11. Limits, Continuity and Differentiability

EE24BTECH11047 - Niketh Prakash Achanta

I. C: MCQs with one correct answer

1) $\lim_{x\to 0} \frac{\sin\left(\pi \cos^2 x\right)}{x^2}$ equals (2001S)
   a) $-\pi$
   b) π
   c) $\pi/2$
   d) 1

2) The left-hand derivative of $f(x) = [x]\sin(\pi x)$ at $x = k$, k an integer, is (2001S)
   a) $(-1)^k(-1)\pi$
   b) $(-1)^{k-1}(k-1)\pi$
   c) $-1^k k\pi$
   d) $-1^{k-1}k\pi$

3) Let $f : \mathbf{R} \to \mathbf{R}$ be a function defined by $f(x) = \max\left\{x, x^3\right\}$. The set of all points where $f(x)$ is NOT differentiable is (2001S)
   a) $\{-1, 1\}$
   b) $\{-1, 0\}$
   c) $\{0, 1\}$
   d) $\{-1, 0, 1\}$

4) Which of the following functions is differentiable at $x = 0$ (2001S)
   a) $\cos(|x|) + |x|$
   b) $\cos(|x|) - |x|$
   c) $\sin(|x|) + |x|$
   d) $\sin(|x|) - |x|$

5) The domain of the derivative of the function $f(x) = \begin{matrix} \tan^{-1} x & \text{if } |x| \leq 0 \\ \frac{1}{2}(|x| - 1) & \text{if } x > 1 \end{matrix}$ is (2002S)
   a) $\mathbf{R} - \{0\}$
   b) $\mathbf{R} - \{1\}$
   c) $\mathbf{R} - \{-1\}$
   d) $\mathbf{R} - \{-1, 1\}$

6) The integer n for which $\lim_{x\to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number is (2002S)
   a) 1
   b) 2
   c) 3
   d) 4

7) Let $f : \mathbf{R} \to \mathbf{R}$ be such that $f(1) = 3$ and $f'(1) = 6$. Then $\lim_{x\to 0} \left(\frac{f(1+x)}{f(1)}\right)^{1/x}$ equals (2002S)

a) 1
b) $e^{1/2}$
c) e^2
d) e^3

8) If $\lim_{x\to 0} \frac{((a-n)nx-\tan x)\sin nx}{x^2} = 0$, where n is nonzero real number, then a is equal to (2003S)

a) 0
b) $\frac{n+1}{n}$
c) n
d) $n + \frac{1}{n}$

9) $\lim_{h\to 0} \frac{f\left(2h+2+h^2\right)-f(2)}{f\left(h-h^2+1\right)-f(1)}$ given that $f'(2) = 6$ and $f'(1) = 4$ (2003S)

a) Does not exist
b) is equal to $-3/2$
c) is equal to $3/2$
d) is equal to 3

10) If f(x) is differentiable and strictly increasing function, then the value of $\lim_{x\to 0} \frac{f\left(x^2\right)-f(x)}{f(x)-f(0)}$ is (2004S)

a) $(-1)^k (k-1)\pi$
b) $(-1)^{k-1} (k-1)\pi$
c) $(-1)^k k\pi$
d) $(-1)^{k-1} k\pi$

11) The function given by $y = ||x| - 1|$ is differentiable for all real numbers except the points (2005S)

a) $\{0, 1, -1\}$
b) ± 1
c) 1
d) -1

12) If $f(x)$ is a continuous and differentiable function and $f(1/n) = 0\ \forall\ n \geq 1$ and$n \in I$, then (2005S)

a) $f(x) = 0, x \in (0, 1]$
b) $f(0) = 0, f'(0) = 0$
c) $f(0) = 0 = f'(0), x \in (0, 1]$
d) $f(0) = 0$ and $f'(0)$ need not be zero

13) The value of $\lim_{x\to 0} \left((\sin)^{1/x} + (1+x)^{sinx}\right)$, where $x > 0$ is (2006 − 3M, −1)

a) 0
b) -1
c) 1
d) 2

14) Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$, and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then $f(x)$ is (2007 − 3*marks*)

a) $\frac{1}{3x} + \frac{2x^2}{3}$

b) $\frac{-1}{3x} + \frac{4x^2}{3}$

c) $\frac{-1}{x} + \frac{2}{x^2}$

d) $\frac{1}{x}$

15) $\lim_{x \to \frac{\pi}{4}} \dfrac{\int_2^{\sec^2 x} f(t)dt}{x^2 - \frac{\pi^2}{16}}$ equals (2007 − 3*marks*)

a) $\frac{8}{\pi} f(2)$
b) $\frac{2}{\pi} f(2)$
c) $\frac{2}{\pi} f\left(\frac{1}{2}\right)$
d) $4f(2)$