Chapter 7

Two Port Network

Terminologies

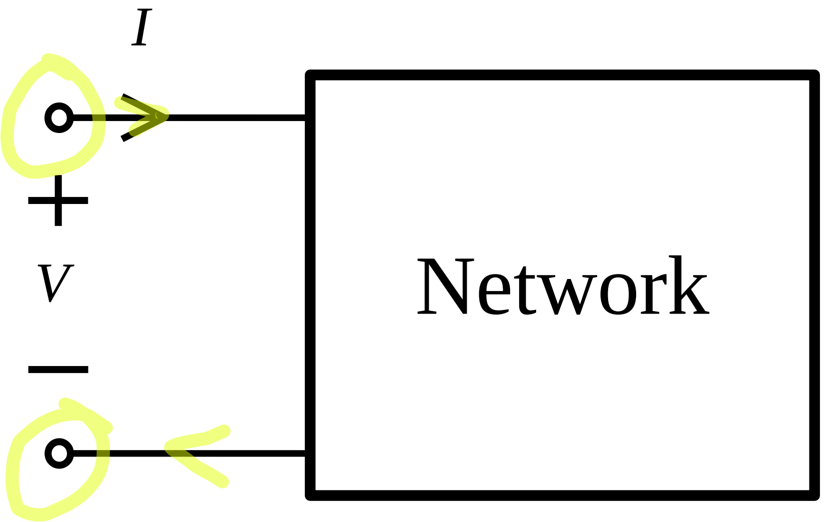
- Terminal

  If a conductor is fastened to any nodes on the network and brought out for access, the end of this conductors is designated as a terminal.

- Port

  A port is a pair of terminal such that the current entering into one terminal is exactly equal to the current leaving from another terminal.

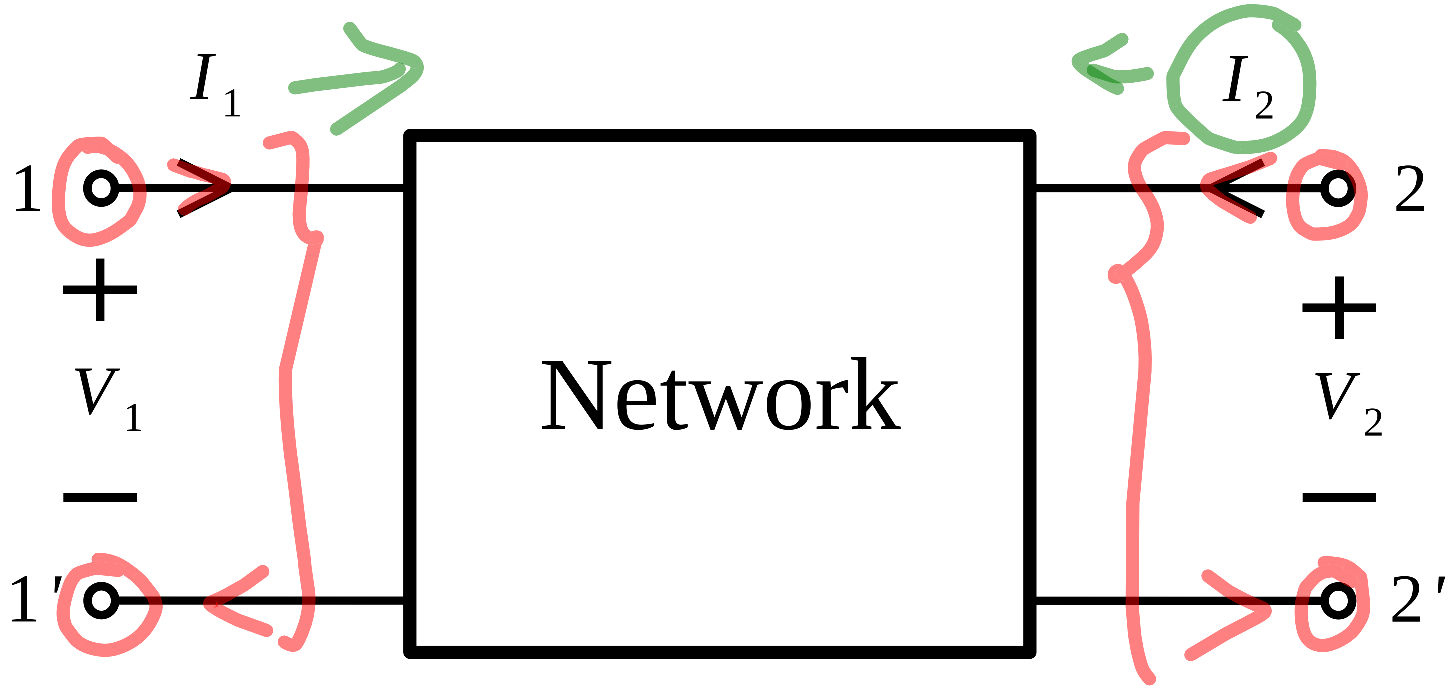
Single Port Network



- Terminal Quantity (V, I)
- Either connect current and measure voltage or connect voltage and measure current.

Note: Application is similar to Thevenin and Norton

Two Port Network



Note: The flow of current is into the network, which is a convention.

Two Port Network

- Terminal Quantities

  Port 1: V_1 , I_1 and Port 2: V_2 , I_2

- We want:
  - a terminal description or,
  - a port description or,
  - an external description

  of the system

Two Port Network

- We can consider any two of the terminal quantities as an independent variable and other shall be dependent variables.

V_1, I_1, V_2, I_2

- Six possible description ($^4C_2$)
  - can set two as independent (by connecting sources) and find another two.

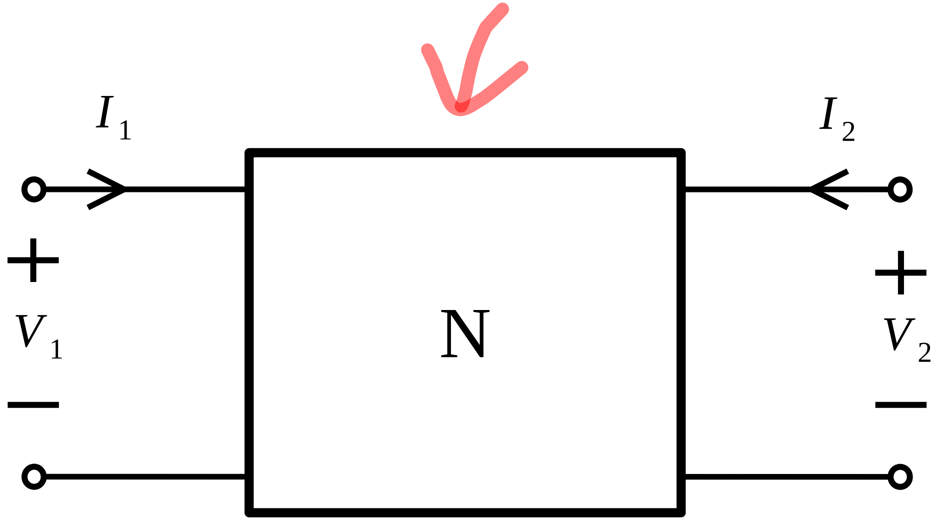
Note: We consider network that are linear and do not have any independent sources. But the network can have dependent sources.

