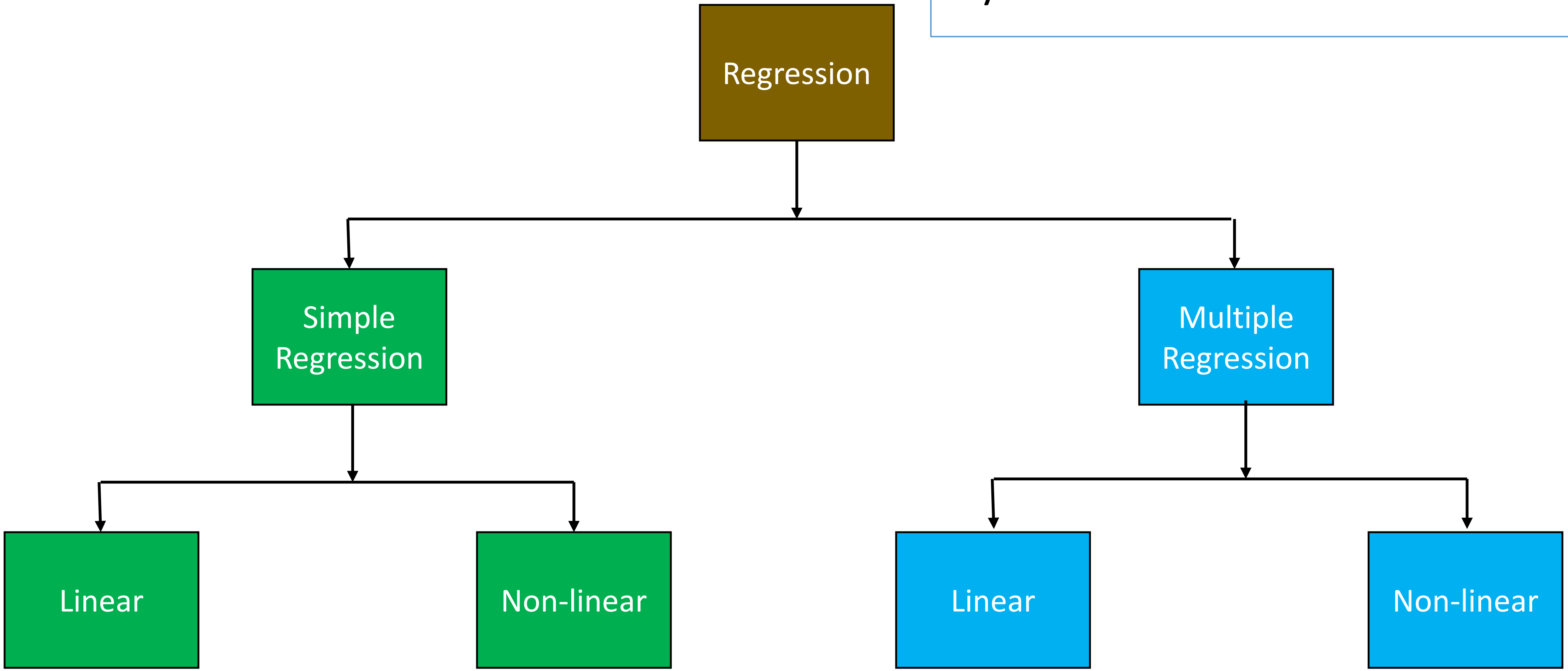
Linear Regression

Introduction

- Linear regression is a powerful and widely used method to estimate values, such as
  - the price of a house
  - the value of a certain stock
  - the life expectancy of an individual
  - the amount of time a user will watch a video or spend on a website

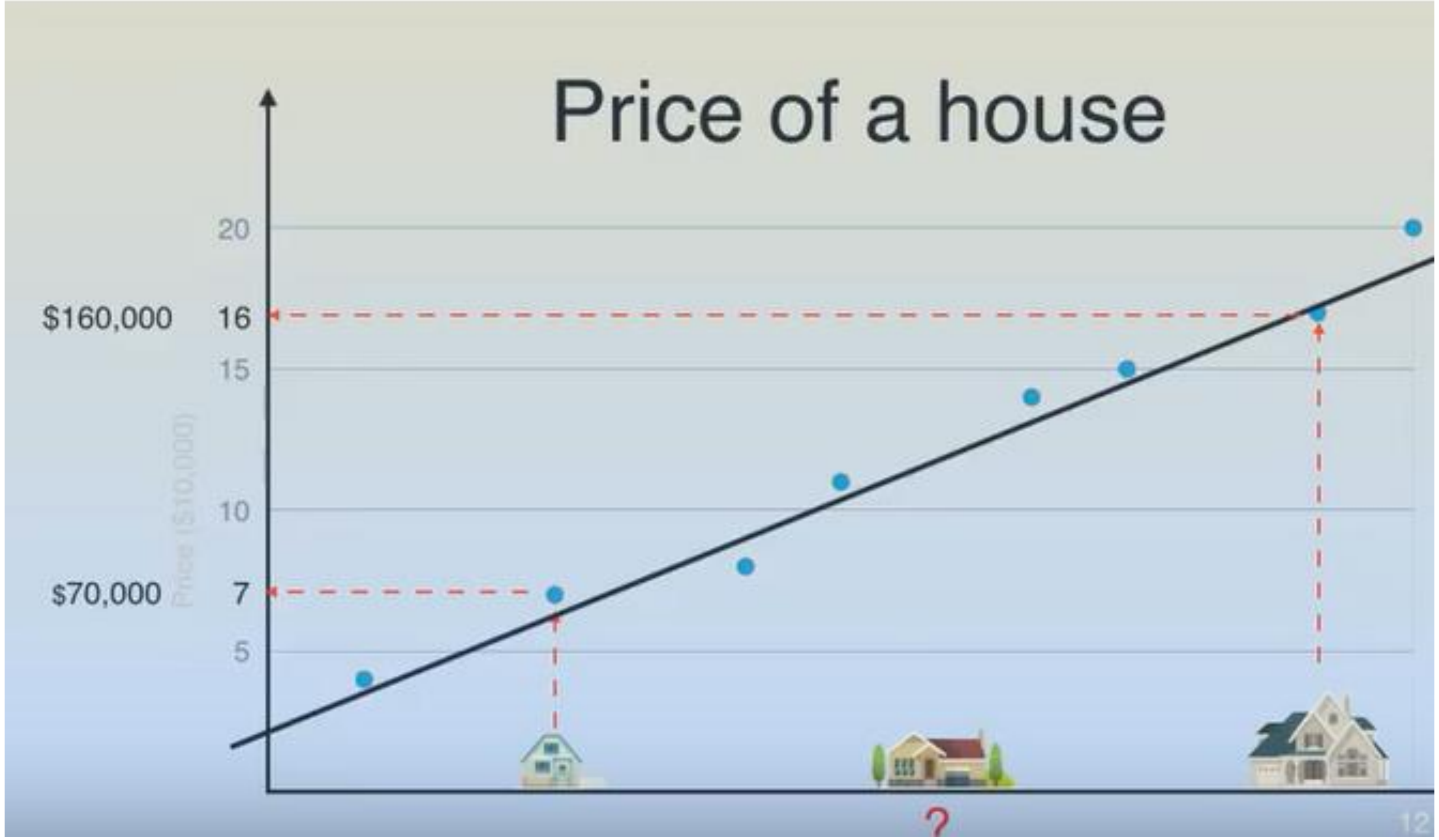
Regression

- It is a supervised learning approach
- If x is given, you can predict y = f(x)
- y is continuous value.



Linear Regression Intuition

House Price Estimation



What is the best estimate for the price of the house?

