27th August 17

UNIT-2

* Methods to find Electric ~~field~~ intensity ($\vec{E}$)

1. Coulombs law (complex integration)
2. Gauss law (simple integration)
3. By dividing a scalar field at all points in the field (only differentiation)

   Ex: Gravitational force, Frictional force, Electrostatic force

   Varying electric field may not be conservative

4. The work done by conservative force is independent of its path

   To check whether a vector is conservative or not

$$\boxed{\nabla \times \vec{V} = 0} \quad \text{or} \quad \oint \vec{V} \cdot \vec{dl} = 0$$

* Show that Electrostatic field is a conservative field:

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a}_\rho}$$

$$\oint \vec{E}\cdot d\vec{l} = \int_{\phi=0}^{2\pi} \frac{\rho_L}{2\pi\epsilon_0 \rho}\hat{a}_\rho \; \rho\, d\phi\, \hat{a}_\phi = 0$$

$$\nabla\times\vec{E} = \begin{vmatrix} \frac{\hat{a}_\rho}{\rho} & \hat{a}_\phi & \frac{\hat{a}_z}{\rho} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ \frac{\rho_L}{2\pi\epsilon_0\rho} & 0 & 0 \end{vmatrix} = 0$$

HW:

Show that $\vec{E}$ due to surface charge and point charge is zero

surface charge:

$$\vec{E} = \frac{\rho_s}{2\epsilon_0}\hat{a}_x \qquad \nabla\times\vec{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\rho_s}{2\epsilon_0} & 0 & 0 \end{vmatrix}$$

$$\boxed{\nabla\times\vec{E} = \vec{0}}$$

due to point charge:

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r \qquad \nabla\times\vec{E} = \begin{vmatrix} \dfrac{\hat{a}_r}{r^2\sin\theta} & \dfrac{\hat{a}_\theta}{r\sin\theta} & \dfrac{\hat{a}_\phi}{r} \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ \dfrac{Q}{4\pi\epsilon_0 r^2} & 0 & 0 \end{vmatrix}$$

$$\boxed{\nabla\times\vec{E} = 0}$$

* Using Null identity, curl of gradient of a scalar is zero, we can represent electro static field $\vec{E}$ has gradient of a scalar field

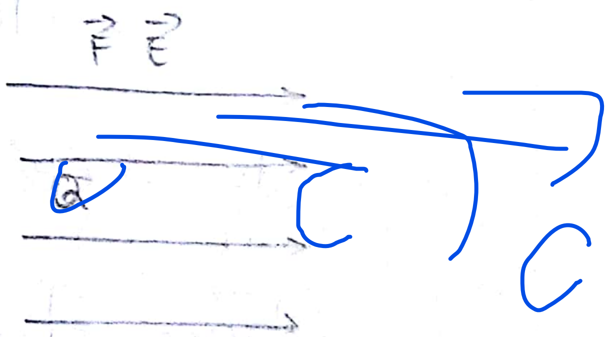
$$\nabla\times\nabla V = 0$$

$$\nabla\times\vec{E} = 0$$

$$\boxed{\vec{E} = -\nabla V}$$

Workdone (W):

units: Joules (J)



$$\vec{F} = \vec{E}Q$$

To move the charged object

$$F_A = -\vec{E}Q$$

$$dW = \text{Force} \times \text{displacement}$$

$$= -\vec{E}Q\,d\ell$$

$$\boxed{W = -Q\int \vec{E}\cdot d\ell}$$

* Potential ($\vec{V}$)

work done per unit positive charge

$$V = \frac{W}{Q} = -\int_{\text{initial}}^{\text{final}} \vec{E}\cdot d\ell$$

Differential potential Difference

$$dv = -\vec{E}\cdot\vec{d\ell}$$

$$= -|\vec{E}||\vec{d\ell}|\cos\theta$$

$$\frac{dv}{d\ell} = -\vec{E}$$

$$\boxed{\nabla V = -\vec{E}}$$

$$\therefore \boxed{\vec{E} = -\nabla V}$$

* Absolute potential:

$$V = -\int_a^b \vec{E}\cdot\vec{dl} = -\int_A^B \vec{E}\cdot\vec{dl} = -\int_A^B \frac{dV}{dl}\cdot dl$$

$$= V_B - V_A$$

$$\boxed{V = V_B}$$

problems:

① Find the workdone to move point charge of 4C from point P(1,0,0) to point A(0,2,0) along $y = 2-2x$, $z=0$ when the electric field is (i) $5\hat{a_x}$ (ii) $5x\hat{a_x}$ (iii) $5x\hat{a_x} + 5y\hat{a_y}$

(i) $W = -Q\int \vec{E}\cdot dl$

$dl = dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z}$

$W = -4\int 5\hat{a_x}\cdot dx\,\hat{a_x}$

$= -20\,[x]_1^0$

$W = 20J$

Scanned by CamScanner

(ii) $W = -Q\int \vec{E}\cdot\vec{dl}$

$dl = dx\,\hat{ax} + dy\,\hat{ay} + dz\,\hat{az}$

$W = -4\int 5x\,\hat{ax}\cdot dx\,\hat{ax}$

$= -20\left[\frac{x^2}{2}\right]_1^0$

$W = 10J$

(iii) $W = -Q\int \vec{E}\cdot\vec{dl}$

$dl = dx\,\hat{ax} + dy\,\hat{ay} + dz\,\hat{az}$

$W = -4\int(5\hat{ax} + 5\hat{ay})\,(dx\,\hat{ax} + dy\,\hat{ay})$

$= -4\left[\int_1^0 5x\,dx + \int_0^2 5y\,dy\right]$

$= -4\left[5\left[\frac{x^2}{2}\right]_1^0 + 5\left[\frac{y^2}{2}\right]_0^2\right]$

