9.4.18

EE24BTECH11047 - Niketh Prakash Achanta

Question: At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point $(-4, -3)$. Find the equation of the curve given that it passes through $(-2, 1)$.

Solution

Slope of tangent = $\frac{dy}{dx}$
Based on the given information, we can form the following differential equation:

$$\frac{dy}{dx} = 2\left(\frac{y+3}{x+4}\right) \tag{0.1}$$

Then, cross-multiply

$$\frac{dy}{y+3} = 2\frac{dx}{x+4} \tag{0.2}$$

Integrate

$$\int \frac{dy}{y+3} = 2\int \frac{dx}{x+4} \tag{0.3}$$

After integration

$$\ln(y+3) = 2\ln(x+4) + k \tag{0.4}$$

Where k is an arbitrary constant of integration.
Now, rewrite $2\ln(x+4)$ as $\ln(x+4)^2$ and take that term to LHS

$$\ln\frac{(y+3)}{(x+4)^2} = k \tag{0.5}$$

$$\frac{(y+3)}{(x+4)^2} = e^k \tag{0.6}$$

Replace e^k with another constant "c"

$$y = c(x+4)^2 - 3 \tag{0.7}$$

Given that the curve passes through $(-2, 1)$ substitute $x = -2, y = 1$ in the current equation
Thus, $c = 1$
$\therefore$ Final Equation of the Curve: $y = (x+4)^2 - 3$

Additionally, a numerical solution is generated using the method of finite differences:

The derivative is approximated using finite differences:

$$\frac{dy}{dx} = \frac{y(x+h) - y(x)}{h} \tag{0.8}$$

$$y(x+h) = y(x) + h\left(\frac{dy}{dx}\right) \tag{0.9}$$

h is a value close to zero, that can be chosen accordingly while creating the program. So we iterate as follows:

$$y_{n+1} = y_n + h\left(\frac{dy}{dx}\right) \tag{0.10}$$

$$x_{n+1} = x_n + h \tag{0.11}$$

$$\text{Here, } \frac{dy}{dx} = 2\frac{(y_n + 3)}{(x_n + 4)} \tag{0.12}$$

So, the final difference equation is

$$y_{n+1} = y_n + 2\frac{(y_n + 3)}{(x_n + 4)}h \tag{0.13}$$

We start with given point $(-2, 1)$. $x_0 = -2$ and $y_0 = 1$. We iterate for a suitable number of times in order to get a solution that is approximately equal to the theoretical solution.

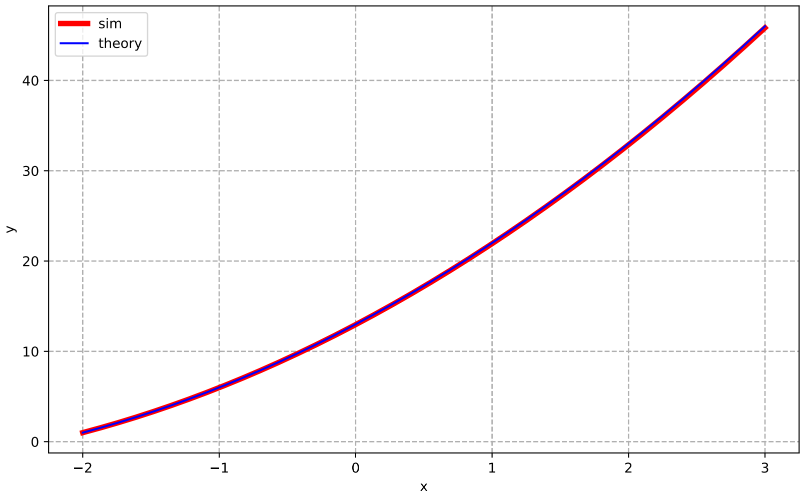

Fig. 0