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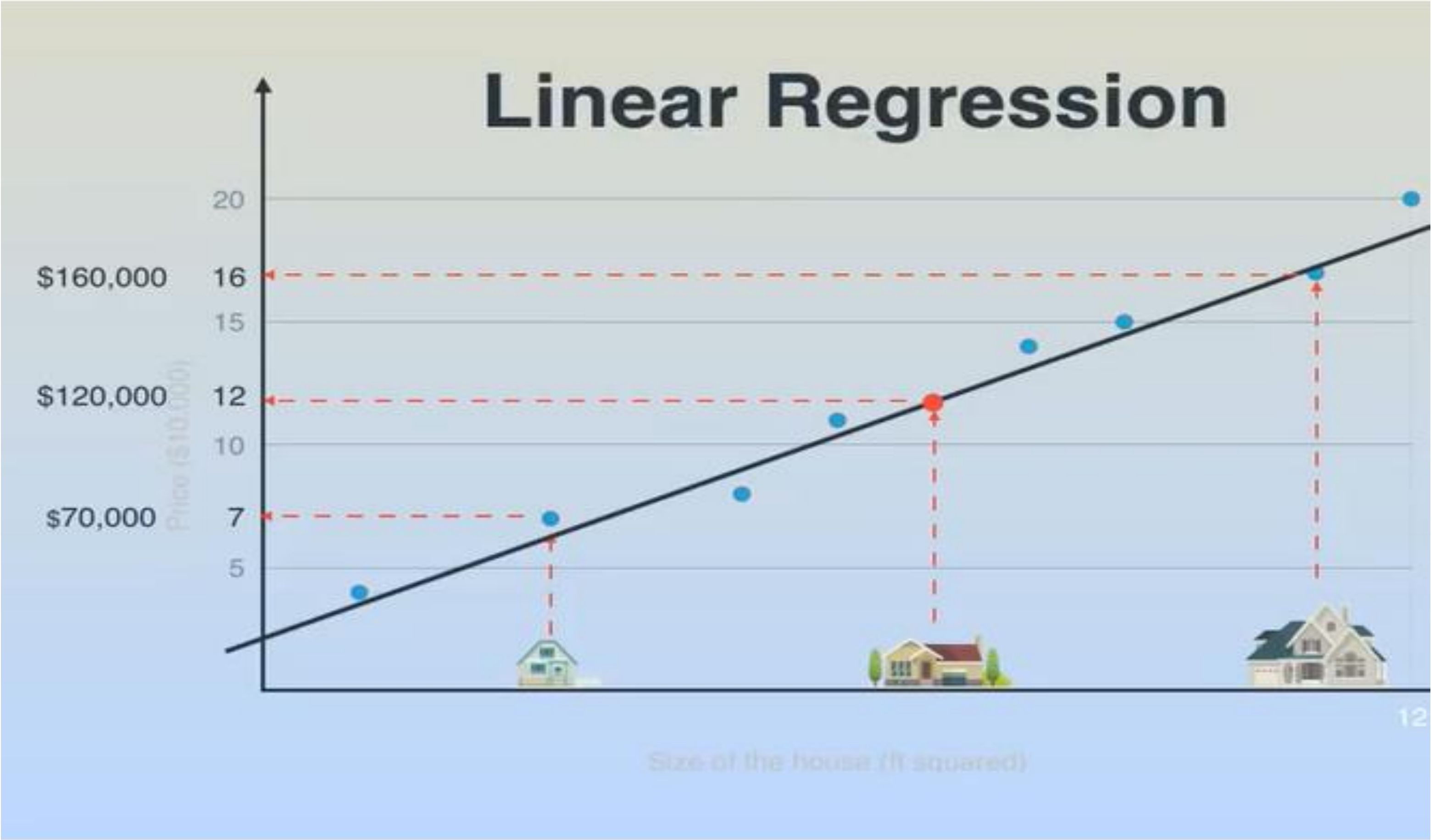
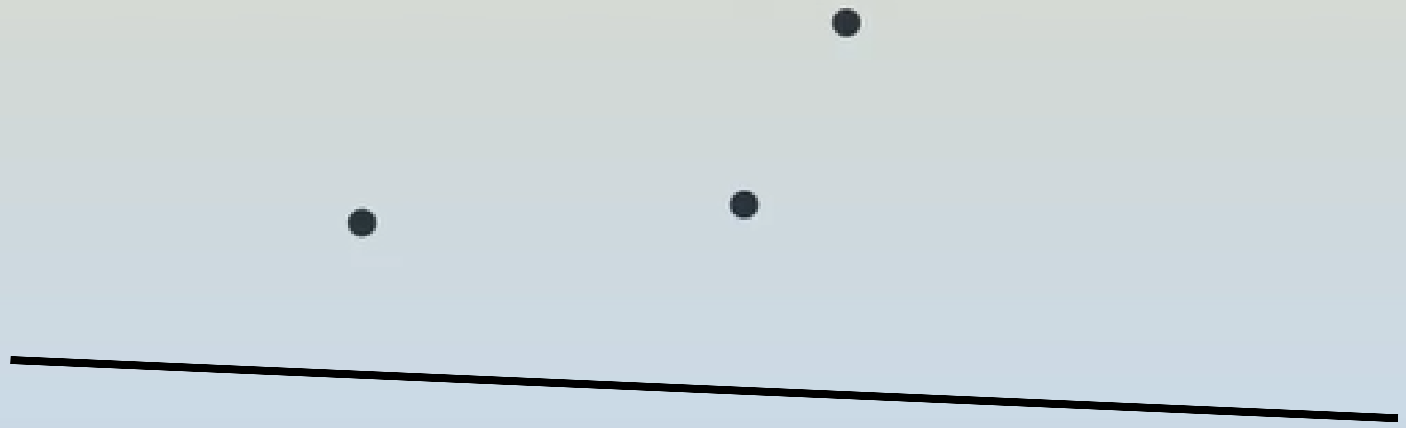
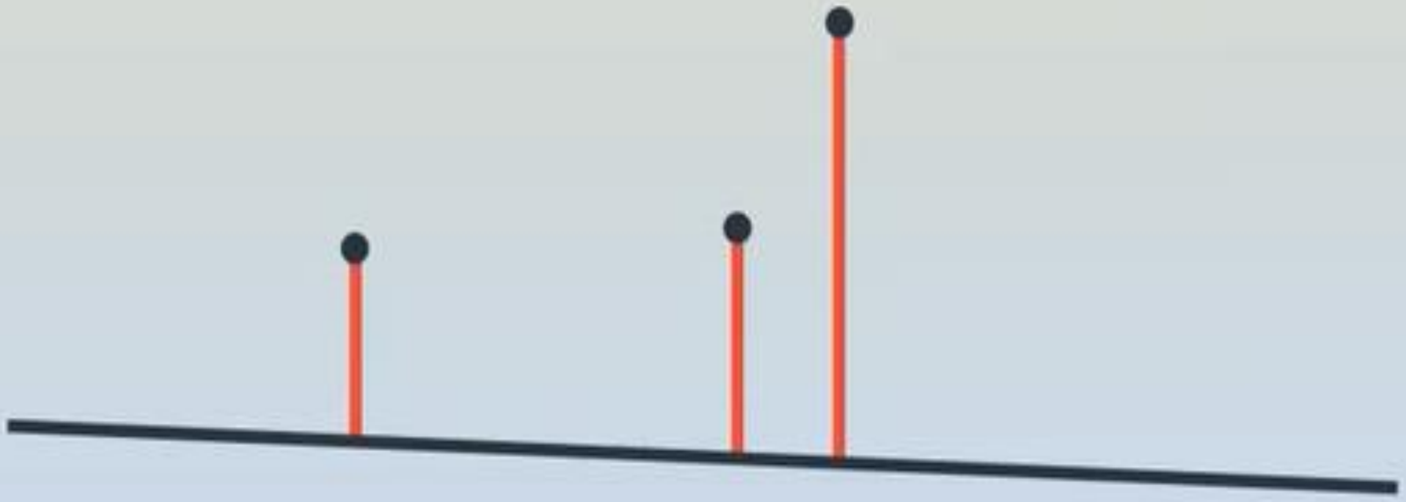


What is the best estimate for the price of the house?

Linear Regression



Linear Regression



Error:

Linear Regression

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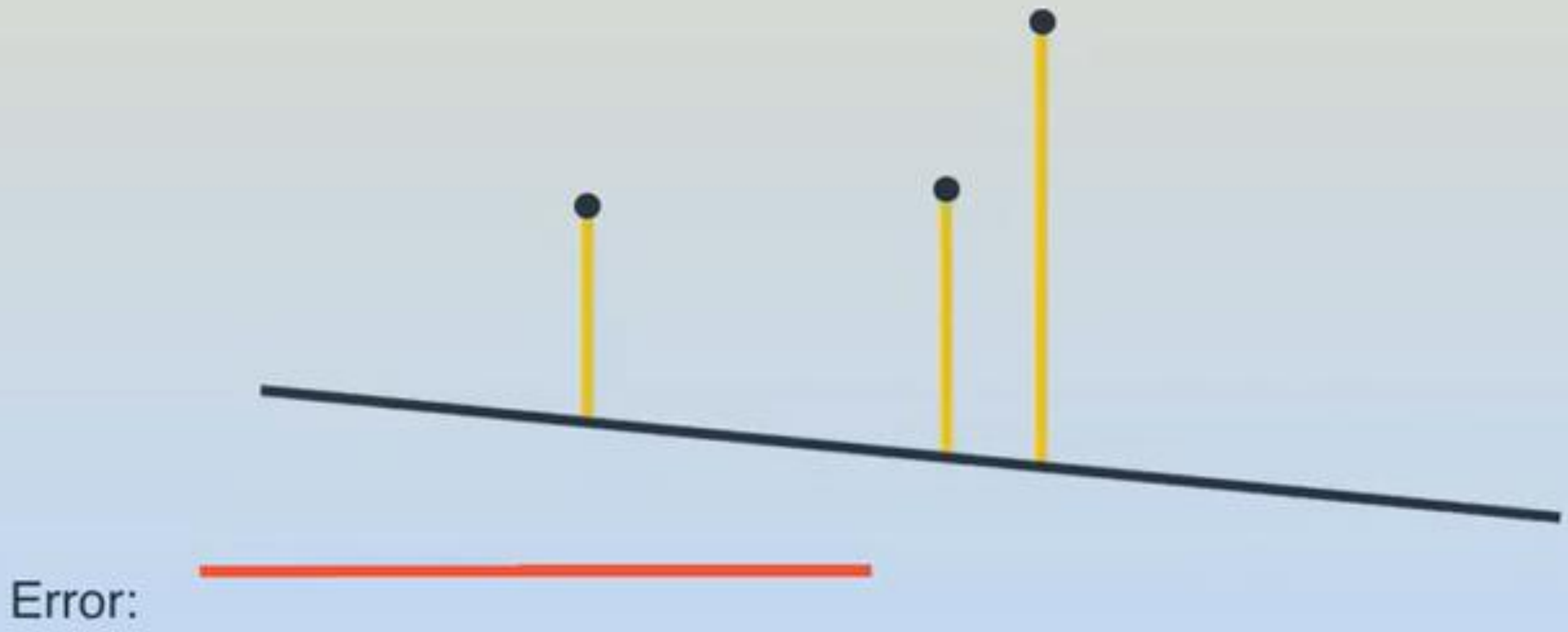
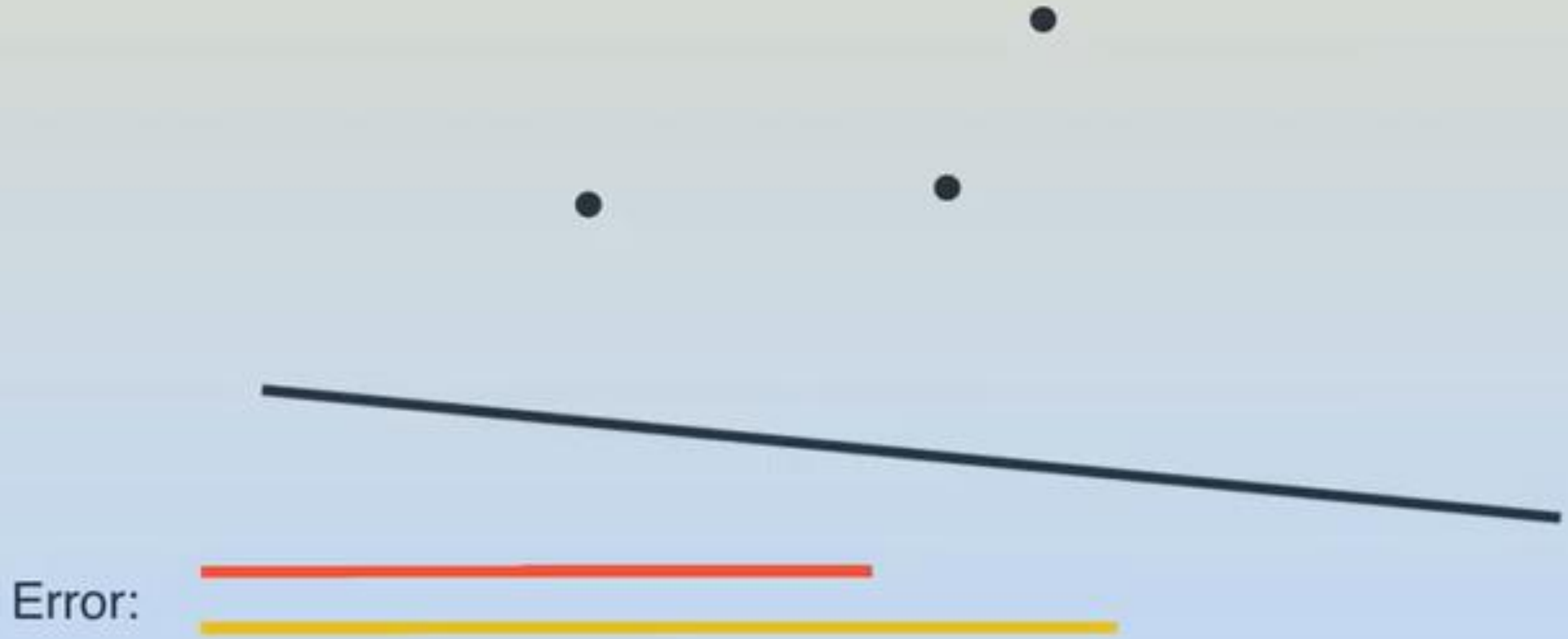
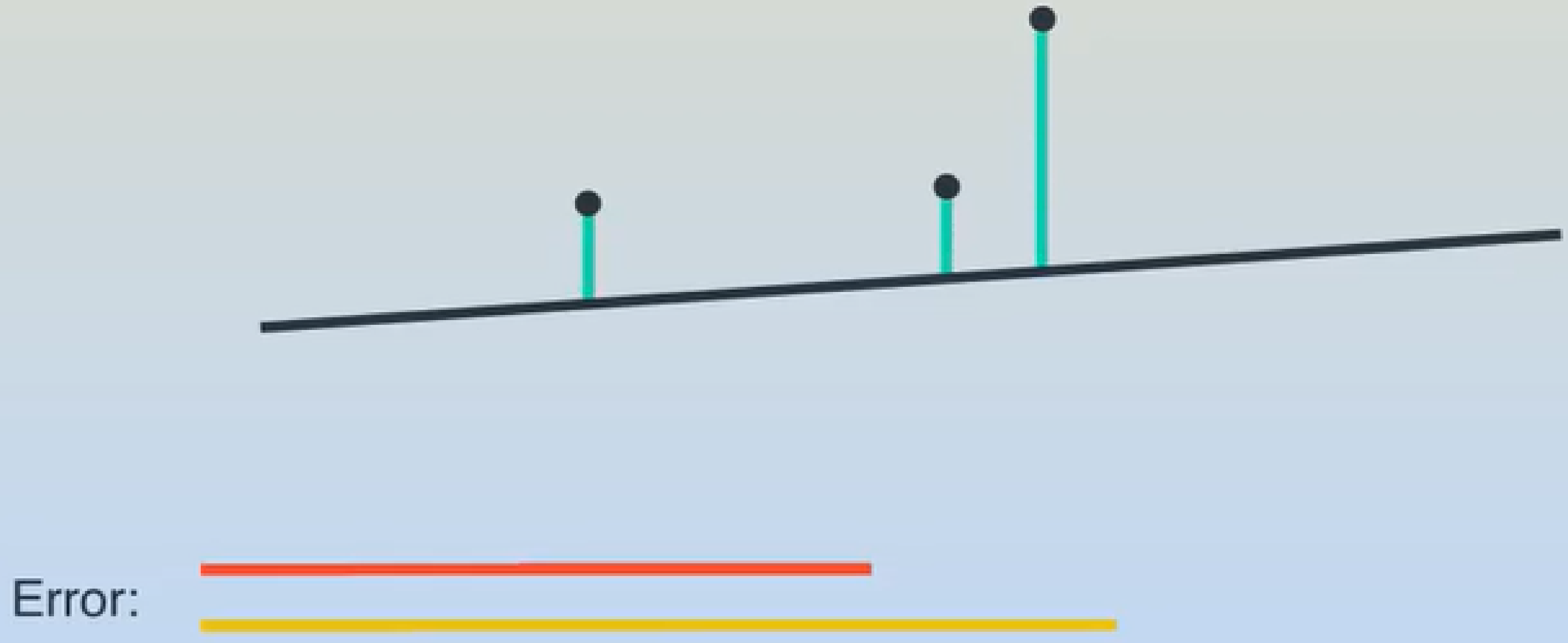
Linear Regression



