

20. Vector Algebra

EE24BTECH11047 - Niketh Prakash Achanta

I. D: MCQs WITH ONE OR MORE THAN ONE CORRECT

- 1) Three lines $L_1 : \mathbf{r} = \lambda \hat{\mathbf{i}}, \lambda \in \mathbb{R}$
 $L_2 : \mathbf{r} = \hat{\mathbf{k}} + \mu \hat{\mathbf{j}}, \mu \in \mathbb{R}$ and
 $L_3 : \mathbf{r} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \nu \hat{\mathbf{k}}, \nu \in \mathbb{R}$
 are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P,Q and R are collinear? (JEE Adv. 2019)
- a) $\hat{\mathbf{k}} - \frac{1}{2}\hat{\mathbf{j}}$
 b) $\hat{\mathbf{k}}$
 c) $\hat{\mathbf{k}} + \hat{\mathbf{j}}$
 d) $\hat{\mathbf{k}} + \frac{1}{2}\hat{\mathbf{j}}$

II. E: SUBJECTIVE PROBLEMS

- 1) From a point O inside the triangle ABC, perpendiculars OD,OE,OF are drawn to the sides BC,CA,AB respectively. Prove that the perpendiculars from A,B,C to the sides EF,FD,DE are concurrent. (1978)
- 2) A_1, A_2, \dots, A_n are the vertices of a regular plane polygon with n sides and O is its centre. Show that

$$\sum_{i=1}^{n-1} (\mathbf{OA}_i \times \mathbf{OA}_{i+1}) = (1 - n) (\mathbf{OA}_2 \times \mathbf{OA}_1)$$
 (1982-2Marks)
- 3) Find all values of λ such that $x, y, z \neq (0, 0, 0)$ and
 $(\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}})_x + (3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}})_y + (-4\hat{\mathbf{i}} + 5\hat{\mathbf{j}})_z = \lambda(\mathbf{x}\hat{\mathbf{i}} + \mathbf{y}\hat{\mathbf{j}} + \mathbf{z}\hat{\mathbf{k}})$ where $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are unit vectors along the coordinate axes. (1982-3Marks)
- 4) A vector \mathbf{A} has components A_1, A_2, A_3 in a right-handed rectangular Cartesian coordinate system oxyz. The coordinate system is rotated about the x-axis through an angle $\frac{\pi}{2}$. Find the components of A in the new coordinate system, in terms of A_1, A_2, A_3 . (1983-2Marks)
- 5) The position vectors of the points A,B,C and D are $(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}), (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}), (-\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ and $(4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \lambda\hat{\mathbf{k}})$, respectively. If the points A,B,C and D lie on a plane, find the value of λ . (1986-2.5Marks)
- 6) If A,B,C,D are any four points in space, prove that- (1987-2Marks)
 $|\mathbf{AB} \times \mathbf{CD} + \mathbf{BC} \times \mathbf{AD} + \mathbf{CA} \times \mathbf{BD}| = 4(\text{area of triangle ABC})$
- 7) Let OACB be a parallelogram with O at the origin and OC a diagonal. Let D be the midpoint of OA. Using vector methods prove that BD and CO intersect in the same ratio. Determine this ratio. (1988-3Marks)
- 8) If vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are coplanar, show that

$$\begin{vmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \\ \mathbf{a} \cdot \mathbf{a} & \mathbf{a} \cdot \mathbf{b} & \mathbf{a} \cdot \mathbf{c} \\ \mathbf{b} \cdot \mathbf{a} & \mathbf{b} \cdot \mathbf{b} & \mathbf{b} \cdot \mathbf{c} \end{vmatrix} = 0$$
 (1989-2Marks)
- 9) In a triangle OAB, E is the midpoint of BO and D is a point on AB such that AD:DB=2:1. If OD and AE intersect at P, determine the ratio OP:PD using vector methods. (1989-4 Marks)
- 10) Let $\mathbf{A} = 2\hat{\mathbf{i}} + \hat{\mathbf{k}}, \mathbf{B} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}},$ and $\mathbf{C} = 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 7\hat{\mathbf{k}}.$ Determine a vector \mathbf{R} satisfying $\mathbf{R} \times \mathbf{B} = \mathbf{C} \times \mathbf{B}$ and $\mathbf{R} \cdot \mathbf{A} = 0$ (1990-3Marks)
- 11) Determine the value of 'c' so that for all real x, the vector $\mathbf{cx}\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\mathbf{x}\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\mathbf{cx}\hat{\mathbf{k}}$ make an obtuse angle with each other. (1991-4Marks)
- 12) In a triangle ABC,D and E are points on BC and AC respectively, such that $BD = 2DC$ and $AE = 3EC.$ Let P be the point of intersection of AD and BE. Find BP/PE using vector methods. (1993-5Marks)
- 13) If the vectors $\mathbf{b}, \mathbf{c}, \mathbf{d}$ are not coplanar, then

prove that the vector $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) + (\mathbf{a} \times \mathbf{c}) \times (\mathbf{d} \times \mathbf{b}) + (\mathbf{a} \times \mathbf{d}) \times (\mathbf{b} \times \mathbf{c})$ is parallel to \mathbf{a} .
(1994-4Marks)

- 14) The position vectors of the vertices A, B and C of a tetrahedron ABCD are $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}}$ and $3\hat{\mathbf{i}}$ respectively. The altitude from vertex D to the opposite face ABC meets the median line through A of the triangle ABC at a point E. If the length of the side AD is 4 and the volume of the tetrahedron is $\frac{2\sqrt{2}}{3}$, find the position vector of the point E for all its possible positions.
(1996-5Marks)