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18) Let a primal linear programming problem admit an optimal solution. Then the corresponding dual problem
    a) does not have a feasible solution
    b) has a feasible solution but does not have any optimal solution
    c) does not have a convex feasible region
    d) has an optimal solution
19) In any system of particles, suppose we do not assume that the internal forces come in pairs. Then the fact that the sum of internal forces is zero follows from
    a) Newton's second law
    b) conservation of angular momentum
    c) conservation of energy
    d) principle of virtual displacement
20) Let $q_1, q_2, \cdots, q_n$ be the generalized coordinates and $\dot{q}_1, \dot{q}_2, \cdots, \dot{q}_n$ be the generalized velocities in a conservative force field. If under a transformation φ, the new coordinate system has the generalized coordinates $Q_1, Q_2, \cdots, Q_n$ and velocities $\dot{Q}_1, \dot{Q}_2, \cdots, \dot{Q}_n$, then the equation $\frac{\partial L}{\partial \dot{q}_k} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right)$ takes the form
    a) $\frac{\partial L}{\partial Q_k} = \varphi \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{Q}_k}\right)$
    b) $\varphi \frac{\partial L}{\partial Q_k} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{Q}_k}\right)$
    c) $\frac{\partial L}{\partial Q_k} = \frac{d}{dt}\varphi\left(\frac{\partial L}{\partial \dot{Q}_k}\right)$
    d) $\frac{\partial L}{\partial Q_k} = \varphi \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{Q}_k}\right)$
21) Let $T : \mathbf{R}^4 \to \mathbf{R}^4$ be the linear map satisfying
    $T(e_1) = e_2, T(e_2) = e_3, T(e_3) = 0, T(e_4) = e_3,$
    where $\{e_1, e_2, e_3, e_4\}$ is the standard basis of $\mathbf{R}^4$. Then
    a) T is idempotent
    b) T is nvertible
    c) Rank $T = 3$
    d) T is nilpotent
22) Let $M = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ and $V = \{Mx : x \in \mathbf{R}^3\}$. Then an orthonormal basis for V is
    a) $\left\{(1,0,0)^T, \frac{1}{\sqrt{5}}(0,2,1)^T, \frac{1}{\sqrt{6}}(2,1,1)^T\right\}$
    b) $\left\{(1,0,0)^T, \frac{1}{\sqrt{2}}(0,1,1)^T\right\}$
    c) $\left\{(1,0,0)^T, \frac{1}{\sqrt{3}}(1,1,1)^T, \frac{1}{\sqrt{6}}(2,1,1)^T\right\}$
    d) $\left\{(1,0,0)^T, (0,0,1)^T\right\}$

23) For any $n \in \mathbf{N}$, let P_n denote the vector space of all polynomials with real coefficients and of degree at most n. Define $T : P_n \to P_{n+1}$ by

$$T(p)(x) = p'(x) - \int_0^x p(t)dt.$$

Then the dimension of the null space of T is

a) 0
b) 1
c) n
d) $n+1$

24) Let $M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$, where $0 < \theta < \frac{\pi}{2}$. Let $V = \left\{u \in \mathbf{R}^3 : Mu^T = u^T\right\}$. Then the dimension of V is

a) 0
b) 1
c) 2
d) 3

25) The number of linearly independent eigenvectors of the matrix $\begin{pmatrix} 2 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 4 \end{pmatrix}$ is

a) 1
b) 2
c) 3
d) 4

26) Let f be a bilinear transformation that maps -1 to 1, i to ∞ and -i to 0. Then f(1) is equal to

a) -2
b) -1
c) i
d) -i

27) Which one of the following does NOT hold for all continuous functions $f : [-\pi, \pi] \to \mathbf{C}$?

