2018 期中

选择

1. B 2. C

3. $P(X\geq 1)=1-P(X<1)=1-P(X=0)=1-(1-p)^2=\frac{5}{9} \Rightarrow 1-p=\frac{2}{3}\Rightarrow p=\frac{1}{3}$.

$P(Y\geq 1)=1-P(Y<1)=1-P(Y=0)=1-(\frac{2}{3})^3=\frac{19}{27}$ B

4. $P_1=P(-2<X<2)$ $P_2=P(-1<X<1)$ $P_3=P(-\frac{7}{3}<X<-1)$

$P_1>P_2>P_3$ A

5. C

填空:

1. $2\cdot\frac{1}{120}=\frac{1}{60}$

2. $0.4\times0.3\times0.5+0.6\times0.3\times0.5+0.4\times0.7\times0.5=\underline{0.29}$

$P(X|B)=0.4\times0.5+0.6\times0.5=0.5$

$P(X|B)\cdot P(B)=P(B|X)\cdot P(X) \Rightarrow P(B|X)=\frac{P(X|B)\cdot P(B)}{P(X)}=\frac{0.5\times0.3}{0.29}=\underline{\frac{15}{29}}$

3. $P_1=(0.6)^3$ $P_2=(0.4)^3$ $P=1-P_1-P_2=0.72$.

4. $P(\bar{A}\cup\bar{B})=1-P(A\cap B)=1-[-P(A\cup B)+P(A)+P(B)]=1-(\frac{4}{3}-1)=\underline{\frac{2}{3}}$

5. $f(x)=\begin{cases}\lambda e^{-\lambda x} & x\geq 0\\ 0 & x<0\end{cases}$

$P(3<X<4)=\int_3^4 \lambda e^{-\lambda x}dx=e^{-3\lambda}-e^{-4\lambda}$ $P(X>2)=\int_2^{\infty}\lambda e^{-\lambda x}dx=e^{-2\lambda}$

$P(3<X<4|X>2)=\underline{e^{-\lambda}-e^{-2\lambda}}=t-t^2=-(t-\frac{1}{2})^2+\frac{1}{4}$ $t=e^{-\lambda}=\frac{1}{2}\Rightarrow\underline{\lambda=\ln 2}$.

6. $P(\max(X,Y)>2)=1-P(\max(X,Y)\leq 2)=1-P(X\leq 2,Y\leq 2)=1-P(X\leq 2)P(Y\leq 2)$

$=1-\frac{1}{2}\cdot\frac{1}{2}=\frac{3}{4}$

7. $X-Y\sim N(-\mu,26^2)$ $-\mu=2\Rightarrow\underline{\mu=-2}$

8. $P(X=k)=\frac{\lambda^k}{k!}e^{-\lambda}$.

$P(Y=k)=P(X=k')=\frac{\lambda^{k^{1/3}}}{(k^{1/3})!}e^{-\lambda}$

大题.

1. $P=P(X\leq\frac{2}{3})=\int_0^{\frac{2}{3}}3x^2dx=(\frac{2}{3})^3=\frac{8}{27}$

$P(Y=1)=C_3^1\cdot p\cdot(1-p)^2=3\cdot\frac{8}{27}\cdot(1-\frac{8}{27})^2=\frac{2888}{6561}$.

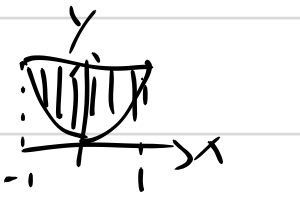
2.

(1) $\frac{1}{6}+a+\frac{1}{18}+\frac{1}{3}+\frac{2}{9}+b=\frac{7}{9}+a+b=1 \Rightarrow a+b=\frac{2}{9}$

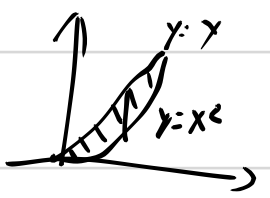
(2) $P(X=1)=\frac{1}{2}$ $P(Y=0)=\frac{2}{9}+a$

$P(X=1,Y=0)=\frac{1}{2}(\frac{2}{9}+a)=\frac{1}{6} \Rightarrow \frac{2}{9}+a=\frac{1}{3} \Rightarrow a=\frac{1}{9}$ $b=\frac{1}{9}$.

3. (1) $\int_{-1}^{1}\int_{x^2}^{1} cx^2y\,dy\,dx=\int_{-1}^{1}\frac{1}{2}cx^2(1-x^4)dx=\frac{4}{21}c=1 \Rightarrow c=\frac{21}{4}$



(2) $\int_0^1\int_{x^2}^{x} cx^2y\,dy\,dx=\int_0^1 \frac{1}{2}cx^2(x^2-x^4)dx=\frac{1}{10}c-\frac{1}{14}c=\frac{3}{20}$



4. $P(X>x)=1-P(X\le x)\le 0.05 \Rightarrow P(X\le x)\ge 0.95 \Rightarrow P(Y\le\frac{x-110}{12})\ge 0.95$

$\Rightarrow P(\frac{x-110}{12})\ge 0.95 \Rightarrow \frac{x-110}{12}\ge 1.645 \Rightarrow x\ge 129.74$ 故最小为129.74

5.

(1) $f(x,y)=\begin{cases} c & (x,y)\in G \\ 0 & \text{其它} \end{cases} \Rightarrow \iint_{(x,y)\in G} f(x,y)dxdy=c\cdot\frac{1}{2}\cdot 2\cdot 1=c=1$

$f_X(x)=\begin{cases}\int_0^x f(x,y)dy=x & 0\le x<1 \\ \int_0^{2-x} f(x,y)dy=2-x & 1\le x\le 2\end{cases}$

(2) $f_Y(y)=\int_y^{2-y} f(x,y)dx=2(1-y)$ $0\le y\le 1$

故当 $0\le y<1$ 时. $f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)}=\frac{1}{2(1-y)}$ $0\le x\le 2$

6.

(1) $P(Y\le 1)=P(Y=1)=P(X\ge 2)=\int_2^3\frac{1}{9}x^2dx=\frac{19}{27}$

$x\in[1,2)$ $P(Y\le x)=P(Y\le 1)+P(1<Y<x)=\frac{19}{27}+\int_1^x\frac{1}{9}t^2dt=\frac{19}{27}+\frac{1}{27}(x^3-1)=\frac{1}{27}x^3+\frac{2}{3}$

$y\ge 2$ $P(Y\le y)=\frac{26}{27}+P(X\le 1)=1$

故 $F_Y(y)=\begin{cases}0 & y<1 \\ \frac{2}{3}+\frac{1}{27}y^3 & 1\le y<2 \\ 1 & 2\le y\end{cases}$

(2) $P\{X\le Y\}=P(Y=2)+P(Y=X)=P(X<2)=\int_0^2\frac{1}{9}x^2dx=\frac{8}{27}$