1.1.8.26

EE24BTECH11047 - Niketh Prakash Achanta

Question:
Find a point on the Y axis which is equidistant from the points $\mathbf{A}\begin{pmatrix}6,5\end{pmatrix}$ $\mathbf{B}\begin{pmatrix}-4,3\end{pmatrix}$.

Solution:

Variable	Description	Formula
A	A Point to be plotted	$A = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$
B	A Point to be plotted	$B = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$
P	Midpoint of A and B	$(A - B)^\top P = \frac{\|A\|^2 - \|B\|^2}{2}$

TABLE 0

If $\mathbf{P}$ is equidistant from the points $\mathbf{A}$ and $\mathbf{B}$

$$\|\mathbf{P} - \mathbf{A}\| = \|\mathbf{P} - \mathbf{B}\| \tag{0.1}$$

$$\|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 \tag{0.2}$$

$$\|\mathbf{P}\|^2 - 2\mathbf{P}^\top\mathbf{A} + \|\mathbf{A}\|^2 = \|\mathbf{P}\|^2 - 2\mathbf{P}^\top\mathbf{B} + \|\mathbf{B}\|^2 \tag{0.3}$$

By simplifying further,

$$(\mathbf{A} - \mathbf{B})^\top \mathbf{P} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \tag{0.4}$$

$$\begin{pmatrix} 10 \\ 2 \end{pmatrix}^\top \mathbf{P} = \frac{\left\|\begin{pmatrix} 6 \\ 5 \end{pmatrix}\right\|^2 - \left\|\begin{pmatrix} -4 \\ 3 \end{pmatrix}\right\|^2}{2} = 18 \tag{0.5}$$

Comparing with $n^\top x = c$

$$\mathbf{n} = \begin{pmatrix} 10 \\ 2 \end{pmatrix} \tag{0.6}$$

$$\mathbf{c} = 18 \tag{0.7}$$

$$10x + 2y = 18 \tag{0.8}$$

$$x = 0, y = 9 \tag{0.9}$$

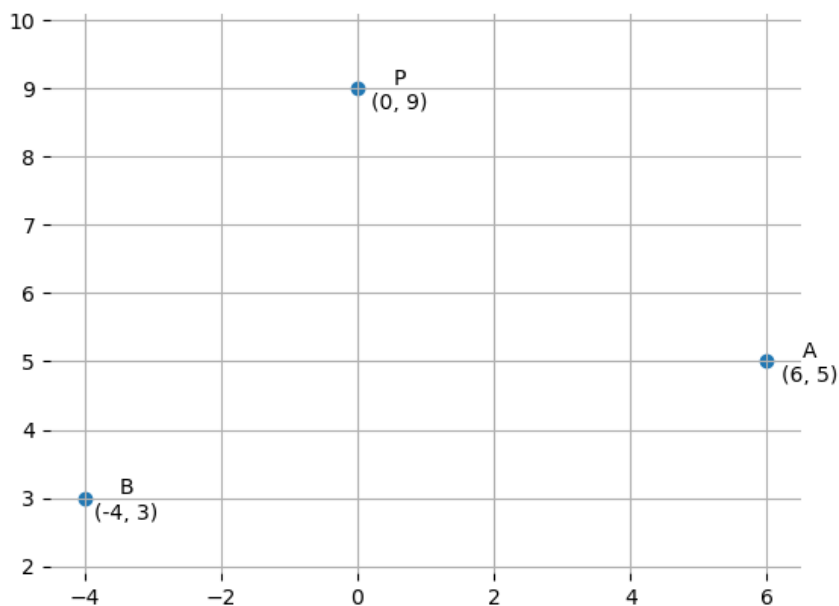$$\mathbf{P} = \begin{pmatrix} 0 \\ 9 \end{pmatrix} \tag{0.10}$$

Fig. 0.1