20. Vector Algebra

EE24BTECH11047 - Niketh Prakash Achanta

I. C:MCQs With One Correct Answer

1) Two adjacent sides of a parallelogram ABCD are given by $\mathbf{AB} = \mathbf{2\hat{i}} + \mathbf{10\hat{j}} + \mathbf{11\hat{k}}$ and $\mathbf{AD} = \mathbf{\hat{i}} + \mathbf{2\hat{j}} + \mathbf{2\hat{k}}$
The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by (2010)
   a) $\frac{8}{9}$
   b) $\frac{\sqrt{17}}{9}$
   c) $\frac{1}{9}$
   d) $\frac{4\sqrt{5}}{9}$

2) Let $\mathbf{a} = \mathbf{\hat{i}} + \mathbf{\hat{j}} + \mathbf{\hat{k}}$, $\mathbf{b} = \mathbf{\hat{i}} - \mathbf{\hat{j}} + \mathbf{\hat{k}}$ and $\mathbf{c} = \mathbf{\hat{i}} - \mathbf{\hat{j}} - \mathbf{\hat{k}}$ be three vectors. A vector $\mathbf{v}$ in the plane of $\mathbf{a}$ and $\mathbf{b}$, whose projection on $\mathbf{c}$ is $\frac{1}{\sqrt{3}}$, is given by (2011)
   a) $\mathbf{\hat{i}} - \mathbf{3\hat{j}} + \mathbf{3\hat{k}}$
   b) $-\mathbf{3\hat{i}} - \mathbf{3\hat{j}} - \mathbf{\hat{k}}$
   c) $\mathbf{3\hat{i}} - \mathbf{\hat{j}} + \mathbf{3\hat{k}}$
   d) $\mathbf{\hat{i}} + \mathbf{3\hat{j}} - \mathbf{3\hat{k}}$

3) The point P is the intersection of the straight line joining the points $\mathbf{Q(2,3,5)}$ and $\mathbf{R(1,-1,4)}$ with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point $\mathbf{T(2,1,4)}$ to QR, then the length of the line segment PS is (2010)
   a) $\frac{1}{\sqrt{2}}$
   b) $\sqrt{2}$
   c) 2
   d) $2\sqrt{2}$

4) The equation of a plane passing through the line of intersection of the planes $x+2y+3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(\mathbf{3},\mathbf{1},\mathbf{-1})$ is 2012
   a) $5x - 11y + z = 17$
   b) $\sqrt{2}x + y = 3\sqrt{2} - 1$
   c) $x + y + z = \sqrt{3}$
   d) $x - \sqrt{2}y = 1 - \sqrt{2}$

5) If $\mathbf{a}$ and $\mathbf{b}$ are vectors such that $|\mathbf{a} + \mathbf{b}| = \sqrt{29}$ and $\mathbf{a} \times (\mathbf{2\hat{i}} + \mathbf{3\hat{j}} + \mathbf{4\hat{k}}) = (\mathbf{2\hat{i}} + \mathbf{3\hat{j}} + \mathbf{4\hat{k}}) \times \mathbf{b}$, then a possible value of $(\mathbf{a}+\mathbf{b})\cdot(-\mathbf{7\hat{i}} + \mathbf{2\hat{j}} + \mathbf{3\hat{k}})$ is (2012)
   a) 0
   b) 3
   c) 4
   d) 8

6) Let P be the image of the point $(\mathbf{3},\mathbf{1},\mathbf{7})$ with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1} = \frac{y}{z} = \frac{z}{1}$ (JEE Adv. 2016)
   a) $x + y - 3z = 0$
   b) $3x + z = 0$
   c) $x - 4y + 7z = 0$
   d) $2x - y = 0$

7) The equation of the plane passing through the point $(\mathbf{1},\mathbf{1},\mathbf{1})$ and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is (JEE Adv. 2017)
   a) $14x + 2y - 15z = 1$
   b) $14x - 2y + 15z = 27$
   c) $14x + 2y + 15z = 31$
   d) $-14x + 2y + 15z = 3$

