

11.16.3.8.3

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1 PROBLEM STATEMENT

Question:

Three coins are tossed once. Find the probability of getting at least 2 heads.

2 SOLUTION

2.1 Random Variables

Let X_1, X_2, X_3 be three independent random variables representing the outcomes of the three coin tosses:

$$X_i = \begin{cases} 1, & \text{if Head appears,} \\ 0, & \text{if Tail appears.} \end{cases} \quad (0.1)$$

Each X_i follows a Bernoulli distribution with:

$$P_X(1) = p = \frac{1}{2}, \quad P_X(0) = q = \frac{1}{2}. \quad (0.2)$$

2.2 Probability Mass Function (PMF) Using Z-Transform

The PMF of X_i is:

$$P_X(n) = p^n q^{1-n}. \quad (0.3)$$

The z-transform of this PMF is:

$$P_{X_i}(z) = \mathbb{E}[z^{X_i}] = P_X(0) + P_X(1)z^{-1} = q + \frac{p}{z}. \quad (0.4)$$

For each X_i :

$$P_{X_i}(z) = \frac{1}{2} + \frac{1}{2}z^{-1}. \quad (0.5)$$

2.3 Distribution of $Y = X_1 + X_2 + X_3$

Using the independence property, the z-transform of Y is:

$$P_Y(z) = P_{X_1}(z) \cdot P_{X_2}(z) \cdot P_{X_3}(z). \quad (0.6)$$

$$P_Y(z) = \left(\frac{1}{2} + \frac{1}{2z} \right)^3. \quad (0.7)$$

Expanding, we get:

$$P_Y(z) = \frac{1}{8} \left(1 + \frac{3}{z} + \frac{3}{z^2} + \frac{1}{z^3} \right). \quad (0.8)$$

Thus, the PMF of Y is:

$$P_Y(k) = \begin{cases} \frac{1}{8}, & k = 0, \\ \frac{3}{8}, & k = 1, \\ \frac{3}{8}, & k = 2, \\ \frac{1}{8}, & k = 3. \end{cases} \quad (0.9)$$

We can also express this PMF in a summation notation:

$$P_Y(k) = \sum_{k=0}^3 P_Y(k) \quad \text{where} \quad P_Y(k) \in \left\{ \frac{1}{8}, \frac{3}{8} \right\}, \quad k = 0, 1, 2, 3. \quad (0.10)$$

2.4 Cumulative Distribution Function (CDF)

The CDF of Y is:

$$F_Y(k) = \begin{cases} 0, & k < 0, \\ \frac{1}{8}, & 0 \leq k < 1, \\ \frac{4}{8}, & 1 \leq k < 2, \\ \frac{7}{8}, & 2 \leq k < 3, \\ 1, & k \geq 3. \end{cases} \quad (0.11)$$

2.5 Probability of At Least Two Heads

To get the probability of a particular event (Say 1 head), we take the coefficient of the corresponding power of z . To find $P(Y \geq 2)$, we use the coefficients of z^{-2} and z^{-3} .

$$\therefore \text{Probability of At Least Two Heads} = \frac{1}{8} + \frac{3}{8} = \frac{1}{2}$$

3 CONCLUSION

The probability that at least two of the three coin tosses result in heads is:

$$P(Y \geq 2) = \frac{1}{2}. \quad (0.12)$$

We get the following plot.