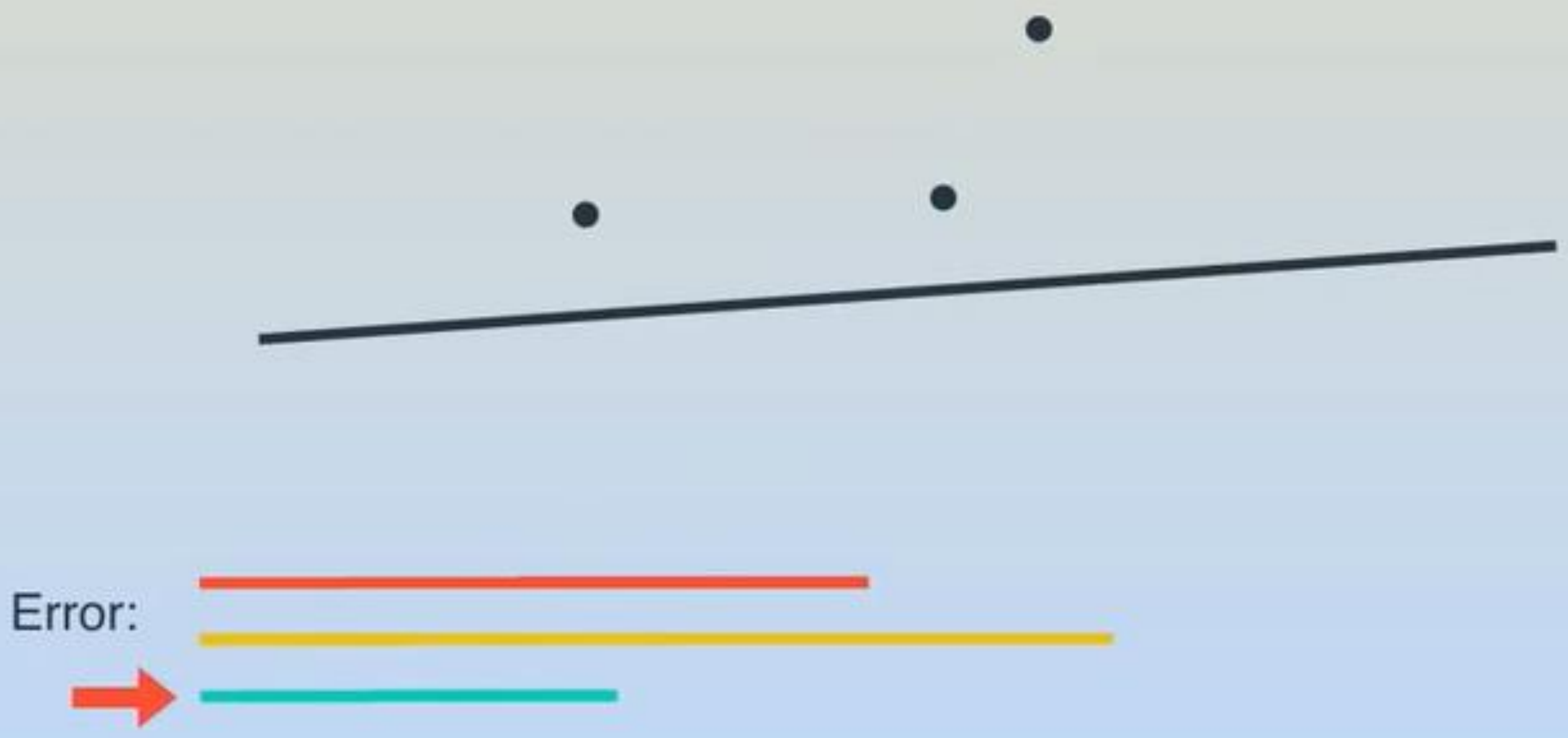
Linear Regression

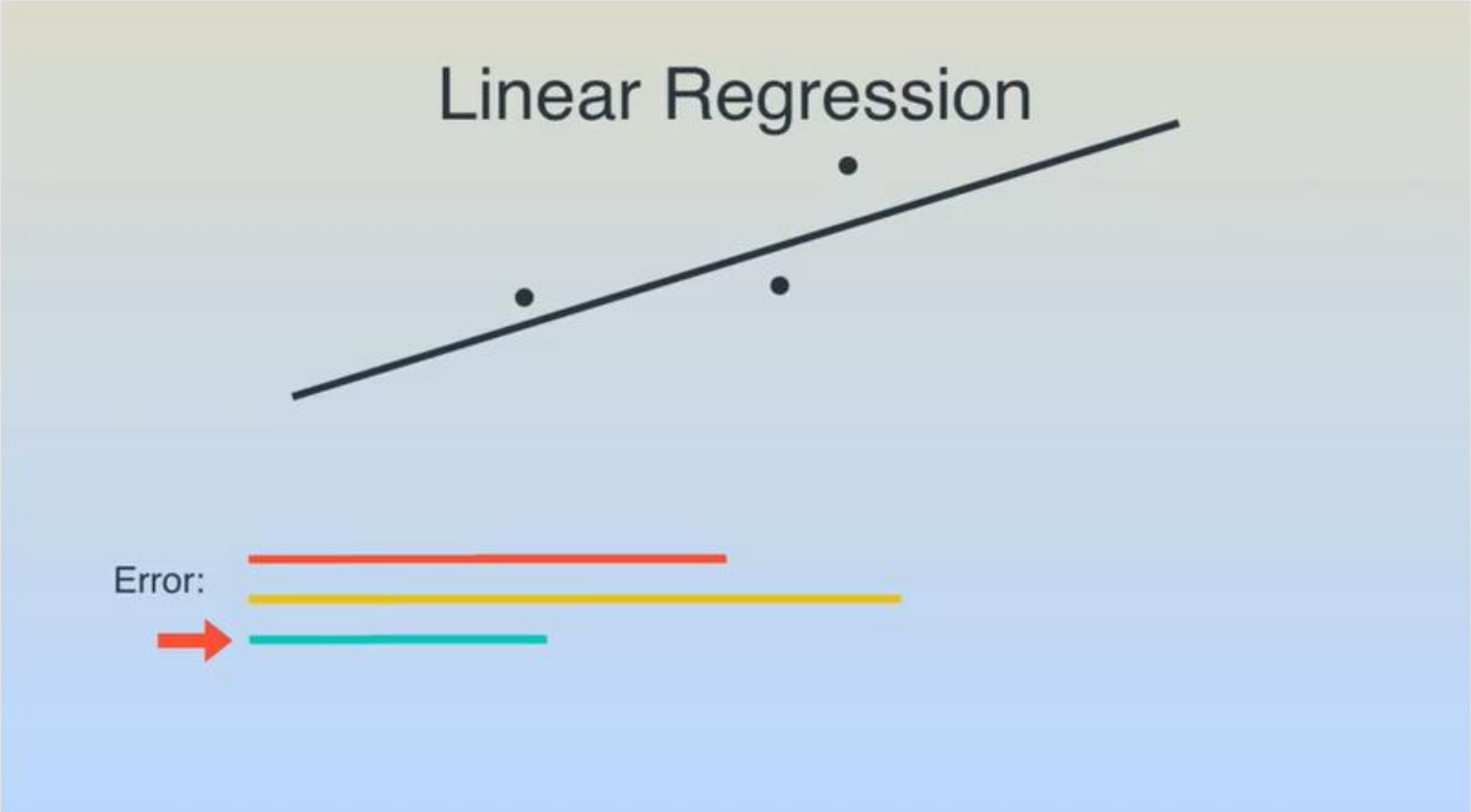


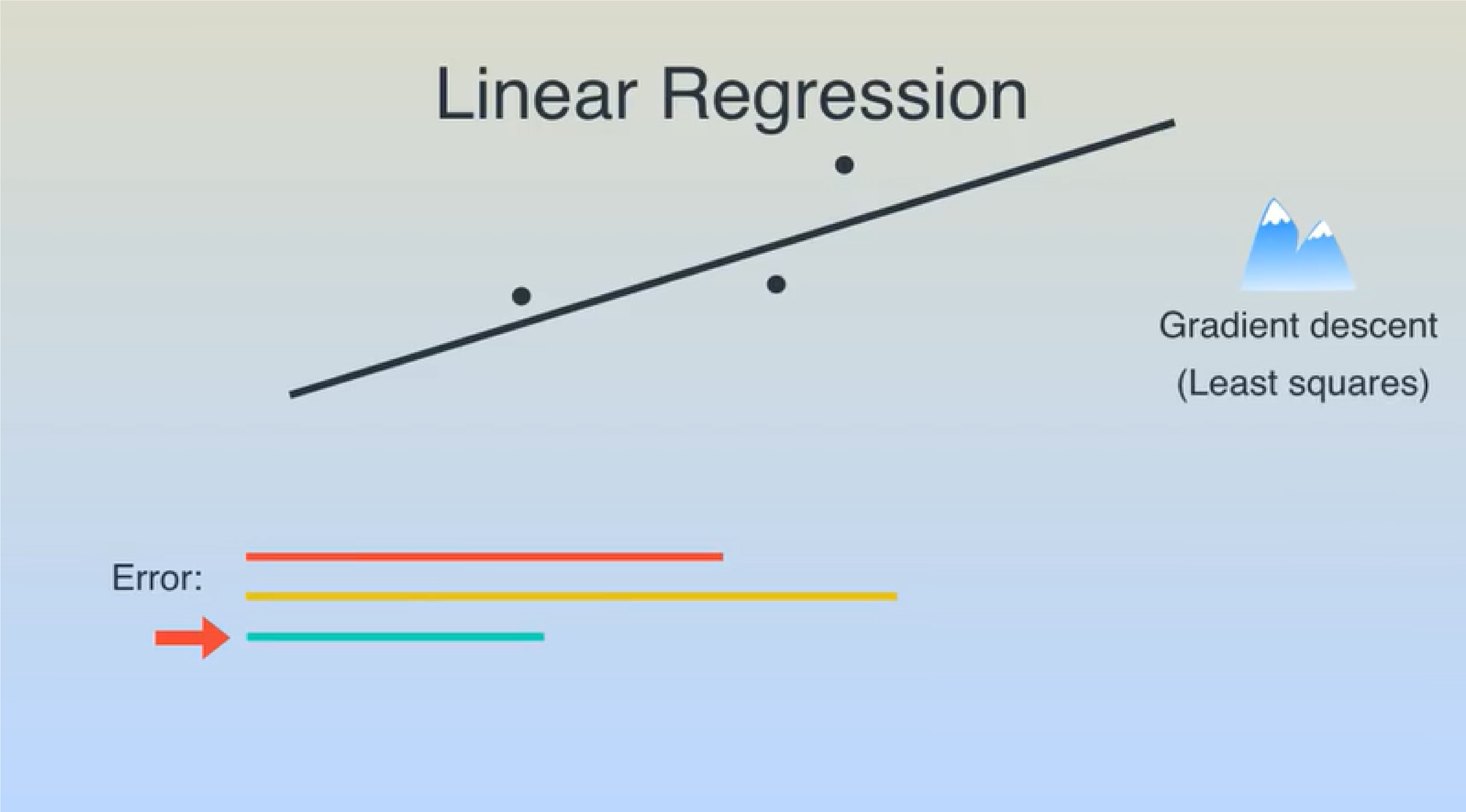
Linear Regression