S.N.	Dependent Quantities (Express)	Independent Quantities (In terms of)	Description
1	V_1 , V_2	I_1 , I_2	Open-circuit impedance parameter (z parameter)
2	I_1 , I_2	V_1 , V_2	Short-circuit admittance parameter (y parameter)
3	V_1 , I_1	V_2 , I_2	Transmission parameter T- param (ABCD parameter)

S.N.	Dependent Quantities (Express)	Independent Quantities (In terms of)	Description
4	V_2 , I_2	V_1 , I_1	Inverse transmission parameter (A'B'C'D' parameter)
5	V_1 , I_2	I_1 , V_2	Hybrid parameter (h parameter)
6	I_1 , V_2	V_1 , I_2	Inverse hybrid parameter (g parameter)

Open-circuit Impedance Parameter
(z parameter)

z parameter



- Express V_1 , V_2 in terms of I_1 and I_2

$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

z_{11}

- From definition:

$$\begin{cases} V_1 = z_{11} I_1 + z_{12} I_2 \\ V_2 = z_{21} I_1 + z_{22} I_2 \end{cases}$$

$$z_{11} = \frac{V_1}{I_1}\bigg|_{I_2 = 0}$$

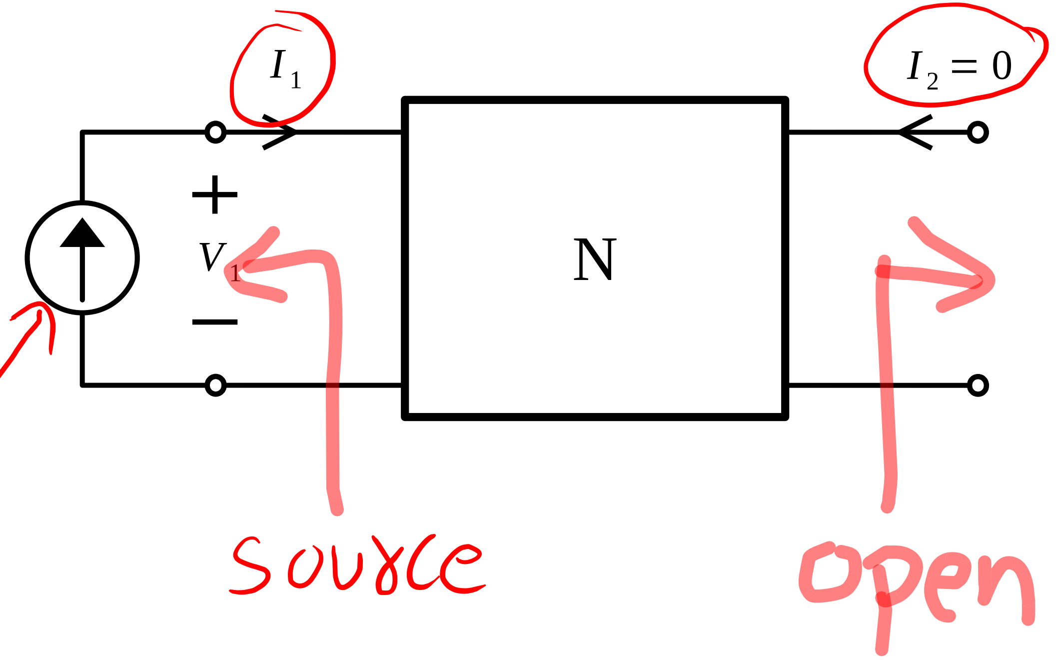
- Driving point impedance at port 1 with port 2 open.
- Also called *open circuit driving point input impedance*.

①

z_{11}

- Driving point impedance at port 1 with port 2 open.

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$



z_{12}

- From definition:

$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

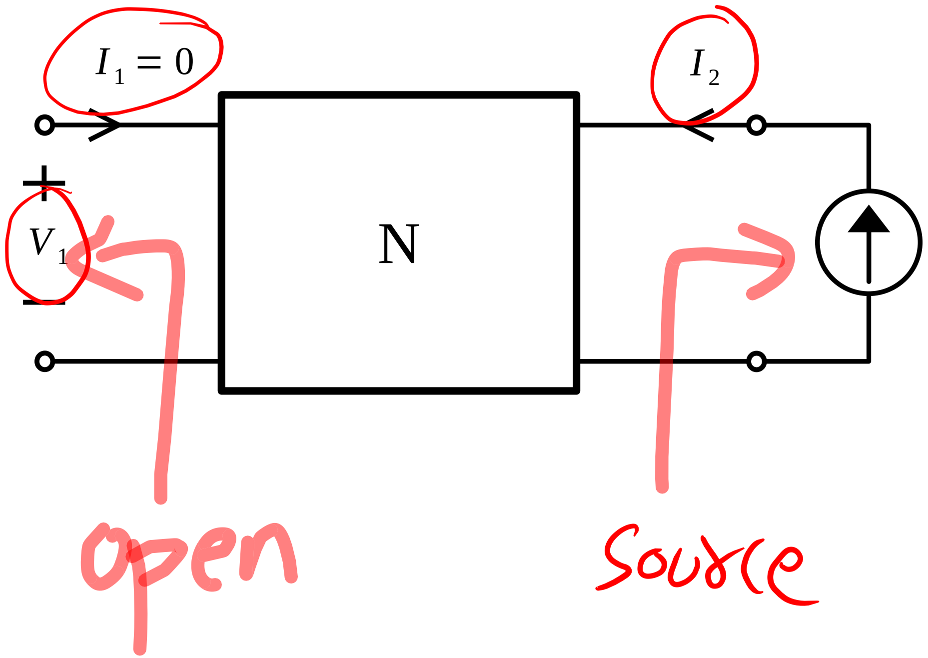
$$z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1 = 0}$$

- Transfer impedance from port 2 to port 1 with port 1 open.
- Also called *open circuit reverse transfer impedance*.

z_{12}

- Transfer impedance from port 2 to port 1 with port 1 open.

$$z_{12} = \frac{V_1}{I_2}\bigg|_{I_1 = 0}$$



z_{21}

- From definition:

$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

2

$$z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2 = 0}$$

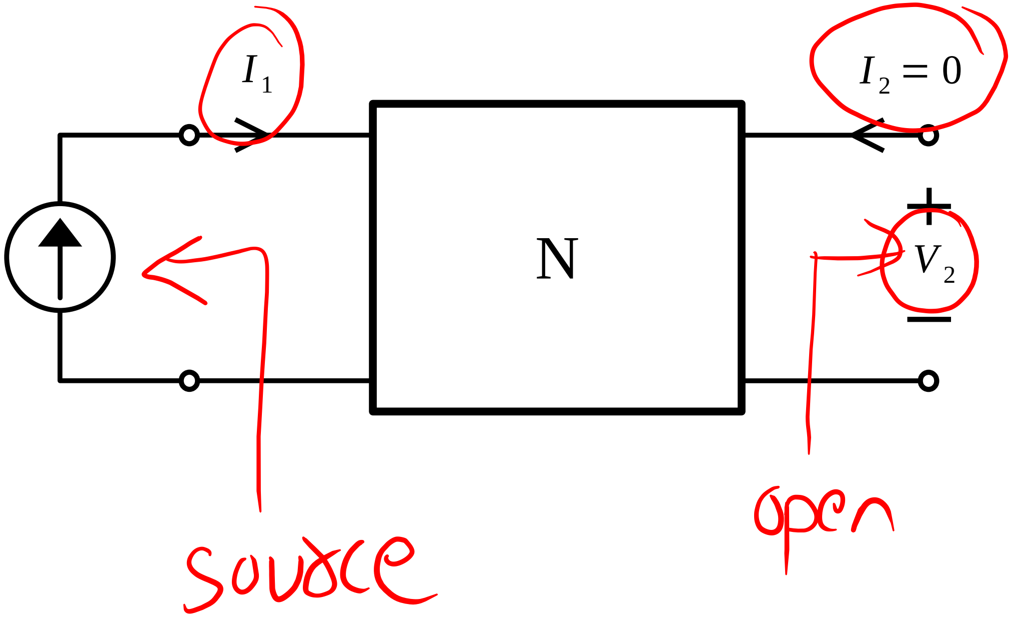
1

- Transfer impedance from port 1 to port 2 with port 2 open.
- Also called *open circuit forward transfer impedance*.

z_{21}

- Transfer impedance from port 1 to port 2 with port 2 open.

