20. Vector Algebra

EE24BTECH11047 - Niketh Prakash Achanta

I. C:MCQs With One Correct Answer

41) Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = \hat{i} + 2\hat{j} + 2\hat{k}$
The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by (2010)

a) $\frac{8}{9}$
b) $\frac{\sqrt{17}}{9}$
c) $\frac{1}{9}$
d) $\frac{4\sqrt{5}}{9}$

42) Let $\overrightarrow{a}=\hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{b}=\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{c}=\hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\overrightarrow{v}$ in the plane of $\overrightarrow{a}$ and $\overrightarrow{b}$, whose projection on $\overrightarrow{c}$ is $\frac{1}{\sqrt{3}}$, is given by (2011)

a) $\hat{i} - 3\hat{j} + 3\hat{k}$
b) $-3\hat{i} - 3\hat{j} - \hat{k}$
c) $3\hat{i} - \hat{j} + 3\hat{k}$
d) $\hat{i} + 3\hat{j} - 3\hat{k}$

43) The point P is the intersection of the straight line joining the points $Q(2, 3, 5)$ and $R(1, -1, 4)$ with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point $T(2, 1, 4)$ to QR, then the length of the line segment PS is (2010)

a) $\frac{1}{\sqrt{2}}$
b) $\sqrt{2}$
c) 2
d) $2\sqrt{2}$

44) The equation of a plane passing through the line of intersection of the planes $x+2y+3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point (3,1,-1) is 2012

a) $5x - 11y + z = 17$
b) $\sqrt{2}x + y = 3\sqrt{2} - 1$
c) $x + y + z = \sqrt{3}$
d) $x - \sqrt{2}y = 1 - \sqrt{2}$

45) If $\overrightarrow{a}$ and $\overrightarrow{b}$ are vectors such that $|\overrightarrow{a} + \overrightarrow{b}| = \sqrt{29}$ and $\overrightarrow{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \overrightarrow{b}$, then a possible value of $(\overrightarrow{a} + \overrightarrow{b}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k})$ is (2012)

a) 0
b) 3
c) 4
d) 8

46) Let P be the image of the point (3,1,7) with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1} = \frac{y}{z} = \frac{z}{1}$ (JEE Adv. 2016)

a) $x + y - 3z = 0$
b) $3x + z = 0$
c) $x - 4y + 7z = 0$
d) $2x - y = 0$

47) The equation of the plane passing through the point $(1, 1, 1)$ and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is (JEE Adv. 2017)

a) $14x + 2y - 15z = 1$
b) $14x - 2y + 15z = 27$
c) $14x + 2y + 15z = 31$
d) $-14x + 2y + 15z = 3$

48) Let O be the origin and let PQR be an arbitrary triangle. The point S is such that $\overrightarrow{OP}\cdot\overrightarrow{OQ}+\overrightarrow{OR}\cdot\overrightarrow{OS}=\overrightarrow{OR}\cdot\overrightarrow{OP}+\overrightarrow{OQ}\cdot\overrightarrow{OS}=\overrightarrow{OQ}\cdot\overrightarrow{OR}+\overrightarrow{OP}\cdot\overrightarrow{OS}$
Then the triangle PQR has S as its (JEE Adv. 2017)

a) Centroid
b) Circumcentre
c) Incentre
d) Orthocenter

II. D: MCQs with One or More than One Correct

1) Let $\overrightarrow{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\overrightarrow{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $\overrightarrow{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ be three non-zero vectors such that $\overrightarrow{c}$ is a unit vector perpendicular to both the vectors $\overrightarrow{a}$ and $\overrightarrow{b}$. If the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$ is $\frac{\pi}{6}$, then

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$$

is equal to (1986-2 Marks)

a) 0
b) 1
c) $\frac{1}{4}\left(a_1^2+a_2^2+a_3^2\right)\left(b_1^2+b_2^2+b_3^2\right)$
d) $\frac{3}{4}\left(a_1^2+a_2^2+a_3^2\right)\left(b_1^2+b_2^2+b_3^2\right)\left(c_1^2+c_2^2+c_3^2\right)$

2) The number of vectors of unit length perpendicular to vectors $\overrightarrow{a}=\{1,1,0\}$ and $\overrightarrow{b}=\{0,1,1\}$ is (1987-2Marks)

a) one
b) two
c) three
d) infinite
e) None of these

3) Let $\overrightarrow{a}=2\hat{i}-\hat{j}+\hat{k}$, $\overrightarrow{b}=\hat{i}+2\hat{j}-\hat{k}$ and $\overrightarrow{c}=\hat{i}+2\hat{j}-2\hat{k}$ be three vectors. A vector in the plane of $\overrightarrow{b}$ and $\overrightarrow{c}$, whose projection on $\overrightarrow{a}$ is of magnitude $\sqrt{2/3}$, is: (1993-2Marks)

a) $2\hat{i}+3\hat{j}-3\hat{k}$
b) $2\hat{i}+3\hat{j}+3\hat{k}$
c) $-2\hat{i}-\hat{j}+5\hat{k}$
d) $2\hat{i}+\hat{j}+5\hat{k}$

4) The vector $\frac{1}{3}(2\hat{i}-2\hat{j}+\hat{k})$ is (1994)

a) a unit vector
b) makes an angle with the vector
c) parallel to the vector $\left\{-\hat{i}+\hat{j}-\frac{1}{2}\hat{k}\right\}$
d) perpendicular to the vector $3\hat{i}+2\hat{j}-2\hat{k}$

5) If $\overrightarrow{a}=\hat{i}+\hat{j}+\hat{k}, \overrightarrow{b}=4\hat{i}+3\hat{j}+4\hat{k}$ and $\overrightarrow{c}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$ are linearly dependent vectors and $|c|=\sqrt{3}$, then (1998-2Marks)

a) $\alpha=1,\beta=-1$
b) $\alpha=1,\beta=\pm1$
c) $\alpha=-1,\beta=\pm1$
d) $\alpha=\pm1,\beta=1$