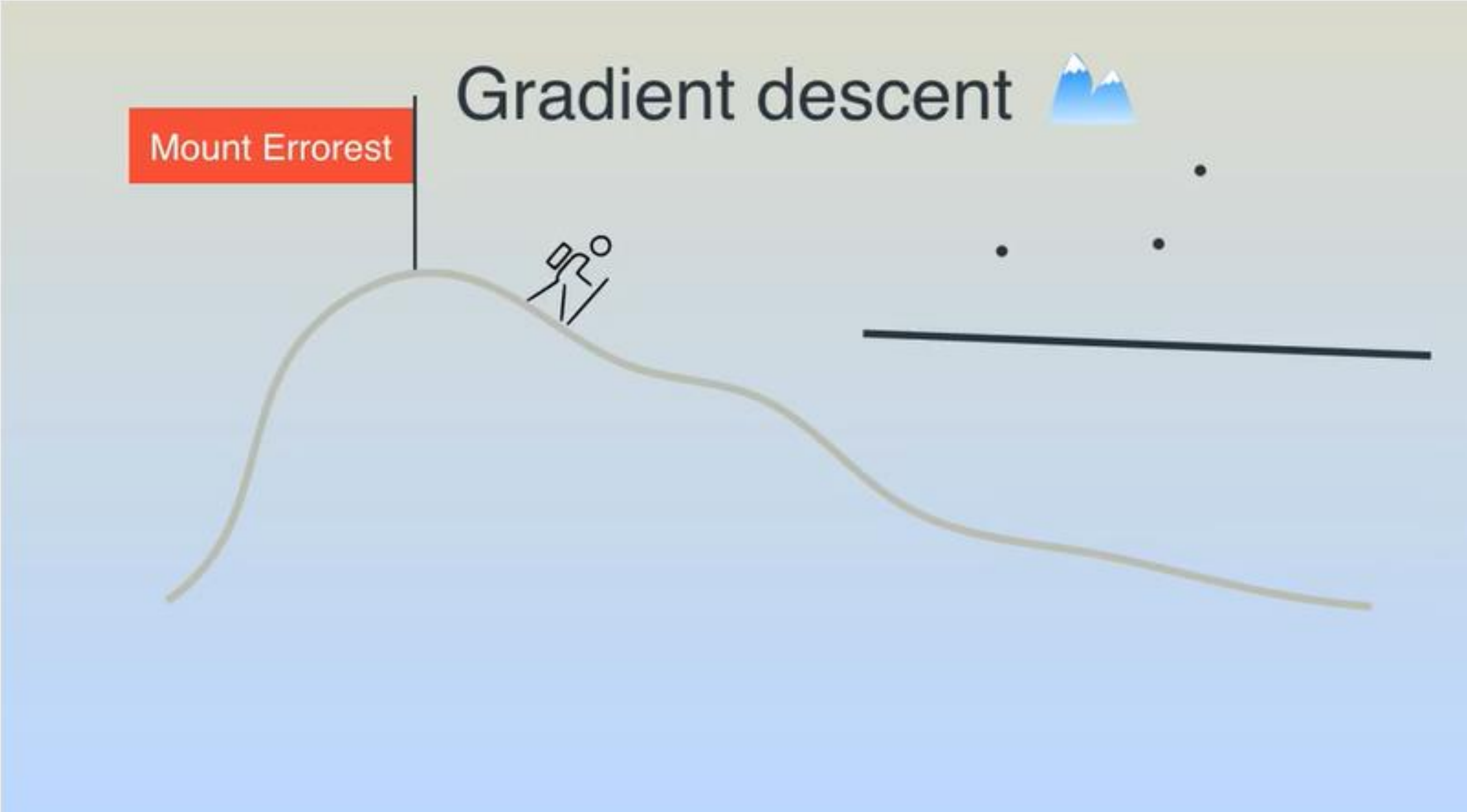


Linear Regression

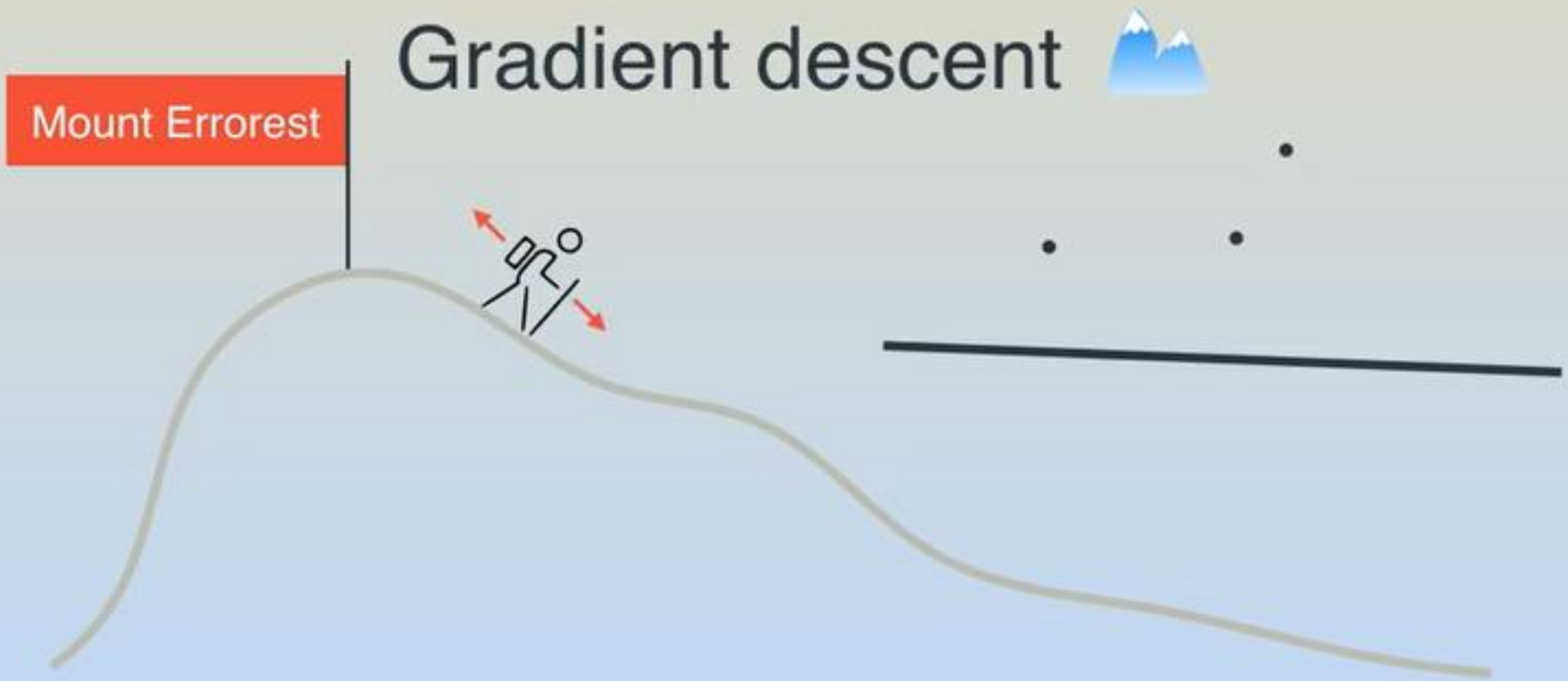



Linear Regression
Error:


Linear Regression
Gradient descent
(Least squares)
Error:


