11.16.3.8.3

EE24BTECH11047 - Niketh Prakash Achanta

Question:
Three coins are tossed once. Find the probability of getting at least 2 heads.

Solution:

Theoretical solution:
To calculate the probability of getting at least 2 heads:

- The total number of outcomes when three coins are tossed is:
$$2^3 = 8$$
- The favorable outcomes for at least 2 heads are: HHT, HTH, THH, HHH. Thus, there are 4 favorable outcomes.
- The probability of getting at least 2 heads is:
$$P(\text{At least 2 heads}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{4}{8} = \frac{1}{2}.$$

Thus, the probability of getting at least 2 heads is:

$$\boxed{\frac{1}{2}}$$

Computational solution:

Z-Transform Computational Method for Coin Toss PMF

PMF for a Single Coin Toss

For a single coin toss, the probability mass function (PMF) is:

$$P_X(x) = \begin{cases} p, & \text{if } x = 1 \text{ (Heads)} \\ 1 - p, & \text{if } x = 0 \text{ (Tails)} \end{cases}$$

where $p = 0.5$ for a fair coin.

Z-Transform Expansion

The Z-transform for the number of heads in n coin tosses is given by:

$$H(z) = (p + (1 - p)z)^n \tag{0.1}$$

where:

- $p = 0.5,\ 1 - p = 0.5,$
- $n = 3.$

Expansion of $H(z)$

Expand the expression $(p + (1 - p)z)^n$ using the binomial theorem:

$$H(z) = \sum_{k=0}^{n} \binom{n}{k} p^{n-k} (1 - p)^k z^k \tag{0.2}$$

Result for $n = 3$

Substituting $n = 3$ and expanding:

$$H(z) = (0.5 + 0.5z)^3 = 0.125 + 0.375z + 0.375z^2 + 0.125z^3 \tag{0.3}$$

The PMF values for heads are:

$$P_X(0) = 0.125, \quad P_X(1) = 0.375, \quad P_X(2) = 0.375, \quad P_X(3) = 0.125 \tag{0.4}$$

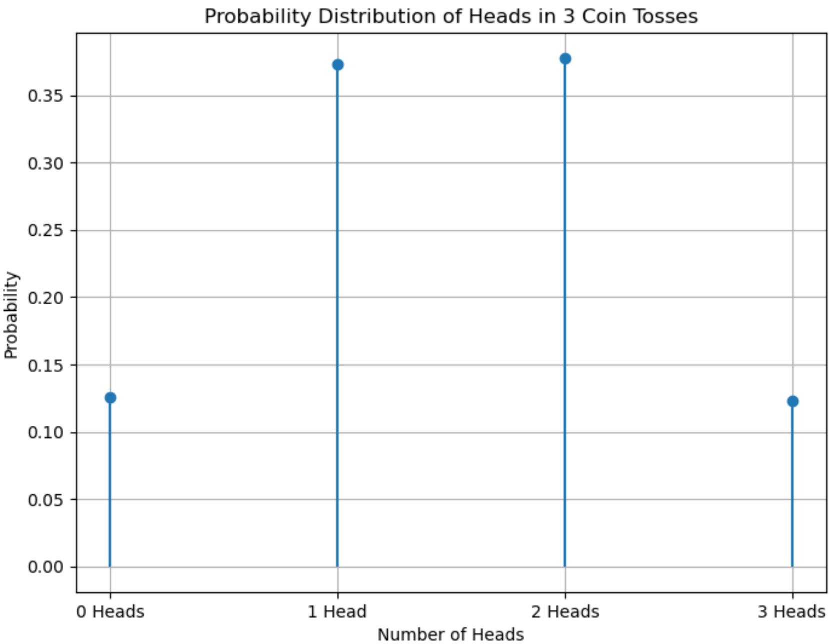
The probability of getting at least 2 heads is:

$$P_X(2) + P_X(3) = 0.375 + 0.125 = 0.5$$

Conclusion

The probability of getting at least 2 heads is:

$$\boxed{0.5}$$


Probability Distribution of Heads in 3 Coin Tosses
Probability
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0 Heads
1 Head
2 Heads
3 Heads
Number of Heads