$= -4\left[\frac{-5}{2} + \frac{20}{2}\right]$

$= -30J$

HW

① Find the workdone to move 2C of charge from point P(1,0,1) to point A (0.8, 0.6, 1) along a straight line when $\vec{E} = y\hat{a}_x + x\hat{a}_y + z\hat{a}_z$

$$W = -Q\int \vec{E}\cdot d\vec{l}$$

$$= -2\int (y\hat{a}_x + x\hat{a}_y + z\hat{a}_z)(dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z)$$

$$= -2\left[\int_1^{0.8} y\,dx + \int_0^{0.6} x\,dy + \int_1^{1} z\,dz\right]$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y = \frac{0.6}{0.8-1}(x-1)$$

$$\boxed{y = -3x + 3} \qquad \boxed{x = \frac{3-y}{3}}$$

$$W = -2\left[\int_1^{0.8} (-3x+3)\,dx + \int_0^{0.6}\left(\frac{3-y}{3}\right) dy\right]$$

$$= -2\left[\left(\frac{-3x^2}{2} + 3x\right)_1^{0.8} + \left(y - \frac{y^2}{6}\right)_0^{0.6}\right]$$

Scanned by CamScanner

$$= -2\left[\left(-0.96+2.4\right)-\left(\frac{-3}{2}+3\right)+\left(0.6-\frac{0.36}{6}-0\right)\right]$$

$$= -2\left[-0.06+0.54\right]$$

$$\boxed{W = -0.96\,J}$$

(2) Find the workdone in the above problem along the arce of a circle $x^2+y^2=1$ & $z=1$

Given: $x^2+y^2=1$; $z=1$

when $\vec{E} = y\hat{a_x} + x\hat{a_y} + z\hat{a_z}$

$$W = -Q\int E \cdot dl$$

$$= -2\int (y\hat{a_x} + x\hat{a_y} + z\hat{a_z})(dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z})$$

$$= -2\left[\int_{1}^{0.8} y\,dx + \int_{0}^{0.6} x\,dy + \int_{1}^{1} z\,dz\right]$$

$x^2+y^2=1$

$x = \sqrt{1-y^2}$ $y = \sqrt{1-x^2}$

Scanned by CamScanner

$$W = -2\left[\int_{1}^{0.8} \sqrt{1-x^2}\, dx + \int_{0}^{0.6} \sqrt{1-y^2}\, dy\right]$$

put $x = \sin\theta$ $\quad\quad$ $y = \cos\theta$

$dx = \cos\theta\, d\theta$ $\quad\quad$ $dy = -\sin\theta\, d\theta$

. $\pi/2$ to $53.13°$ $\quad\quad$ $\theta = \pi/2$ to 53.13

$$W = -2\left[\int_{\pi/2}^{53.13} \cos\theta \cdot \cos\theta\, d\theta + \int_{\pi/2}^{53.13} \sin\theta\,(-\sin\theta\, d\theta)\right]$$

$$= -2\left[\int_{90}^{53.13} (\cos^2\theta - \sin^2\theta)\, d\theta\right]$$

$$= -2\left[\int_{90}^{53.13} \cos 2\theta\, d\theta\right]$$

$$= -2 \times \left[\frac{\sin 2\theta}{2}\right]_{90}^{53.13}$$

$$\boxed{W = -0.96 J}$$

$\Rightarrow$ It is proved that the work done by conservative field is independent of path followed.

③ Find the electric potential due to point charge, $\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{a_r}$ using $V = -\int \vec{E} \cdot dl$

$$V = -\int \vec{E} \cdot dl$$

$$= -\int \frac{Q}{4\pi\epsilon_0 r^2}\hat{a_r} \cdot dr\hat{a_r}$$

$$= \frac{-Q}{4\pi\epsilon_0}\left[\frac{-1}{r}\right]_{\infty}^{r}$$

$$\boxed{V = \frac{Q}{4\pi\epsilon_0}}$$

30 August 17

Maxwell's second equations for electrostatic field:

Based on the conservative property of electrostatic field

$$\oint E \cdot dl = 0$$

By using stokes theorem

$$\oint \vec{E} \cdot dl = \iint \nabla \cdot E \cdot ds$$

$$\boxed{\nabla \cdot E = 0}$$

$$\nabla \times E = 0$$

Scanned by CamScanner

* Given the potential field $V = 2x^2y - 5z$. Find the electric field intensity, direction of $\vec{E}$ ρ_v at a point $(-4, 3, 6)$

$$\vec{E} = -\nabla V$$

$$\nabla V = \frac{1}{h_1}\frac{\partial}{\partial u_1} V \hat{a}_{u_1} + \frac{1}{h_2}\frac{\partial}{\partial u_2} V \hat{a}_{u_2} + \frac{1}{h_3}\frac{\partial V}{\partial u_3} \hat{a}_{u_3}$$

$$\nabla V = \frac{\partial}{\partial x}(2x^2y - 5z)\hat{a}_x + \frac{\partial}{\partial y}(2x^2y - 5z)\hat{a}_y + \frac{\partial}{\partial z}(2x^2y - 5z)\hat{a}_z$$

$$\boxed{\nabla V = 4xy\hat{a}_x + 2x^2\hat{a}_y - 5\hat{a}_z}$$

$$\vec{E} = -\nabla V$$

$$\vec{E} = -4xy\hat{a}_x - 2x^2\hat{a}_y + 5\hat{a}_z$$

$$\vec{D} = \epsilon_0 \vec{E}$$

$$\vec{D} = \epsilon_0(-4xy\hat{a}_x - 2x^2\hat{a}_y + 5\hat{a}_z)$$

$$\rho_v = \nabla \cdot \vec{D}$$

$$= \epsilon_0\left(\frac{\partial}{\partial x}(-4xy) + \frac{\partial}{\partial y}(-2x^2) + \frac{\partial}{\partial z}(5)\right)$$