Gradient descent
Mount Errorest

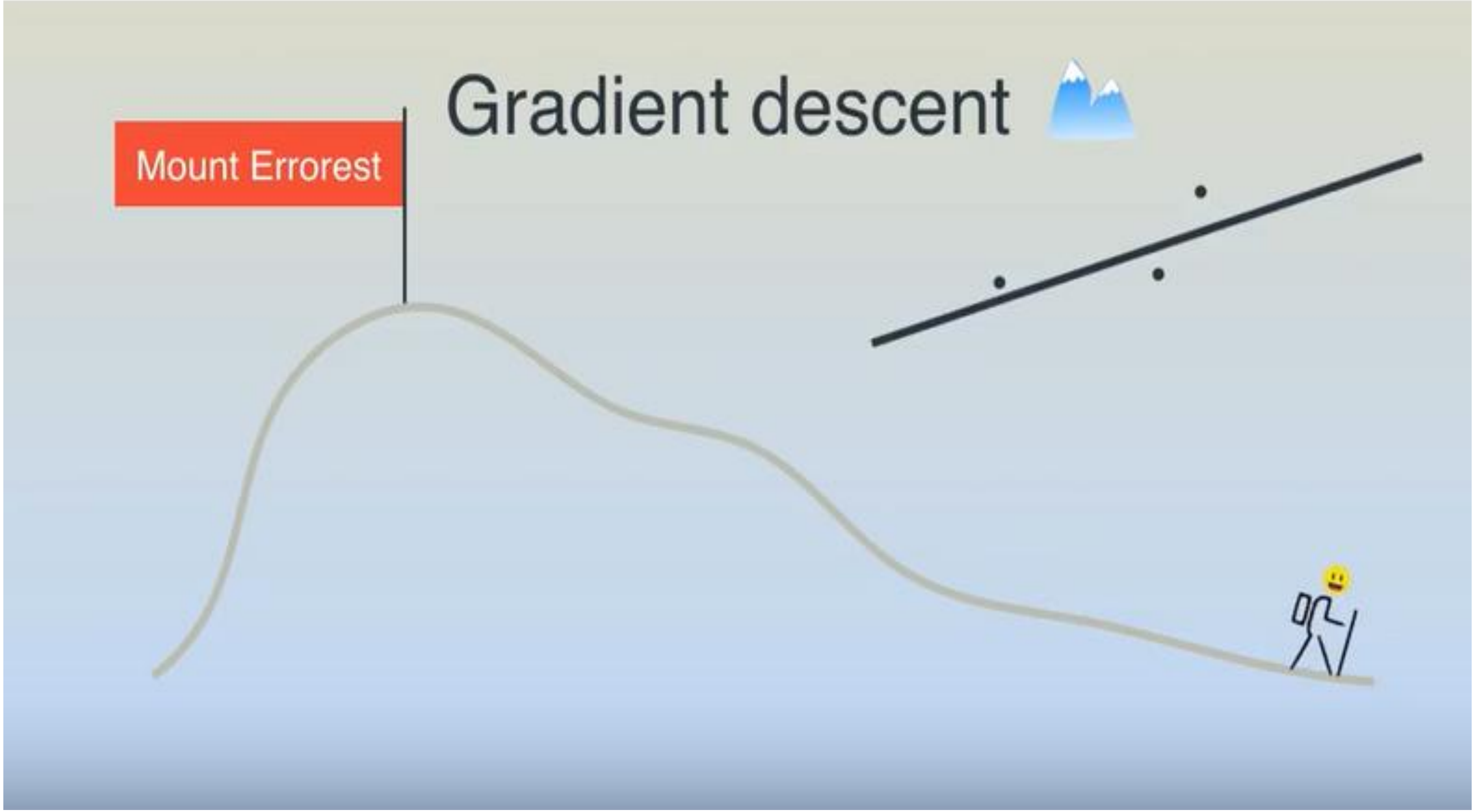
Gradient descent



Gradient descent 🏔️

Gradient descent 🏔

Gradient descent 🏔️


Gradient descent
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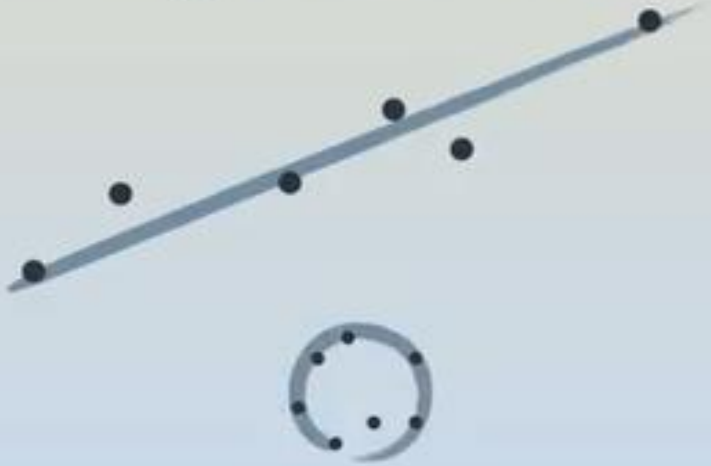
Linear and Polynomial Regression

Linear and Polynomial Regression

Linear and Polynomial Regression



Linear and Polynomial Regression

Introduction

- **Regression models** describe the relationship between variables by fitting a line to the observed data.
- Linear regression models use a straight line, while logistic and nonlinear regression models use a curved line.
- Regression allows you to estimate how a dependent variable changes as the independent variable(s) change.

Simple Linear Regression

- **Simple linear regression** is used to estimate the relationship between **two quantitative variables**.

- You can use simple linear regression when you want to know:
  - **The relationship is between two variables**
  - e.g. the relationship between salary and experience.
  - The value of the dependent variable at a certain value of the independent variable
  - e.g. the salary at a certain level of experience.

Experience (years) (IDV)	Salary (DV)
1.1	39,343
1.3	46,205
1.5	37,731
2	43,525
2.2	39,891
2.9	56,642
3	60,150

Simple Linear Regression:

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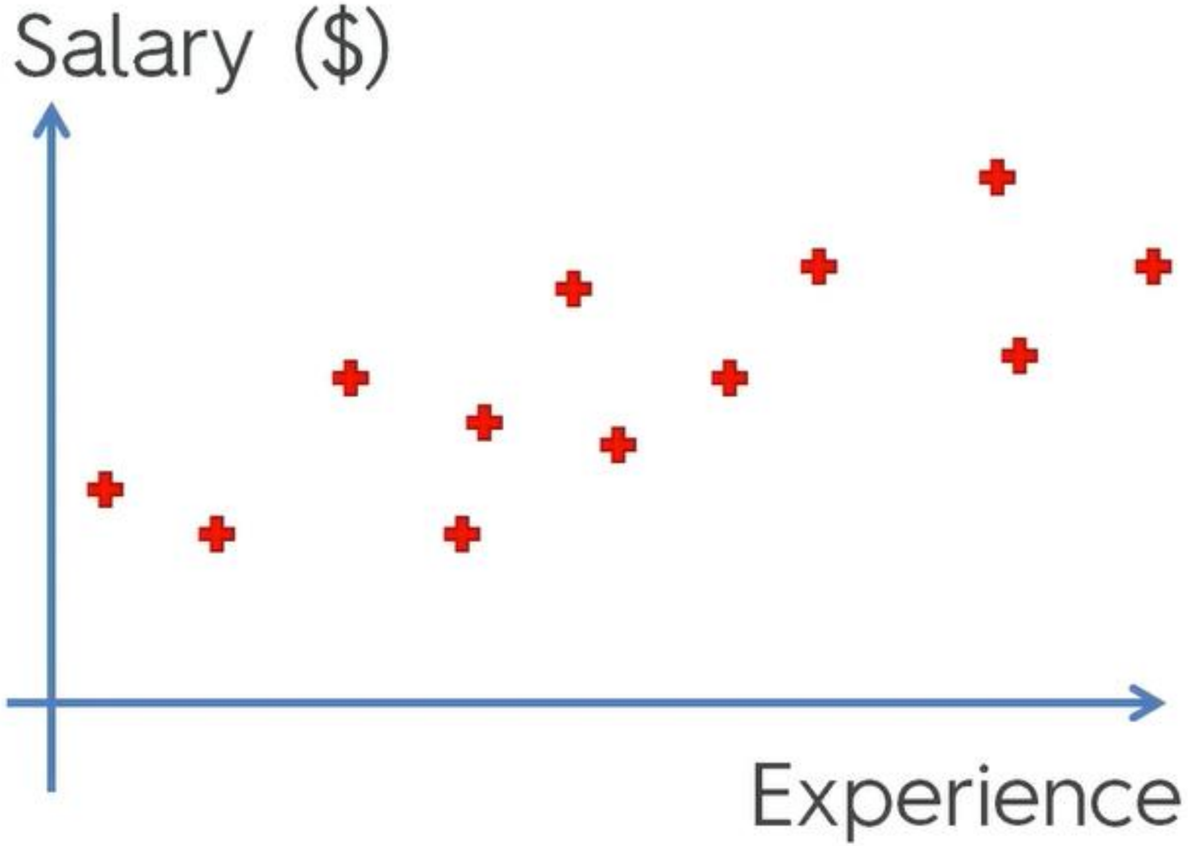
Simple Linear Regression:



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:	:
:	:

How to perform a simple linear regression?

$$y = b_0 + b_1 * x_1$$

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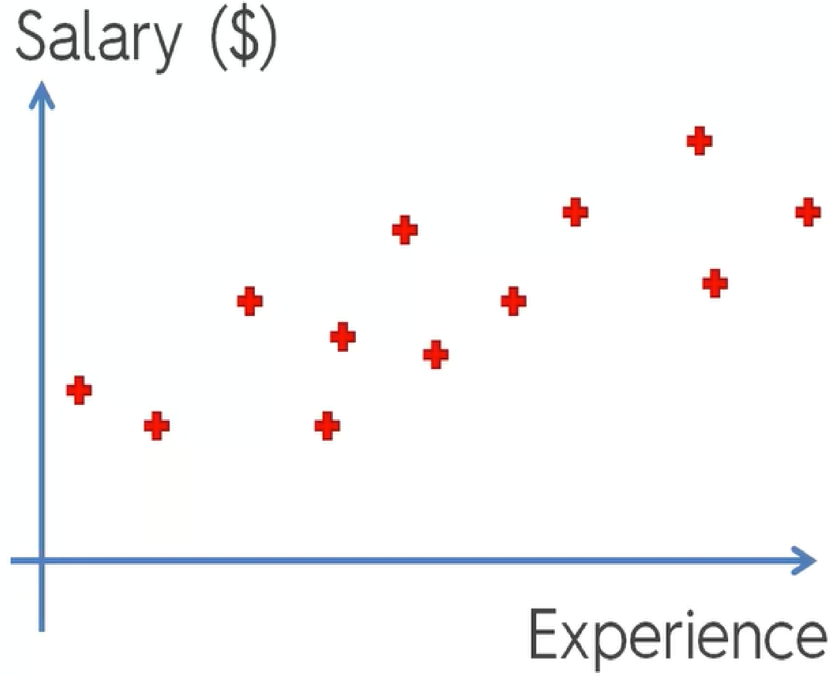
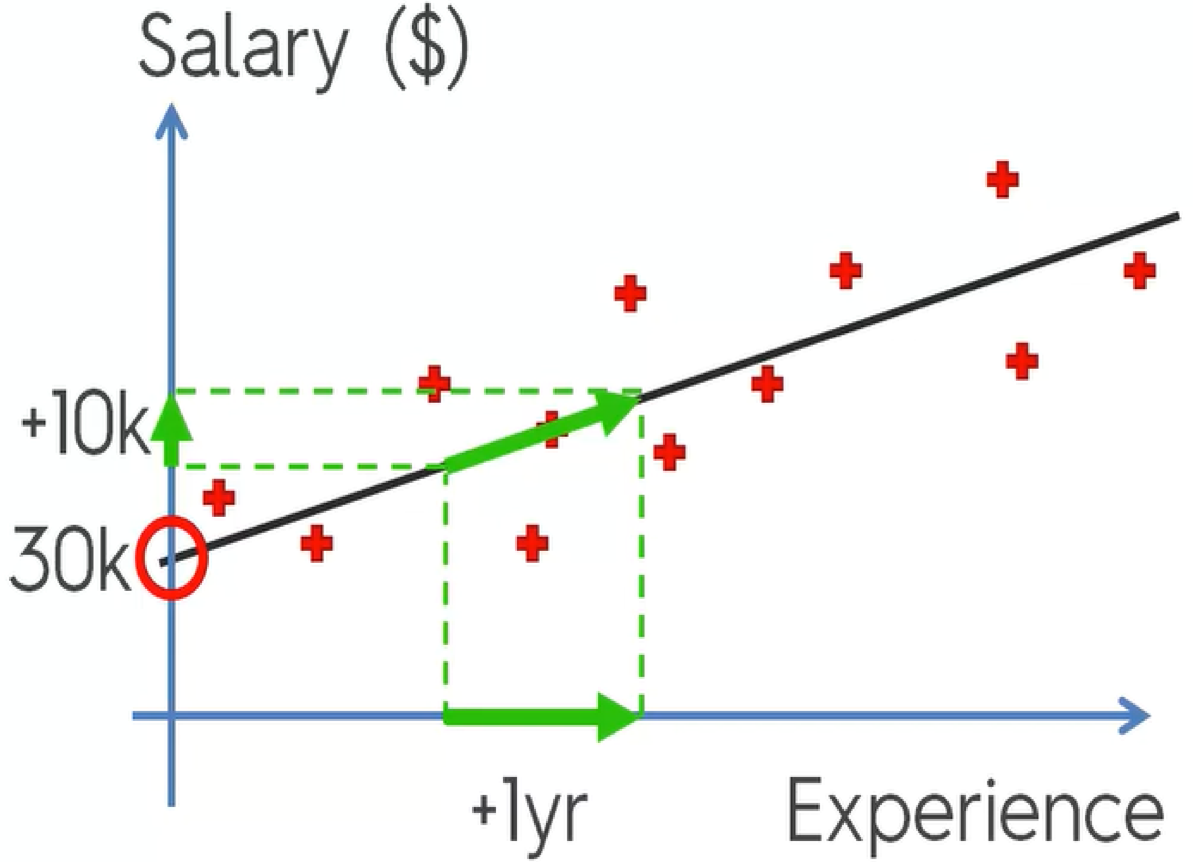
Dependent variable (DV) Independent variable (IV)