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$



z_{22}

- From definition:

$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

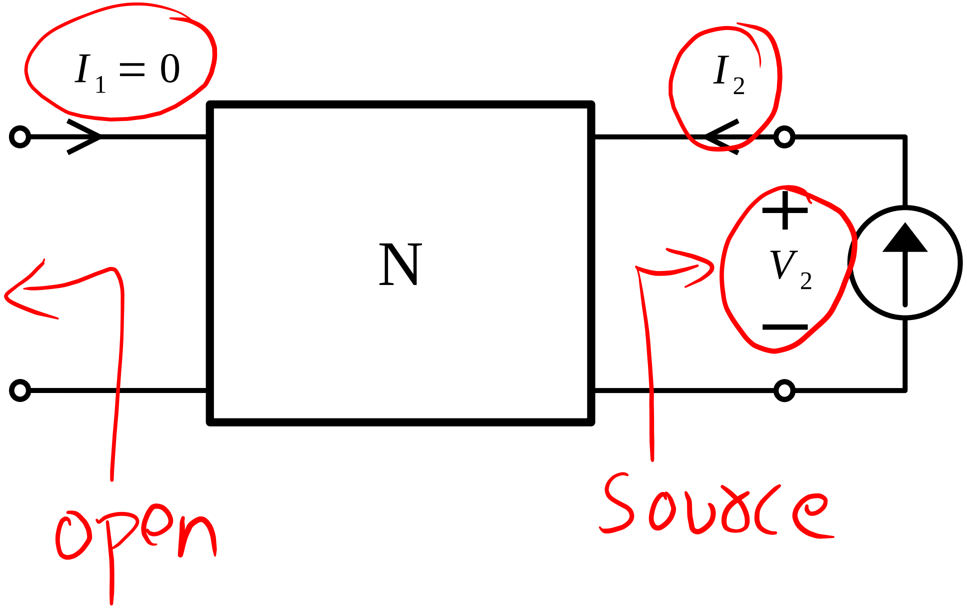
$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0}$$

- Driving point impedance at port 2 with port 1 open.
- Also called *open circuit driving point output impedance*.

z_{22}

- Driving point impedance at port 2 with port 1 open.

$$z_{22} = \frac{V_2}{I_2}\bigg|_{I_1 = 0}$$



Open-circuit Impedance Parameter

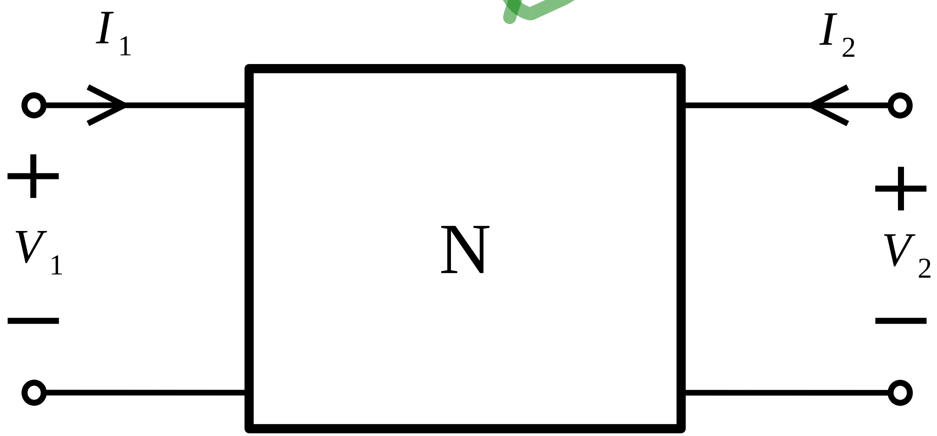
- z parameter are also called open-circuit impedance parameter because, each parameter is determined by open circuiting either port 1 or port 2.

$$z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} \qquad z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

$$z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} \qquad z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

Short-circuit Admittance Parameter (y parameter)

y parameter



- Express I_1 , I_2 in terms of V_1 and V_2

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

y_{11}

- From definition:

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

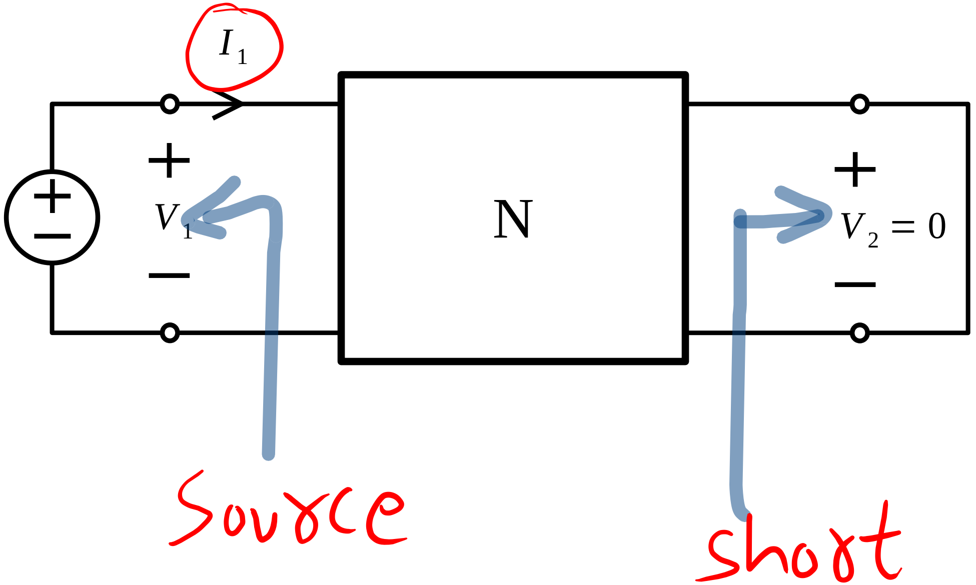
$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2 = 0}$$

- Driving point admittance at port 1 with port 2 short circuited.
- Also called short *circuit driving point input admittance*.

y_{11}

- Driving point admittance at port 1 with port 2 short circuited.

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$



y_{12}

- From definition:

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

$$y_{12} = \frac{I_1}{V_2}\bigg|_{V_1 = 0}$$

1

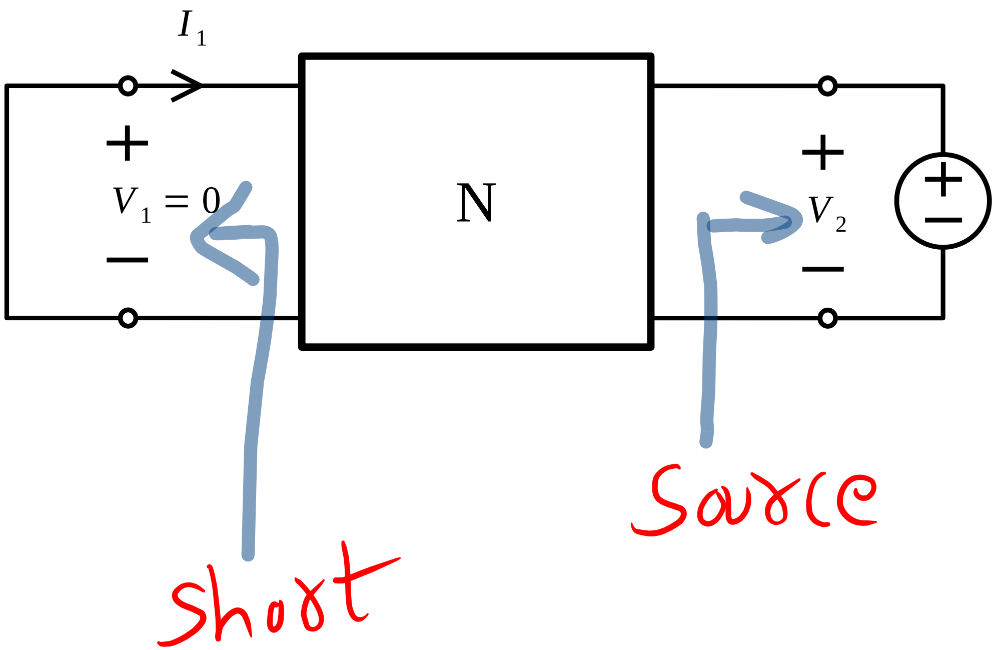
2

- Transfer admittance from port 2 to port 1 with port 1 short circuited.
- Also called short *circuit reverse transfer admittance*.