$$= \epsilon_0(-4y + 0 + 0)$$

$$\boxed{\rho_v = -4\epsilon_0 y}$$

At $(-4,3,6)$

$$\vec{E} = -4(-4)(3)\,\hat{a}_x - 2(-4)^2\,\hat{a}_y + 5\hat{a}_z$$

$$\vec{E} = 48\hat{a}_x - 32\hat{a}_y + 5\hat{a}_z$$

$$\vec{D} = \epsilon_0 \vec{E} = 8.85\times10^{-12}(48\hat{a}_x - 32\hat{a}_y + 5\hat{a}_z)$$

$$\rho_v = -4\times 8.85\times10^{-3}\times 3$$

$$\boxed{\rho_v = -106.2\ \text{pC/m}^3}$$

1st sept17

* $\vec{V}$ due to infinite length line charge.

$$V = -\int_a^b \vec{E}\cdot\vec{d\ell}$$

$$= -\int_a^b \frac{\rho_L}{2\pi\epsilon_0 \rho}\hat{a}_\rho \cdot d\rho\,\hat{a}_\rho$$

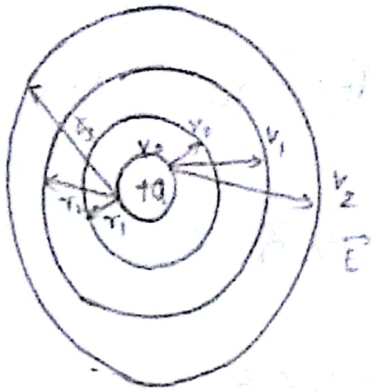
$$= \frac{-\rho_L}{2\pi\epsilon_0}\left[\log \rho\right]_a^b$$

$$= \frac{-\rho_L}{2\pi\epsilon_0}\left[\log b - \log a\right]$$

$$\boxed{V = \frac{\rho_L}{2\pi\epsilon_0}\log\left(\frac{a}{b}\right)}$$

Equipotential surfaces:

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{a_r}$$



$$V = \frac{Q}{4\pi\epsilon_0 r}$$

$$\boxed{\vec{E} = -\nabla V}$$

Laplace and Poissons equations:

consider gauss law in differential form

$$\nabla \cdot \vec{D} = \rho_v$$

$$\nabla \cdot \epsilon \vec{E} = \rho_v$$

$$\epsilon \left(\nabla \cdot (-\nabla V)\right) = \rho_v$$

$$\nabla \cdot \nabla V = \frac{-\rho_v}{\epsilon}$$

$$\boxed{\nabla^2 V = \frac{-\rho_v}{\epsilon}}$$ (poissons equation)

$$\nabla V = 0$$

↙

Laplacian of a scalar

① $$\nabla V = \frac{1}{h_1}\frac{\partial V}{\partial u_1} \cdot \hat{a}_{u_1} + \frac{1}{h_2}\frac{\partial V}{\partial u_2}\hat{a}_{u_2} + \frac{1}{h_3}\frac{\partial V}{\partial u_3}\hat{a}_{u_3}$$

② $$\nabla \cdot \vec{D} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1} h_2 h_3 D_{u_1} + \frac{\partial}{\partial u_2} h_1 h_3 D_{u_2} + \frac{\partial}{\partial u_3} h_1 h_2 D_{u_3}\right]$$

③ $$\nabla \cdot \nabla V = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}\frac{h_2 h_3}{h_1}\frac{\partial V}{\partial u_1} + \frac{\partial}{\partial u_2}\frac{h_1 h_3}{h_2}\frac{\partial V}{\partial u_2} + \frac{\partial}{\partial u_3}\frac{h_1 h_2}{h_3}\frac{\partial V}{\partial u_3}\right]$$

method - 2

$$\vec{E} = \nabla V$$

apply divergence on both sides

$$\nabla \cdot (-\nabla V) = \nabla \cdot \vec{E}$$

$$\nabla \cdot \nabla V = -\nabla \cdot \vec{E}$$

$$= -\frac{\nabla \cdot \vec{D}}{\epsilon}$$

$$\boxed{\nabla^2 V = \frac{-\rho_v}{\epsilon}}$$

Laplacian of scalar is a scalar.

cartesian:

$$\nabla \cdot \nabla V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

cylindrical:

$$\nabla \cdot \nabla V = \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{\partial}{\partial \phi}\left(\frac{1}{\rho}\frac{\partial V}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(\rho \cdot \frac{\partial V}{\partial z}\right)\right]$$

spherical:

$$\nabla \cdot \nabla V = \frac{1}{r^2 \sin\theta}\left[\frac{\partial}{\partial r}\left(r^2 \sin\theta \frac{\partial V}{\partial r}\right) + \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{\partial}{\partial \phi}\left(\frac{1}{\sin\theta}\frac{\partial V}{\partial \phi}\right)\right]$$

Scanned by CamScanner

Importance of Laplacians equation :

* To find the information in field thery at a point, potential is the gateway

* If we know the electric potential, we can determine the electric fied intensity using

$$E = -\nabla V$$

Similarly $D = \epsilon E$

$$\rho_v = \nabla \cdot D$$

$$\rho_s = |\vec{D_n}|$$

* To find the quantities, electro static potential energy, capacitence, Resistance we need to use integration

* Practically the evaluable parameter is potential

* It is very difficult to know charge distribution of a charged body practically

* Laplace equation is a second order partial differential equation

* Helmholtzs equation is also second order partial differential eqn in time & space

Calculate the numerical values of V & ρ_v, ρ_s at a point P in free space.

