11.16.3.8.3

EE24BTECH11047 - Niketh Prakash Achanta

1 Problem Statement

Question:
Three coins are tossed once. Find the probability of getting at least 2 heads.

2 Solution

2.1 Random Variables

Let X_1, X_2, X_3 be three independent Bernoulli random variables:

$$X_i = \begin{cases} 1, & \text{if Head appears,} \\ 0, & \text{if Tail appears.} \end{cases} \tag{0.1}$$

Each X_i follows a Bernoulli distribution with:

$$P_X(1) = p = \frac{1}{2}, \quad P_X(0) = 1 - p. \tag{0.2}$$

2.2 Moment-Generating Function (MGF) Using Z-Transform

The moment-generating function (MGF) for X_i is given by:

$$M_{X_i}(z) = \sum_{k=-\infty}^{\infty} P_{X_i}(k) z^{-k} = (1-p) + pz^{-1}. \tag{0.3}$$

Using independence, the MGF of $Y = X_1 + X_2 + X_3$ is:

$$M_Y(z) = M_{X_1}(z) M_{X_2}(z) M_{X_3}(z) = \left((1-p) + pz^{-1}\right)^3. \tag{0.4}$$

Expanding using the binomial theorem:

$$M_Y(z) = \sum_{k=0}^{3} \binom{3}{k} (1-p)^{3-k} p^k z^{-k}. \tag{0.5}$$

Applying the inverse Z-transform, the probability mass function (PMF) is:

$$p_Y(k) = \binom{3}{k} (1-p)^{3-k} p^k, \quad k = 0, 1, 2, 3. \tag{0.6}$$

Substituting $p = \frac{1}{2}$:

$$p_Y(k) = \binom{3}{k} \left(\frac{1}{2}\right)^3. \tag{0.7}$$

2.3 Cumulative Distribution Function (CDF)

The cumulative distribution function (CDF) is given by:

$$F_Y(k) = \sum_{j=-\infty}^{k} p_Y(j). \tag{0.8}$$

Substituting values:

$$F_Y(k) = \begin{cases} 0, & k < 0, \\ \frac{1}{8}, & 0 \leq k < 1, \\ \frac{4}{8}, & 1 \leq k < 2, \\ \frac{7}{8}, & 2 \leq k < 3, \\ 1, & k \geq 3. \end{cases} \tag{0.9}$$

2.4 Probability of At Least Two Heads

$$p(Y \geq 2) = 1 - F_Y(1) = 1 - \frac{4}{8} = \frac{1}{2}. \tag{0.10}$$

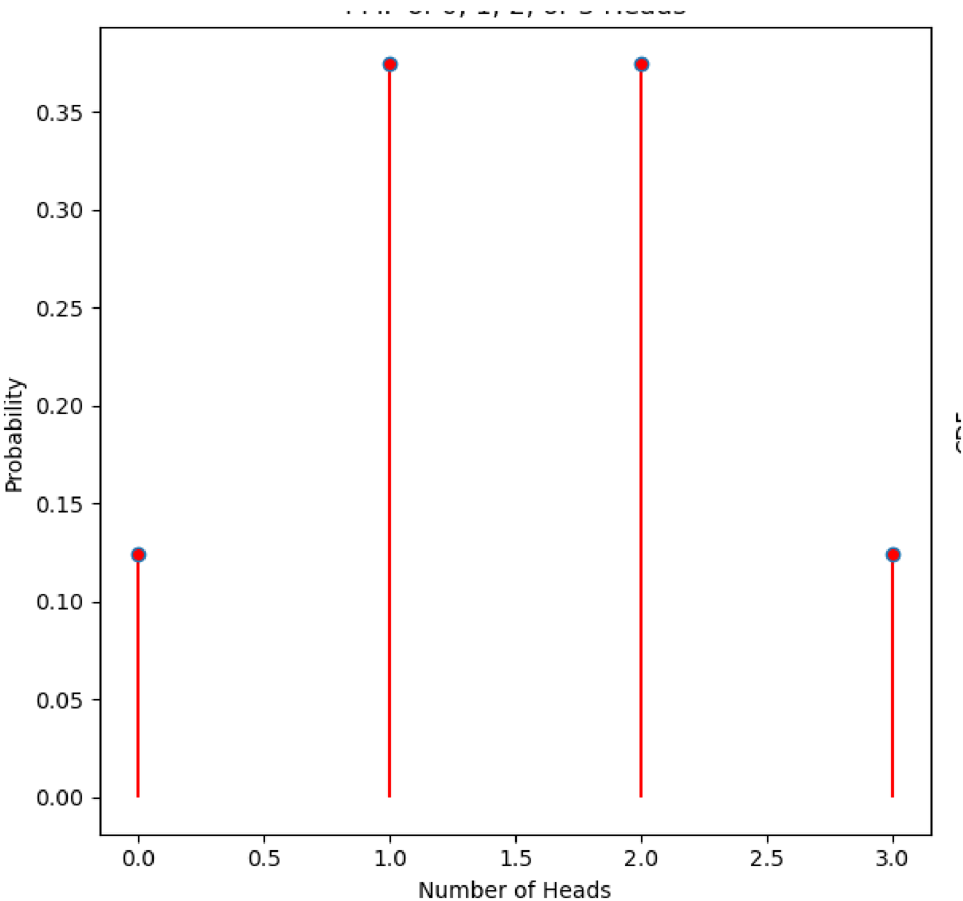
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Probability
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Number of Heads