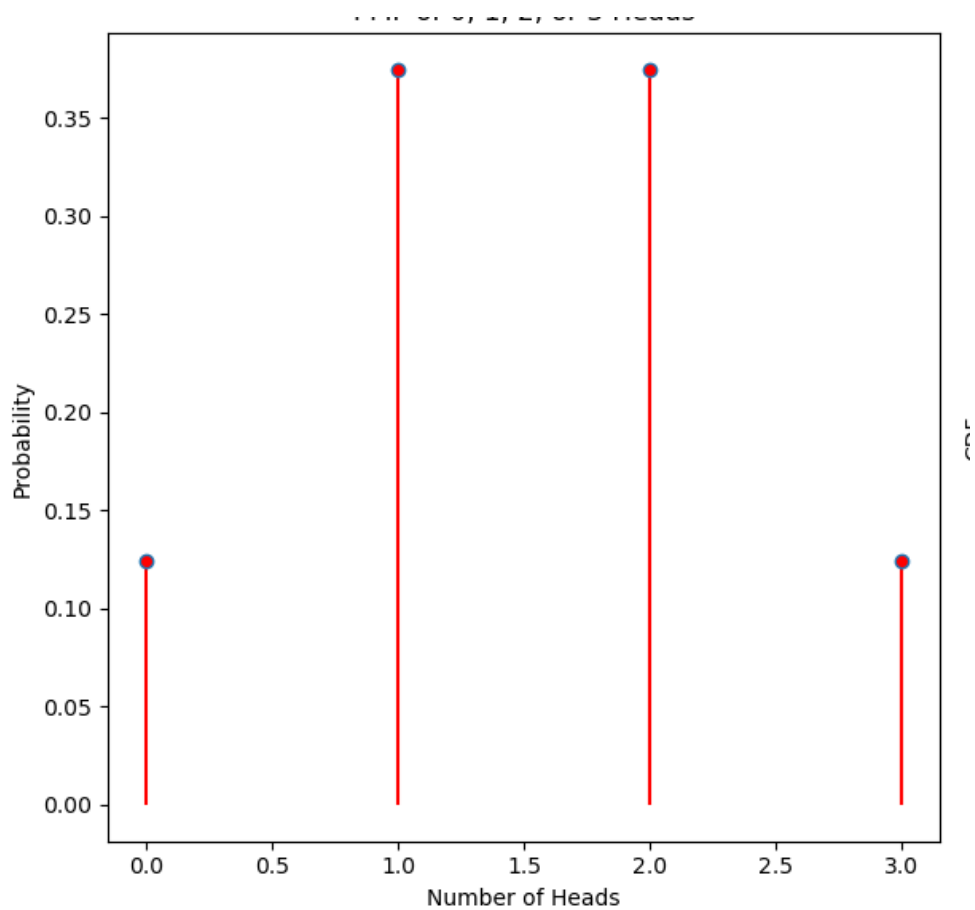


Fig. 0.1

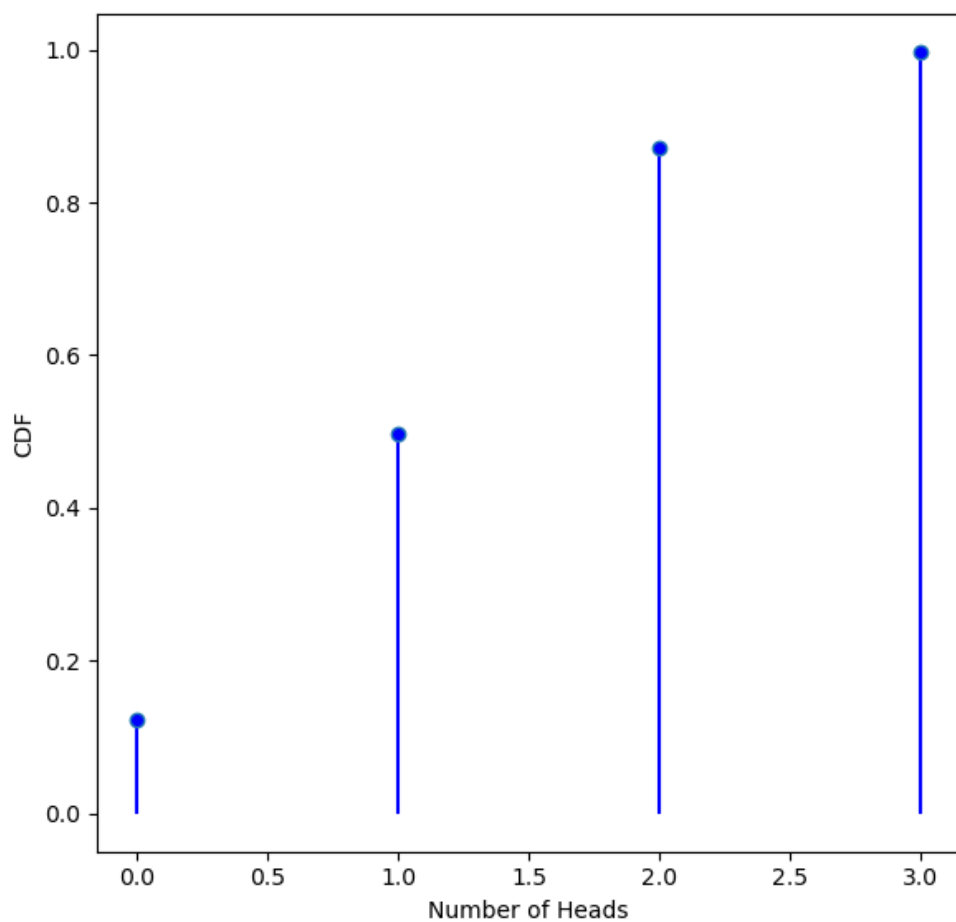


Fig. 0.2