a) $V = \frac{4yz}{x^2+1}$ at a point $P(1,2,3)$

b) $V = 5\rho^2 \cos 2\phi$ at $P(3, \pi/3, 2)$

c) $V = \frac{2\cos\phi}{r^2}$ at $P(0.5, 45^\circ, 60^\circ)$

check whether the laplacian equation satisfies or not at all three cases

$$E_v = -\nabla V$$

$$-\nabla V = -\left[\frac{\partial}{\partial x} V \hat{a}_x + \frac{\partial}{\partial y} V \hat{a}_y + \frac{\partial}{\partial z} V \hat{a}_z\right]$$

$$= -\left[\frac{\partial}{\partial x}\frac{4yz}{x^2+1}\hat{a}_x + \frac{\partial}{\partial y}\frac{4yz}{x^2+1}\hat{a}_y + \frac{\partial}{\partial z}\frac{4yz}{x^2+1}\hat{a}_z\right]$$

$$\vec{E} = -\left[\frac{-4yz}{(x^2+1)^2}\times 2x\,\hat{a}_x + \frac{4z}{x^2+1}\hat{a}_y + \frac{4y}{x^2+1}\hat{a}_z\right]$$

$$\rho_v = \nabla\cdot D$$

$$= \nabla\cdot(-\nabla V)\epsilon$$

$$= -\epsilon\nabla^2 V$$

Scanned by CamScanner

$$\rho_v = -\epsilon\left[\frac{\partial^2 V}{\partial x^2} + \frac{\partial V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right]$$

$$= -\epsilon\left[\frac{\partial}{\partial x}\left[\frac{-4yz}{(x^2+1)^2} \times 2x\right] + \frac{\partial}{\partial y}\left[\frac{4z}{x^2+1}\right] + \frac{\partial}{\partial z}\left[\frac{4y}{x^2+1}\right]\right]$$

$$= -\epsilon\left[(-8yz)\left[\frac{x(2)(x^2+1)(2x) - (x^2+1)^2}{(x^2+1)^4}\right]\right]$$

$$= 8yz\epsilon\left[\frac{4x^2(x^2-1) - (x^2+1)^2}{(x^2+1)^4}\right]$$

$$= 8yz\epsilon\left[\frac{4x^2 - (x^2-1)}{(x^2+1)^3}\right]$$

$$\rho_v = \frac{8yz(3x^2-1)\epsilon}{(x^2+1)^3}$$

$$= \frac{8 \times 2 \times 3\,[2]\,\epsilon}{8}$$

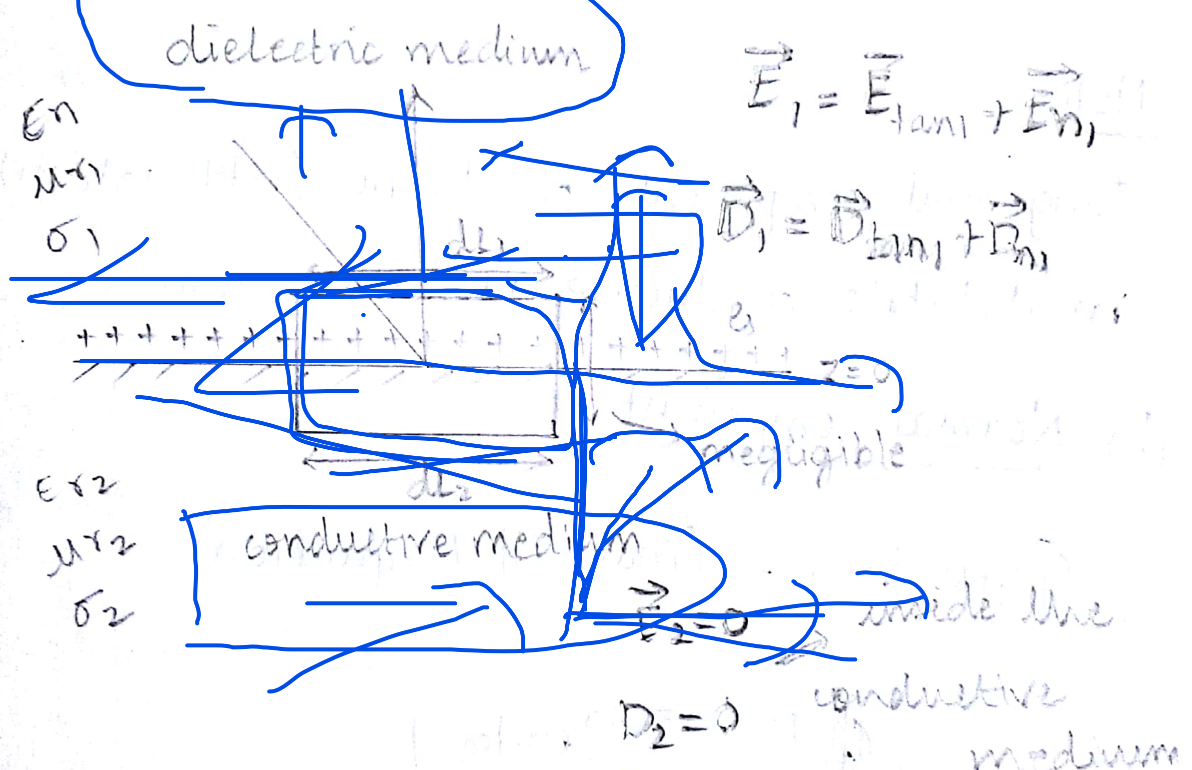
$$\boxed{\rho_v = 12\epsilon}$$

Boundary Conditions:

6th sept 17

If we know Electric field intensity ($\vec{E}$) and Electric flux density ($\vec{D}$) in one medium, by using boundary conditions, we can predics $\vec{E}$ and $\vec{D}$ in medium 2

When external field is applied all charges move towards exterior surface of the conductive medium with a time duration of Relaxation time



$$\vec{E}_1 = \vec{E}_{tan1} + \vec{E}_{n1}$$

$$\vec{D}_1 = \vec{D}_{tan1} + \vec{D}_{n1}$$

(1) Dielectric - Conductive medium

(a) Tangential Component:

Using maxwell's equation based on conservative property of electrostatic fields

$$\oint \vec{E}\cdot dl = 0$$

$$\int_1 \vec{E}_{tan_1} dl_1 + \int_2 \cancel{E\, dn_1}^{\,0} + \int_3 E_{tan_2} \vec{dl_2} + \int_4 \cancel{E\, dn_2}^{\,0} = 0$$

$$\int \vec{E}_{tan_1} dl_1 - \int \vec{E}_{tan_2} dl_2 = 0$$

$$E_{tan_2} = 0$$

$$E_{tan_1} = 0$$

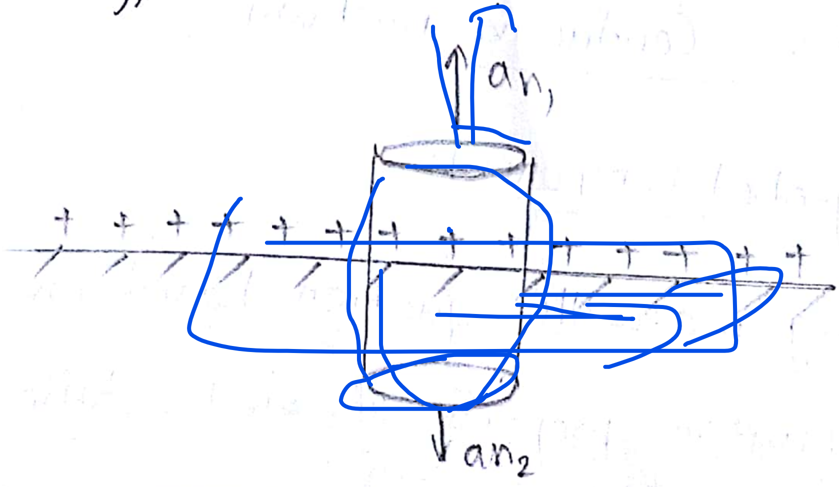
Note:

Tangential component of electric field intensity in dielectric - conductive medium is zero

(b) Normal component:

Gauss law - (Maxwell I equation)

$$\oiint \vec{D}\cdot\vec{ds} = Q_{enclosed}$$



For dielectric - conductive medium, normal components of electric flux densities are discontinued by surface charge density (ρ_s)

$$\iint_{Top} D_{n_1} ds_1 + \iint_{Bottom} D_{n_2} ds_2 + \iint \cancel{D} ds = \rho_s A$$

$$D_{n_1} A - D_{n_2} A = \rho_s A$$

$$\boxed{D_{n_1} - D_{n_2} = \rho_s}$$

Dielectric - Dielectric medium:

Tangential component:

$$\oint \vec{E} \cdot \vec{dl} = 0$$

$$\int E_{tan_1} dl_1 - \int E_{tan_2} dl_2 = 0$$

$$E_{tan_1} - E_{tan_2} = 0$$

For D-D medium tangential components of electric field intensity are equal, continuous

$$\frac{D_{tan_1}}{\epsilon_1} - \frac{D_{tan_2}}{\epsilon_2} = 0$$

Normal Components:

Gauss law - I Maxwell's equation

$$\iint_{Top} D_{n_1}\, ds_1 + \iint_{Bottom} D_{n_2}\, ds_2 + \iint D \cdot ds = 0$$

$$D_{n_1} A - D_{n_2} A = 0$$

$$\boxed{D_{n_1} - D_{n_2} = 0}$$

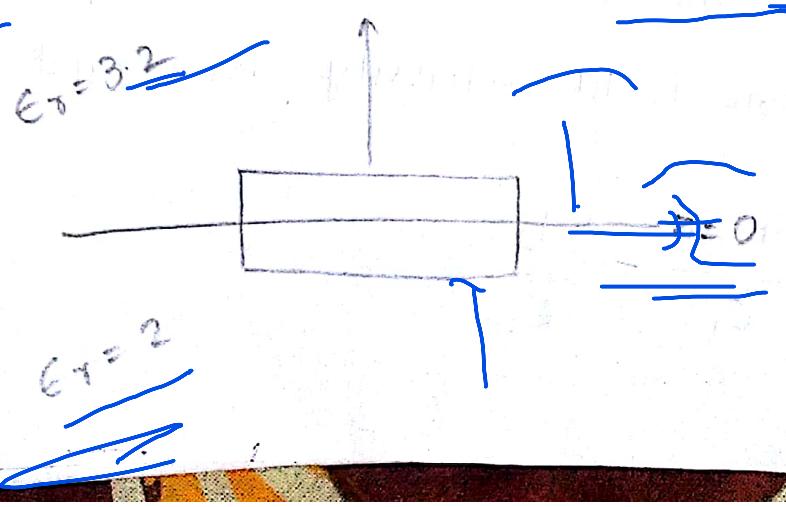
$$\epsilon_1 E_{n_1} - \epsilon_2 E_{n_2} = 0$$

Problem:

The region $z>0$ is composed of uniform dielectric medium with $\epsilon_r = 3.2$ and $z<0$ is composed of dielectric medium $\epsilon_r = 2$

Let $\vec{D} = -30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_z \ C/m^2$. Find