Fig. 0.1: PMF

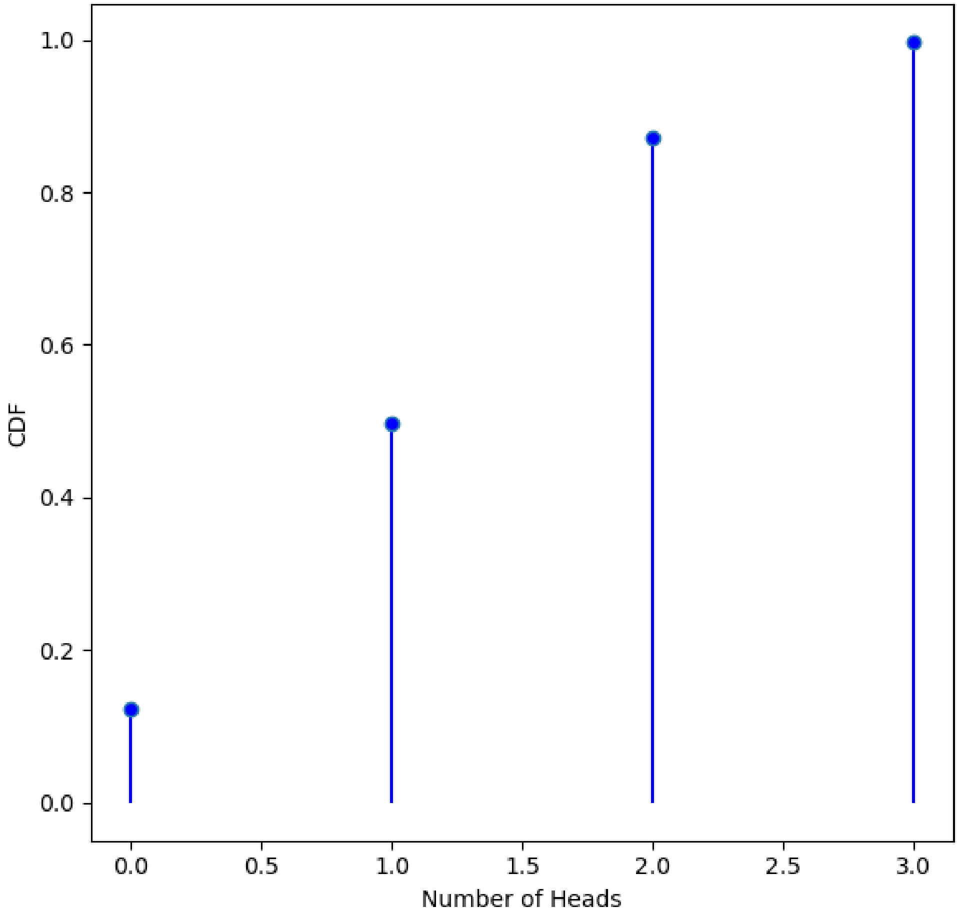

Fig. 0.2: CDF