,

$$\epsilon_r = \epsilon/\epsilon_0$$

$$\epsilon_r \geq 1.$$

For eg:- Value of ϵ_r
Vaccum : 1
Air : 1.0012
water : 82.

Hence,

$$\boxed{\vec{F} = \frac{1}{4\pi\epsilon_0\epsilon_r}\frac{q_1 q_2}{r^2}\hat{r}}$$

⇒ For point charge application.
→ Principle of superposition of force.

Net force, $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \dots$

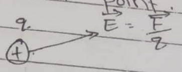
⇒ Electric Field:-

Electric field is the space around the charge in which another test charge experienced force at attraction or repulsion.
Test charge:- Very small, always +ve charge

Electric field intensity :- 'E' :-

~~At~~ Electric field intensity at any point in space may be defined as the force experienced per unit test charge hold at that point.

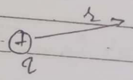
$\vec{E} = \vec{F}/q$.



Unit - Newton/columb.

Electric field due to point charge :-

$$F = \frac{1}{4\pi\epsilon_0}\frac{qq'}{r^2}$$



From definition,

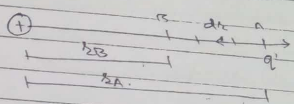
$$\boxed{E = \frac{F}{q} = \frac{q}{4\pi\epsilon_0 r^2}}$$

Principle of superposition of fields,

$$\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + \dots$$

Electric potential :-

Potential difference (Pd) between any two points to another



$$V_B - V_A = \frac{W_{AB}}{q'}$$

$$= \int_A^B \frac{\vec{F}}{q'} \cdot d\vec{r}$$

$$= -\int_A^B \vec{E} \cdot d\vec{r}$$

$$\therefore V_B - V_A = \int_{r_B}^{r_A} \frac{q}{4\pi\epsilon_0 r^2} dr$$

$$= -\frac{q}{4\pi\epsilon_0}\int_{r_A}^{r_B} \frac{dr}{r^2}$$

$$= -\frac{q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_{r_A}^{r_B}$$

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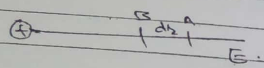
$$\therefore V_B - V_A = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

If $r_A \to \infty$, $V_A \to \infty$

$r_B \to r$, $V_B = V$

$$\therefore \boxed{V = \frac{q}{4\pi\epsilon_0 r}}$$ Imp

Relation between P.d. and electric field.



$$dV = -\vec{E} \cdot \vec{dr}$$

$$= -E\, dr \cos\theta.$$

If $\theta = 0$,

$$dV = -E\, dr.$$

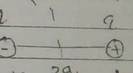
$$\therefore \vec{E} = \frac{-dV}{dr}$$

vector quantity — scalar quantity

Calculation of Electric field and Potential.

① Diapole:-

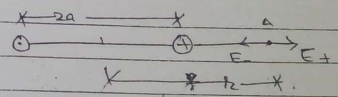
Two equal and opposite charges separated by a distance form a diapole. The two charges $+q$ and $-q$ are separated by distance $2a$ which is called diapole length.



Diapole moment :

$$\vec{P} = q \times 2\vec{a}$$

• Electric field on axial line,



To find electric field intensity at an axial line at distance r from the centre of the diapole. We use principle of superposition of field.

$$\therefore \vec{E} = \vec{E_+} + \vec{E_-}$$

In ~~case of~~, our case,

$$E = E_+ - E_-$$

$$= \frac{q}{4\pi\epsilon_0(r-a)^2} - \frac{q}{4\pi\epsilon_0(r+a)^2}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{(r+a)^2-(r-a)^2}{(r-a)^2\,(r+a)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{r^2+2ar+a^2-r^2+2ar-a^2}{(r^2-a^2)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{4ar}{(r^2-a^2)^2}\right]$$

or, $E = \dfrac{2Pr}{4\pi\epsilon_0(r^2-a^2)^2}$ $\quad \left[\because \vec{P} = q \times 2\vec{a}\right]$

