Assignment 3:

1. If $\frac{d\vec{r}}{dt} = t^2\,\vec{i} + (6t+1)\,\vec{j} + 8t^3\,\vec{k}$ and $\vec{r}(0) = 2\,\vec{i} - 3\,\vec{j} + \vec{k}$, find $\vec{r}$.

2. If $\vec{r}$ is the unit vectors, prove that $\left|\vec{r} \times \frac{d\vec{r}}{dt}\right| = \left|\frac{d\vec{r}}{dt}\right|$

3. If $\vec{a}$ any vector then prove that $\vec{i} \times (\vec{a} \times \vec{i}) + \vec{j} \times (\vec{a} \times \vec{j}) + \vec{k} \times (\vec{a} \times \vec{k}) = 2\vec{a}$ where $\vec{i}, \vec{j}, \vec{k}$ mutually perpendicular unit vectors along the co-ordinate are axes.

4. For the curve $x = 3t,\ y = 3t^2, z = 2t^3$, prove that $[\dot{\vec{r}}\ \ddot{\vec{r}}\ \dddot{\vec{r}}] = 216$.

5. If $\frac{d\vec{a}}{dt} = \vec{c} \times \vec{a}$, $\frac{d\vec{b}}{dt} = \vec{c} \times \vec{b}$ show that $\frac{d(\vec{a}\times\vec{b})}{dt} = \vec{c} \times (\vec{a} \times \vec{b})$

6. Prove that the necessary and sufficient conditions for a vector function $\vec{a}$ of a scalar variable t to have a constant direction is $\vec{a} \times \frac{d\vec{a}}{dt} = 0$.

7. Prove that the necessary and sufficient conditions for a vector function $\vec{a}$ of a scalar variable t to have a constant magnitude is $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$.

8. A particle moves along the curve $x = a\cos t,\ y = a\sin t, z = bt$. Find the velocity and acceleration at $t = 0$ and $t = \frac{\pi}{2}$. Also find their magnitude.

9. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, -1, 2).

10. Find the angle between the normal to the surfaces $x\ logz = y^2 - 1$ and $x^2y + z = 2$ at the point (1, 1, 1).

11. In what direction from the point (3,1,-2) is the directional derivative of $\phi(x, y, z) = x^2y^2z^4$ maximum? Find also the magnitude of this maximum.

12. Find the constant 'a' such that the vector $(ax^2y + yz)\vec{i} + (xy^2 - xz^2)\vec{j} + (2xyz - 2x^2y^2)\vec{k}$ is solenoidal.

13. Find the constants a, b, c so that the vector $\vec{v} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational..

14. If $\vec{a}$ is a constant vector and $\vec{r}$ be the position vector then prove that

$$\nabla \times (\vec{a} \times \vec{r}) = 2\vec{a}$$

15. Define gradient of a scalar point function and divergence, curl of vector point function. Find the gradient, divergence and curl (whichever possible) of the following scalar and vector point functions.

    i. $\vec{v} = 3x^2\vec{i} + 5xy^2\vec{j} + xyz^3\vec{k}$ at the point (1,2,3)

ii. $\vec{v} = \frac{x\vec{i}+y\vec{j}+z\vec{k}}{\sqrt{x^2+y^2+z^2}}$

iii. $\emptyset = 3x^2y - y^3z^2$ at the point (1,-2,-1).