E_{tan1}, E_{tan2}, D_{tan1}, D_{tan2}, E_{n_1}, E_{n_2}, D_{n_1}, D_{n_2}



Dielectric - Dielectric medium

Given $\vec{D} = -30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_z$

$D_{n_1} = 70\hat{a}_z$ $\quad D_{tan1} = -30\hat{a}_x + 50\hat{a}_y$

$D_{n_1} = D_{n_2} = 70\hat{a}_z$

$D_{tan1} = D_{tan2} = -30\hat{a}_x + 50\hat{a}_y$

$$\vec{E}_1 = \frac{1}{3.2\epsilon_0}(-30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_z)$$

$$E_{n_2} = E_{n_1} = \frac{70\hat{a}_z}{3.2\epsilon_0}$$

$$E_{tan2} = E_{tan1} = \frac{-30\hat{a}_x + 50\hat{a}_y}{3.2\epsilon_0}$$

Capacitance: The capacity of storing electric energy in the form of electric charges in a system is known as capacitence

Structure: Two conductive surfaces separate by a distance d and the gap between two conductors is filled with electric medium

$C \propto Q$

$C = Q/V$

Capacitence of a Parallel Plate Capacitor:

charge on each plate

$$Q = \iint \rho_s \, ds$$

$$= \rho_s A$$

$\vec{E}$ due to two sheets $= \frac{\rho_s}{\epsilon_0} \hat{a_x}$

$$V = \frac{W}{Q} = \frac{-Q\int \vec{E}\cdot\vec{dl}}{Q} = -\int_{\text{initial}}^{\text{final}} E \cdot dl$$

$$= -\int \vec{E}\, \hat{a_x} \cdot dx \, \hat{a_x}$$

$$= \int_{x=0}^{x=d} \frac{\rho_s}{\epsilon_0} \hat{a_x} \, dx \, \hat{a_x}$$

Scanned by CamScanner

$$C = \frac{Q}{V} = \frac{\rho_s A}{\left(\frac{\rho_s d}{\epsilon}\right)} = \frac{\epsilon A}{d}$$

$$\boxed{C = \frac{\epsilon A}{d}}$$

where A = area of the plate

d = distance b/w two parallel plates

- Capacitence depends on its geometrical dimensions of the system

problem

$d = 1mm$ $A = 10mm^2$ $\epsilon_r = 1$ $C = ?$

$$C = \frac{\epsilon A}{d}$$

$$= \frac{\epsilon_r \epsilon_0 \, 10 \, (10^{-3})^2}{1 \times 10^{-3}}$$ $\epsilon_0 = 8.854 \times 10^{-12}$

$$= 8.854 \times 10^{-12} \times 10 \times 10^{-3}$$

$$C = 8.854 \times 10^{-14} \, F$$

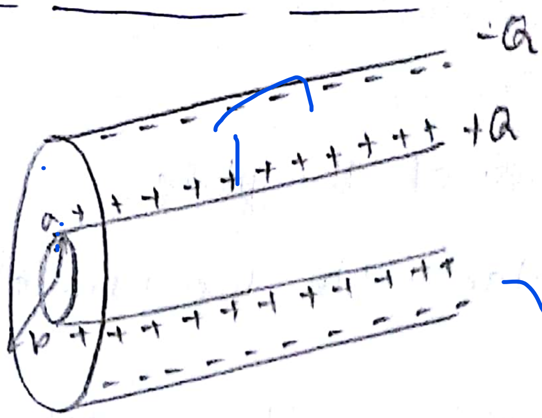
Steps to find capacitence

1. Calculate charge on each plate (surface)
2. Find the elect~~ri~~ric field intensity b/w two plates

Scanned by CamScanner

3. Find the potential diff b/w two plates using formulae $V = -\int E \cdot dl$

Capacitence of coaxial cable:



Step-1 $\quad Q = \rho_L L$

Step-2: $\quad E = \dfrac{\rho_L}{2\pi \epsilon \rho} \hat{a}_\rho$

Step-3 $\quad V = -\int_A^B E \cdot dl$

$$= -\int_A^B \frac{\rho_L}{2\pi \epsilon_0 \rho} \hat{a}_\rho \cdot d\rho \, \hat{a}_\rho$$

$$= \frac{-\rho_L}{2\pi \epsilon_0} \left[\log \rho\right]_A^B$$

$$V = \frac{+\rho_L}{2\pi \epsilon_0} \log \frac{A}{B}$$

Step-4 $\quad C = \dfrac{Q}{V} = \dfrac{\cancel{\rho_L} L}{\dfrac{\cancel{\rho_L}}{2\pi \epsilon_0} \log \dfrac{A}{B}} = \dfrac{L \, 2\pi \epsilon_0}{\log \dfrac{A}{B}}$

Q

F

$V = \int E . M$

ρ

$C = Q$

$$\boxed{\frac{C}{L} = \frac{2\pi\epsilon_0}{\log\frac{A}{B}}}$$

capacitence of a pspherical capacitor

Electric field Intensity due to dipole:

Two point charges with same charge on each in ~~magr~~ magnitude but opposite in direction

$$\nabla V = \frac{\partial V}{\partial r} \hat{a_r} + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{a_\theta} + \frac{1}{r \sin\theta} \frac{\partial V}{\partial \phi} \hat{a_\phi}$$

$$V = \frac{Qd\cos\theta}{4\pi\epsilon r^2}$$

Electric potential energy

Energy: work done to move point charge from infinite distance to a point in the given region

Energy = workdone

Units: joules

$$\text{workdone} = F \times d$$
$$= -\int F \cdot dl$$
$$= -\int Q E\, dl$$
$$= -Q \int E \cdot dl$$
$$= QV$$

$$V = \frac{W}{Q}$$

$W_1 = 0$ (no charge inside the volume)

$$W_2 = Q_2 V_{12}$$
$$= \frac{Q_2 Q_1}{4\pi\epsilon R_{12}} \quad (R_{12} \approx R_2)$$

$$W_3 = Q_3 [V_{31} + V_{32}] = \frac{Q_3 Q_1}{4\pi\epsilon R_{13}} + \frac{Q_3 Q_2}{4\pi\epsilon R_{23}}$$

$$W_{total} = 0 + Q_2 V_{21} + Q_3 [V_{31} + V_{32}] + Q_4 [V_{41} + V_{42} + V_{43}]$$

$$W_{total} = 0 + \frac{Q_3 Q_4}{4\pi\epsilon R_{34}} + Q_2 \left[\frac{Q_3}{4\pi\epsilon R_{32}} + \frac{Q_4}{4\pi\epsilon R_{24}}\right]$$
$$+ Q_1 \left[\frac{Q_2}{4\pi\epsilon R_{12}} + \frac{Q_3}{4\pi\epsilon R_{13}} + \frac{Q_4}{4\pi\epsilon R_{14}}\right]$$