For short diapole $a << r$

$$\therefore E = \frac{2Pr}{4\pi\epsilon_0 r^4} = \frac{2P}{4\pi\epsilon_0 r^3}$$

$$\therefore \boxed{E = \frac{P}{2\pi\epsilon_0 r^3}}$$

→ To find potential at any point due to dipole at a distance 'r' which makes angle θ with dipole axis,
Here,

We simply subtract two potential i.e.

$$V = V_+ - V_-$$

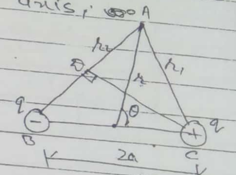


Fig (i)

or, $V = \dfrac{q}{4\pi\epsilon_0 r_1} - \dfrac{q}{4\pi\epsilon_0 r_2}$

or, $V = \dfrac{q}{4\pi\epsilon_0}\left[\dfrac{1}{r_1} - \dfrac{1}{r_2}\right]$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{r_2-r_1}{r_1 r_2}\right] \longrightarrow (i)$$

To evalule the quantity inside the bracket, we draw a circle having radius $= r_1$ and at centre A. For short diapole, 'a' is very far from the diapole axis so that the arc CD will be a straight line which cut the line AB at D normally as in figure (i)

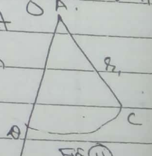


Fig (ii)

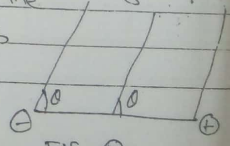


Fig. (iii)

Further. A is very far so that line BQ and OE will be parallel to each other. as in figure (iii).

From ΔBCQ.

$$\cos\theta = \frac{BQ}{BC}$$

$\therefore BQ = BC\cos\theta$

$$r_2 - r_1 = 2a\cos\theta \longrightarrow (ii)$$

But r_1 and r_2 are very large as compared to 'a', so, $r_1 r_2 \approx r^2 \longrightarrow$ (iii)

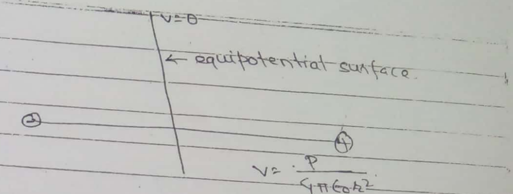
From (i), (ii) and (iii),

$$V = \frac{q.2a\cos\theta}{4\pi\epsilon_0 r^2}$$

$$\therefore \boxed{V = \frac{P\cos\theta}{4\pi\epsilon_0 r^2}}$$

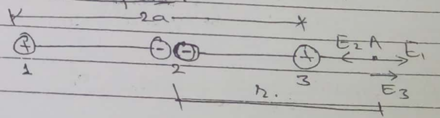
case I: For axial line, $\theta = 0$, $\boxed{V = \frac{P}{4\pi\epsilon_0 r^2}}$

case II: For equitorial line, $\theta = \pi/2$, $\therefore V = 0$



The surface made by equal potential is called equipotential surface.

(ii) Quadrupole:-



Two diapoles connected form a quadrupole. A linear quadrupole is shown in figure. To find electric field intensity at an axial line, we use principle of superposition of field.

$$\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3}$$

In our case

$$E = E_1 - E_2 + E_3$$

or, $E = \frac{q}{4\pi\epsilon_0 (r+a)^2} - \frac{2q}{4\pi\epsilon_0 r^2} + \frac{q}{4\pi\epsilon_0 (r-a)^2}$

$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r+a)^2} - \frac{2}{r^2} + \frac{1}{(r-a)^2}\right]$

$= \frac{q}{4\pi\epsilon_0}\left[\frac{r^2(r-a)^2 - 2(r^2-a^2)^2 + r^2(r+a)^2}{r^2(r^2-a^2)^2}\right]$

