GATE-2020-MA-53-65

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53) Let $0 < p < 1$ and let

$$X = \{f : \mathbb{R} \to \mathbb{R} \text{ is continuous and } \int_{\mathbb{R}} |f(x)|^p \, dx < \infty\}.$$

For $f \in X$, define

$$\|f\|_p = \left(\int_{\mathbb{R}} |f(x)|^p \, dx\right)^{\frac{1}{p}}.$$

Then

a) $\|\cdot\|_p$ defines a norm on X,
b) $\|f+g\|_p \leq \|f\|_p + \|g\|_p$ for all $f, g \in X$,
c) $\|f+g\|_p^p \leq \|f\|_p^p + \|g\|_p^p$ for all $f, g \in X$,
d) If f_n converges to f pointwise on $\mathbb{R}$, then $\lim_{n\to\infty} \|f_n\|_p = \|f\|_p$.

54) Suppose that ϕ_1 and ϕ_2 are linearly independent solutions of the differential equation

$$2x^2y'' - (x + x^2)y' + (x^2 - 2)y = 0$$

and $\phi_1(0) = 0$. Then the smallest positive integer n such that

$$\lim_{x\to 0} x^n \frac{\phi_2(x)}{\phi_1(x)} = 0$$

is...

55) Suppose that $f(z) = \prod_{n=1}^{17}(z - \frac{\pi}{n})$, $z \in \mathbb{C}$, and $\gamma(t) = e^{2it}$, $t \in [0, 2\pi]$. If

$$\int_\gamma \frac{f'(z)}{f(z)} dz = a\pi i$$

then the value of a is equal to...

56) If $\gamma(t) = \frac{1}{2}e^{3\pi it}$, $t \in [0, 2]$ and

$$\int_\gamma \frac{1}{z^2(e^z - 1)} dz = \beta\pi i$$

(correct up to one decimal place), then β is equal to...

57) Let $K = \mathbb{Q}(\sqrt{3 + 2\sqrt{2}}, \omega)$, where ω is a primitive cube root of unity. Then the degree of extension of K over $\mathbb{Q}$ is...

58) Let $a \in \mathbb{R}$. If $(3, 0, 0, \beta)$ is an optimal solution of the linear programming problem

$$\begin{aligned} \text{minimize } & x_1 + x_2 + x_3 - ax_4 \\ \text{subject to } & 2x_1 - x_2 + x_3 = 6 \\ & -x_1 + x_2 + x_4 = 3 \\ & x_1, x_2, x_3, x_4 \geq 0, \end{aligned}$$

then the maximum value of $|\beta - a|$ is...

59) Suppose that $T : \mathbb{R}^4 \to \mathbb{R}[x]$ is a linear transformation over $\mathbb{R}$ satisfying

$$\begin{aligned} T(-1, 1, 1, 1) &= x^2 + 2x^4 \\ T(1, 2, 3, 4) &= 1 - x^2 \\ T(2, -1, -1, 0) &= x^3 - x^4. \end{aligned}$$

Then the coefficient of x in $T(-3, 5, 6, 6)$ is...

60) Let $\mathbf{F}(x, y, z) = (2x - 2y \cos x)\mathbf{i} + (2y - y^2 \sin x)\mathbf{j} + 4z\mathbf{k}$ and let S be the surface of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$. If $\mathbf{n}$ is the unit outward normal to the tetrahedron, then the value of

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$$

(rounded off to two decimal places) is...

61) Let $\mathbf{F} = (x + 2y)e^x\mathbf{i} + (ye^x + x^2)\mathbf{j} + y^2x\mathbf{k}$ and let S be the surface $x^2 + y^2 + z = 1$, $x \geq 0$. If $\mathbf{n}$ is a unit normal to S and

$$\left| \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS \right| = \alpha\pi,$$

then α is equal to...

62) Let G be a non-cyclic group of order 57. Then the number of elements of order 3 in G is...

63) The coefficient of $(x - 1)^5$ in the Taylor expansion about $x = 1$ of the function

$$F(x) = \int_1^x \frac{\log_e t}{t - 1}\, dt, \quad 0 < x < 2$$

(correct up to two decimal places) is...

64) Let $u(x, y)$ be the solution of the initial value problem

$$\begin{aligned} \frac{\partial u}{\partial x} + \sqrt{u}\frac{\partial u}{\partial y} &= 0, \\ u(x, 0) &= 1 + x^2. \end{aligned}$$

Then the value of $u(0,1)$ is (rounded off to three decimal places)...

65) The value of

$$\lim_{n\to\infty} \int_0^1 nx^n e^{x^2}\, dx$$

is...