y_{12}

- Transfer admittance from port 2 to port 1 with port 1 short circuited.

$$y_{12} = \frac{I_1}{V_2}\bigg|_{V_1 = 0}$$



y_{21}

- From definition:

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

$$y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2 = 0}$$

2

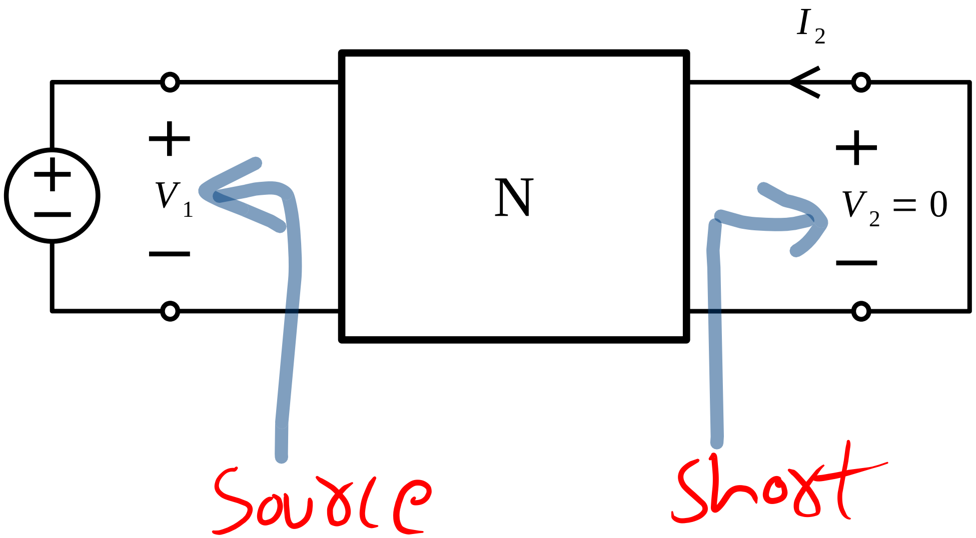
1

- Transfer admittance from port 1 to port 2 with port 2 short circuited.
- Also called short *circuit forward transfer admittance*.

y_{21}

- Transfer admittance from port 1 to port 2 with port 2 short circuited.

$$y_{21} = \frac{I_2}{V_1}\bigg|_{V_2 = 0}$$



y_{22}

- From definition:

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

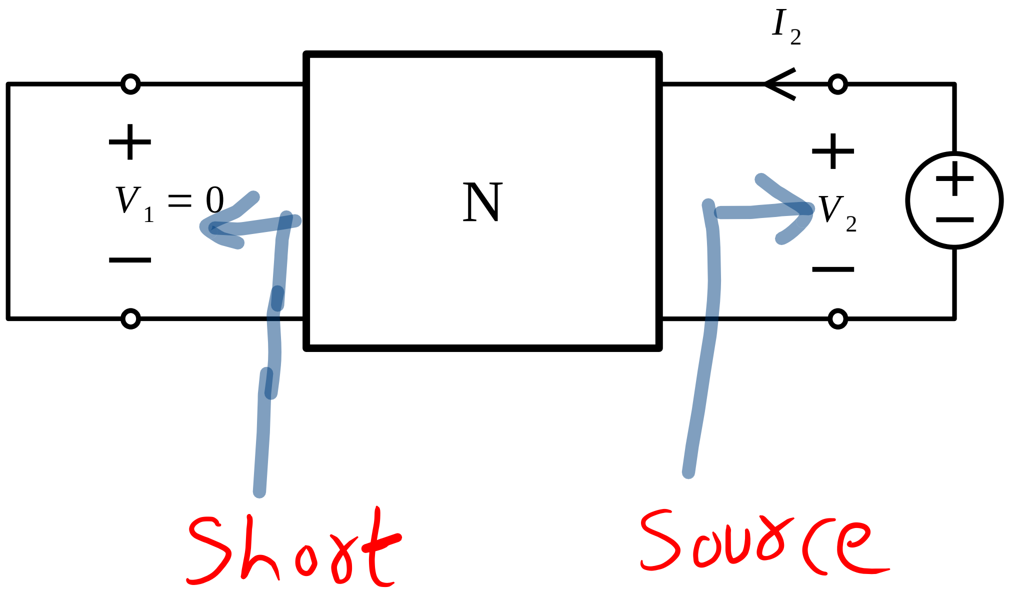
$$y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1 = 0}$$

- Driving point admittance at port 2 with port 1 short circuited.
- Also called short *circuit driving point output admittance*.

y_{22}

- Driving point admittance at port 2 with port 1 short circuited.

$$y_{22} = \frac{I_2}{V_2}\Bigg|_{V_1 = 0}$$



Short-circuit Admittance Parameter

- y parameter are also called short-circuit admittance parameter because, each parameter is determined by short circuiting either port 1 or port 2.

$$y_{11} = \frac{I_1}{V_1}\bigg|_{V_2 = 0} \quad \text{sc} \qquad y_{12} = \frac{I_1}{V_2}\bigg|_{V_1 = 0}$$

$$y_{21} = \frac{I_2}{V_1}\bigg|_{V_2 = 0} \quad \text{sc} \qquad y_{22} = \frac{I_2}{V_2}\bigg|_{V_1 = 0}$$

Communication

cascade

Transmission Parameter (ABCD parameter)

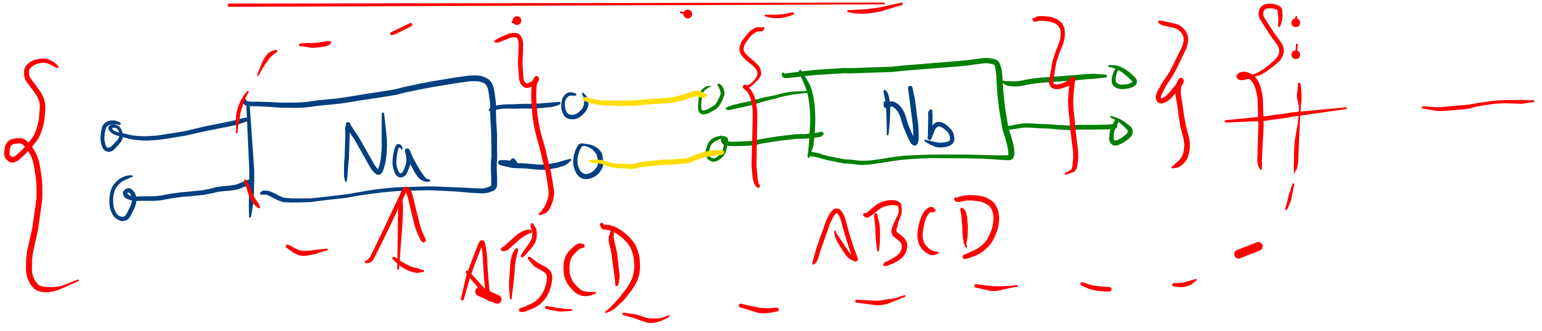
$$\begin{cases} V_1 = AV_2 + B(-I_2) \\ I_1 = CV_2 + D(-I_2) \end{cases}$$