$= \frac{q}{4\pi\epsilon_0}\left[\frac{6a^2r^2 - 2a^4}{r^2(r^2-a^2)^2}\right]$

or, $E = \frac{q\,2a^2}{4\pi\epsilon_0}\left[\frac{3r^2-a^2}{r^2(r^2-a^2)^2}\right]$

Let quadrupole moment, $Q = q \cdot 2a^2$

so,

$E = \frac{Q}{4\pi\epsilon_0}\left[\frac{3r^2-a^2}{r^2(r^2-a^2)^2}\right]$

$\therefore$ For short quadrupole, $r >> a$

$\therefore E = \frac{Q \cdot 3r^2}{4\pi\epsilon_0 r^2 r^4}$

$\therefore \boxed{E = \frac{3Q}{4\pi\epsilon_0 r^4}}$

⇒ Potential:-

$V = V_1 - V_2 + V_3$

$= \frac{q}{4\pi\epsilon_0 (r+a)} - \frac{2q}{4\pi\epsilon_0 r} + \frac{q}{4\pi\epsilon_0 (r-a)}$

$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r+a)} - \frac{2}{r} + \frac{1}{(r-a)}\right]$

$= \frac{q}{4\pi\epsilon_0}\left[\frac{r(r-a) - 2(r^2-a^2)}{r(r^2-a^2)}\right]$

$= \frac{q}{4\pi\epsilon_0} \frac{2a^2}{r(r^2-a^2)}$

$= \frac{q}{4\pi\epsilon_0} \frac{2a^2}{r(r^2-a^2)}$

or, $V = \frac{Q}{4\pi\epsilon_0} \frac{1}{r(r^2-a^2)}$

For short quadrupole. $r >> a$.

$\boxed{V = \frac{Q}{4\pi\epsilon_0 r^3}}$

And,

$E = -\frac{dv}{dr}$

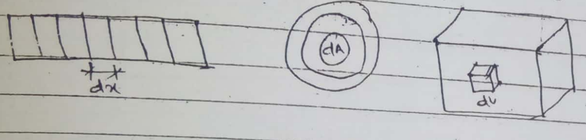
$\therefore \boxed{E = \frac{3Q}{4\pi\epsilon_0 r^4}}$

P.116

Calculation of electric field and potential; Continuous charge distribution:-

General method:-

(i) Divide the charge distribution into number of charge element each of length or area or volume as dx, dA and dv respectively in according to the problem.



(ii) Calculate charge contained in the charge element e.g. $dq = \lambda dx$ or $dq = \sigma dA$ or $dq = \rho dV$.

(iii) Calculate electric field & potential as. dE or dV

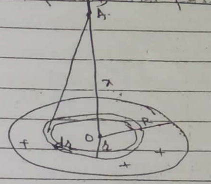
(iv) If the direction of dE and the given problem is same then,

$$E = \oint dE \quad \text{or} \quad V = \oint dV$$

(v) If not, resolve dE into two components then, total field will be,

$E = \oint dE \cos\theta$ or $E = \oint dE \sin\theta$ but not both.

(iii) Electric field and potential of a charged disk:-



Consider a charged disk of radius 'R' carrying uniformly distributed charges. Let σ be the surface charge density.

To find electric field intensity at an axial distance 'x' from the centre of disc. We divide it into no. of charge ring as shown in figure.

Electric field intensity at A due to the ring of radius r and thickness dr is given by,

$$dE = \frac{(\sigma \cdot 2\pi r dr) \cdot x}{4\pi\epsilon_0 (x^2+r^2)^{3/2}} \quad \text{along AB}$$

$$\therefore E = \frac{\sigma x}{2\epsilon_0}\int_0^R \frac{r\,dr}{(x^2+r^2)^{3/2}}$$

