11. Limits, Continuity and Differentiability

EE24BTECH11047 - Niketh Prakash Achanta

I. C: MCQs with one correct answer

1) $\lim_{x\to 0} \frac{sin(\pi \cos^2 x)}{x^2}$ equals (2001S)
   a) $-\pi$
   b) π
   c) $\pi/2$
   d) 1

2) The left-hand derivative of f(x)=[x]sin(π x) at x=k, k an integer, is (2001S)
   a) $(-1)^k(k-1)\pi$
   b) $(-1)^{k-1}(k-1)\pi$
   c) $-1^k k\pi$
   d) $-1^{k-1} k\pi$

3) Let $f : R \to R$ be a function defined by f(x)=max$\{x, x^3\}$. The set of all points where f(x) is NOT differentiable is (2001S)
   a) $\{-1, 1\}$
   b) $\{-1, 0\}$
   c) $\{0, 1\}$
   d) $\{-1, 0, 1\}$

4) Which of the following functions is differentiable at x=0 (2001S)
   a) $cos(|x|) + |x|$
   b) $cos(|x|) - |x|$
   c) $sin(|x|) + |x|$
   d) $sin(|x|) - |x|$

5) The domain of the derivative of the function $f(x) = \begin{cases} tan^{-1}x & \text{if } |x| \leq 0 \\ \frac{1}{2}(|x|-1) & \text{if } x > 1 \end{cases}$ is (2002S)
   a) $R - \{0\}$
   b) $R - \{1\}$
   c) $R - \{-1\}$
   d) $R - \{-1, 1\}$

6) The integer n for which $\lim_{x\to 0} \frac{(cosx-1)(cosx-e^x)}{x^n}$ is a finite non-zero number is (2002S)
   a) 1
   b) 2
   c) 3
   d) 4

7) Let $f : R \to R$ be such that $f(1) = 3$ and $f'(1) = 6$. Then $\lim_{x\to 0} \left(\frac{f(1+x)}{f(1)}\right)^{1/x}$ equals (2002S)
   a) 1
   b) $e^{1/2}$
   c) e^2
   d) e^3

8) If $\lim_{x\to 0} \frac{((a-n)nx-tanx)sinnx}{x^2} = 0$, where n is nonzero real number, then a is equal to (2003S)
   a) 0
   b) $\frac{n+1}{n}$
   c) n
   d) $n + \frac{1}{n}$

9) $\lim_{h\to 0} \frac{f(2h+2+h^2)-f(2)}{f(h-h^2+1)-f(1)}$ *given that* $f'(2) = 6$ *and* $f'(1) = 4$ (2003S)
   a) Does not exist
   b) is equal to -3/2
   c) is equal to 3/2
   d) is equal to 3

10) If f(x) is differentiable and strictly increasing function, then the value of $\lim_{x\to 0} \frac{f(x^2)-f(x)}{f(x)-f(0)}$ is (2004S)
    a) $(-1)^k(k-1)\pi$
    b) $(-1)^{k-1}(k-1)\pi$
    c) $-1^k k\pi$
    d) $-1^{k-1} k\pi$

11) The function given by $y = ||x| - 1|$ is differentiable for all real numbers except the points (2005S)
    a) $\{0, 1, -1\}$
    b) ± 1
    c) 1
    d) -1

12) If f(x) is a continuous and differentiable function and $f(1/n) = 0 \ \forall \ n \geq 1$ *and* $n \in I$, then (2005S)
   a) $f(x) = 0, x \in (0, 1]$
   b) $f(0) = 0, f'(0) = 0$
   c) $f(0) = 0 = f'(0), x \in (0, 1]$
   d) $f(0) = 0$ and $f'(0)$ need not be zero

13) The value of $\lim_{x\to 0} \left((sin)^{1/x} + (1 + x)^{sinx}\right)$, where $x > 0$ is (2006-3M,-1)
   a) 0
   b) -1
   c) 1
   d) 2

14) Let f(x) be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$, and $\lim_{t\to x} \frac{t^2 f(x) - x^2 f(t)}{t-x} = 1$ for each $x > 0$. Then f(x) is (2007-3marks)
   a) $\frac{1}{3x} + \frac{2x^2}{3}$
   b) $\frac{-1}{3x} + \frac{4x^2}{3}$
   c) $\frac{-1}{x} + \frac{2}{x^2}$
   d) $\frac{1}{x}$

15) $\lim_{x\to \frac{\pi}{4}} \dfrac{\int_2^{sec^2 x} f(t)dt}{x^2 - \frac{\pi^2}{16}}$ equals (2007-3marks)
   a) $\frac{8}{\pi} f(2)$
   b) $\frac{2}{\pi} f(2)$
   c) $\frac{2}{\pi} f(\frac{1}{2})$
   d) $4f(2)$