Example Q1. Draw the bode plot of,

$$G(s)H(s) = \frac{2\left(s+\frac{1}{4}\right)}{s^2(s+1)\left(s+\frac{1}{2}\right)}$$

Soln :

Step 1: Convert the given transfer function into standard form for bode plot ie.

$$G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)\,---}{s^N(1+sT_1)\,---\,(s^2+2\xi\omega_n s+\omega_n^2)}$$

The T.f^n can be rewritten as,

$$T.F = \frac{\cancel{2}\times\cancel{\frac{1}{4}}(1+4s)}{s^2(1+s)(1+2s)\times\cancel{\frac{1}{2}}} = \frac{1(1+4s)}{s^2(1+2s)(1+s)}$$

On comparing we get $K=1$, & $N=2$, (Note → Initial slope depends No. of pole or zero at origin)

Step 2 → find corner frequency from the T.f^n for pole and zero & put order from smallest to largest

Term	P/Z	Factors	Time	Frequency	
$(1+sT_a)$	Z	$(1+4s)$	$T_a=4$	$\omega_1 = 1/T_a = \frac{1}{4}$	→ 0.25
$(1+sT_1)$	P	$(1+2s)$	$T_1=2$	$\omega_2 = \frac{1}{T_1} = \frac{1}{2}$	→ 0.5
$(1+sT_2)$	P	$(1+s)$	$T_2=1$	$\omega_3 = 1/T_3 = 1/1$	→ 1

$\boxed{\text{Note}}$ Reciprocal of Time is frequency

$\boxed{\text{Step 3}}$ For magnitude plot : " Draw initial line with slope $(-20N$ dB/decade) such that the line cuts $\log(\omega)$ - Axis at $\omega_0 = (K)^{1/N}$. For $N=0$ the line gives straight line parallel to $\log(\omega)$ with value

$$\boxed{M_{dB} = 20 \log K}$$

Type 0 $\longrightarrow \frac{1}{s^0} \longrightarrow$ zero pole at Origin $\longrightarrow$ 0 dB/dec

Type 1 $\longrightarrow \frac{1}{s^1} \longrightarrow$ 1 pole at origin $\longrightarrow$ -20 dB/dec

Type 2 $\longrightarrow \frac{1}{s^2} \longrightarrow$ 2 pole at origin $\longrightarrow$ -40 dB/dec

Type 3 $\longrightarrow \frac{1}{s^3} \longrightarrow$ 3 pole at origin $\longrightarrow$ -60 dB/dec

Starting magnitude → M(dB) → (i) $20 \log K$ | (ii) $20 \log\left(\frac{K}{s^N}\right)_{s=}$ | (iii) $20 \log\left(\frac{s^N}{K}\right)$

magnitude plot Analysis : $K = 1 \Rightarrow 20 \log(K) \Rightarrow 20 \log 1$

$\Rightarrow 0$ dB

Term		Corner freq (ascending order)	Slope	change in slope
$1/s^2$	$\xrightarrow{\text{pole}}$	it don't've any Corner freq, since this is 1st Term	-40 dB/dec	-40 dB/dec
$(1+4s)$	$\xrightarrow{\text{zero}}$	0.25 rad/sec	$+20$ dB/dec	-20 dB/dec
$\frac{1}{(1+2s)}$	$\xrightarrow{\text{pole}}$	0.5 rad/se	-20 dB/dec	-40 dB/dec
$\frac{1}{(1+s)}$	$\xrightarrow{\text{pole}}$	1 rad/sec	-20 dB/dec	-60 dB/dec

2nd part : phase plot analysis

step 1: Replace $s = j\omega$

step 2: Write each pole and zero & find angle

Term

→ $\frac{1}{j^2\omega^2} \Rightarrow -\tan^{-1}\left(\frac{0}{-\omega^2}\right) \Rightarrow -180^\circ$

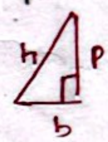
→ $1+j4\omega \Rightarrow \tan^{-1}\left(\frac{4\omega}{1}\right) \Rightarrow \tan^{-1}(4\omega)$

→ $1+j2\omega \Rightarrow -\tan^{-1}\left(\frac{2\omega}{1}\right) \Rightarrow -\tan^{-1}(2\omega)$

→ $1+j\omega \Rightarrow -\tan^{-1}\left(\frac{\omega}{1}\right) \Rightarrow -\tan^{-1}(\omega)$

[−ve $\tan^{-1}$ for pole
+ve $\tan^{-1}$ for zero

$\tan\theta = \frac{P}{b} = \frac{I}{R} = \frac{y}{x}$]



Resultant phase angle is given by

$$\phi_R = \angle G(j\omega)H(j\omega) = \tan^{-1}(4\omega) - 180^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$$

$\boxed{\text{Step}}$ 3 : We have 3 corner frequency (0.25, 0.5, 1),
Now for phase analysis take some extra frequency near to corner frequency & we find phase angle.