~~Put~~

$$\cancel{x^2 + r^2 = y}$$

$$\cancel{2r\,dr = dy}$$

$$\cancel{\int \frac{r\,dr}{(x^2+r^2)^{3/2}} = \int y^{3/2}\,dy}$$

On Integration,

$$E = \frac{\sigma_0}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$$

$$\therefore E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]$$

If we let $R \to \infty$, plane sheet,

$$E = \frac{\sigma}{2\epsilon_0}$$

Potential,

$$dV = \frac{\text{charge}}{\text{distance}},$$

$$= \int \frac{\sigma \cdot 2\pi r\,dr}{4\pi\epsilon_0 (x^2+r^2)^{1/2}}$$

$$= \frac{\sigma r\,dr}{2\epsilon_0 (x^2+r^2)^{1/2}}$$

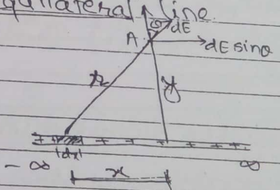
$$\therefore V = \int dV$$

$$= \frac{\sigma}{2\epsilon_0}\int_0^R \frac{r\,dr}{(x^2+r^2)^{1/2}}$$

$$= \frac{\sigma}{2\epsilon_0}\Big[(x^2+r^2)^{1/2}\Big]_0^R$$

$$\boxed{\therefore V = \frac{\sigma}{2\epsilon_0}\left[(x^2+R^2)^{1/2} - x\right]}$$

(iv) Electric field of a long charged rod at an equilateral line



Net electric field, $E = \oint dE\cos\theta$

$$= \int_{-\infty}^{\infty} \frac{\lambda\,dx}{4\pi\epsilon_0 r^2}\,\frac{y}{r}$$

$$\therefore E = \frac{2\lambda}{4\pi\epsilon_0}\int_0^{\infty} \frac{y\,dx}{r^3}$$

$$= \frac{\lambda}{2\pi\epsilon_0}\int_0^{\infty} \frac{y\,dx}{(x^2+y^2)^{3/2}}.$$

From figure, $\tan\theta = x/y$

$$\Rightarrow x = y\tan\theta$$

$$\therefore E = \frac{\lambda}{2\pi\epsilon_0}\int_0^{\pi/2} \frac{y^2\sec^2\theta\, d\theta}{(y^2\tan^2\theta + y^2)^{3/2}}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 y}\int_0^{\pi/2} \cos\theta\, d\theta.$$

$$= \frac{\lambda}{2\pi\epsilon_0 y}\left[\sin\theta\right]_0^{\pi/2}$$

$$\therefore \boxed{E = \frac{\lambda}{2\pi\epsilon_0 y}}$$

which is the required expression for electric field of a long charged rod at equilateral.

Gauss Law:-

Total electric flux from any closed surface is equal to $1/\epsilon_0$ times the charge enclosed by the surface.

$$\phi = q/\epsilon_0 \longrightarrow \text{(i)}$$

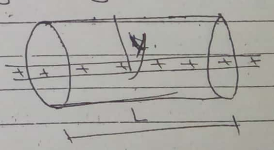
$$\phi = \oint \vec{E}\cdot d\vec{A} \longrightarrow \text{(ii)} \text{ or } \int E\,dA\cos\theta$$

From (i) and (ii), we get,

$$\boxed{\oint \vec{E}\cdot d\vec{A} = q/\epsilon_0}$$

Application:-

(i) Electric field on the equitorial line of long charged rod:-



Draw a Gaussian cylinder of radius y, that encloses the given charged rod.

From Gauss law i.e.

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

> for Complicated Case, we use Gaussian law over Coloumb law.

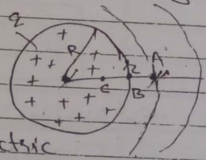
In our case,

$$E \oint dA = \frac{\lambda L}{\epsilon_0}$$

or, $E \cdot 2\pi r L = \frac{\lambda L}{\epsilon_0}$

$$\therefore \boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$$

(ii) Electric field of a dielectric charged sphere:-



Consider a non-conducting charged sphere of radius R'. To find electric field intensity outside, at the surface and inside the sphere. We use Gauss Law.