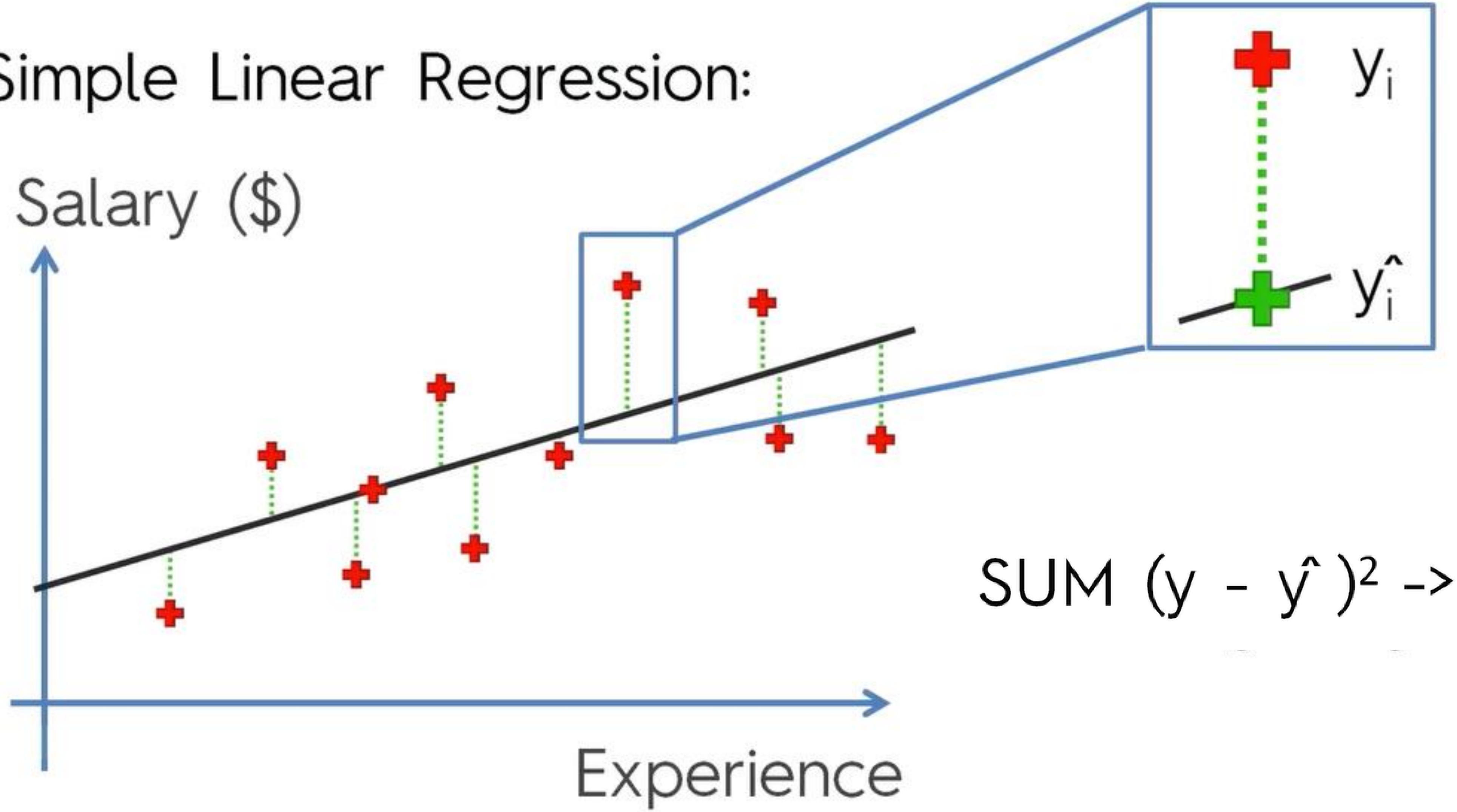
Simple Linear Regression:



$$y = b_0 + b_1{*}x$$

$$\text{Salary} = b_0 + b_1{*}\text{Experience}$$

Simple Linear Regression:



SUM $(y - \hat{y})^2$ -> min

Population versus Sample

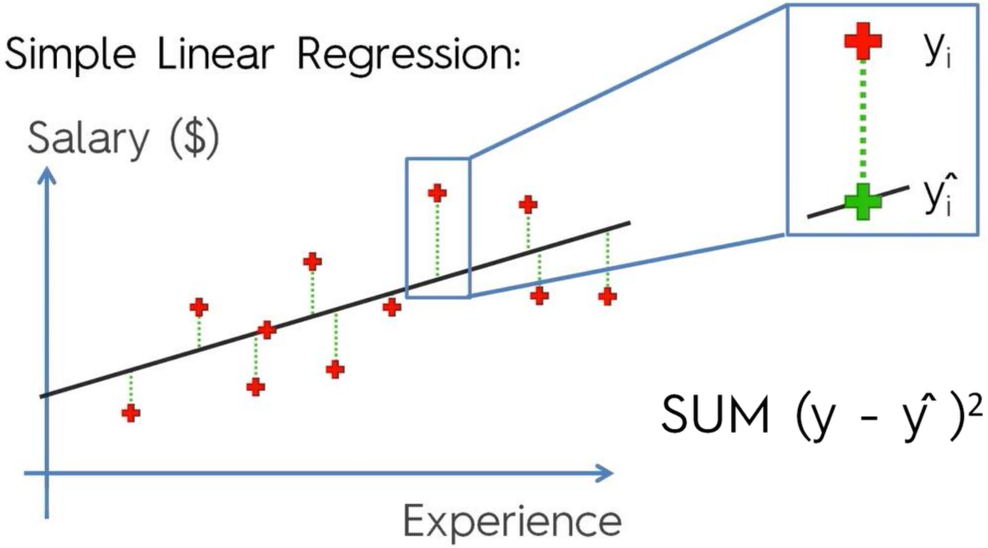
- Population Regression Function

  - Deterministic Component

$$y = f(x)$$

  - Stochastic Component

$$y = f(x) + \epsilon$$



Population versus Sample

- Normally distributed error component
- Univariate function $y = wx + b + \epsilon$
- Multivariate function

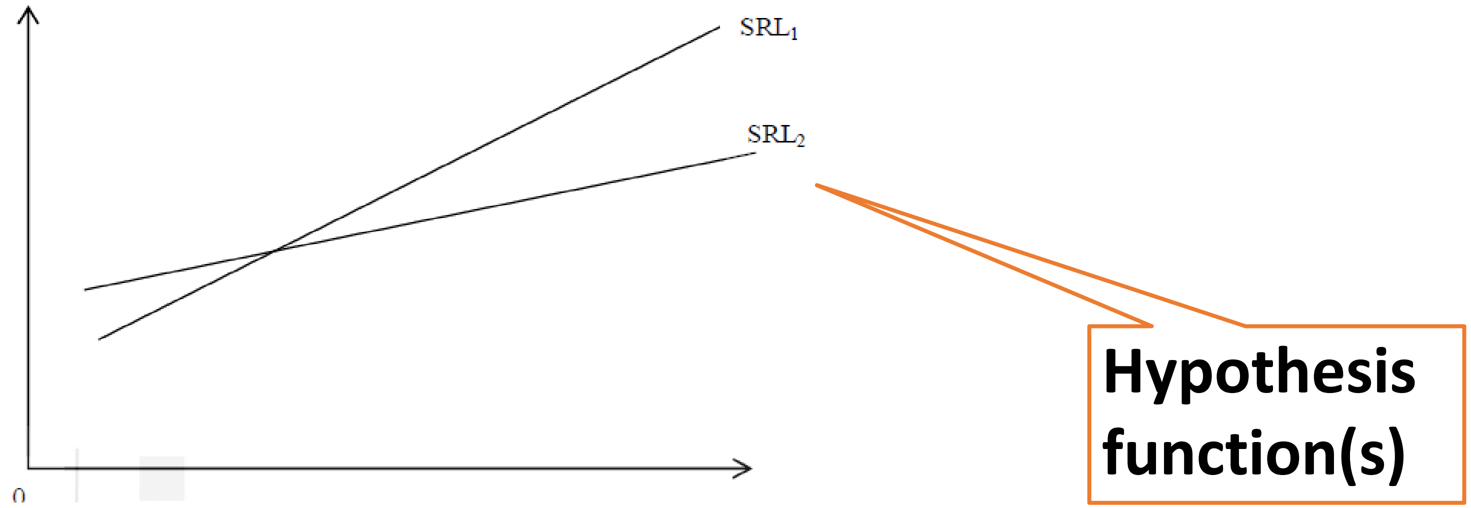
$$y = w_n x_n + ...w_1 x_1 + w_0 + \epsilon$$
$$= \mathbf{w}^T \mathbf{x} + \epsilon$$

$$\mathbf{w} = \begin{bmatrix} w_0 \\ w_1 \\ .. \\ w_n \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} 1 \\ x_1 \\ .. \\ x_n \end{bmatrix}$$

