* Conversion from finite automata to regular expression:- (Arden's Theorem)

Let P and q be the regular expression over alphabet Σ, if P doesnot contain empty string then $r = q + rp$ has a unique solution $r = qp^*$

Proof:

Here, $r = q + rp$. — ①

Let us put value of $r = q + rp$ in eqn ①

$$r = q + (q + rp)p$$

$$r = q + qp + rp^2 \quad - ②$$

Again, put the value of r in eqn ②

$$r = q + qp + (q + rp)p^2$$

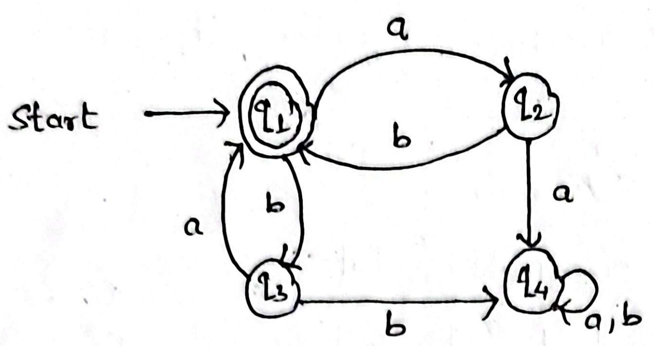
$$r = q + qp + qp^2 + rp^3 \quad - ③$$

Similarly, $r = q + qp + qp^2 + qp^3 + \ldots$

$$r = q(\varepsilon + p + p^2 + p^3 + \ldots)$$

$$r = qp^*$$ Hence proved.

Example find the regular expression for following DFA.



Now, lets from the equation for each state

$$q_1 = q_2 b + q_3 a + \epsilon \quad - (1)$$

$$q_2 = q_1 a \quad - (2)$$

$$q_3 = q_1 b \quad - (3)$$

$$q_4 = q_2 a + q_3 b + q_4 a + q_4 b \quad - (4)$$

Now, putting the value of q_2 and q_3 in eqn (1)

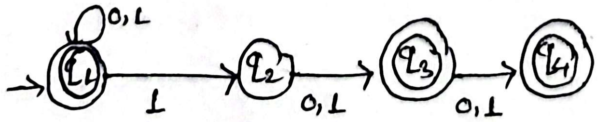
$$q_1 = q_1 ab + q_3 a + \epsilon$$

$$q_1 = q_1 ab + q_1 ba + \epsilon$$

$$q_1 = \epsilon + q_1(ab+ba)$$

$$= \epsilon(ab+ba)^*$$

Example 2 Given the following NFA, configure the equivalent Regular Expression



Solution   let's form the equation for each state

$q_1 = q_1 0 + q_1 1 + \varepsilon$ - (1)

$q_2 = q_1 1$ - (2)

$q_3 = q_2 0 + q_2 1$ - (3)

$q_4 = q_3 0 + q_3 1$ - (4)

from eqn (1)

$q_1 = q_1(0+1) + \varepsilon$

$q_1 = \varepsilon + q_1(0+1)$

$q_1 = \varepsilon.(0+1)^*$

Now, putting the value of eqn (1). in (2)

$q_2 = q_1 1$

$= (0+1)^* 1$

Now, again put the value. of q_2 in eqn (3)

$q_3 = q_2(0+1)$

$= (0+1)^* 1 (0+1)$

Now, again put the value of q_3 in (4)

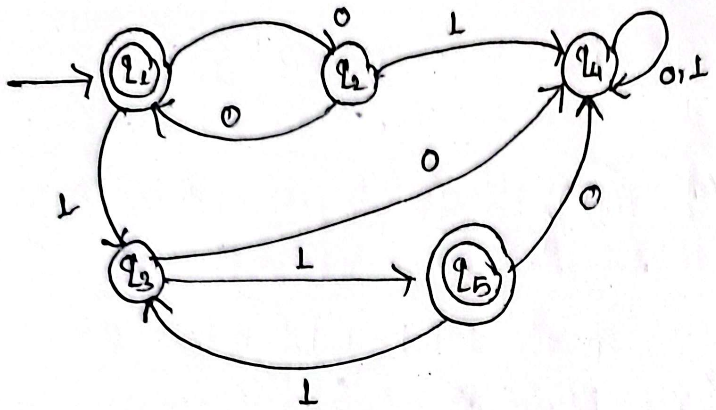
$q_4 = q_3(0+1)$

$= (0+1)^* 1 (0+1).(0+1)$

Since we have q_1 q_3 and q_4 are final state so regular expression for given automata is,

$(0+1)^*\{(0+1)^* 1 (0+1) + (0+1)^* 1 (0+1)(0+1)\}$

Example Configure the following regular expression for following finite Aut



Solution Let form the equation for each state.

$q_1 = q_2 0 + \varepsilon$ - ①

$q_2 = q_1 0$ - ②

$q_3 = q_1 1 + q_5 1$ - ③

$q_4 = q_2 1 + q_3 0 + q_4 0 + q_5 0$ - ④

$q_5 = q_3 1$ - ⑤

Solving equation ①

$q_1 = q_2 0 + \varepsilon$

$= q_1 00 + \varepsilon$

$q_1 = \varepsilon + q_1 00$

$q_1 = \varepsilon \cdot (00)^*$

$= (00)^*$

Now solving eqn ③

$q_3 = (00)^* 1 + q_3 11$

$q_3 = (00)^* 1 (11)^*$

Now put the value of q_3 in ⑤

$$q_5 = (00)^* 1 (11)^* 1$$

Hence, the regular expression for given DFA is,

$$(00)^* + (00)^* 1 (11)^* 1$$