$$\begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

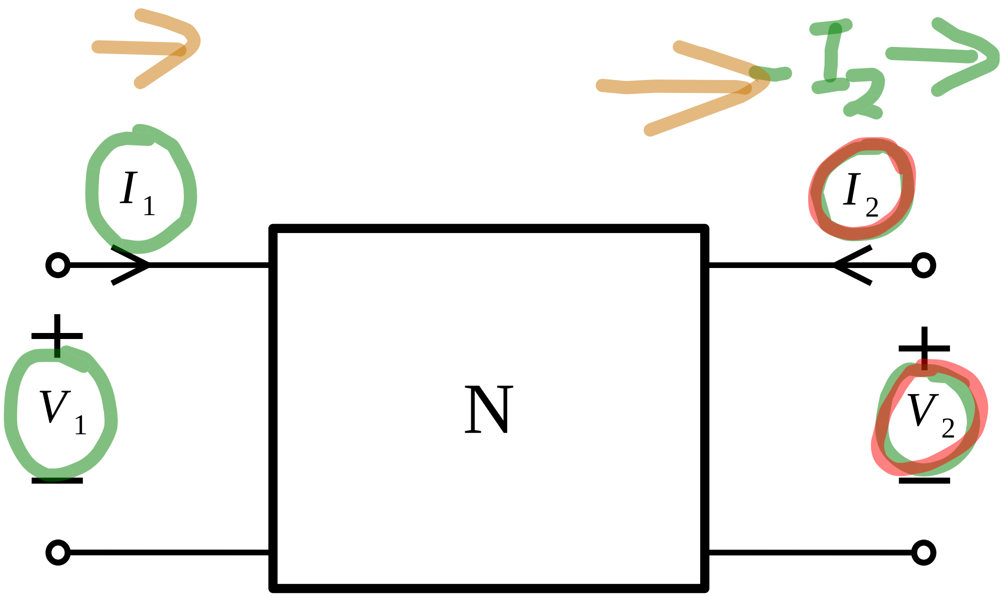
T- parameter

T'

inverse



Transmission Parameter



- Express V_1 , I_1 in terms of V_2 and $-I_2$

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

A

- From definition:

$$\begin{cases} V_1 = A\,V_2 - B\,I_2 \\ I_1 = C\,V_2 - D\,I_2 \end{cases}$$

V_1 $V_2\ I_2$

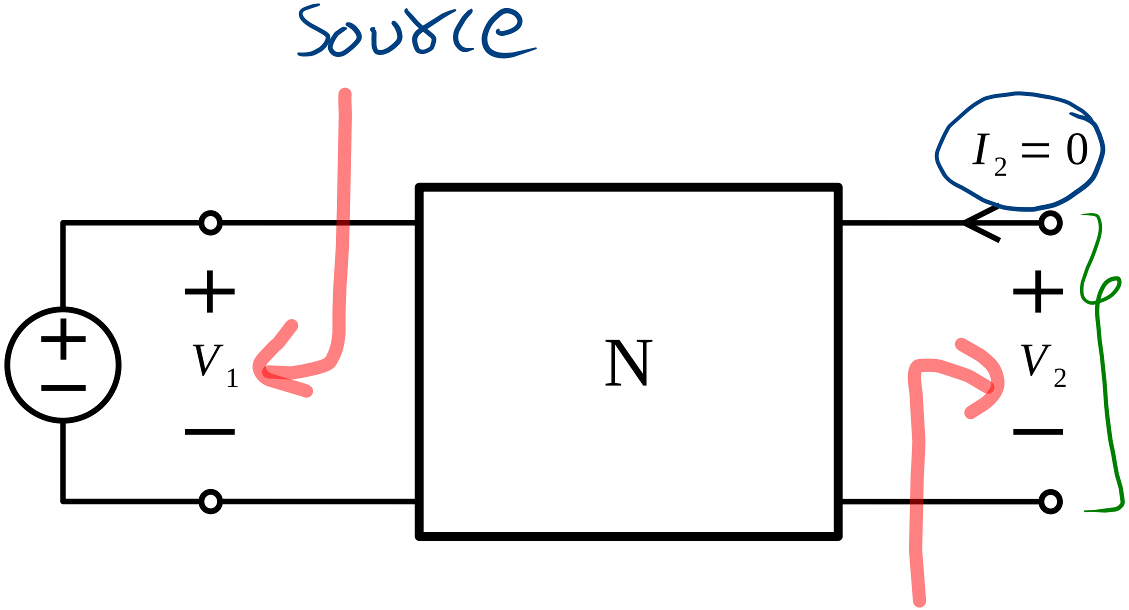
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{\left.\frac{V_2}{V_1}\right|_{I_2=0}}$$

1 2

- Reciprocal of open circuit forward voltage transfer function

A

$$A = \frac{1}{\left.\dfrac{V_2}{V_1}\right|_{I_2 = 0}}$$



- Reciprocal of open circuit forward voltage transfer function

B

- From definition:

$$V_1 = A\,V_2 - B\,I_2$$
$$I_1 = C\,V_2 - D\,I_2$$

- Reciprocal of short circuit forward transfer admittance function

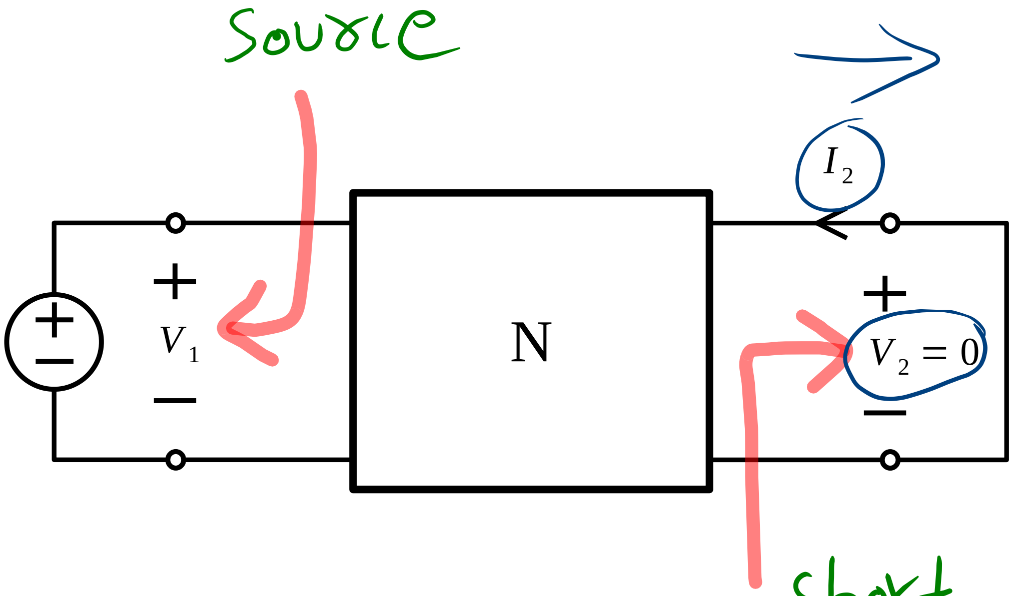
$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = \frac{1}{-\left.\frac{I_2}{V_1}\right|_{V_2=0}}$$