8) Let O be the origin and let PQR be an arbitrary triangle. The point S is such that $\mathbf{OP} \cdot \mathbf{OQ} + \mathbf{OR} \cdot \mathbf{OS} = \mathbf{OR} \cdot \mathbf{OP} + \mathbf{OQ} \cdot \mathbf{OS} = \mathbf{OQ} \cdot \mathbf{OR} + \mathbf{OP} \cdot \mathbf{OS}$
Then the triangle PQR has S as its (JEE Adv. 2017)

a) Centroid
b) Circumcentre
c) Incentre
d) Orthocenter

II. D: MCQs with One or More than One Correct

1) Let $\mathbf{a} = \mathbf{a_1}\hat{\mathbf{i}} + \mathbf{a_2}\hat{\mathbf{j}} + \mathbf{a_3}\hat{\mathbf{k}}$, $\mathbf{b} = \mathbf{b_1}\hat{\mathbf{i}} + \mathbf{b_2}\hat{\mathbf{j}} + \mathbf{b_3}\hat{\mathbf{k}}$ and $\mathbf{c} = \mathbf{c_1}\hat{\mathbf{i}} + \mathbf{c_2}\hat{\mathbf{j}} + \mathbf{c_3}\hat{\mathbf{k}}$ be three non-zero vectors such that $\mathbf{c}$ is a unit vector perpendicular to both the vectors $\mathbf{a}$ and $\mathbf{b}$. If the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{6}$, then

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$$

is equal to (1986-2 Marks)

a) 0
b) 1
c) $\frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$
d) $\frac{3}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)\left(c_1^2 + c_2^2 + c_3^2\right)$

2) The number of vectors of unit length perpendicular to vectors $\mathbf{a} = \{\mathbf{1}, \mathbf{1}, \mathbf{0}\}$ and $\mathbf{b} = \{\mathbf{0}, \mathbf{1}, \mathbf{1}\}$ is (1987-2Marks)

a) one
b) two
c) three
d) infinite
e) None of these

3) Let $\mathbf{a} = \mathbf{2}\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} + \mathbf{2}\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{c} = \hat{\mathbf{i}} + \mathbf{2}\hat{\mathbf{j}} - \mathbf{2}\hat{\mathbf{k}}$ be three vectors. A vector in the plane of $\mathbf{b}$ and $\mathbf{c}$, whose projection on $\mathbf{a}$ is of magnitude $\sqrt{2/3}$, is: (1993-2Marks)

a) $\mathbf{2}\hat{\mathbf{i}} + \mathbf{3}\hat{\mathbf{j}} - \mathbf{3}\hat{\mathbf{k}}$
b) $\mathbf{2}\hat{\mathbf{i}} + \mathbf{3}\hat{\mathbf{j}} + \mathbf{3}\hat{\mathbf{k}}$
c) $-\mathbf{2}\hat{\mathbf{i}} - \hat{\mathbf{j}} + \mathbf{5}\hat{\mathbf{k}}$
d) $\mathbf{2}\hat{\mathbf{i}} + \hat{\mathbf{j}} + \mathbf{5}\hat{\mathbf{k}}$

4) The vector $\frac{1}{3}(\mathbf{2}\hat{\mathbf{i}} - \mathbf{2}\hat{\mathbf{j}} + \hat{\mathbf{k}})$ is (1994)

a) a unit vector
b) makes an angle with the vector
c) parallel to the vector $\left\{-\hat{\mathbf{i}} + \hat{\mathbf{j}} - \frac{1}{2}\hat{\mathbf{k}}\right\}$
d) perpendicular to the vector $\mathbf{3}\hat{\mathbf{i}} + \mathbf{2}\hat{\mathbf{j}} - \mathbf{2}\hat{\mathbf{k}}$

5) If $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \mathbf{b} = \mathbf{4}\hat{\mathbf{i}} + \mathbf{3}\hat{\mathbf{j}} + \mathbf{4}\hat{\mathbf{k}}$ and $\mathbf{c} = \hat{\mathbf{i}} + \alpha\hat{\mathbf{j}} + \beta\hat{\mathbf{k}}$ are linearly dependent vectors and $|c| = \sqrt{3}$, then (1998-2Marks)

a) $\alpha = 1, \beta = -1$
b) $\alpha = 1, \beta = \pm 1$
c) $\alpha = -1, \beta = \pm 1$
d) $\alpha = \pm 1, \beta = 1$