(a) Outside:-

Draw a Gaussian sphere of radius 'r' that encloses the given charged sphere. From Gauss Law,

$$\oint \vec{E} \cdot \vec{dA} = q/\epsilon_0$$

for conducting sphere

(i) Outside $\rightarrow \frac{q}{4\pi\epsilon_0 r^2}$

(ii) on Surface $\rightarrow \frac{q}{4\pi\epsilon_0 R^2}$ (maxm)

(iii) Inside $\rightarrow$ 0 (minm)

In our case,

$$E \oint dA = q/\epsilon_0$$

$$E \cdot 4\pi r^2 = q/\epsilon_0$$

$$\therefore \boxed{E = \frac{q}{4\pi\epsilon_0 r^2}}$$

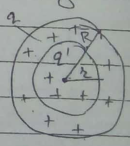
(b) At the surface,

$$\boxed{E = \frac{q}{4\pi\epsilon_0 R^2}}$$

(c) Inside:-

Draw a Gaussian surface of radius r (r<R) which encloses charge q'.

From Gauss law,



$$\oint \vec{E} \cdot \vec{dA} = q'/\epsilon_0$$

or, $E \cdot 4\pi r^2 = \frac{q'}{\epsilon_0} \longrightarrow (1)$

Assuming charges are uniformly distributed so that charge density ρ

remains constant.

$\rho = \dfrac{\text{Total charge}}{\text{Total volume}}$

or, $\rho = \dfrac{q}{4/3 \pi R^3} = \dfrac{q'}{4/3 \pi r^3}$

$\therefore q' = \dfrac{q r^3}{R^3} \longrightarrow$ (ii)

From (i) and (ii),

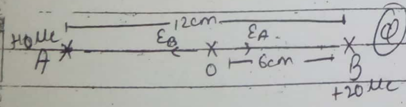
$E \cdot 4\pi r^2 = \dfrac{q r^3}{\epsilon_0 R^3}$

$\therefore \boxed{E = \dfrac{q r}{4\pi\epsilon_0 R^3}}$

which implies,

$\Rightarrow E \propto r$

Q



(a) find E at O.

$E = |E_B - E_A|$

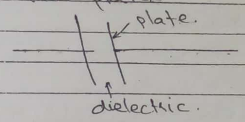
$= \dfrac{20}{4\pi\epsilon_0 \cdot 6^2} - \dfrac{10}{4\pi\epsilon_0 \cdot 6^2}$

$= \dfrac{10}{4\pi\epsilon_0 6^2}(2-1)$

$= 2.49 \times 10^7$

Capacitor:-

It is a device to store charges and energy. It consist of two conductors separated by an insulator called dielectric plates.



Types:-

(a) According to dielectric:-
(i) Paper
(ii) Polyster
(iii) Electrolytic.

(b) According to shape of plates
(i) Parallel plates.
(ii) Spherical plates.
(iii) Cylindrical.

Capacitance, C

$C = q/V$

- Capacity to store charge.
- C is property of the plates.
- independent of q and V

Calculation of capacitance:-

General method:

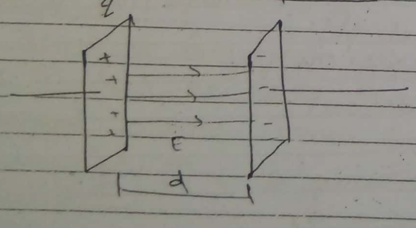
(i) Assume charges on the plates of the capacitor say q.

(ii) calculate electric field in between the plates by using Gauss law.

(iii) Calculate p.d between the plates by
$$V = \int \vec{E} \cdot \vec{dl}$$

(iv) calculate C by $C = q/V$.

(a) Parallel plate capacitor.



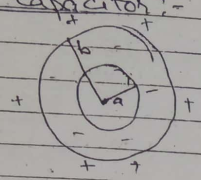
Two parallel plates having surface area A separated in air by distance 'd' form a parallel plate capacitor as shown in figure. 'q' be the charges on the capacitor. Let E be the electric field in the capacitor which is obtained by Gauss Law.