2

1

B

$$B = \frac{1}{-\dfrac{I_2}{V_1}\bigg|_{V_2 = 0}}$$



short

- Reciprocal of short circuit forward transfer admittance function

C

- From definition:

$$V_1 = A\,V_2 - B\,I_2$$

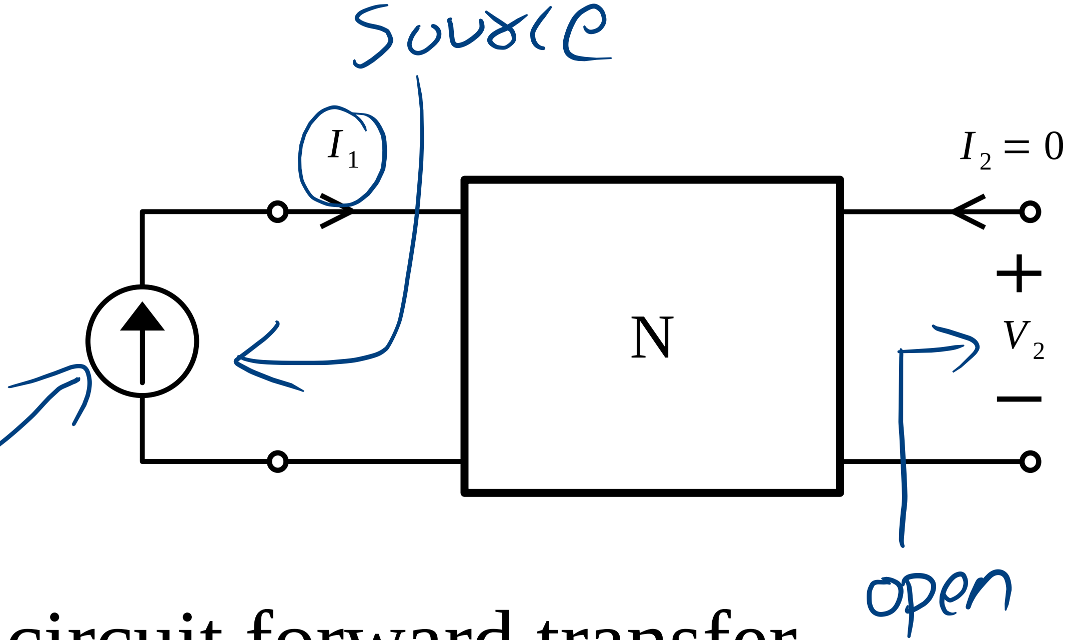
$$I_1 = C\,V_2 - D\,I_2$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2 = 0} = \frac{1}{\left.\dfrac{V_2}{I_1}\right|_{I_2 = 0}}$$

- Reciprocal of open circuit forward transfer impedance function

C

$$C = \frac{1}{\left.\dfrac{V_2}{I_1}\right|_{I_2=0}}$$



- Reciprocal of open circuit forward transfer impedance function

D

- From definition:

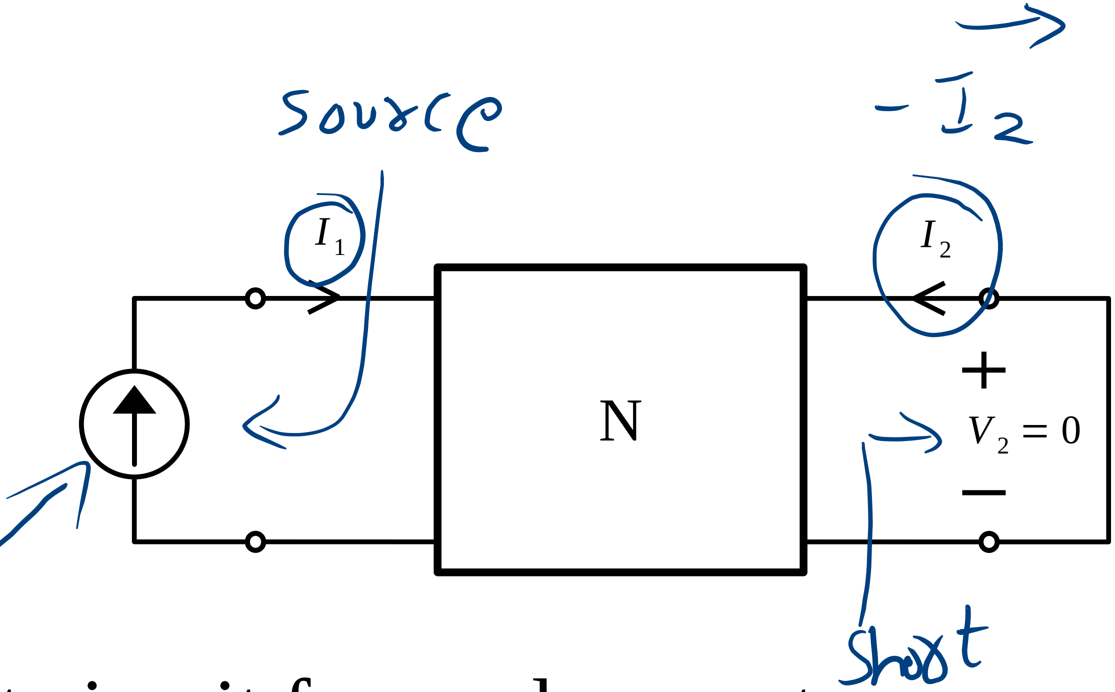
$$V_1 = A\,V_2 - B\,I_2$$
$$I_1 = C\,V_2 - D\,I_2$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{1}{-\left.\dfrac{I_2}{I_1}\right|_{V_2=0}}$$

- Reciprocal of short circuit forward current transfer function

D

$$D = \frac{1}{-\frac{I_2}{I_1}\bigg|_{V_2 = 0}}$$



- Reciprocal of short circuit forward current transfer function

Transmission Parameter

- From definition:

$$V_1 = A V_2 - B I_2$$
$$I_1 = C V_2 - D I_2$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2 = 0}$$ transfer

$$B = \left. \frac{V_1}{-I_2} \right|_{V_2 = 0}$$ transfer

$$C = \left. \frac{I_1}{V_2} \right|_{I_2 = 0}$$ transfer

$$D = \left. \frac{I_1}{-I_2} \right|_{V_2 = 0}$$ transfer

Transmission Parameter

A	Reciprocal of open circuit forward voltage transfer function
B	Reciprocal of short circuit forward transfer admittance function
C	Reciprocal of open circuit forward transfer impedance function
D	Reciprocal of short circuit forward current transfer function

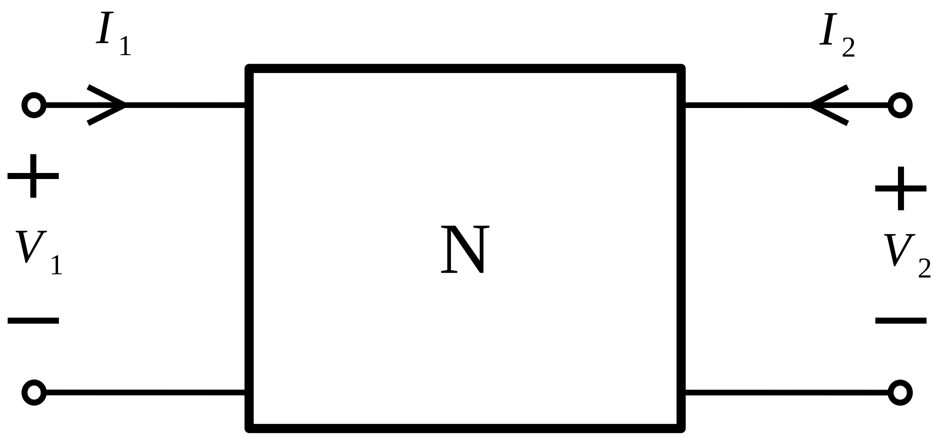
Note: ABCD parameter are all transfer functions relating quantity of one port to another port.