Adding ① & ②

$$2W_{total} = Q_1\left[\frac{Q_2}{4\pi\epsilon R_{12}} + \frac{Q_3}{4\pi\epsilon R_{13}} + \frac{Q_4}{4\pi\epsilon R_{14}}\right] +$$

$$Q_2\left[\frac{Q_1}{4\pi\epsilon R_{12}} + \frac{Q_3}{4\pi\epsilon R_{13}} + \frac{Q_4}{4\pi\epsilon R_{14}}\right] +$$

$$Q_3\left[\frac{Q_1}{4\pi\epsilon R_{12}} + \frac{Q_2}{4\pi\epsilon R_{23}} + \frac{Q_4}{4\pi\epsilon R_{34}}\right] +$$

$$Q_4\left[\frac{Q_1}{4\pi\epsilon R_{14}} + \frac{Q_2}{4\pi\epsilon R_{24}} + \frac{Q_3}{4\pi\epsilon R_{34}}\right]$$

$$2W_{Total} = Q_1V_1 + Q_2V_2 + Q_3V_3 + Q_4V_4$$

$$\boxed{W_{Total} = \frac{1}{2}\sum_{m=1}^{n} Q_m V_m}$$ point charge

$$W_{total} = \frac{1}{2}\int \rho_L V\, dl$$

$$= \frac{1}{2}\iint \rho_s V\, ds$$

$$= \frac{1}{2}\iiint \rho_v V\, dv$$

$$W_{Total} = \frac{1}{2}\iiint \rho_v V\, dv$$

$$= \frac{1}{2}\iiint \nabla\cdot D\, V\, dv \quad (\because \rho_v = \nabla\cdot V)$$

$$\nabla D V = V \nabla \vec{D} - \vec{D} \cdot \nabla V$$

$$W_T = \frac{1}{2}\iiint (V \nabla \cdot \vec{D})\, dv - \frac{1}{2}\iiint \vec{D} \cdot \nabla V\, dv$$

using divergence theorem

$$= \frac{1}{2}\oint\!\!\oint V \cdot \vec{D} \cdot ds + \frac{1}{2}\iiint \vec{D} \cdot (E)\, dv$$

$$\boxed{W_E = \frac{1}{2}\iiint \epsilon E^2 dv}$$

Energy density $= \frac{1}{2}\epsilon E^2\ J/m^3$

prob:

Find the energy stored in free space. For the region $2 < r < 3$ $0 < \theta < 90°$ $0 < \phi < 90°$. $V = \frac{200}{r}$

$$E = -\nabla V$$

$$\nabla V = \frac{\partial V}{\partial r}\hat{a_r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a_\phi}$$

$$= \frac{\partial}{\partial r}\left(\frac{200}{r}\right)\hat{a_r} + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{200}{r}\right)\hat{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left(\frac{200}{r}\right)\hat{a_\phi}$$

$$= -\frac{200}{r^2}\hat{a_r} + \frac{1}{r}(0)\hat{a_\theta} + \frac{1}{r\sin\theta}(0)\hat{a_\phi}$$

$$\boxed{E = \frac{200}{r^2}\hat{a_r}}$$

$$W_E = \frac{1}{2}\iiint \epsilon E^2 \, dv$$

$$= \frac{1}{2}\int_2^3\int_0^{\pi/2}\int_0^{\pi/2} \epsilon \left(\frac{200}{r^2}\right)^2 \hat{a_r} \; r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{\epsilon}{2} 4\times 10^4 \int_0^{\pi/2}\int_0^{\pi/2}\int_2^3 \frac{1}{r^2} \sin\theta \, dr \, d\theta \, d\phi$$

$$= 2\epsilon \times 10^4 \left[\frac{-1}{r}\right]_2^3 \left[-\cos\theta\right]_0^{\pi/2} \left[\phi\right]_0^{\pi/2}$$

$$= \cancel{2}\epsilon\times 10^4 \times \frac{1}{6} \times \frac{\pi}{\cancel{2}} \times (1)$$

$$= \frac{10^4 \pi \epsilon}{6}$$

(2) Given potential $V = 100(x^2-y^2)$v

(a) Find the potential at $(2,-1,3)$

$$E = -\nabla V$$

$$\nabla V = \frac{\partial V}{\partial x}\hat{a_x} + \frac{\partial V}{\partial y}\hat{a_y} + \frac{\partial V}{\partial z}\hat{a_z}$$

$$= \frac{\partial}{\partial x} 100(x^2-y^2)\hat{a_x} + \frac{\partial}{\partial y} 100(x^2-y^2)\hat{a_y} + \frac{\partial}{\partial z} 100(x^2-y^2)\hat{a_z}$$

$E = -200x\hat{a_x} + 200y\hat{a_y}$

$V = -\int E \cdot dl$

$= \int (200x\hat{a_x} - 200y\hat{a_y})(dx\hat{a_x} + dy\hat{a_y} + dz\hat{a_z})$

$= \int 200x\,dx - \int 200y\,dy$

$= 200\frac{x^2}{2} - 200\frac{y^2}{2}$

$V = 100x^2 - 100y^2$

potential at $(2,-1,3)$

$V = 100(4) - 100(1)$

$\boxed{V = 300}$

Current:

Flow of charges into a point / line / surface / volume for certain time duration is called current

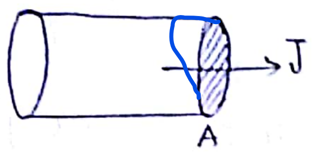
$$\boxed{I = \frac{dq}{dt}}$$

Surface charge density ($\vec{J}$) current per unit surface area

$$I = \oiint \vec{J}\cdot ds$$

$$= JA$$

Amp/m^2



Note: current density is defined as current passing through unit normal area at that point.

