MA321 Optimization

Jan-May, 2020

End-semester Exam

Date: 14th June Time: 14th June, 10:00am -15th June, 2:00pm Maximum marks: 18

Answers given without proper justification/calculation as the case may be will not be given any credit

1. Consider a linear programming problem (P) of the form,

$$\text{Minimize } \mathbf{c}^T\mathbf{x}$$
$$\text{subject to } A_{3\times 2}\mathbf{x} \leq \mathbf{b},\ \mathbf{x} \geq \mathbf{0}.$$

Let the simplex table of the optimal BFS $[\mathbf{x}_0^T, \mathbf{s}_0^T]^T$ of the corresponding problem with equality constraints be given by:

$c_j - z_j$			2		3	
	$B^{-1}\tilde{\mathbf{a}_1}$	$B^{-1}\tilde{\mathbf{a}_2}$	$B^{-1}\mathbf{s}_1$	$B^{-1}\mathbf{s}_2$	$B^{-1}\mathbf{s}_3$	$B^{-1}\mathbf{b}$
$\tilde{\mathbf{a}_1}$			-1		*	5
$\tilde{\mathbf{a}_2}$			2		*	3
$\mathbf{s}_2$			-3		*	1

Fill up the missing entries according to the following rule:
If your birthday is say on the 24th of June, 24/06 then take the largest three numbers from 2,4,0,6 written in increasing order with alternating sign the first entry being positive, to fill the missing entries of that column. So in this example the column corresponding to $B^{-1}\mathbf{s}_3$ will be $[2, -4, 6]^T$. If your birthday is on the 1st of June that is 01/06 then the column corresponding to $B^{-1}\mathbf{s}_3$ will be $[0, -1, 6]^T$. If your birthday is on the 12th of December that is 12/12 then $B^{-1}\mathbf{s}_3$ will be $[1, -2, 2]^T$.

(a) Add the constraint $x_1 + 2x_2 \leq 7$ to (P) and by using the above table find the new optimal solution **only** by using Dual Simplex Algorithm.

(b) Change c_2 to $c_2 + 1$ and check whether the above BFS is optimal for the changed problem (everything else remaining same as (P)).
(You do not have to find the optimal solution if the BFS in part(b) is not optimal).

(All parts in the above question are independent) [4]

2. Consider a linear programming problem (P) of the form,

$$\text{Maximimize } \mathbf{c}^T\mathbf{x} \quad (\mathbf{c} \neq \mathbf{0})$$
$$\text{subject to } A_{m\times n}\mathbf{x} \leq \mathbf{b},\ \mathbf{x} \geq \mathbf{0}.$$

(a) Let $\mathbf{x}_0$ be an optimal extreme point of (P) and let a constraint $\mathbf{a}_{m+1}^T\mathbf{x} \leq b_{m+1}$ be added to (P) such that $\mathbf{x}_0$ is no longer optimal for the changed problem (same as (P) with the added constraint). Then if possible give a basic **or** a nonbasic variable of an optimal BFS of the changed problem (P') (the added variables being s_i, $i = 1, \ldots, m$ and s_{m+1}, in order to apply simplex) and a basic **or** a non nonbasic variable of an optimal BFS of the Dual of (P') (the added variables to the Dual being s'_j, $j = 1, \ldots, n$, in order to apply simplex).

(b) Let $[\mathbf{x}^T, \mathbf{s}^T]^T$ be an optimal BFS of (P) with $m = 3$ and basis matrix B such that $x_1 > 0$. Let $\tilde{\mathbf{a}}_1$ (the column in A corresponding to the variable x_1) be changed to $\tilde{\mathbf{a}'}_1 = \tilde{\mathbf{a}}_1 + [-1, 0, -2]^T$ such that $[\mathbf{x}^T, \mathbf{s}^T]^T$ is not a BFS of the changed problem (P') (same as (P) except that the first column of A is changed) then if possible give an entry of $B^{-1}\tilde{\mathbf{a}'}_1$ (also indicate its position).

(c) If $\mathbf{x}, \mathbf{y}$ are optimal solutions of (P) and the Dual (D) of (P) respectively, such that for exactly one $i \in \{1, \ldots, n\}$, $x_i = 0$ and $(A^T\mathbf{y})_i = c_i$,
and for all $j \in \{1, 2, \ldots, m\}$, $y_j = 0$ implies $(A\mathbf{x})_j < b_j$,
then what can you say about the number of optimal solutions of (P) and the Dual (D) of (P), respectively?

(d) If you are solving (P) by Big (M) method let P(M) be the modified problem obtained from (P) by first adding the variables s_i, $i = 1, \ldots, m$ then the artificial variables with cost $-M$ where M is large. Then if P(M) for $M = 1000$ does not have an optimal solution is it true that (P) also does not have an optimal solution? Can you suggest 10 other values of (M) for which P(M) does not have an optimal solution?

(All parts in the above question are independent) **[9]**

3. (a) Construct a transportation problem with supply a_1, a_2, a_3 and demand d_1, d_2, d_3 such that $a_1 + a_2 + a_3 = d_1 + d_2 + d_3 = 100$, and $\{(1,2), (1,3), (2,3)\} \subset \mathcal{B}$ where $\mathcal{B}$ is the basic set of cells of the corresponding BFS $\mathbf{x}_0$ of your transportation problem. Check whether $\mathbf{x}_0$ is optimal for your problem (the c_{ij}'s are given below), if not then find an optimal solution of your transportation problem.
Fill the costs c_{ij} in the corresponding transportation array according to the following rule:
If your roll number is say 170101061,
then the associated c_{ij}'s will be 1,7,[2],1,[3],1,[4],6,1, that is keep all the nonzero digits in your roll number just as it is and replace the zeros by an increasing sequence of consecutive numbers starting with 2 (marked by square brackets) and write the c_{ij}'s in that order by starting with the leftmost cell in every row starting with the first row and moving towards the right. Then in the above example $c_{11} = 1, c_{12} = 7, c_{13} = 2, c_{21} = 1, ..., c_{32} = 6, c_{33} = 1$.
For example if your roll number is 170123030
then the associated c_{ij}'s will be 1,7,[2],1,2,3,[3],3,[4], then $c_{11} = 1, c_{12} = 7, c_{13} = 2, c_{21} = 1, c_{22} = 2, ..., c_{32} = 3, c_{33} = 4$ (the square brackets mark the replaced numbers).
So every student will have a different set of c_{ij}'s customized according to his/her roll number and the a_i, d_j's depending on the students choice may also vary.

(b) If possible find a balanced transportation problem with m supply stations, n destinations ($m, n \geq 3$) and a BFS of that transportation problem such that every nonbasic column in its corresponding simplex table (leaving the $B^{-1}\mathbf{b}$ column and the $c_j - z_j$ row) has exactly three nonzero entries. (Give the picture of the transportation array together with the basic cells marked and the corresponding simplex table if it is possible. If the above situation is not possible then justify) Note that a_i, d_j's are not specified.

(c) Given $m = 4$, find all possible values of $n \geq 2$ such that there exists a balanced transportation problem with m supply stations, n destinations and a BFS such that the simplex table corresponding to that BFS has atleast one column (leaving the $B^{-1}\mathbf{b}$ column and the $c_j - z_j$ row) with all nonzero entries.

(All parts in the above question are independent) [**7**]

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