Probability

1. EXPERIMENT:

An operation which results in some well-defined outcomes is called an experiment.

2. RANDOM EXPERIMENT:

The experimenet about which no confirm prediction is possible, is called the random experiment.

e.g. (i) Tossing a coin, (ii) Throwing a die etc.

3. SAMPLE SPACE:

The set of all possible outcomes of an experiment is called the sample space, which is denoted by S.

e.g. (i) On throwing a die, sample space is

$5 = \{1, 2, 3, 4, 5, 6\}$ and $n(S) = 6$

(ii) On tossing two coins, sample space is

$S = \{H, T\} \times \{H, T\} = \{HH, HT, TH, TT\}$.

$n(S) = 4$

4. EVENT:

Every subset of sample space is defined as the event. It is denoted by E.

e.g. (i) The event of appearing odd numbers in throwing a die is

$E = \{1, 3, 5\}$

and $E \subseteq S$, where $S = \{1, 2, 3, 4, 5, 6\}$

5. TYPES OF EVENTS :

(i) Null Event or Impossible Event: It has no member and is denoted by ϕ.

e.g.(a) On throwing a die the event of appearing member 7 is an impossible event.

(ii) Sure Event: Since $S \subseteq S$, so S is an event which is called sure event.

(iii) Simple Event or Elementary Event: It has only one member.

e.g. (b) On throwing a die the event of appearing prime even number is $E = \{2\}$ which is a simple event.

(iv) Mixed Event or Composite event or
Compound Event: This event has two or more than 2 members.

e.g.: On throwing a die the event of appearing even numbers is
$E = \{2, 4, 6\}$ which is a compound event.

(v) Mutually Exclusive Events: Events E_1 and E_2 are mutually exclusive if $E_1 \cap E_2 = \phi$

e.g.: In drawing a card from a pack of 52 cards, the events

A $\to$ the event when the card drawn is spade

B $\to$ the event when the card drawn is a heart are mutually exclusive events.

(vi) Mutually Exclusive and Exhaustive Events:
Events $E_1, E_2, E_3, ..., E_n$ are mutually exclusive and exhaustive events if

(a) $E_i \cap E_j = \phi$, where $i \neq j$ and i, j = 1, 2, 3, ..., n.

(b) $E_1 \cup E_2 \cup E_3 \cup ......E_n = S$

e.g. On throwing a die if

A = the event of appearing odd numbers = {1, 3, 5}

B = The event of appearing even numbers = {2, 4, 6}

Then $A \cap B = \phi$

and $A \cup B = S = \{1, 2, 3, 4, 5, 6\}$

$\therefore$ A and B are mutually exclusive and exhaustive events.

(v) Complement of an Event E:

It is denoted by E' or E^C or $\overline{E}$ and it is the set of all sample points of sample space not of event E.

On throwing a , die, S = {1, 2, 3, 4, 5, 6}

If event be E = {1, 2, 3}

then $\overline{E}$ = {4, 5, 6}

Note : (a) $A \cup \overline{A} = S$ (b) $A \cap \overline{A} = \phi$.

6. PROBABILITY OF AN EVENT:

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{no. of favourable cases}}{\text{no. of favourable and infavourable cases}}$$

Remarks:

- $P(S) = 1$
- $P(\phi) = 0$
- $0 \le P(E) \le 1$
- If $A \subseteq B$ then $P(A) \le P(B)$
- $P(A) + P(\overline{A}) = 1$

7. ODDS IN FAVOUR OF EVENT E:

Odds is favour of event $E = \frac{P(E)}{P(E')} = \frac{P(E)}{1 - P(E)}$

= a : b, say.

If odds is favour of event E be a : b, then $P(E) = \frac{a}{a+b}$

8. ODDS AGAINST EVENT E:

Odds against event $E = \frac{P(E')}{P(E)} = \frac{1 - P(E)}{P(E)} = c : d$, say

It odds against event E be c : d, then

$$P(E)=\frac{d}{c+d}.$$

9. IDEAS ABOUT A PACK OF 52 CARDS:

A Pack of 52 Cards

- Red (26 cards)
  - Hearts (13)
  - Dramonds (13)
- Black (26 cards)
  - Spades (13)
  - Clubs (13)

Aces $\rightarrow$ 4
KinGS $\rightarrow$ 4
Queens $\rightarrow$ 4
Jacks $\rightarrow$ 4

Four suits: Heart, Diamond, Spade, Clubs.

