

- 1) Let the range of the function $f(x) = \frac{1}{2 + \sin 3x + \cos 3x}$, $x \in \mathbf{R}$ be $[a, b]$. If α and β are respectively the arithmetic mean and the geometric mean of a and b , then $\frac{\alpha}{\beta}$ is equal to:
 - a) π
 - b) $\sqrt{\pi}$
 - c) 2
 - d) $\sqrt{2}$
- 2) If an unbiased die is rolled thrice, then the probability of getting a greater number in the i^{th} roll than the number obtained in the $(i-1)^{\text{th}}$ roll for $i = 2, 3$, is equal to:
 - a) $2/54$
 - b) $5/54$
 - c) $1/54$
 - d) $3/54$
- 3) Let the foci of a hyperbola H coincide with the foci of the ellipse $E: \frac{(x-1)^2}{100} + \frac{(y-1)^2}{75} = 1$ and the eccentricity of H be the reciprocal of the eccentricity of the ellipse E . If the length of the transverse axis of H is α and the length of its conjugate axis is β , then $3\alpha^2 + 2\beta^2$ is equal to
 - a) 225
 - b) 205
 - c) 237
 - d) 242
- 4) Let $\int_0^x \sqrt{1 - (y'(t))^2} dt$, $0 \leq x \leq 3$, $y \geq 0$, $y(0) = 0$. Then at $x = 2$, $y'' + y + 1$ is equal to
 - a) 2
 - b) $\sqrt{2}$
 - c) $1/2$
 - d) 1
- 5) The sum of the coefficients of $x^{2/3}$ and $x^{-2/5}$ in the binomial expansion of $\left(x^{2/3} + \frac{1}{2}x^{-2/5}\right)^9$ is
 - a) $19/4$
 - b) $69/16$
 - c) $63/16$
 - d) $21/4$
- 6) The value of the integral $\int_{-1}^2 \log(x + \sqrt{x^2 + 1}) dx$ is
 - a) $\sqrt{5} - \sqrt{2} + \log\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

- b) $\sqrt{2} - \sqrt{5} + \log\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$
 c) $\sqrt{5} - \sqrt{2} + \log\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$
 d) $\sqrt{2} - \sqrt{5} + \log\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

7) $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\int_{x^3}^{\left(\frac{\pi}{2}\right)^3} (\sin(2t^{1/3}) + \cos(t^{1/3})) dt}{\left(x - \frac{\pi}{2}\right)^2} \right)$ is equal to

- a) $\frac{9\pi^2}{8}$
 b) $\frac{3\pi^2}{2}$
 c) $\frac{11\pi^2}{10}$
 d) $\frac{5\pi^2}{9}$

8) Let a, ar, ar^2, \dots be an infinite G.P. If $\sum_{n=0}^{\infty} ar^n = 57$ and $\sum_{n=0}^{\infty} a^3 r^{3n} = 9747$, then $a + 18r$ is equal to

- a) 27
 b) 31
 c) 46
 d) 38

9) If $\log_e y = 3 \arcsin x$, then $(1 - x^2)y'' - xy'$ at $x = 1/2$ is equal to:

- a) $3e^{\pi/2}$
 b) $9e^{\pi/6}$
 c) $9e^{\pi/2}$
 d) $3e^{\pi/6}$

10) $\lim_{x \rightarrow 0} \frac{e - (1+2x)^{1/2x}}{x}$ is equal to

- a) 0
 b) $-2/e$
 c) e
 d) $e - e^2$

11) Let $\mathbf{a} = 2\hat{i} + \alpha\hat{j} + \hat{k}$, $\mathbf{b} = -\hat{i} + \hat{k}$, $\mathbf{c} = \beta\hat{j} - \hat{k}$, where α and β are integers and $\alpha\beta = -6$. Let the values of the ordered pair (α, β) , for which the area of the parallelogram of diagonals $\mathbf{a} + \mathbf{b}$ and $\mathbf{b} + \mathbf{c}$ is $\frac{\sqrt{21}}{2}$, be (α_1, β_1) and (α_2, β_2) . Then $\alpha_1^2 + \beta_1^2 - \alpha_2\beta_2$ is equal to

- a) 17
 b) 24
 c) 19
 d) 21

12) Between the following two statements:

Statement 1: Let $\mathbf{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\mathbf{b} = 2\hat{i} + \hat{j} - \hat{k}$. Then the vector \mathbf{r} satisfying $\mathbf{a} \times \mathbf{r} = \mathbf{a} \times \mathbf{b}$ and $\mathbf{a} \cdot \mathbf{r} = 0$ is of magnitude $\sqrt{10}$.

Statement 2: In a triangle ABC, $\cos 2A + \cos 2B + \cos 2C \geq -\frac{3}{2}$.

- a) Both Statement 1 and Statement 2 are correct.
 b) Both Statement 1 and Statement 2 are incorrect.
 c) Statement 1 is correct but Statement 2 is incorrect.

d) Statement 1 is incorrect but Statement 2 is correct.

- 13) Let z be a complex number such that the real part of $\frac{z-2i}{z+2i}$ is zero. Then, the maximum value of $|z - (6 + 8i)|$ is equal to

- a) 10
- b) ∞
- c) 8
- d) 12

- 14) If the variance of the frequency distribution

| x | c | $2c$ | $3c$ | $4c$ | $5c$ | $6c$ |
|-----|-----|------|------|------|------|------|
| f | 2 | 1 | 1 | 1 | 1 | 1 |

is 160, then the value of $c \in \mathbf{N}$ is

- a) 5
- b) 6
- c) 8
- d) 7

- 15) Let $a, b; a > b$, be the roots of the equation $x^2 - \sqrt{2}x - \sqrt{3} = 0$. Let $P_n = a^n - b^n$, $n \in \mathbf{N}$. Then $(11\sqrt{3} - 10\sqrt{2})P_{10} + (11\sqrt{2} + 10)P_{11} - 11P_{12}$ is equal to:

- a) $10\sqrt{2}P_9$
- b) $10\sqrt{3}P_9$
- c) $11\sqrt{2}P_9$
- d) $11\sqrt{3}P_9$