6.6.4

EE24BTECH11047 - Niketh Prakash Achanta

Question: Find the equation of the normal to the curve $x^2 = 4y$ that passes through the point $(1, 2)$.

Solution

Theoretical solution:

The equation of the curve is

$$x^2 = 4y. \tag{0.1}$$

Step 1: Slope of the tangent Differentiating $\left(x^2 = 4y\right)$ with respect to x, we get:

$$2x = 4\frac{dy}{dx}, \tag{0.2}$$

$$\frac{dy}{dx} = \frac{x}{2}. \tag{0.3}$$

Thus, the slope of the tangent at a point (x_1, y_1) is:

$$\text{slope of tangent} = \frac{x_1}{2}. \tag{0.4}$$

Step 2: Slope of the normal The slope of the normal, being the negative reciprocal of the slope of the tangent, is:

$$\text{slope of normal} = -\frac{2}{x_1}. \tag{0.5}$$

Step 3: Equation of the normal The equation of the normal passing through (x_1, y_1) is:

$$y - y_1 = -\frac{2}{x_1}(x - x_1), \tag{0.6}$$

$$x_1 y - x_1 y_1 = -2(x - x_1), \tag{0.7}$$

$$x_1 y + 2x = 2x_1 + x_1 y_1. \tag{0.8}$$

Step 4: Using the condition that the normal passes through (1, 2)

Substitute $(x = 1, y = 2)$ into the normal equation:

$$x_1(2) + 2(1) = 2x_1 + x_1 y_1, \tag{0.9}$$

$$2x_1 + 2 = 2x_1 + x_1\left(\frac{x_1^2}{4}\right), \tag{0.10}$$

$$2 = \frac{x_1^3}{4}, \tag{0.11}$$

$$x_1^3 = 8, \tag{0.12}$$

$$x_1 = 2. \tag{0.13}$$

Step 5: Finding y_1 Substitute $x_1 = 2$ into $x_1^2 = 4y_1$ to find y_1:

$$(2)^2 = 4y_1, \tag{0.14}$$

$$y_1 = 1. \tag{0.15}$$

Conclusion: The foot of the normal is $(2, 1)$.
Computational solution :

1 Introduction

This document describes the computation performed in the given C and Python code, which involves generating function values and performing gradient descent optimization.

2 Point Generation

The C function computes a set of points $(x, f(x))$ over a specified range. The function follows these steps:

- Define an initial value $x_0 = -10$.
- Compute the step size as:

$$h = \frac{2x_0}{n} \tag{0.16}$$

  where n is the number of points.
- Iteratively update x and compute $f(x) = x^2/4$.

3 Gradient Descent Algorithm

The C function **run_gradient_descent** implements the gradient descent algorithm to find the minimum of a function $f(a)$. The process involves:

- Initializing the guess a_0.
- Iteratively updating a using the formula:

$$a_{n+1} = a_n - \alpha f'(a_n) \tag{0.17}$$

  where α is the step size.
- The process stops when $|f'(a_n)|$ is below a specified tolerance.

4 Python Implementation

The Python script uses (*ctypes*) to interface with the compiled C functions. The key components include:

- Calling **generate_points** to compute function values.
- Defining the derivative function:

$$f'(x_n) = x_n/2 \tag{0.18}$$

- Running **run_gradient_descent** to find the minimum of the function.

5 Results and Visualization

A scatter plot highlights the computed minimum.

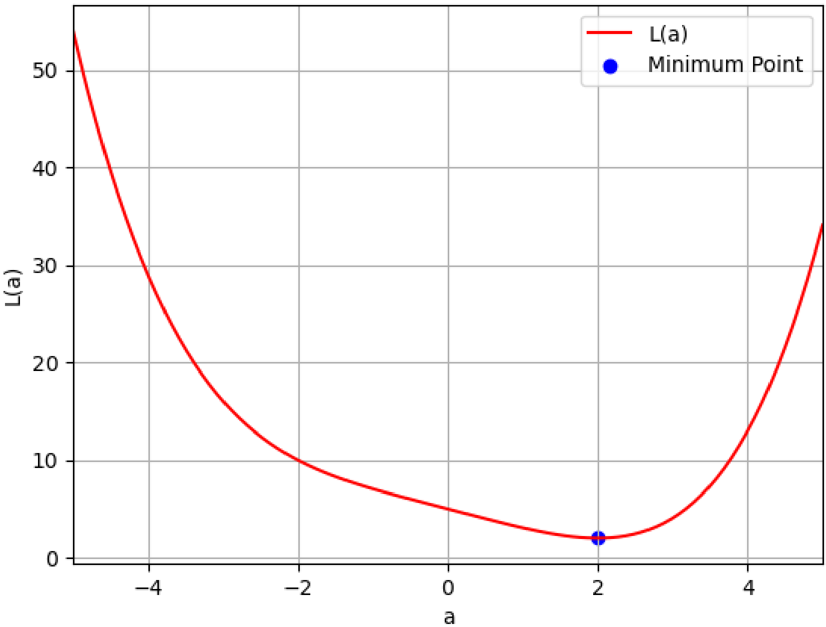

Fig. 0.1