

11N501

Roll No. _____

Total No. of Pages: **3****11N501****B. Tech. I - Sem. (New Scheme) (Main) Exam., May - 2023****All Branch****1FY1 – 01 Engineering Mathematics – I****Common to all Branches****Time: 3 Hours****Maximum Marks: 70****Instructions to Candidates:**

Part – A: Short answer questions (up to 25 words) 10×2 marks = 20 marks.
All ten questions are compulsory.

Part – B: Analytical/Problem solving questions 5×4 marks = 20 marks.
Candidates have to answer five questions out of seven.

Part – C: Descriptive/Analytical/Problem Solving/Design questions 3×10 marks = 30 marks. Candidates have to answer three questions out of five.

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination.
(Mentioned in form No. 205)

1. NIL2. NIL**PART – A**

Q.1 What is the range of x for which the function - [2]

$$y = x^4 - 6x^3 + 12x^2 + 5x + 7$$

is concave upwards?

Q.2 Find the radius of curvature at the origin for the curve $x^3 + y^3 - x^2 + y = 0$ [2]

Q.3 Find the asymptotes of $y^2(x - b) = x^3 + a^3$. [2]

Q.4 Provide order and classification of the partial differential equation - [2]

$$x(y^2 + z^2 q^2) = z p y^2.$$

Q.5 If $y_1 = x^2$ is one of the independent solutions of the differential equation - [2]

$$x^2 \frac{d^2 y}{dx^2} - 2y = 0$$

Then, find the second linearly independent solution.

Q.6 Solve the initial value problem - [2]

$$(1 + y) dx + (x + 2y) dy = 0, y(0) = 1$$

Q.7 Solve $\frac{dy}{dx} = \cos(x + y) + \sin(x + y)$. [2]

Q.8 Write a short note on the symmetry of Cartesian curves. [2]

Q.9 Solve $(D^4 + a^4)y = 0, a > 0$. [2]

Q.10 If $u = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$, Show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$

PART - B

Q.1 If $\theta = t^n \exp\left(\frac{-r^2}{4t}\right)$, find the value of n for which the relation [4]

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \theta}{\partial r} \right) = \frac{\partial \theta}{\partial t}$$

is true.

Q.2 Trace the polar curve $r = 3 + 2 \cos \theta$. [4]

Q.3 Find the equation of the cubic which has the same asymptotes as the curve [4]

$$x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y + 1 = 0$$

and which touches the axis of y at origin and passes through the point $(3, 2)$.

Q.4 If $u = \sin^{-1} \left(\frac{x+y}{\sqrt{x}+\sqrt{y}} \right)$, show that $\left(x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \right) = \frac{1}{2} \tan u$. [4]

Q.5 Find the solution of the differential equation - [4]

$$\frac{d^3 y}{dx^3} - 9 \frac{dy}{dx} = 10 \cos x.$$

Q.6 Solve the differential equation - [4]

$$(x+a)^2 \frac{d^2y}{dx^2} - 4(x+a) \frac{dy}{dx} + 6y = x, a > 0.$$

Q.7 Reduce the partial differential equation - [4]

$$y + 2zq = q(4xp + yq)$$

To Clairaut's form and hence solve it.

PART - C

Q.1 Find the minimum value of the function $x + y + z$ subject to the condition [10]

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1.$$

Q.2 Solve the differential equation - [10]

$$\cos x \frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} - 2y \cos^3 x = 2 \cos^3 x.$$

Q.3 Show that the coordinates (α, β) of the center of curvature of the curve

$$27ay^2 = 4x^3 \text{ at } (x, y) \text{ are given by } 3a(\alpha + x) + 2x^2 = 0 \text{ and } \beta = 4y + \frac{9ay}{x}. \quad [10]$$

Q.4 Solve by method of variation of parameters - [10]

$$\frac{d^2y}{dx^2} + (1 - \cot x) \frac{dy}{dx} - y \cot x = \sin^2 x.$$

Q.5 Write Charpit's equations. Find a general solution of $p^2 x + q^2 y = u$, $u = (x, y)$ using Charpit's equations. [10]

