Kurtosis - Tail Behavior and Peaks

Detailed Study Notes

Kurtosis measures the "tailedness" of a probability distribution - how much of the data is in the tails versus the peak compared to a normal distribution.

Definition:
Kurtosis quantifies whether data has heavy tails (outlier-prone) or light tails (outlier-resistant) relative to a normal distribution.

Formula:
Kurtosis = [Σ(x - μ)■ / n] / σ■

Types of Kurtosis:

1. Mesokurtic (Kurtosis ≈ 3):
- Normal distribution
- Moderate tails and peak
- Baseline for comparison

2. Leptokurtic (Kurtosis > 3):
- Heavy tails and sharp peak
- More outliers than normal
- Higher probability of extreme values
- Example: Financial returns during crisis

3. Platykurtic (Kurtosis < 3):
- Light tails and flat peak
- Fewer outliers than normal
- More uniform distribution
- Example: Uniform distribution

Excess Kurtosis:
Often reported as Excess Kurtosis = Kurtosis - 3
- Excess > 0: Leptokurtic (heavy tails)
- Excess ≈ 0: Mesokurtic (normal)
- Excess < 0: Platykurtic (light tails)

Detailed Example:
Dataset A (Normal-like): 2, 3, 4, 5, 6, 7, 8

- Moderate spread, few extremes
- Kurtosis ≈ 3 (mesokurtic)

Dataset B (With outliers): 2, 4, 5, 5, 6, 6, 6, 20, 25
- Same mean but with extreme values
- Kurtosis > 3 (leptokurtic)

Real-World Applications:

Finance:
- Stock returns: Often leptokurtic (fat tails)
- Risk assessment: High kurtosis = higher risk of extreme losses
- Option pricing: Accounts for tail risk

Quality Control:
- Process monitoring: High kurtosis may indicate occasional extreme defects
- Product testing: Understanding failure patterns

Insurance:
- Claim distributions: Often leptokurtic
- Catastrophic events: Captured in the tails
- Premium calculation: Must account for tail risk

Common Misunderstanding:
Kurtosis is NOT about the peak's height!
It's about the weight in the tails vs. the shoulders of the distribution.

Why Kurtosis Matters:
1. Risk Management: Heavy tails = more extreme events
2. Model Selection: Normal distribution may be inadequate
3. Outlier Detection: High kurtosis suggests outliers
4. Process Control: Changes in kurtosis indicate process changes

Interpreting Together with Skewness:
- Skewness tells direction of asymmetry
- Kurtosis tells about extreme values
- Together they describe distribution shape beyond mean and variance

Example in Practice:
Daily stock returns:
- Often near-zero skewness (symmetric)
- High kurtosis (fat tails)
- Interpretation: Returns are symmetric but have more extreme movements than normal distribution would predict