Population versus Sample

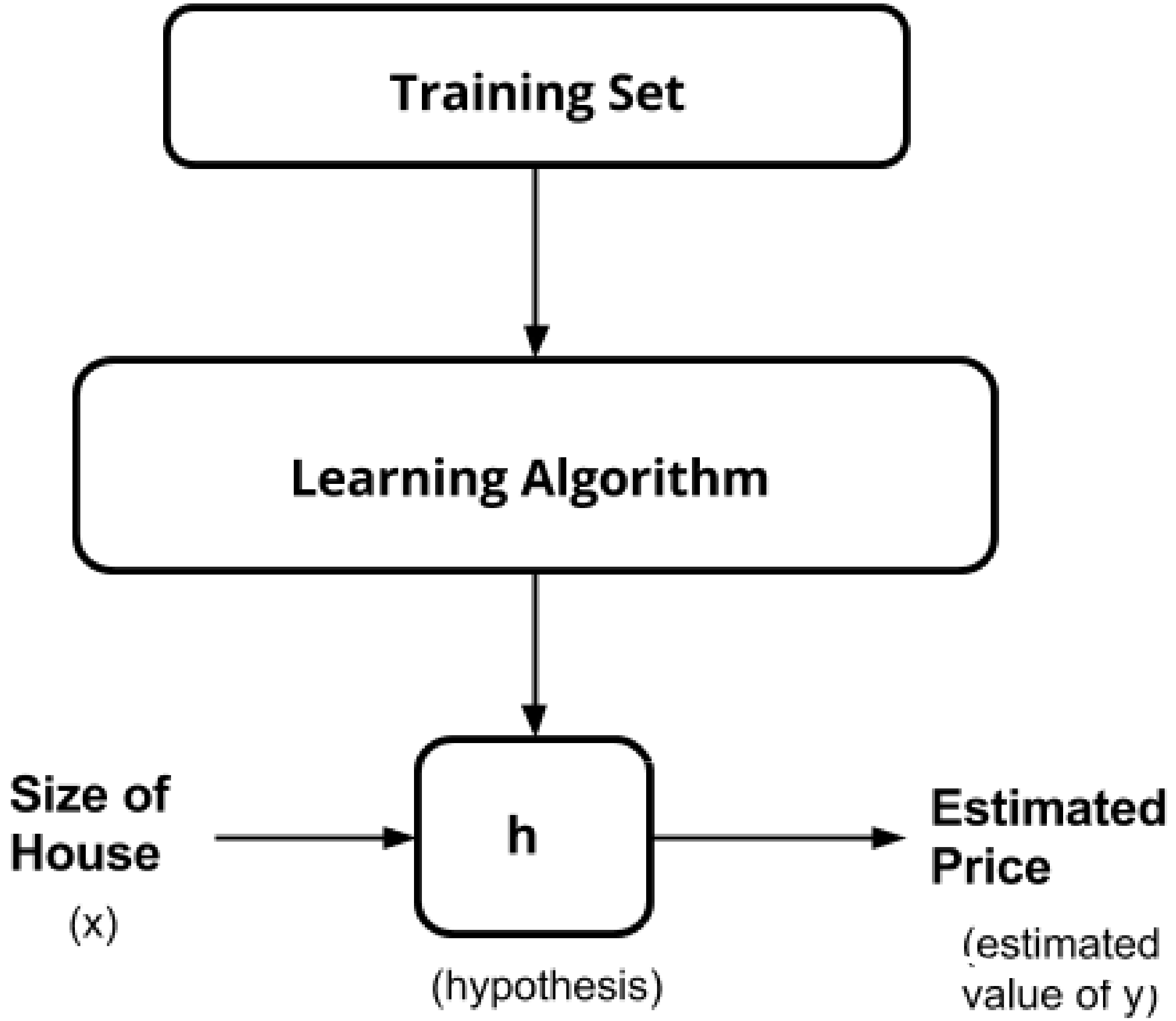
- Sample Regression Functions
  - Different Regression Line/Plane/Hyperplane



- Univariate function $\hat{y} = h(x) = wx + b$

- Multivariate function $\hat{y} = h(x) = \mathbf{w}^T\mathbf{x}$

Hypothesis function(s)



Linear Regression algorithm

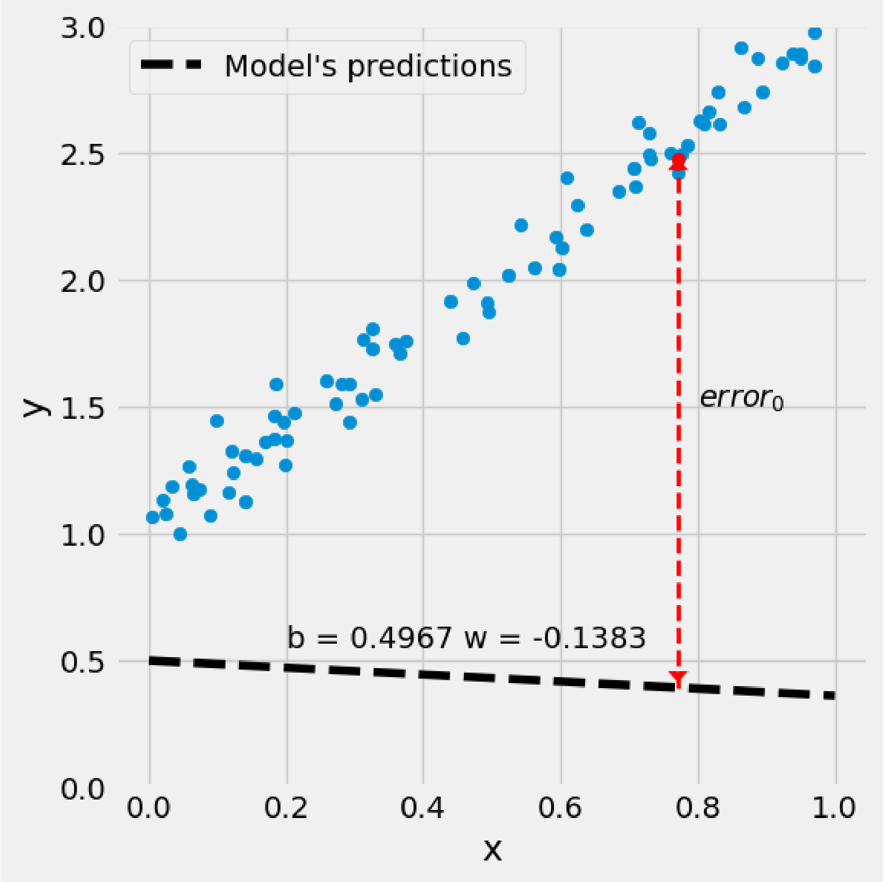
Step 1. Initialization

- Assume parametric form $\hat{y} = wx + b$

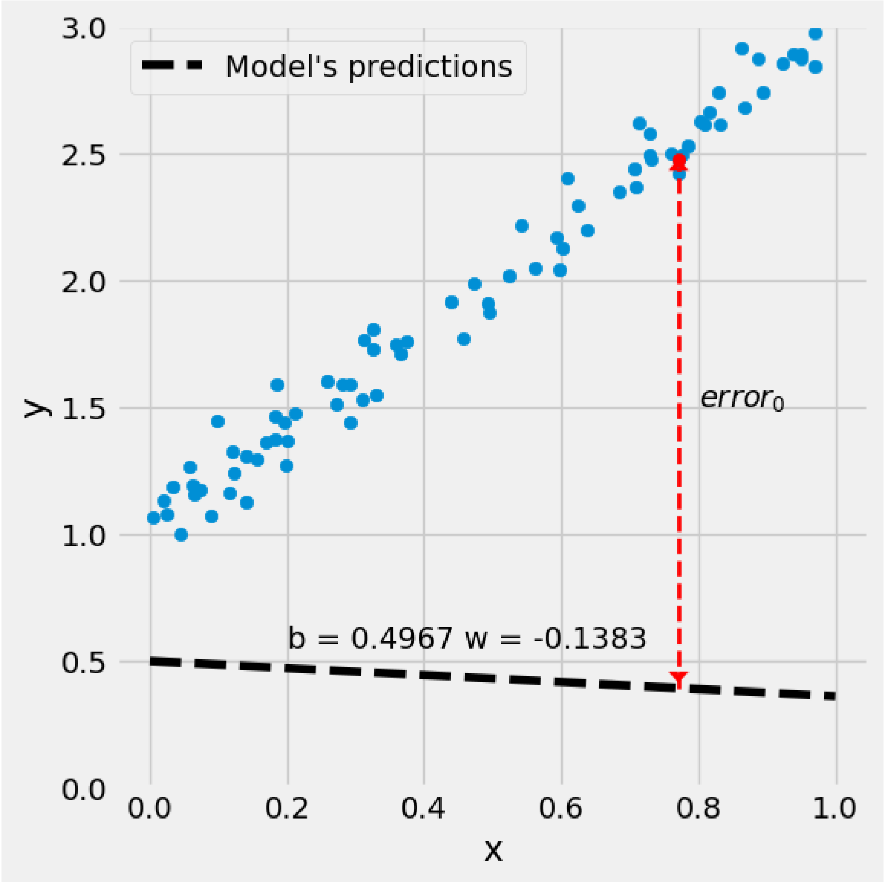
Parametric form of the hypothesis function

- Calculate error using the initial w & b

$$error^{(i)} = \hat{y}^{(i)} - y^{(i)}$$



Formulate Objective function



$$error^{(i)} = \hat{y}^{(i)} - y^{(i)}$$

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n} \mathrm{error}^{(i)^2}$$

$$= \frac{1}{n}\sum_{i=1}^{n} \left(\hat{y}^{(i)} - y^{(i)}\right)^2$$

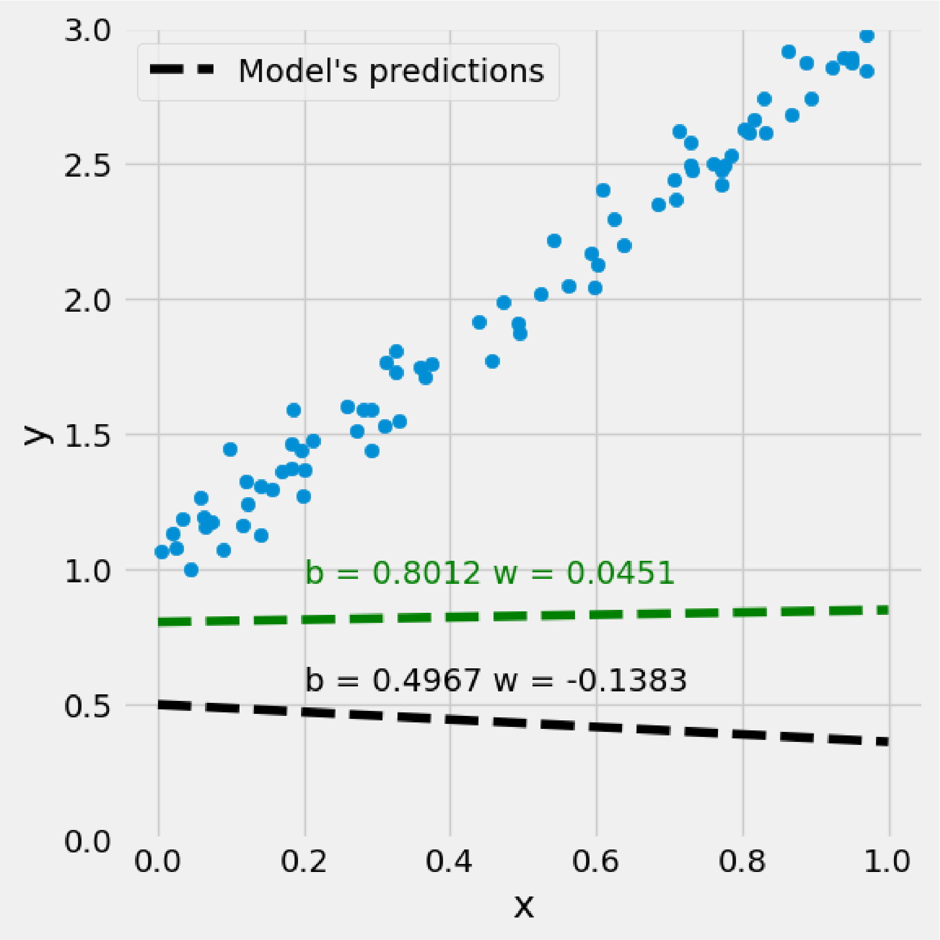
$$\mathcal{J}(w, b) = \frac{1}{n}\sum_{i=1}^{n} \left(b + wx^{(i)} - y^{(i)}\right)^2$$