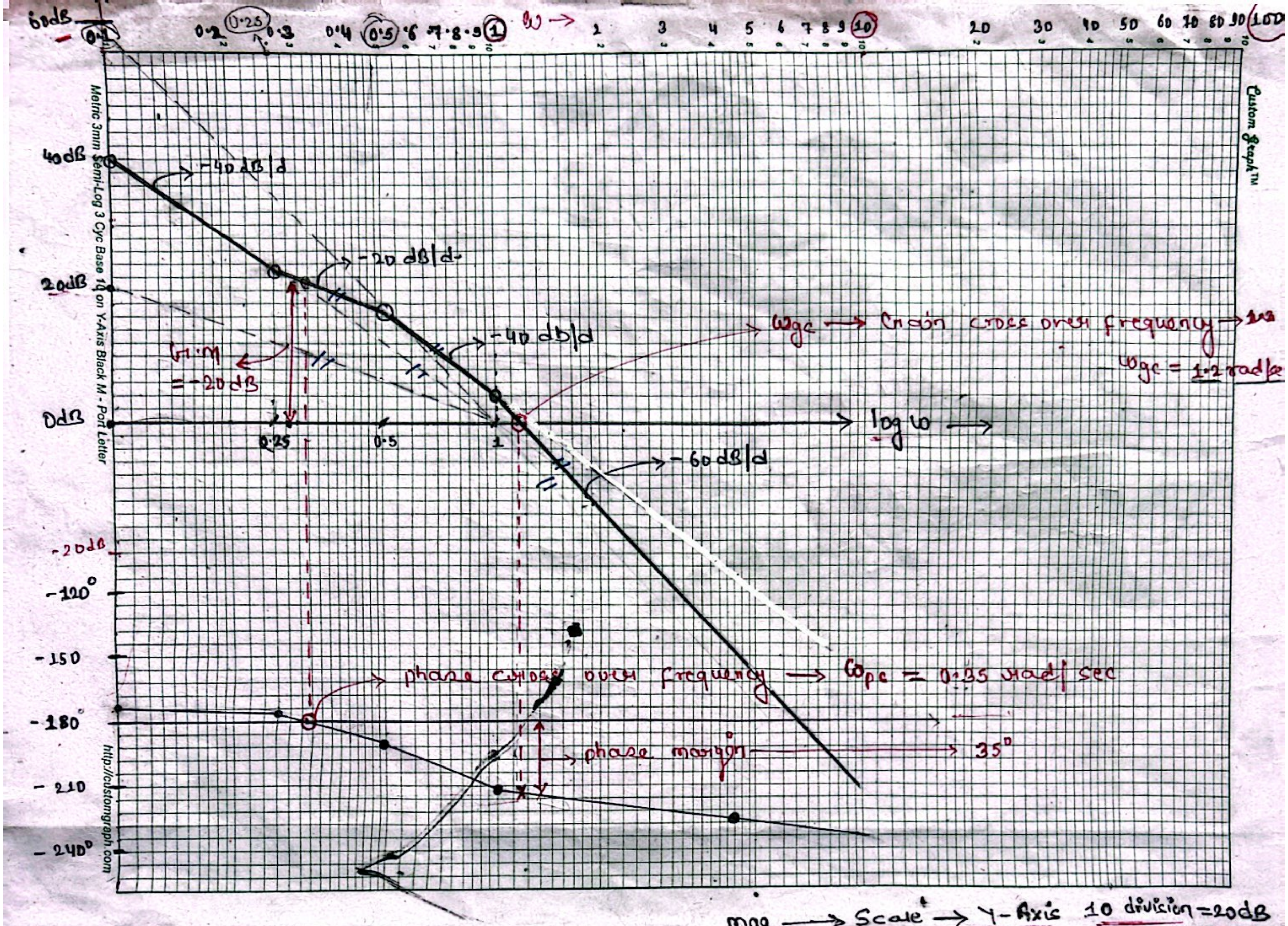
ω (rad/sec)	0.1	0.25	0.5	1	5 → Corner freq
$\angle G(j\omega)H(j\omega)$	−175.2	−175.6	−188	−212.4	−225 → Angle

Step: 5: plotting the Bode plot in semi-log graph paper

(i) System given is Type-2 $\left(\frac{1}{s^2}\right)$, Hence Initial slope of Bode plot $= -40$ dB/dec and intersect 0 dB axis at $\omega_0 = (k)^{1/2}$ $\left[\because \omega_0 = (k)^{1/N}\right]$

$$\omega_0 = (k)^{1/2} = (1)^{1/2}$$

$$\omega_0 = 1 \text{ rad/sec.}$$



mag → Scale → Y-Axis 10 division = 20dB

→ phase ↳ Y-Axis 5 division = 30°

NOTE

The initial slope of bode plot depends on Type of O.L.T.F. → $G(s)$
↳ open loop Transfer function

T.F Type	Initial slope of B.P	Remarks
0	0 dB/dec	A line $\lvert G(j\omega)\rvert_{dB} = 20\log_{10}k$ parallel to ω axis
1	-20 db/dec	Intersect 0 dB axis at $\omega = k$
2	-20×2 dB/dc	Int. 0 dB axis at $\omega = (k)^{1/2}$
3	-20×3 dB/dec	Int. 0 dB axis at $\omega = k^{1/3}$
N	$-20 \times N \frac{dB}{dc}$	Intersect 0 dB axis at $\left(\omega = k^{1/N}\right)$

Starting magnitude

a. $M_{dB} = 20\log_{10} k$ dB (For Type 0)

b. $M_{dB} = 20\log_{10}\left(\frac{k}{s^N}\right)$ dB (for pole)

c. $M_{dB} = 20\log_{10}(s^N k)$ dB (For zero)

$\boxed{\text{THEORY}}$

① Mag plot of $G(s)$ in dB → For plotting Mag v/s $\log_{10}\omega$
$20\log_{10}\lvert G(s)\rvert$ v/s $\log_{10}\omega$

② phase angle $\angle G(j\omega)$ v/s $\log_{10}\omega$

~~plot~~ Each term is considered seprately ~~are~~ graphs are drawn. To obtain Final plot the contribution due to each term are added.