Court Cards: 12 (4 kings, 4 queens, 4 jacks)

Honour Cards: 16 (4 aces, 4 kings, 4 queens, 4 jacks)

Face Cards: 12 (4 kings, 4 queens, 4 jacks)

10. SOME OTHER TYPES OF EVENTS:

Independent Events: Two or more events are raid to be independent if occurrence or non-occurrence of any one of them does not affect the occurrence or non-occurrence of other.

e.g. If a bag contains 3 red and 2 black balls and if two balls are drawn one by one with replacements, then events are independent events.

Dependent Events: Two or more events are said to be dependent, if occurrence or non-occurrence of any one of them affects occurrence or non-occurrence of others.

e.g. Let a bag contains 3 red and 2 black balls. Two balls are drawn one by one without replacement. Then these events are dependent events.

Equally Likely Events: Outcomes are said to be equally likely which we have no reason to believe that one is more likely to occur then the other.

e.g. When an unbiased die is thrown all the six faces 1, 2, 3, 4, 5, 6 are equally likely to come up.

11. ADDITION THEOREM FOR PROBABILITY:

(i) If A and B the any two events, then $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

(ii) If A and B are mutually exclusive events, then $P(A\cup B)=P(A)+P(B)$

(iii) If A, B, C are any three vents, then

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$$

(iv) If A, B and C are mutually exclusive events, then

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)$$

(v) $P(A \cup B) \le P(A) + P(B)$

Some other results:

(i) $P(\bar{A} \cap B) = P(B) - P(A \cap B) = P(B - A)$

(ii) $P(A \cap \bar{B}) = P(A) - P(A \cap B) = P(A - B)$

(iii) $P\{(A \cap \bar{B}) \cup (\bar{A} \cap B)\} = P(A) + P(B) - 2P(A \cap B)$

(iv) $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

(v) $P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$

(vi) If A, B, C be three events the

(a) P(atleast two of A, B, C occur)

$$= P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$$

(b) P(exactly two of A, B, C occur) $= P(A \cap B) + P(B \cap C) + P(A \cap C) - 3P(A \cap B \cap C)$

(c) P(exactly one of A, B, C, occurs)

$$= P(A) + P(B) + P(C) - 2P(A \cap B) - 2P(B \cap C) - 2P(A \cap C) - 3P(A \cap B \cap C)$$

12. CONDITIONAL PROBABILITIES

Let A and B be any two events such that $B \neq \phi$ or $n(B) \neq 0$ or $P(B) \neq 0$, then P(A/B) denotes the conditional probability of occurrence of event A when B has been already occured.
e.g. A coin is lossed thrice. If E be the event of showing atleast two heads and F the event of showing head in the first throw, then

$$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{3/8}{4/8} = \frac{3}{4}$$

13. MULTIPLICATION THEOREM:

(i) If A and B be any two events then

$$P(A \cap B) = P(A) \cdot P(B/A)$$

$$= P(B) \cdot P(A/B)$$

(ii) If events A and B are independent events, then

$$P(A \cap B) = P(A) \cdot P(B) = P(AB)$$

14. BAYE'S THEOREM:

Let S be the sample space and let $E_1, E_2, \ldots\ldots\ldots\ldots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or .... E_n, then

$$P(E_i / A) = \frac{P(E_i) \times P(A/E_i)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2) + \ldots\ldots + P(E_n).P(A/E_n)}$$ for n = 1, 2, 3, ...n.

15. BINOMIAL PROBABILITY DISTRIBUTION:

If for a sample space S, n = total no. of trial
p = probability of event of success for one - trial
q = probability of event of failure for one trial
so that p + q = 1, then probability for r success trial is
$P(X = r) = {}^nC_r \cdot p^r \cdot q^{n-r}$.

16. MEAN AND VARIANCE OF BINOMIAL DISTRIBUTION:

Mean = np and Variance = npq
and mean > variance
Also standard deviation = $\sqrt{npq}$