

12. Limits, Continuity and Differentiability

EE24BTECH11047 - Niketh Prakash Achanta

I. C: MCQs WITH ONE CORRECT ANSWER

- 1) $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ equals (2001S)
 - a) $-\pi$
 - b) π
 - c) $\pi/2$
 - d) 1
- 2) The left-hand derivative of $f(x) = [x] \sin(\pi x)$ at $x = k$, k an integer, is (2001S)
 - a) $(-1)^k (-1)\pi$
 - b) $(-1)^{k-1} (k-1)\pi$
 - c) $-1^k k\pi$
 - d) $-1^{k-1} k\pi$
- 3) Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be a function defined by $f(x) = \max\{x, x^3\}$. The set of all points where $f(x)$ is NOT differentiable is (2001S)
 - a) $\{-1, 1\}$
 - b) $\{-1, 0\}$
 - c) $\{0, 1\}$
 - d) $\{-1, 0, 1\}$
- 4) Which of the following functions is differentiable at $x = 0$ (2001S)
 - a) $\cos(|x|) + |x|$
 - b) $\cos(|x|) - |x|$
 - c) $\sin(|x|) + |x|$
 - d) $\sin(|x|) - |x|$
- 5) The domain of the derivative of the function $f(x) = \begin{cases} \tan^{-1} x, & \text{if } |x| \leq 0 \\ \frac{1}{2}(|x| - 1), & \text{if } x > 1 \end{cases}$ (2002S)
 - a) $\mathbf{R} - \{0\}$
 - b) $\mathbf{R} - \{1\}$
 - c) $\mathbf{R} - \{-1\}$
 - d) $\mathbf{R} - \{-1, 1\}$
- 6) The integer n for which $\lim_{x \rightarrow 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number is (2002S)
 - a) 1
 - b) 2
 - c) 3
 - d) 4

- 7) Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be such that $f(1) = 3$ and $f'(1) = 6$. Then $\lim_{x \rightarrow 0} \left(\frac{f(1+x)}{f(1)} \right)^{1/x}$ equals (2002S)
- 1
 - $e^{1/2}$
 - e^2
 - e^3
- 8) If $\lim_{x \rightarrow 0} \frac{((a-n)nx - \tan x) \sin nx}{x^2} = 0$, where n is nonzero real number, then a is equal to (2003S)
- 0
 - $\frac{n+1}{n}$
 - n
 - $n + \frac{1}{n}$
- 9) $\lim_{h \rightarrow 0} \frac{f(2h+2+h^2)-f(2)}{f(h-h^2+1)-f(1)}$ given that $f'(2) = 6$ and $f'(1) = 4$ (2003S)
- Does not exist
 - is equal to $-3/2$
 - is equal to $3/2$
 - is equal to 3
- 10) If $f(x)$ is differentiable and strictly increasing function, then the value of $\lim_{x \rightarrow 0} \frac{f(x^2)-f(x)}{f(x)-f(0)}$ is (2004S)
- $(-1)^k (k-1)\pi$
 - $(-1)^{k-1} (k-1)\pi$
 - $(-1)^k k\pi$
 - $(-1)^{k-1} k\pi$
- 11) The function given by $y = ||x| - 1|$ is differentiable for all real numbers except the points (2005S)
- $\{0, 1, -1\}$
 - ± 1
 - 1
 - 1
- 12) If $f(x)$ is a continuous and differentiable function and $f(1/n) = 0 \forall n \geq 1$ and $n \in I$, then (2005S)
- $f(x) = 0, x \in (0, 1]$
 - $f(0) = 0, f'(0) = 0$
 - $f(0) = 0 = f'(0), x \in (0, 1]$
 - $f(0) = 0$ and $f'(0)$ need not be zero
- 13) The value of $\lim_{x \rightarrow 0} ((\sin)^{1/x} + (1+x)^{\sin x})$, where $x > 0$ is (2006 - 3M, -1)
- 0
 - 1
 - 1
 - 2

14) Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$, and $\lim_{t \rightarrow x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then $f(x)$ is (2007 – 3marks)

a) $\frac{1}{3x} + \frac{2x^2}{3}$

b) $\frac{-1}{3x} + \frac{4x^2}{3}$

c) $\frac{-1}{x} + \frac{2}{x^2}$

d) $\frac{1}{x}$

15) $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\int_{\frac{\pi}{2}}^{\sec^2 x} f(t) dt}{x^2 - \frac{\pi^2}{16}}$ equals (2007 – 3marks)

a) $\frac{8}{\pi} f(2)$

b) $\frac{2}{\pi} f(2)$

c) $\frac{2}{\pi} f\left(\frac{1}{2}\right)$

d) $4f(2)$