a) if $f(-t) = f(t)$ for each $t \in [-/pi, /pi]$, then $\int_{-\pi}^{\pi} f(t)\,dt = 2\int_0^{\pi} f(t)\,dt$
b) If $f(-t) = -f(t)$ for each $t \in [-/pi, /pi]$, then $\int_{-\pi}^{\pi} f(t)\,dt = 0$
c) $\int_{-\pi}^{\pi} f(-t)\,dt = -\int_{-\pi}^{\pi} f(t)\,dt =$
d) There is an α with $-\pi < \alpha < \pi$ such that $\int_{-\pi}^{\pi} f(t)\,dt = 2\pi f(\alpha)$

28) Let S be the positively oriented circle given by $|z - 3i| = 2$. Then the value of $\int_S \frac{dz}{z^2+4}$ is

a) $-\frac{\pi}{2}$
b) $\frac{\pi}{2}$
c) $-\frac{i\pi}{2}$

d) $\frac{i\pi}{2}$

29) Let T be the closed unit disk and ∂T be the unit circle. Then which one of the following holds for every analytic function $f : T \to \mathbf{C}$.

a) $|f|$ attains its minimum and its maximum on ∂T
b) $|f|$ attains its minimum on ∂T but need not attain its maximum on ∂T
c) $|f|$ attains its maximum on $|\partial T|$ but need not attain its minimum on $|\partial T|$
d) $|f|$ need not attain its maximum on ∂T and also it need not attain its minimum on ∂T

30) Let S be the disk $|z| < 3$ in the complex plane and let $f : S \to \mathbf{C}$ be an analytic function such that $f\left(1 + \frac{\sqrt{2}}{n} i\right) = -\frac{2}{n^2}$ for each natural number n. Then $f(\sqrt{2})$ is equal to

a) $3 - 2\sqrt{2}$
b) $3 + 2\sqrt{2}$
c) $2 - 3\sqrt{2}$
d) $2 + 3\sqrt{2}$

31) Which one of the following statements holds?

a) The series $\sum_{n=0}^{\infty} x^n$ converges for each $x \in [-1, 1]$
b) The series $\sum_{n=0}^{\infty} x^n$ converges uniformly in $(-1, 1)$
c) The series $\sum_{n=1}^{\infty} \frac{x^n}{n}$ converges for each $x \in [-1, 1]$
d) The series $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ converges uniformly in $(-1, 1)$

32) For $x \in [-\pi, \pi]$, let

f(x) $= (\pi + x)(\pi - x)$ and $g(x) = \begin{cases} \cos(1/x) & if x \neq 0, \\ 0 & if x = 0. \end{cases}$

Consider the statements: P: The Fourier series of f converges uniformly to f on $[-\pi, \pi]$. Q: The Fourier series of g converges uniformly to g on $[-\pi, \pi]$. Then

a) P and Q are true
b) P is true but Q is false
c) P is false but Q is true
d) Both P and Q are false

33) Let $W = \left\{(x, y, z) \in \mathbf{R}^3 : 1 \leq x^2 + y^2 + z^2 \leq 4\right\}$ and $F : W \to \mathbf{R}^3$ be defined by $F(x, y, z) = \frac{(x,y,z)}{\left[x^2+y^2+z^2\right]^{3/2}}$ for $(x, y, z) \in$ W. If ∂W denotes the boundary of W oriented by the outward normal n to W, then $\iint_{\partial W} F \cdot n \, dS$ is equal to

a) 0
b) 4π
c) 8π
d) 12π

34) For each $n \in \mathbf{N}$, let $f_n : [0, 1] \to \mathbf{R}$ be a measurable function such that $|f_n(t)| \leq \frac{1}{\sqrt{t}}$ for all t $\in$ (0,1]. Let $f : [0, 1] \to \mathbf{R}$ be defined by $f(t) = 1$ if t is irrational and $f(t) = -1$ if t is rational. Assume that $f_n(t) \to f(t)$ as $n \to \infty$ for all $t \in [0, 1]$. Then

a) f is not measurable

b) f is measurable and $\int_0^1 f_n d\mu \to 1$ as $n \to \infty$

c) f is measurable and $\int_0^1 f_n d\mu \to 0$ as $n \to \infty$

d) f is measurable and $\int_0^1 f_n d\mu \to -1$ as $n \to \infty$