

20. Vector Algebra

EE24BTECH11047 - Niketh Prakash Achanta

I. C:MCQs WITH ONE CORRECT ANSWER

- 1) Two adjacent sides of a parallelogram ABCD are given by $\mathbf{AB} = 2\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$ and $\mathbf{AD} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by (2010)
 - a) $\frac{8}{9}$
 - b) $\frac{\sqrt{17}}{9}$
 - c) $\frac{1}{9}$
 - d) $\frac{4\sqrt{5}}{9}$
- 2) Let $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{c} = \hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$ be three vectors. A vector \mathbf{v} in the plane of \mathbf{a} and \mathbf{b} , whose projection on \mathbf{c} is $\frac{1}{\sqrt{3}}$, is given by (2011)
 - a) $\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$
 - b) $-3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$
 - c) $3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$
 - d) $\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$
- 3) The point \mathbf{P} is the intersection of the straight line joining the points $\mathbf{Q}(2, 3, 5)$ and $\mathbf{R}(1, -1, 4)$ with the plane $5x - 4y - z = 1$. If \mathbf{S} is the foot of the perpendicular drawn from the point $\mathbf{T}(2, 1, 4)$ to QR, then the length of the line segment PS is (2010)
 - a) $\frac{1}{\sqrt{2}}$
 - b) $\sqrt{2}$
 - c) 2
 - d) $2\sqrt{2}$
- 4) The equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3, 1, -1)$ is (2012)
 - a) $5x - 11y + z = 17$
 - b) $\sqrt{2}x + y = 3\sqrt{2} - 1$
 - c) $x + y + z = \sqrt{3}$
 - d) $x - \sqrt{2}y = 1 - \sqrt{2}$
- 5) If \mathbf{a} and \mathbf{b} are vectors such that $|\mathbf{a} + \mathbf{b}| = \sqrt{29}$ and $\mathbf{a} \times (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \times \mathbf{b}$, then a possible value of $(\mathbf{a} + \mathbf{b}) \cdot (-7\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ is (2012)
 - a) 0
 - b) 3
 - c) 4
 - d) 8
- 6) Let \mathbf{P} be the image of the point $(3, 1, 7)$ with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through \mathbf{P} and containing the straight line $\frac{x}{1} = \frac{y}{z} = \frac{z}{1}$ (JEEAdv.2016)
 - a) $x + y - 3z = 0$
 - b) $3x + z = 0$
 - c) $x - 4y + 7z = 0$
 - d) $2x - y = 0$
- 7) The equation of the plane passing through the point $(1, 1, 1)$ and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is (JEEAdv.2017)
 - a) $14x + 2y - 15z = 1$
 - b) $14x - 2y + 15z = 27$
 - c) $14x + 2y + 15z = 31$
 - d) $-14x + 2y + 15z = 3$

- 8) Let \mathbf{O} be the origin and let PQR be an arbitrary triangle. The point \mathbf{S} is such that $\mathbf{OP} \cdot \mathbf{OQ} + \mathbf{OR} \cdot \mathbf{OS} = \mathbf{OR} \cdot \mathbf{OP} + \mathbf{OQ} \cdot \mathbf{OS} = \mathbf{OQ} \cdot \mathbf{OR} + \mathbf{OP} \cdot \mathbf{OS}$. Then the triangle PQR has \mathbf{S} as its (JEEAdv.2017)

- a) Centroid
- b) Circumcentre
- c) Incentre
- d) Orthocenter

- c) parallel to the vector $\{-\hat{i} + \hat{j} - \frac{1}{2}\hat{k}\}$
 d) perpendicular to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$
 5) If $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$, $\mathbf{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\mathbf{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors and $|\mathbf{c}| = \sqrt{3}$, then (1998 – 2Marks)

- a) $\alpha = 1, \beta = -1$
- b) $\alpha = 1, \beta = \pm 1$
- c) $\alpha = -1, \beta = \pm 1$
- d) $\alpha = \pm 1, \beta = 1$

II. D: MCQs WITH ONE OR MORE THAN ONE CORRECT

- 1) Let $\mathbf{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\mathbf{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\mathbf{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that \mathbf{c} is a unit vector perpendicular to both the vectors \mathbf{a} and \mathbf{b} . If the angle between \mathbf{a} and \mathbf{b} is $\frac{\pi}{6}$, then

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 \text{ is equal to } \quad (1986 - 2Marks)$$

- a) 0
 - b) 1
 - c) $\frac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$
 - d) $\frac{3}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)(c_1^2 + c_2^2 + c_3^2)$
- 2) The number of vectors of unit length perpendicular to vectors $\mathbf{a} = \{1, 1, 0\}$ and $\mathbf{b} = \{0, 1, 1\}$ is (1987 – 2Marks)

- a) one
- b) two
- c) three
- d) infinite
- e) None of these

- 3) Let $\mathbf{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\mathbf{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\mathbf{c} = \hat{i} + 2\hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of \mathbf{b} and \mathbf{c} , whose projection on \mathbf{a} is of magnitude $\sqrt{2/3}$, is: (1993 – 2Marks)

- a) $2\hat{i} + 3\hat{j} - 3\hat{k}$
- b) $2\hat{i} + 3\hat{j} + 3\hat{k}$
- c) $-2\hat{i} - \hat{j} + 5\hat{k}$
- d) $2\hat{i} + \hat{j} + 5\hat{k}$

- 4) The vector $\frac{1}{3}(2\hat{i} - 2\hat{j} + \hat{k})$ is (1994)

- a) a unit vector
- b) makes an angle with the vector