jee-main-maths-06-04-2023-shift-2

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1) Let the range of the function $f(x) = \frac{1}{2+\sin 3x+\cos 3x}, x \in \mathbf{R}$ be $[a, b]$. If α and β are respectively the arithmetic mean and the geometric mean of a and b, then $\frac{\alpha}{\beta}$ is equal to:
   a) π
   b) $\sqrt{\pi}$
   c) 2
   d) $\sqrt{2}$
2) If an unbiased die is rolled thrice, then the probability of getting a greater number in the i^{th} roll than the number obtained in the $(i-1)^{th}$ roll for $i = 2, 3$, is equal to:
   a) $2/54$
   b) $5/54$
   c) $1/54$
   d) $3/54$
3) Let the foci of a hyperbola H coincide with the foci of the ellipse E:$\frac{(x-1)^2}{100} + \frac{(y-1)^2}{75} = 1$ and the eccentricity of H be the reciprocal of the eccentricity of the ellipse E. If the length of the transverse axis of H is α and the length of its conjugate axis is β, then $3\alpha^2 + 2\beta^2$ is equal to
   a) 225
   b) 205
   c) 237
   d) 242
4) Let $\int_0^x \sqrt{1-(y'(t))^2}dt, 0 \leq x \leq 3, y \geq 0, y(0) = 0$. Then at $x = 2, y^n + y + 1$ is equal to
   a) 2
   b) $\sqrt{2}$
   c) 1/2
   d) 1
5) The sum of the coefficients of $x^{2/3}$ and $x^{-2/5}$ in the binomial expansion of $\left(x^{2/3} + \frac{1}{2}x^{-2/5}\right)^9$ is
   a) $19/4$
   b) $69/16$
   c) $63/16$
   d) $21/4$
6) The value of the integral $\int_{-1}^{2} \log\left(x + \sqrt{x^2+1}\right)dx$ is
   a) $\sqrt{5} - \sqrt{2} + \log\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

b) $\sqrt{2}-\sqrt{5}+\log\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$
c) $\sqrt{5}-\sqrt{2}+\log\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$
d) $\sqrt{2}-\sqrt{5}+\log\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

7) $\lim_{x\to\frac{\pi}{2}}\left(\frac{\int_{x^3}^{\left(\frac{\pi}{2}\right)^3}\left(\sin\left(2t^{1/3}\right)+\cos\left(t^{1/3}\right)\right)dt}{\left(x-\frac{\pi}{2}\right)^2}\right)$ is equal to
a) $\frac{9\pi^2}{8}$
b) $\frac{3\pi^2}{2}$
c) $\frac{11\pi^2}{10}$
d) $\frac{5\pi^2}{9}$

8) Let $a, ar, ar^2, \cdots$ be an infinite G.P. If $\sum_{n=0}^{\infty} ar^n = 57$ and $\sum_{n=0}^{\infty} a^3r^{3n} = 9747$, then $a+18r$ is equal to
a) 27
b) 31
c) 46
d) 38

9) If $\log_e y = 3\arcsin x$, then $\left(1-x^2\right)y''-xy'$ at $x=1/2$ is equal to:
a) $3e^{\pi/2}$
b) $9e^{\pi/6}$
c) $9e^{\pi/2}$
d) $3e^{\pi/6}$

10) $\lim_{x\to 0}\frac{e-(1+2x)^{1/2x}}{x}$ is equal to
a) 0
b) $-2/e$
c) e
d) $e-e^2$

11) Let $\mathbf{a}=2\hat{\imath}+\alpha\hat{\jmath}+\hat{k}, \mathbf{b}=-\hat{\imath}+\hat{k}, \mathbf{c}=\beta\hat{\jmath}-\hat{k}$, where α and β are integers and $\alpha\beta=-6$. Let the values of the ordered pair(α,β), for which the area of the parallelogram of diagonals $\mathbf{a}+\mathbf{b}$ and $\mathbf{b}+\mathbf{c}$ is $\frac{\sqrt{21}}{2}$, be (α_1,β_1) and (α_2,β_2). Then $\alpha_1^2+\beta_1^2-\alpha_2\beta_2$ is equal to
a) 17
b) 24
c) 19
d) 21

12) Between the following two statements:
Statement 1: Let $\mathbf{a}=\hat{\imath}+2\hat{\jmath}-3\hat{k}$ and $\mathbf{b}=2\hat{\imath}+\hat{\jmath}-\hat{k}$. Then the vector $\mathbf{r}$ satisfying $\mathbf{a}\times\mathbf{r}=\mathbf{a}\times\mathbf{b}$ and $\mathbf{a}\cdot\mathbf{r}=0$ is of magnitude $\sqrt{10}$.
Statement 2: In a triangle ABC, $\cos 2A+\cos 2B+\cos 2C\geq -\frac{3}{2}$.
a) Both Statement 1 and Statement 2 are correct.
b) Both Statement 1 and Statement 2 are incorrect.
c) Statement 1 is correct but Statement 2 is incorrect.

d) Statement 1 is incorrect but Statement 2 is correct.

13) Let z be a complex number such that the real part of $\frac{z-2i}{z+2i}$ is zero. Then, the maximum value of $|z-(6+8i)|$ is equal to

a) 10
b) ∞
c) 8
d) 12

14) If the variance of the frequency distribution

x	c	2c	3c	4c	5c	6c
f	2	1	1	1	1	1

is 160, then the value of $c \in \mathbf{N}$ is

a) 5
b) 6
c) 8
d) 7

15) Let $a, b; a > b$, be the roots of the equation $x^2 - \sqrt{2}x - \sqrt{3} = 0$. Let $P_n = a^n - b^n$, $n \in \mathbf{N}$. Then $(11\sqrt{3} - 10\sqrt{2})P_{10} + (11\sqrt{2} + 10)P_{11} - 11P_{12}$ is equal to:

a) $10\sqrt{2}P_9$
b) $10\sqrt{3}P_9$
c) $11\sqrt{2}P_9$
d) $11\sqrt{3}P_9$