Here,
$$\oint \vec{E} \cdot \vec{dA} = q/\epsilon_0$$
$$EA = q/\epsilon_0$$
$$E = q/\epsilon_0 A$$

Let V be p.d between the plates which is given by,
$$V = \int_0^d E \cdot dl = \int_0^d \frac{q}{\epsilon_0 A} dl.$$
$$\therefore V = \frac{qd}{\epsilon_0 A}.$$

We know,
capacitance, $C = q/V = \frac{\epsilon_0 A}{d}$.
$$\therefore \boxed{C = \epsilon_0 L} \quad ; \left[\because L = A/d\right]$$

(b) Spherical capacitor:-



Let q be the changes on the spherical capacitor.
For E we draw Gaussian surface which encloses negatively charged. surface

$$\oint \vec{E} \cdot \vec{dA} = q/\epsilon_0$$

$$E \cdot A = q/\epsilon_0$$

$$E \cdot 4\pi r^2 = q/\epsilon_0$$

$$\therefore E = q/4\pi\epsilon_0 r^2$$

Let V be the p.d between the plates which is,

$$V = \int_a^b E \cdot dr = \int_a^b \frac{q}{4\pi r^2 \epsilon_0} dr$$

or, $$V = \frac{q}{4\pi\epsilon_0} \int_a^b r^{-2} dr$$

$$= \frac{q}{4\pi\epsilon_0} \left[-\frac{1}{r} \right]_a^b$$

$$= \frac{q}{4\pi\epsilon_0} \left[-\frac{1}{b} + \frac{1}{a} \right]$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{b-a}{ab} \right]$$

Now,

$$C = q/V = \frac{4\pi ab\epsilon_0}{b-a}.$$

$$\therefore C = \epsilon_0 L \quad ; \quad L = \frac{4\pi ab}{b-a}.$$

Isolated spherical capacitor.

$$b \to \infty$$

$$C = \frac{4\pi\epsilon_0 ab}{b(1-a/b)}$$

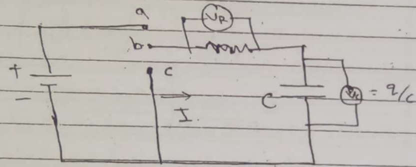
$$= 4\pi\epsilon_0 a \frac{1}{(1-a/\infty)}$$

$$= 4\pi\epsilon_0 a.$$

$$\therefore \boxed{C = 4\pi\epsilon_0 a}$$

Charging & Discharging of Capacitors:-

① Charging of capacitors:



Fig①:- RC circuit.

In figure, let the point 'a' is joined with point 'b' for charging of the given capacitor:

Energy supplied by cell,

$E \cdot dq = I^2 R\, dt + \frac{q}{C} dq$

or, $E \cdot \frac{dq}{dt} = I^2 R + \frac{q}{C} \frac{dq}{dt}$

or, $E \cancel{I} = I^{\cancel{2}} R + \frac{q}{C} \cancel{I}$

or, $\left(E - \frac{q}{C}\right) = IR$

or, $\left[E - \frac{q}{C}\right] = R \frac{dq}{dt}$

or, $(EC - q) = RC \frac{dq}{dt}$

or, $\frac{dq}{(EC - q)} = \frac{dt}{RC}$

Integrating both sides, we get,

$\int \frac{dq}{(EC - q)} = \frac{1}{RC} \int dt + A$

or, $-\log_e [EC - q] = \frac{t}{RC} + A$

For $t = 0$, $q = 0$, then above relation becomes,

$-\log_e EC = A$

i.e. $-\log_e [EC - q] = \frac{t}{RC} - \log_e EC$

or, $\log_e [EC - q] - \log_e EC = -\frac{t}{RC}$

or, $\log_e \left[\frac{EC - q}{EC}\right] = -\frac{t}{RC}$

Taking antilog both sides, we get,

$\left(\frac{EC - q}{EC}\right) = e^{-t/RC}$

or, $\left[1 - \frac{q}{EC}\right] = e^{-t/RC}$

or, $\frac{q}{EC} = 1 - e^{-t/RC}$

$\therefore q = EC\left[1 - e^{-t/RC}\right]$

Here, $EC = q_0$ is called maximum or equilibrium charged in the circuit. Then above relation becomes

Imp. $\boxed{q = q_0\left[1 - e^{-t/RC}\right]}$

This is required relation for charging of a capacitor.

