20. Vector Algebra

EE24BTECH11047 - Niketh Prakash Achanta

I. C:MCQs With One Correct Answer

1) Two adjacent sides of a parallelogram ABCD are given by $AB = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $AD = \hat{i} + 2\hat{j} + 2\hat{k}$
The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD′. If AD′ makes a right angle with the side AB, then the cosine of the angle α is given by (2010)

   a) $\frac{8}{9}$
   b) $\frac{\sqrt{17}}{9}$
   c) $\frac{1}{9}$
   d) $\frac{4\sqrt{5}}{9}$

2) Let $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$, $\mathbf{b} = \hat{i} - \hat{j} + \hat{k}$ and $\mathbf{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\mathbf{v}$ in the plane of $\mathbf{a}$ and $\mathbf{b}$, whose projection on $\mathbf{c}$ is $\frac{1}{\sqrt{3}}$, is given by (2011)

   a) $\hat{i} - 3\hat{j} + 3\hat{k}$
   b) $-3\hat{i} - 3\hat{j} - \hat{k}$
   c) $3\hat{i} - \hat{j} + 3\hat{k}$
   d) $\hat{i} + 3\hat{j} - 3\hat{k}$

3) The point $\mathbf{P}$ is the intersection of the straight line joining the points $\mathbf{Q}(2, 3, 5)$ and $\mathbf{R}(1, -1, 4)$ with the plane $5x - 4y - z = 1$. If $\mathbf{S}$ is the foot of the perpendicular drawn from the point $\mathbf{T}(2, 1, 4)$ to QR, then the length of the line segment PS is (2010)

   a) $\frac{1}{\sqrt{2}}$
   b) $\sqrt{2}$
   c) 2
   d) $2\sqrt{2}$

4) The equation of a plane passing through the line of intersection of the planes $x+2y+3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3, 1, -1)$ is (2012)

   a) $5x - 11y + z = 17$
   b) $\sqrt{2}x + y = 3\sqrt{2} - 1$
   c) $x + y + z = \sqrt{3}$
   d) $x - \sqrt{2}y = 1 - \sqrt{2}$

5) If $\mathbf{a}$ and $\mathbf{b}$ are vectors such that $|\mathbf{a} + \mathbf{b}| = \sqrt{29}$ and $\mathbf{a} \times \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) = \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) \times \mathbf{b}$, then a possible value of $(\mathbf{a} + \mathbf{b}) \cdot \left(-7\hat{i} + 2\hat{j} + 3\hat{k}\right)$ is (2012)

   a) 0
   b) 3
   c) 4
   d) 8

6) Let $\mathbf{P}$ be the image of the point $(3, 1, 7)$ with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through $\mathbf{P}$ and containing the straight line $\frac{x}{1} = \frac{y}{z} = \frac{z}{1}$ $(JEEAdv.2016)$

   a) $x + y - 3z = 0$
   b) $3x + z = 0$
   c) $x - 4y + 7z = 0$
   d) $2x - y = 0$

7) The equation of the plane passing through the point $(1, 1, 1)$ and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is $(JEEAdv.2017)$

   a) $14x + 2y - 15z = 1$
   b) $14x - 2y + 15z = 27$
   c) $14x + 2y + 15z = 31$
   d) $-14x + 2y + 15z = 3$

8) Let **O** be the origin and let PQR be an arbitrary triangle. The point **S** is such that $OP \cdot OQ + OR \cdot OS = OR \cdot OP + OQ \cdot OS = OQ \cdot OR + OP \cdot OS$
Then the triangle PQR has **S** as its $(JEEAdv.2017)$

a) Centroid
b) Circumcentre
c) Incentre
d) Orthocenter

II. D: MCQs with One or More than One Correct

1) Let $\mathbf{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\mathbf{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\mathbf{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that **c** is a unit vector perpendicular to both the vectors **a** and **b**. If the angle between **a** and **b** is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to $(1986 - 2Marks)$

a) 0
b) 1
c) $\frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$
d) $\frac{3}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)\left(c_1^2 + c_2^2 + c_3^2\right)$

2) The number of vectors of unit length perpendicular to vectors $\mathbf{a} = \{1, 1, 0\}$ and $\mathbf{b} = \{0, 1, 1\}$ is $(1987 - 2Marks)$

a) one
b) two
c) three
d) infinite
e) None of these

3) Let $\mathbf{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\mathbf{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\mathbf{c} = \hat{i} + 2\hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of **b** and **c**, whose projection on **a** is of magnitude $\sqrt{2/3}$, is: $(1993 - 2Marks)$

a) $2\hat{i} + 3\hat{j} - 3\hat{k}$
b) $2\hat{i} + 3\hat{j} + 3\hat{k}$
c) $-2\hat{i} - \hat{j} + 5\hat{k}$
d) $2\hat{i} + \hat{j} + 5\hat{k}$

4) The vector $\frac{1}{3}\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$ is (1994)

a) a unit vector
b) makes an angle with the vector
c) parallel to the vector $-\hat{i} + \hat{j} - \frac{1}{2}\hat{k}$
d) perpendicular to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

5) If $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}, \mathbf{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\mathbf{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors and $|c| = \sqrt{3}$, then $(1998 - 2Marks)$

a) $\alpha = 1, \beta = -1$
b) $\alpha = 1, \beta = \pm 1$
c) $\alpha = -1, \beta = \pm 1$
d) $\alpha = \pm 1, \beta = 1$