Inverse Transmission Parameter

(A' B' C' D' parameter)

T'

Inverse Transmission Parameter



- Express V_2 , I_2 in terms of V_1 and $-I_1$

$$V_2 = A' V_1 - B' I_1$$
$$I_2 = C' V_1 - D' I_1$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

Inverse Transmission Parameter

- From definition:

$$\begin{cases} V_2 = A' V_1 - B' I_1 \\ I_2 = C' V_1 - D' I_1 \end{cases}$$

$$A' = \left. \frac{V_2}{V_1} \right|_{I_1 = 0}$$

transfer

$$B' = \left. \frac{V_2}{-I_1} \right|_{V_1 = 0}$$

transfer

$$C' = \left. \frac{I_2}{V_1} \right|_{I_1 = 0}$$

transfer

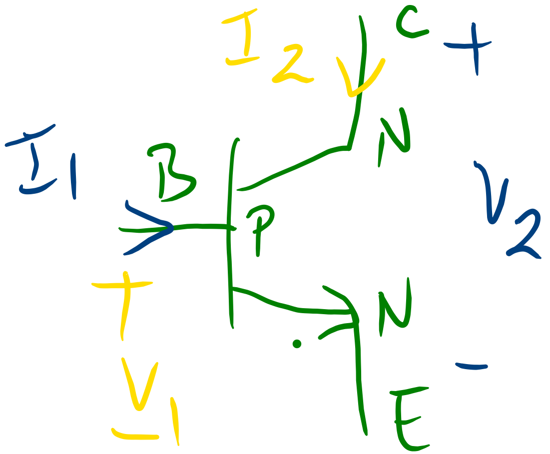
$$D' = \left. \frac{I_2}{-I_1} \right|_{V_1 = 0}$$

transfer

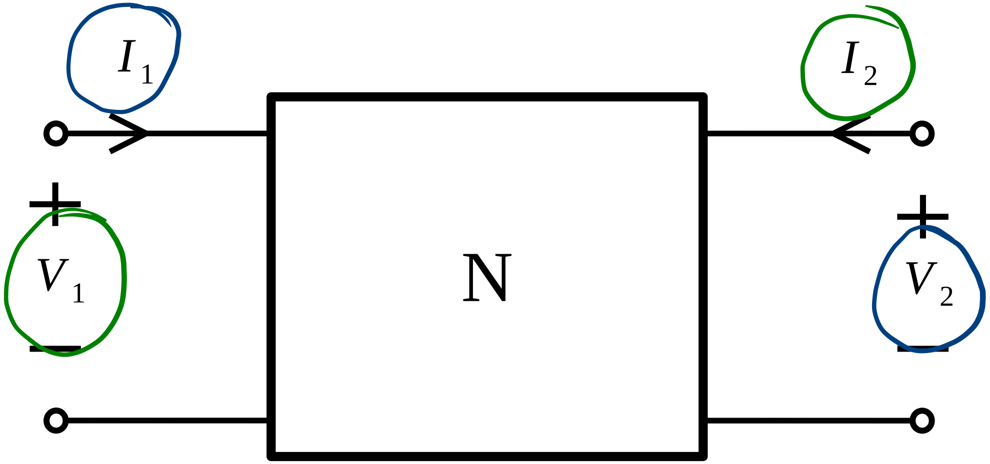
Electronics
circuit

transistor

Hybrid Parameter
(h parameter)



h parameter



- Express V_1 , I_2 in terms of I_1 and V_2

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

h_{11}

- From definition:

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

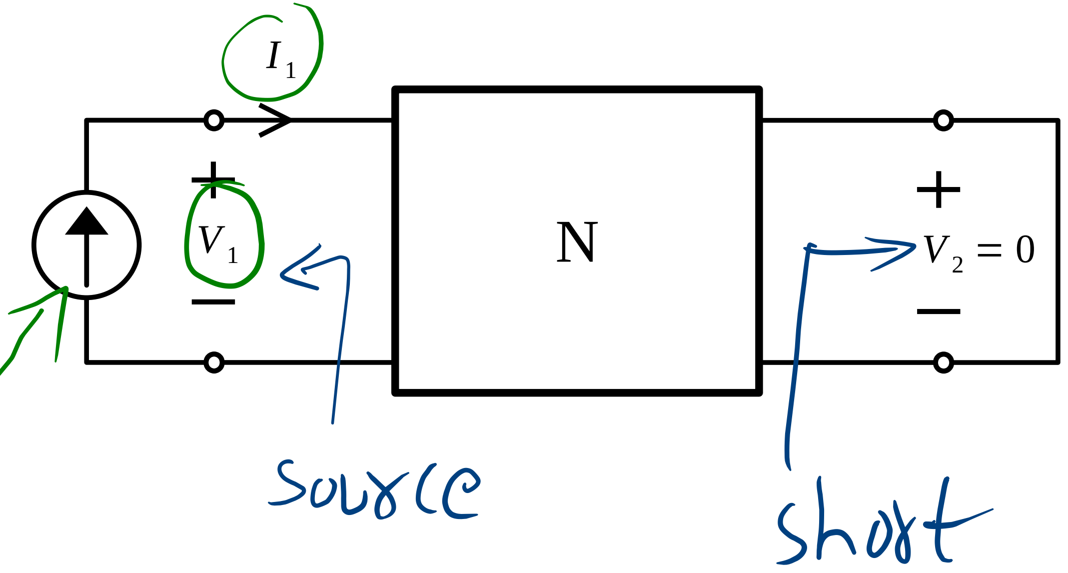
$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2 = 0}$$

- Driving point impedance at port 1 with port 2 short circuited.
- Also called short *circuit driving point input impedance.*

h_{11}

- Driving point impedance at port 1 with port 2 short circuited.

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$



h_{12}

- From definition:

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

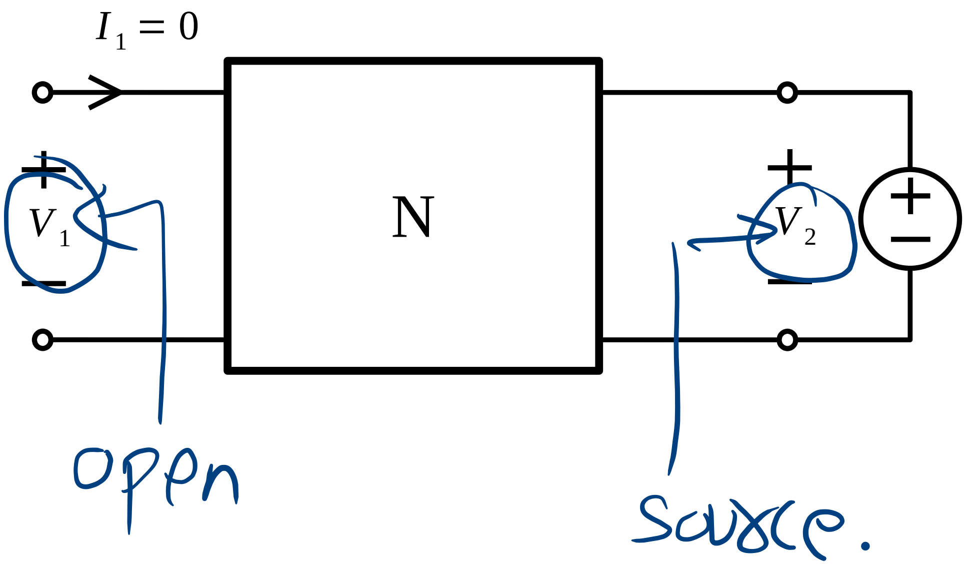
$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0}$$

- Voltage transfer from port 2 to port 1 with port 1 open circuited.
- Also called *open circuit reverse voltage transfer function.*

h_{12}

- Voltage transfer from port 2 to port 1 with port 1 open circuited.

