01-09-22

Delta

* Relation b/w Step and Impulse Functions —

$$\Rightarrow \delta[n] = u[n] - u[n-1]$$

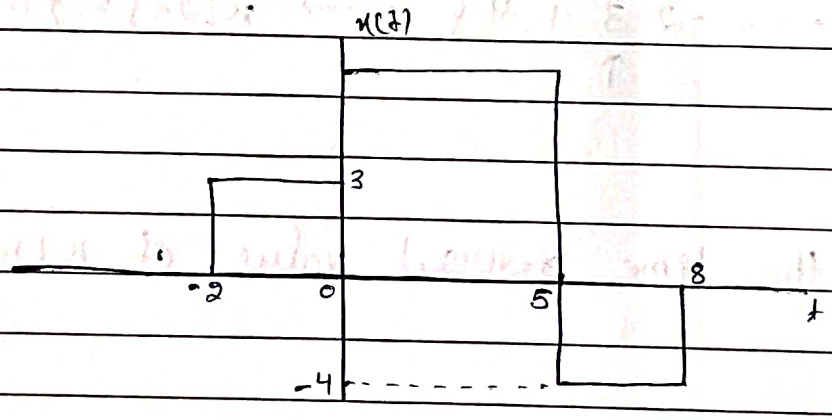
$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

or, $$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

Similarly, $$\delta(t) = \frac{d\,u(t)}{dt}$$

$$\Rightarrow u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

e.g. →



we can write as —

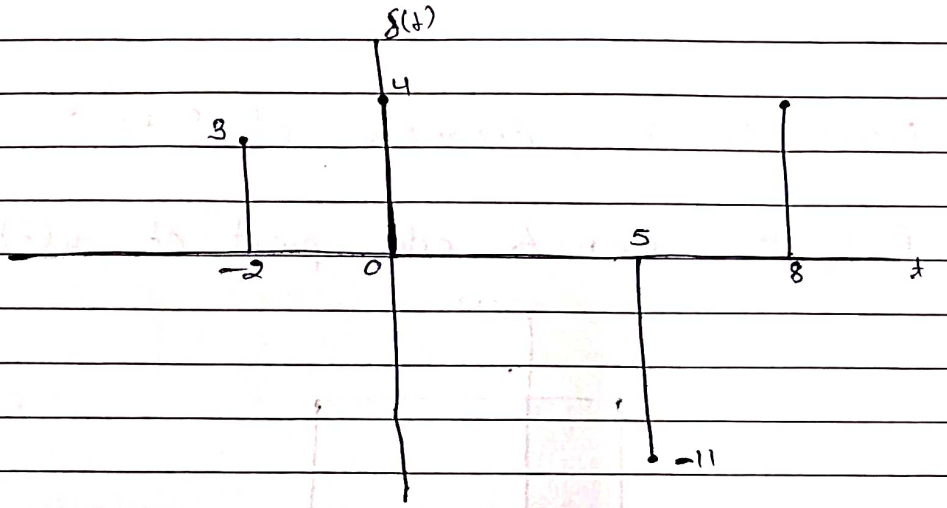
$$x(t) = 3\,u(t+2) + 4\,u(t) - 11\,u(t-5) + 4\,u(t-8)$$

[if, finite duration & all values are non-zero then, sum of coefficients are zero.

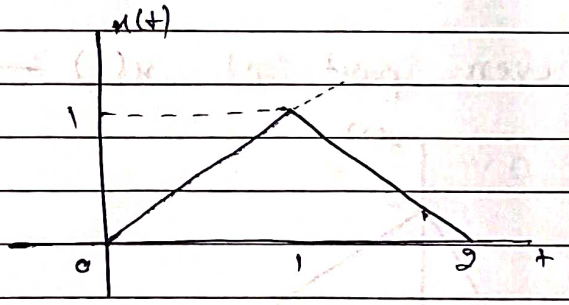
→ $3+4-11+4 = 0$]

Now an impulse function —

$$\frac{d}{dt}x(t) = 3\,\delta(t+2) + 4\,\delta(t) - 11\,\delta(t-5) + 4\,\delta(t-8)$$



e.g. →



$$r(t) = t\,u(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

we can write in form of ramp as —

at $0 \to 1$, slope $= 1$
at $1 \to 2$, slope $= -1 - 1 = -2$ (as previous slope $= 1$)
at $2 \to$, slope $= +1$

$$\therefore x(t) = 1\,r(t) - 2\,r(t-1) + 1\,r(t-2)$$
$$= r(t) - 2r(t-1) + r(t-2)$$

Delta

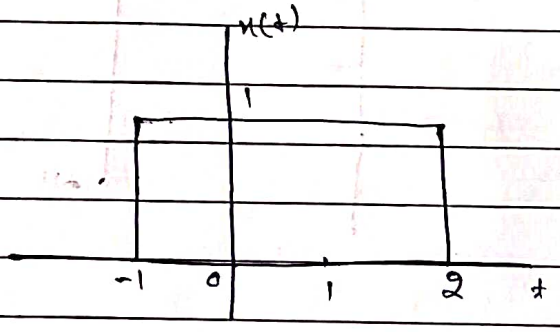
* Sampling Property —

$$x[n]\,\delta[n] = x[0]\,\delta[n]$$
$$x[n]\,\delta[n-n_0] = x[n_0]\,\delta[n-n_0]$$

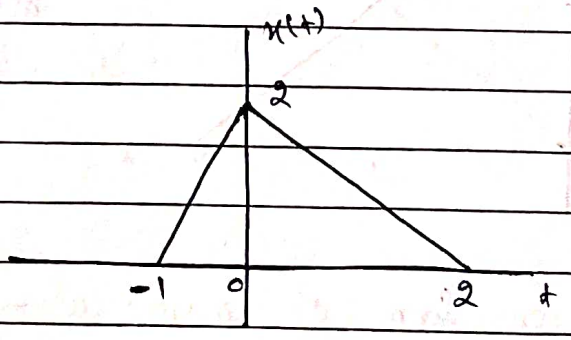
where $n_0 \rightarrow$ integer

Q4 ⇒ What is the derivative of $\delta(t)$?

Q5 ⇒ Plot the even & odd part of $x(t)$ —



Q6 ⇒ Find the even part of $x(t)$ —



Q7 ⇒ Find the energy of the signal $x(-at+b)$ if the energy of signal $x(t)$ is E.

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$