If $t = RC = \tau_c$ (say) called time constant in RC circuit, then,

$q = q_0\left[1 - e^{-1}\right]$

$q = 0.638\, q_0$

Thus, time constant of RC circuit is equal to time in which capacitor approaches 0.638 times the equilibrium charge.

Also,

$I = \frac{dq}{dt} = \frac{d}{dt}\left[q_0\left(1 - e^{-t/RC}\right)\right]$

or, $I = \frac{q_0}{RC} e^{-t/RC}$.

or, $I = \frac{E}{R} e^{-t/RC}$

$\therefore I = I_0 e^{-t/RC}$

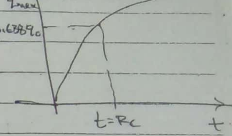


Fig:- Charging of capacitor

(ii) **Discharging of capacitor:**

Let the terminal a & b is disconnected and b is connected to c in above fig. Then we get,

$0 = I^2R\,dt + \frac{q}{c}\,dq$

or, $0 = I^2R + \frac{q}{c}\left(\frac{dq}{dt}\right)$

or, $0 = I^2R + \frac{q}{c} I$.

$\therefore IR + \frac{q}{c} = 0$

or, $IR = -\frac{q}{c}$

or, $\frac{dq}{dt}R = -\frac{q}{c}$

or, $\frac{dq}{q} = -\frac{dt}{RC}$

Impt

Capacitance of a cylindrical capacitor :-

Let us consider a cylinder of outer diameter '2b' and inner radius 'a' as shown in fig.

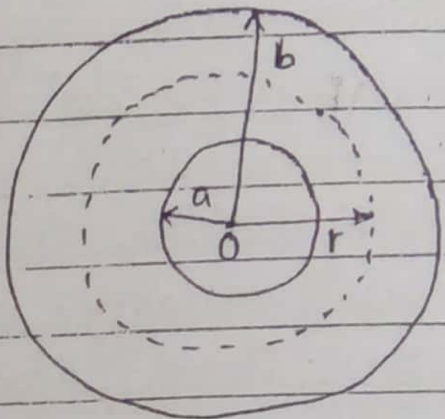


Fig: Cross-section of a long cylindrical capacitor

Figure shows a cross-section of cylindrical capacitor of length 'L' formed by two axial cylinder of radii a and b. We assume $L >> B$, so that we can neglect the fringing of electric field that occurs at the end of the cylinders. Each plates contains a charge of magnitude Q.

Assume a Gaussian surface, we chose a cylinder of length 'l' and radius 'r', closed by end caps. The we write,

$$Q = \epsilon_0 EA = \epsilon_0 E (2\pi r l)$$

where $A = 2\pi r l$ is the area of curved part of Gaussian surface. There is no flux through the end cap.

Then

The potential difference,

$$V = \int_a^b E\,ds = \frac{Q}{2\pi\epsilon_0 l}\int_a^b \frac{dr}{r}$$

$$\text{or, } V = \frac{Q}{2\pi\epsilon_0 l}\ln\left(\frac{b}{a}\right) \quad \text{------ (2)}$$

As integration is done radially inward, we chose $ds = -dr$.

Now,

$$C = \frac{Q}{V}$$

$$\text{or, } \boxed{C = 2\pi\epsilon_0 \cdot \frac{l}{\ln(b/a)}}$$

which is the required expression for the capacitance of a cylindrical capacitor which depends only on geometrical factors.

—— x —— x ——

"Best Of Luck"

- Bikal Khanal

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