10.3.3.2

EE24BTECH11047 - Niketh Prakash Achanta

Question: Solve the system of equations

$$2x + 3y = 11$$
$$2x - 4y = -24$$

and hence find the value of m for which

$$y = mx + 3$$

Solution

Theoretical solution:

Step 1: Solving the system of equations

We can use the elimination method. First, subtract equation (2) from equation (1) to eliminate x:

$$(2x + 3y) - (2x - 4y) = 11 - (-24) \tag{1}$$
$$2x + 3y - 2x + 4y = 11 + 24 \tag{2}$$
$$7y = 35 \tag{3}$$
$$y = 5 \tag{4}$$

Step 2: Substituting $y = 5$ into one of the original equations

Substitute $y = 5$ into equation (1):

$$2x + 3(5) = 11 \tag{5}$$
$$2x + 15 = 11 \tag{6}$$
$$2x = 11 - 15 \tag{7}$$
$$2x = -4 \tag{8}$$
$$x = -2 \tag{9}$$

Thus, the solution to the system of equations is $x = -2$ and $y = 5$.

Step 3: Finding the value of m for the equation $y = mx + 3$

We are given the equation $y = mx + 3$. Since we know that $y = 5$ when $x = -2$, substitute these values into the equation:

$$5 = m(-2) + 3 \tag{10}$$
$$5 = -2m + 3 \tag{11}$$
$$5 - 3 = -2m \tag{12}$$
$$2 = -2m \tag{13}$$
$$m = -1 \tag{14}$$

Thus, the value of m is $\boxed{-1}$.

Computational Solution:

Solution using LU Factorization

Given the system of linear equations:

$$2x + 3y = 11, \tag{15}$$

$$2x - 4y = -24. \tag{16}$$

We rewrite the equations as:

$$x_1 = x, \tag{17}$$

$$x_2 = y, \tag{18}$$

giving the system:

$$2x_1 + 3x_2 = 11, \tag{19}$$

$$2x_1 - 4x_2 = -24. \tag{20}$$

Step 1: Convert to Matrix Form

We write the system as:

$$A\mathbf{x} = \mathbf{b}, \tag{21}$$

where:

$$A = \begin{bmatrix} 2 & 3 \\ 2 & -4 \end{bmatrix}, \tag{22}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{23}$$

$$\mathbf{b} = \begin{bmatrix} 11 \\ -24 \end{bmatrix}. \tag{24}$$

Step 2: LU factorization using update equations

Given a matrix $\mathbf{A}$ of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows:

Step-by-Step Procedure:

1. Initialization: - Start by initializing $\mathbf{L}$ as the identity matrix $\mathbf{L} = \mathbf{I}$ and $\mathbf{U}$ as a copy of $\mathbf{A}$.

2. Iterative Update: - For each pivot $k = 1, 2, \ldots, n$: - Compute the entries of U using the first update equation. - Compute the entries of L using the second update equation.

3. Result: - After completing the iterations, the matrix $\mathbf{A}$ is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where $\mathbf{L}$ is a lower triangular matrix with ones on the diagonal, and $\mathbf{U}$ is an upper triangular matrix.

1. Update for $U_{k,j}$ (Entries of U)

For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k. \tag{25}$$

This equation computes the elements of the upper triangular matrix $\mathbf{U}$ by eliminating the lower triangular portion of the matrix.

2. Update for $L_{i,k}$ (Entries of L)

For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k. \tag{26}$$

This equation computes the elements of the lower triangular matrix $\mathbf{L}$, where each entry in the column is determined by the values in the rows above it.

Step 2: LU Factorization of Matrix A

We decompose A as:

$$A = LU, \tag{27}$$

where L is a lower triangular matrix and U is an upper triangular matrix. By running the iteration code, we get the L and U matrices:

$$L = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \tag{28}$$

$$U = \begin{bmatrix} 2 & 3 \\ 0 & -7 \end{bmatrix}. \tag{29}$$

Step 3: Solve $L\mathbf{y} = \mathbf{b}$ (Forward Substitution)

We solve:

$$L\mathbf{y} = \mathbf{b} \quad \text{or} \quad \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 11 \\ -24 \end{bmatrix}. \tag{30}$$

From the first row:

$$y_1 = 11. \tag{31}$$

From the second row:

$$y_1 + y_2 = -24 \quad \implies \quad 11 + y_2 = -24 \quad \implies \quad y_2 = -35. \tag{32}$$

Thus:

$$\mathbf{y} = \begin{bmatrix} 11 \\ -35 \end{bmatrix}. \tag{33}$$

Step 4: Solve $U\mathbf{x} = \mathbf{y}$ (Backward Substitution)

We solve:

$$U\mathbf{x} = \mathbf{y} \quad \text{or} \quad \begin{bmatrix} 2 & 3 \\ 0 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 11 \\ -35 \end{bmatrix}. \tag{34}$$

From the second row:

$$-7x_2 = -35 \quad \Longrightarrow \quad x_2 = 5. \tag{35}$$

From the first row:

$$2x_1 + 3x_2 = 11 \quad \Longrightarrow \quad 2x_1 + 3(5) = 11, \tag{36}$$

$$2x_1 + 15 = 11 \quad \Longrightarrow \quad 2x_1 = -4, \tag{37}$$

$$x_1 = -2. \tag{38}$$

Thus:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}. \tag{39}$$

Final Solution

The solution is:

$$x = -2, \tag{40}$$

$$y = 5. \tag{41}$$

Step 5: Finding the Value of m

We are given the equation $y = mx + 3$, and we know that $x = -2$ and $y = 5$ from the solution of the system. Substituting these values into the equation:

$$5 = m(-2) + 3. \tag{42}$$

Solving for m:

$$5 - 3 = -2m, \tag{43}$$

$$2 = -2m, \tag{44}$$

$$m = -1. \tag{45}$$

Thus, the value of m is $\boxed{-1}$.

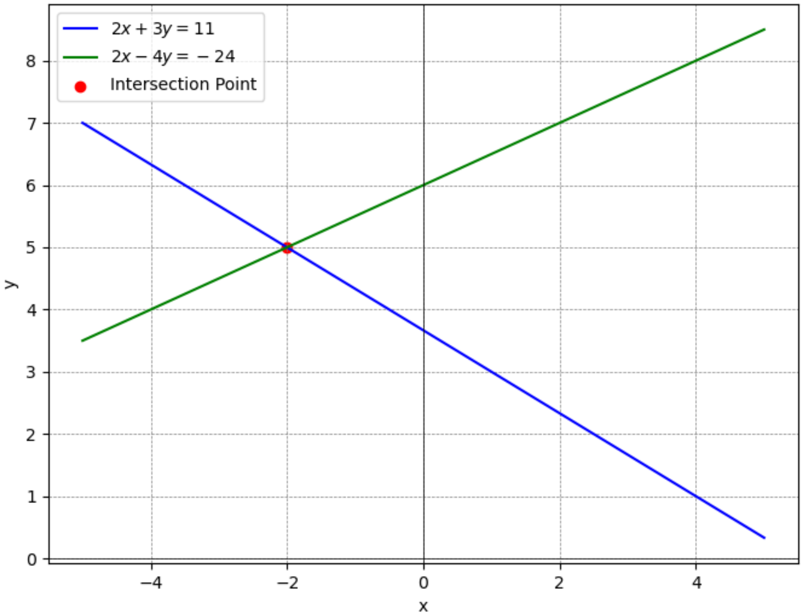
Final Solution

The solution is:

$$x = -2, \tag{46}$$

$$y = 5, \tag{47}$$

$$m = -1. \tag{48}$$



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