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1 = 0}$$



h_{21}

- From definition:

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

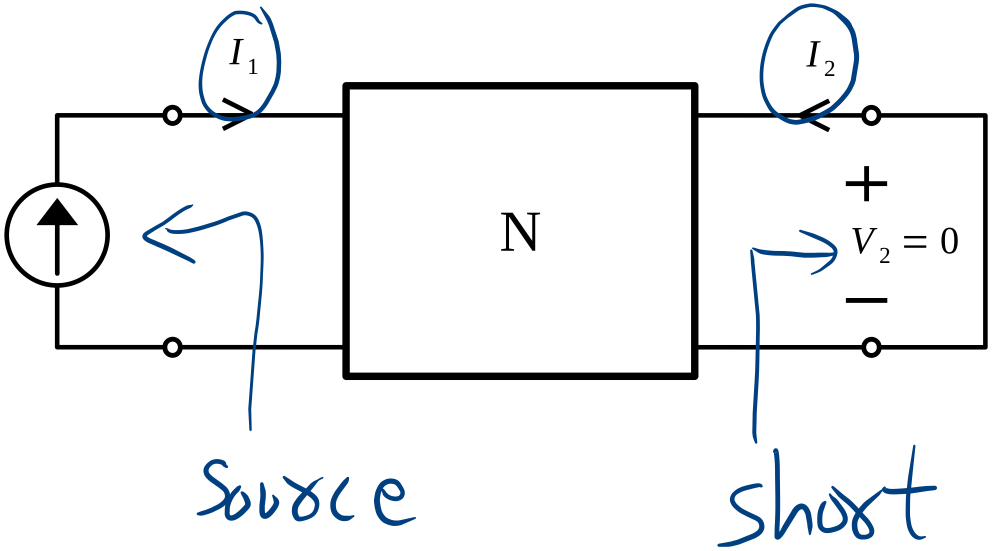
$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0}$$

- Current transfer from port 1 to port 2 with port 2 short circuited.
- Also called short *circuit forward current transfer function.*

h_{21}

- Current transfer from port 1 to port 2 with port 2 short circuited.

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0}$$



h_{22}

- From definition:

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

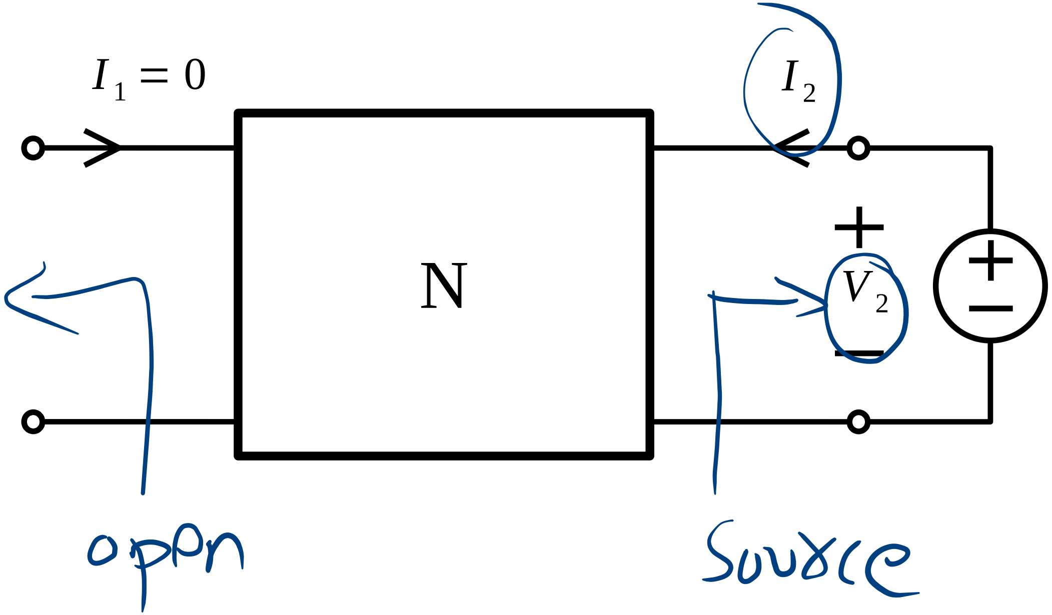
$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1 = 0}$$

- Driving point admittance at port 2 with port 1 open circuited.
- Also called open *circuit driving point output admittance.*

h_{22}

- Driving point admittance at port 2 with port 1 open circuited.

$$h_{22} = \frac{I_2}{V_2}\Bigg|_{I_1 = 0}$$



h Parameter

- From definition:

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

impedance

d.p.

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0}$$ s.c.

ratio

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0}$$ o.c.

ratio

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0}$$ s.c.

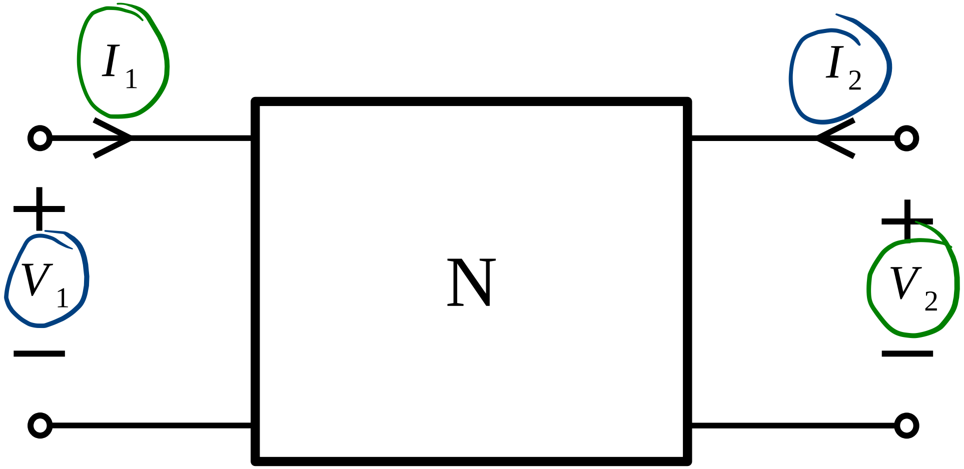
admittance

d.p.

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0}$$ o.c.

Inverse Hybrid Parameter
(g parameter)

g parameter



- Express I_1 , V_2 in terms of V_1 and I_2

$$I_1 = g_{11} V_1 + g_{12} I_2$$
$$V_2 = g_{21} V_1 + g_{22} I_2$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

g Parameter

- From definition:

$$I_1 = g_{11} V_1 + g_{12} I_2$$
$$V_2 = g_{21} V_1 + g_{22} I_2$$

$$g_{11} = \frac{I_1}{V_1}\bigg|_{I_2=0}$$ admittance o.c.

$$g_{12} = \frac{I_1}{I_2}\bigg|_{V_1=0}$$ ratio s.c.

$$g_{21} = \frac{V_2}{V_1}\bigg|_{I_2=0}$$ ratio o.c.

$$g_{22} = \frac{V_2}{I_2}\bigg|_{V_1=0}$$ impedance s.c.