Statistical Moments - Distribution Characteristics

Detailed Study Notes

Moments are quantitative measures that describe the shape and characteristics of a probability distribution. They provide a complete mathematical description of a distribution.

Definition:
The nth moment about the mean is the expected value of the nth power of the deviation from the mean.

Formula:
nth moment = $E[(X - \mu)^n] = \Sigma(x - \mu)^n / n$

The Four Main Moments:

1. First Moment (Mean):
$\mu = \Sigma x / n$
Describes the center or location of the distribution

2. Second Moment (Variance):
$\sigma^2 = \Sigma(x - \mu)^2 / n$
Describes the spread or dispersion

3. Third Moment (Skewness):
Related to asymmetry of the distribution
Skewness = $[\Sigma(x - \mu)^3 / n] / \sigma^3$

4. Fourth Moment (Kurtosis):
Related to the tailedness of the distribution
Kurtosis = $[\Sigma(x - \mu)^4 / n] / \sigma^4$

Detailed Example:
Dataset: 2, 4, 6, 8, 10

Mean (1st moment):
$\mu = (2 + 4 + 6 + 8 + 10) / 5 = 6$

Variance (2nd moment):
$\sigma^2 = [(2-6)^2 + (4-6)^2 + (6-6)^2 + (8-6)^2 + (10-6)^2] / 5$
$= [16 + 4 + 0 + 4 + 16] / 5 = 8$

Standard Deviation:
$\sigma = \sqrt{8} \approx 2.83$

Third Moment (for skewness):
$\Sigma(x - \mu)^3 = [(2-6)^3 + (4-6)^3 + (6-6)^3 + (8-6)^3 + (10-6)^3]$
$= [-64 + -8 + 0 + 8 + 64] = 0$
This indicates a symmetric distribution.

Fourth Moment (for kurtosis):
Σ(x - μ)■ = [(2-6)■ + (4-6)■ + (6-6)■ + (8-6)■ + (10-6)■]
$= [256 + 16 + 0 + 16 + 256] = 544$

Applications of Moments:
- Complete distribution characterization
- Comparing distributions
- Parameter estimation
- Model validation
- Risk assessment in finance

Why Moments Matter:
Mean and variance alone don't fully describe a distribution. Skewness and kurtosis provide additional information about shape that's crucial for:
- Understanding data behavior
- Choosing appropriate statistical tests
- Risk management in finance
- Quality control processes