20. Vector Algebra

EE24BTECH11047 - Niketh Prakash Achanta

I. D: MCQs with One or More than One Correct

1) Three lines $L_1 : \mathbf{r} = \lambda\hat{\mathbf{i}},\ \lambda \in \mathrm{R}$
$L_2 : \mathbf{r} = \hat{\mathbf{k}} + \mu\hat{\mathbf{j}},\ \mu \in R$ and
$L_3 : \mathbf{r} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \nu\hat{\mathbf{k}},\ \nu \in R$
are given. For which point(s) $\mathbf{Q}$ on L_2 can we find a point $\mathbf{P}$ on L_1 and a point $\mathbf{R}$ on L_3 so that $\mathbf{P}, \mathbf{Q}$and$\mathbf{R}$ are collinear? $(JEEAdv.2019)$

   a) $\hat{\mathbf{k}} - \frac{1}{2}\hat{\mathbf{j}}$
   b) $\hat{\mathbf{k}}$
   c) $\hat{\mathbf{k}} + \hat{\mathbf{j}}$
   d) $\hat{\mathbf{k}} + \frac{1}{2}\hat{\mathbf{j}}$

II. E: Subjective Problems

1) From a point $\mathbf{O}$ inside the triangle ABC, perpendiculars OD,OE,OF are drawn to the sides BC,CA,AB respectively. Prove that the perpendiculars from $\mathbf{A}, \mathbf{B}, \mathbf{C}$ to the sides EF,FD,DE are concurrent. (1978)

2) $A_1, A_2, ...A_n$ are the vertices of a regular plane polygon with n sides and $\mathbf{O}$ is its centre. Show that $\sum_{i=1}^{n-1} (\mathbf{OA_i} \times \mathbf{OA_{i+1}}) = (1-n)(\mathbf{OA_2} \times \mathbf{OA_1})$ $(1982 - 2Marks)$

3) Find all values of λ such that $x, y, z \neq (0,0,0)$ and $\left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right)x + \left(3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}\right)y + \left(-4\hat{\mathbf{i}} + 5\hat{\mathbf{j}}\right)z = \lambda\left(\mathbf{x}\hat{\mathbf{i}} + \mathbf{y}\hat{\mathbf{j}} + \mathbf{z}\hat{\mathbf{k}}\right)$ where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the coordinate axes. $(1982 - 3Marks)$

4) A vector $\mathbf{A}$ has components A_1, A_2, A_3 in a right-handed rectangular Cartesian coordinate system oxyz. The coordinate system is rotated about the x-axis through an angle $\frac{\pi}{2}$. Find the components of A in the new coordinate system, in terms of A_1, A_2, A_3. $(1983 - 2Marks)$

5) The position vectors of the points $\mathbf{A}, \mathbf{B}, \mathbf{C}and\mathbf{D}$ are $\left(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}\right), \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}\right), \left(-\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right)$ and $\left(4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}\right)$, respectively. If the points $\mathbf{A}, \mathbf{B}, \mathbf{C}\,(\mathbf{and})\,\mathbf{D}$ lie on a plane, find the value of λ. $(1986 - 2.5Marks)$

6) If $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are any four points in space, prove that- $(1987 - 2Marks)$
$|\mathbf{AB} \times \mathbf{CD} + \mathbf{BC} \times \mathbf{AD} + \mathbf{CA} \times \mathbf{BD}| = 4(areaoftriangleABC)$

7) Let OACB be a parallelogram with $\mathbf{O}$ at the origin and OC a diagonal. Let $\mathbf{D}$ be the midpoint of OA. Using vector methods prove that BD and CO intersect in the same ratio. Determine this ratio. $(1988 - 3Marks)$

8) If vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are coplanar, show that $\begin{vmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \\ \mathbf{a}\cdot\mathbf{a} & \mathbf{a}\cdot\mathbf{b} & \mathbf{a}\cdot\mathbf{c} \\ \mathbf{b}\cdot\mathbf{a} & \mathbf{b}\cdot\mathbf{b} & \mathbf{b}\cdot\mathbf{c} \end{vmatrix} = \mathbf{0}$ $(1989 - 2Marks)$

9) In a triangle OAB, $\mathbf{E}$ is the midpoint of BO and $\mathbf{D}$ is a point on AB such that AD:DB=2:1. If OD and AE intersect at $\mathbf{P}$, determine the ratio OP:PD using vector methods. $(1989 - 4Marks)$

10) Let $\mathbf{A} = 2\hat{\mathbf{i}} + \hat{\mathbf{k}}, \mathbf{B} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, and $\mathbf{C} = 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$. Determine a vector $\mathbf{R}$ satisfying $\mathbf{R} \times \mathbf{B} = \mathbf{C} \times \mathbf{B}$ and $\mathbf{R} \cdot \mathbf{A} = 0$ $(1990 - 3Marks)$

11) Determine the value of 'c' so that for all real x, the vector $\mathbf{cx}\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\mathbf{x}\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\mathbf{cx}\hat{\mathbf{k}}$ make an obtuse angle with each other. $(1991 - 4Marks)$

12) In a triangle ABC,$\mathbf{D}$ and $\mathbf{E}$ are points on BC and AC respectively, such that $BD = 2DC$ and $AE = 3EC$. Let $\mathbf{P}$ be the point of intersection of AD and BE. Find BP/PE using vector methods. $(1993 - 5Marks)$

13) If the vectors $\mathbf{b}, \mathbf{c}, \mathbf{d}$ are not coplanar, then

prove that the vector $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d})$ + $(\mathbf{a} \times \mathbf{c}) \times (\mathbf{d} \times \mathbf{b})$ + $(\mathbf{a} \times \mathbf{d}) \times (\mathbf{b} \times \mathbf{c})$ is parallel to $\mathbf{a}$. (1994 – 4*Marks*)

14) The position vectors of the vertices $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ of a tetrahedron ABCD are $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \hat{\mathbf{i}}$ and $\mathbf{3}\hat{\mathbf{i}}$ respectively. The altitude from vertex **D** to the opposite face ABC meets the median line through **A** of the triangle ABC at a point **E**. If the length of the side AD is 4 and the volume of the tetrahedron is $\frac{2\sqrt{2}}{3}$, find the position vector of the point **E** for all its possible positions. (1996 – 5*Marks*)