* Current Continuity equation:

When the surface area and position are constant then normal differentation can be converted into partial differentiation

point form.

$$I_{out} = -\frac{dq}{dt} \text{ (General Form)}$$

$$\oiint J\cdot ds = -\frac{d}{dt}\iiint \rho_v \, dv \text{ (Integral Form)}$$

$$\iiint \nabla \cdot J \, dv = \iiint -\frac{d\rho_v}{dt} \, dv$$

$$\boxed{\nabla \cdot J = -\frac{d\rho_v}{dt}}$$ point form.

→ This eqn is based on law of conservation of energy

→ Divergence of current density (surface) is equal to the negative time rate of volume charge density at the same point.

Ohms law: (Relation b/w $\vec{J}$ & $\vec{E}$)

$$\vec{J} = n e V_d$$

$$= \rho_v V_d$$

$$= \rho_v \mu \vec{E}$$

$q = \iiint \rho_v \, dv$

$$\boxed{\vec{J} = \sigma \vec{E}}$$ (microscopic, high frequency)

$$\boxed{V = IR}$$ macroscopic

When applied electric field is $\vec{E}$ for conductor then the corresponding conductivity is $\vec{J}$ at the same point, conductivity of the conductor is σ

Relaxation time of charge:

The time taken to travel by the charges inside the conductor to exterior surface of a conductor is called Relaxation time

current continuity equation

$$\nabla \cdot J = -\frac{d\rho_v}{dt}$$

$$\nabla \cdot (\sigma \vec{E}) = -\frac{d\rho_v}{dt}$$

$$\sigma \, \nabla \cdot \frac{\vec{D}}{\epsilon} = -\frac{d\rho_v}{dt}$$

For a linear dielectric medium $\frac{\sigma}{\epsilon} = \text{constant}$

$$\nabla \cdot \vec{D} = -\frac{d\rho_v}{dt}$$

$$\nabla \cdot D = \rho_v$$

$$\frac{\sigma}{\epsilon}\rho_v = -\frac{d\rho_v}{dt}$$

$$\therefore -\int \frac{\sigma}{\epsilon} dt = \int \frac{1}{\rho_v} d\rho_v$$

$$-\frac{\sigma}{\epsilon}(t_2 - t_1) = \log \rho_v - \log \rho_{v_0}$$

$$\rho_v = \rho_{v_0} \, e^{-\frac{\sigma}{\epsilon}(t_2 - t_1)}$$

$$\boxed{\rho_v = \rho_{v_0} e^{-t/\tau}}$$

where τ is the relaxation time, $\boxed{e^{-1} = 0.368}$

$$\boxed{\tau = \frac{\epsilon}{\sigma}}$$

Problems: Find τ for the following

(1) For distilled water $\epsilon_r = 80$ $\sigma = 2\times10^{-4}\ S/m$

(2) For copper $\sigma = 5\times10^{7}\ S/m$, $\epsilon_r = 1$

(3) for wood $\sigma = 1\times10^{-14}\ S/m$, $\epsilon_r = 1.2$

(2) $\tau = \dfrac{\epsilon}{\sigma} = \dfrac{\epsilon_0 \epsilon_r}{\sigma}$ $\quad\epsilon_0 = 8.854\times10^{-12}$

$$= \frac{8.854\times10^{-12}\times 1}{5\times10^{7}}$$

$$= 1.7708\times10^{-19}\ sec$$

The current passing through spherical shell $r = 0.02\,m$ $0<\theta<\pi$ $0<\phi<2\pi$ with current density $J = 2\times10^{3}\dfrac{\sin\theta}{r}\hat{a}_r$. Find the total current passing through the spherical shell

$$I = \iiint \nabla\cdot J\, dv$$

Scanned by CamScanner

$$\nabla \cdot A = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1} h_2 h_3 A_{u_1} + \frac{\partial}{\partial u_2} h_1 h_3 A_{u_2} + \frac{\partial}{\partial u_3} h_1 h_2 \ldots\right.$$

$$= \frac{1}{r^2 \sin\theta}\left[\frac{\partial}{\partial r} r^2 \sin\theta \left[2\times10^3 \frac{\sin\theta}{\cancel{r}}\right]\right]$$

$$= \frac{1}{r^2\sin\theta}\left[2\times10^3 \sin^2\theta \cdot \frac{\partial}{\partial r}(r)\right]$$

$$\boxed{\nabla\cdot J = \frac{2\times10^3 \sin\theta}{r^2}}$$

$$I = \iiint \nabla\cdot J \, dv$$

$$= \int_0^{0.02}\int_0^{\pi}\int_0^{2\pi} \frac{2\times10^3 \sin\theta}{\cancel{r^2}} \times \cancel{r^2}\sin\theta \, dr\, d\theta\, d\phi$$

$$= 2\times10^3 \int_0^{\pi}\int_0^{2\pi} \sin^2\theta \, [0.02] \, d\theta \, d\phi$$

$$= 40\,[2\pi] \int_0^{\pi} \sin^2\theta \, d\theta$$

$$= 40\times2\pi \times \frac{1}{2}\int_0^{\pi} 1-\cos2\theta \, d\theta$$

$$= 40\pi \left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= 40\pi\left[\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 0}{2}\right]$$

$$\boxed{I = 40\pi^2}$$