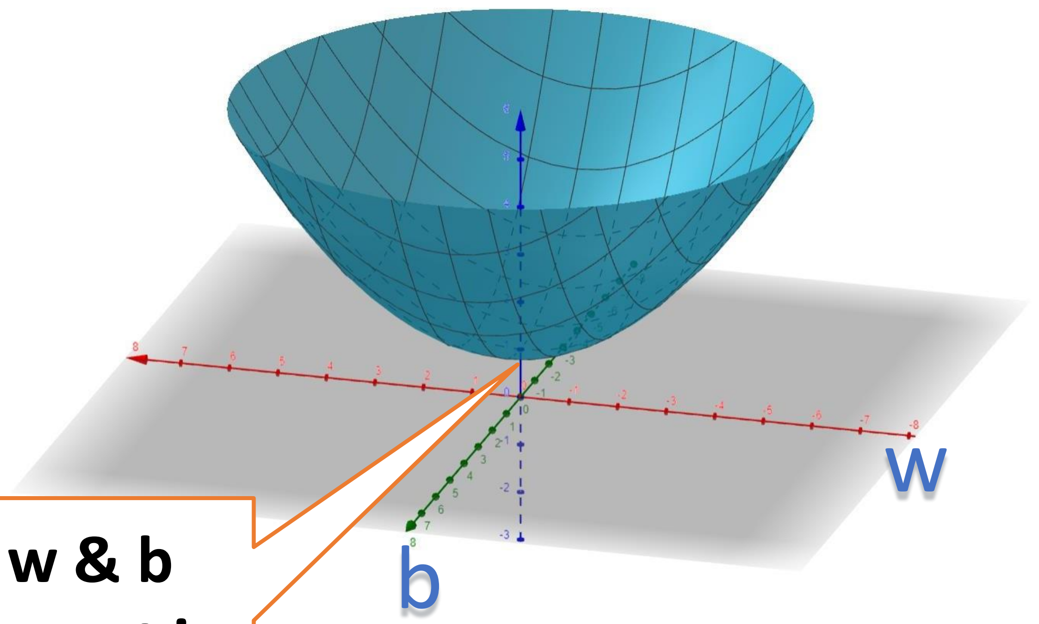
Also called cost / loss function

Feature space versus parameter space

$$\hat{y} = wx + b$$

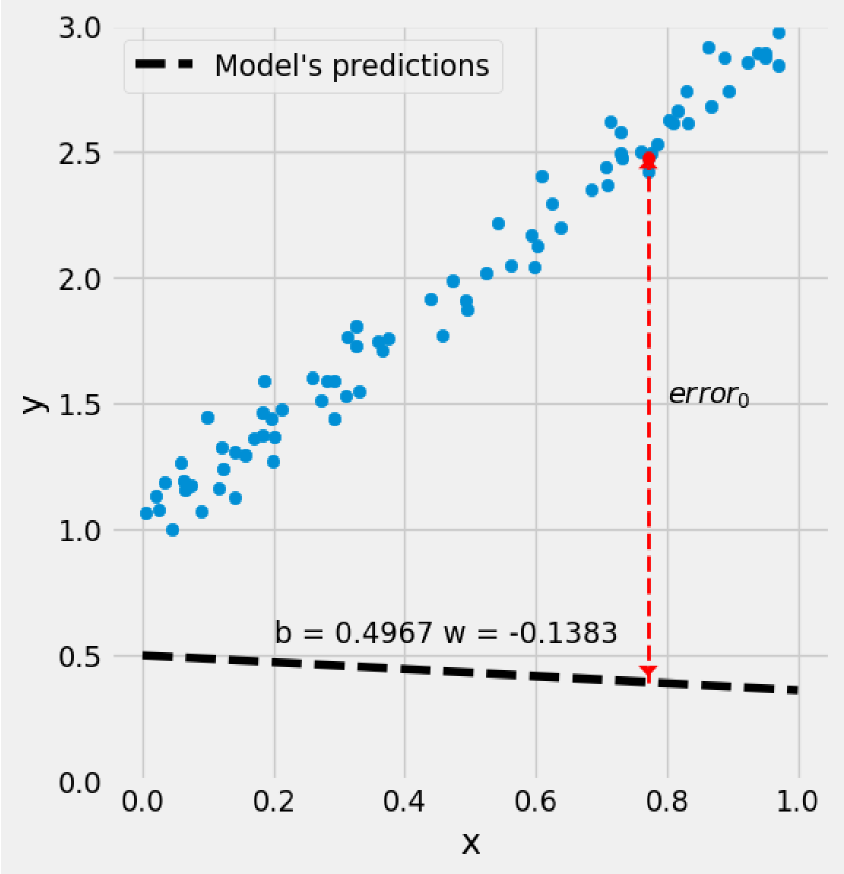
$$\mathcal{J}(w,b) = \frac{1}{n}\sum_{i=1}^{n}\left(b + wx^{(i)} - y^{(i)}\right)^2$$





Find the w & b when the cost is minimum

Step 2. Evaluate y-hat & evaluate objective function

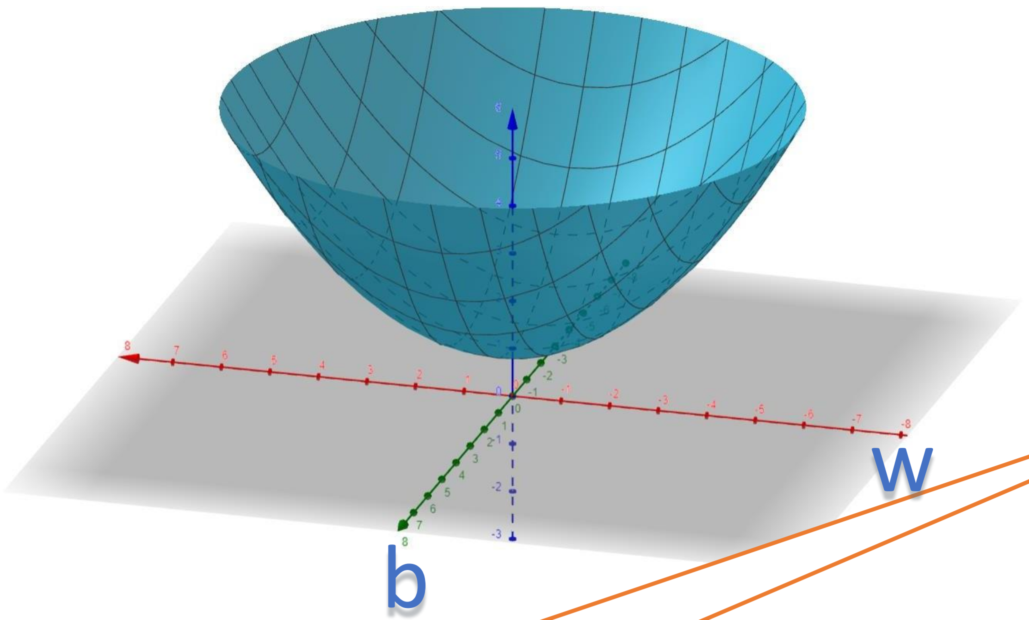


- Evaluate y-hat $\hat{y} = wx + b$
- Evaluate objective function (also known as forward pass)

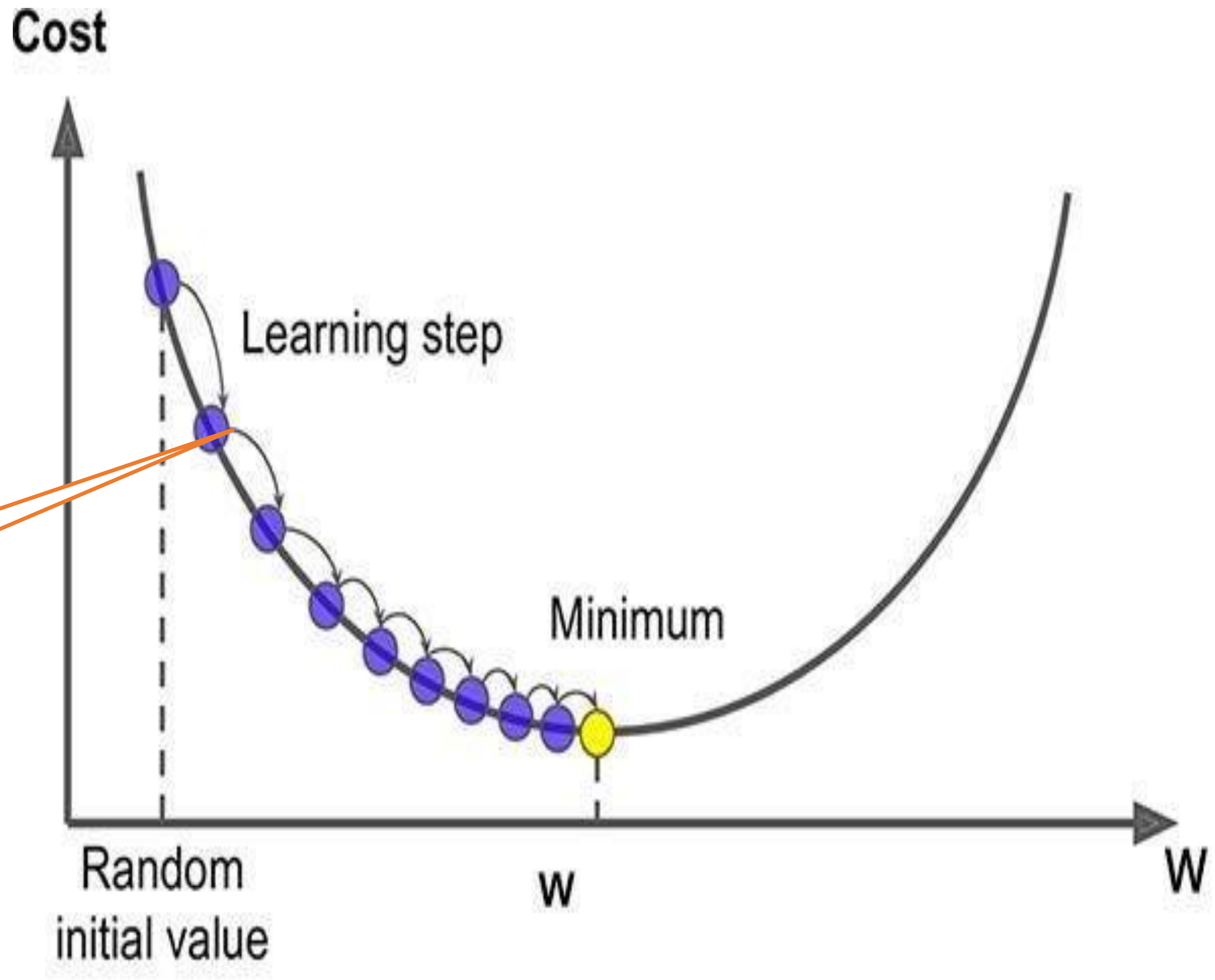
$$\mathcal{J}(w,b) = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{y}^{(i)} - y^{(i)}\right)^2$$

$$\mathcal{J}(w,b) = \frac{1}{n}\sum_{i=1}^{n}\left(b + wx^{(i)} - y^{(i)}\right)^2$$

Objective function plot



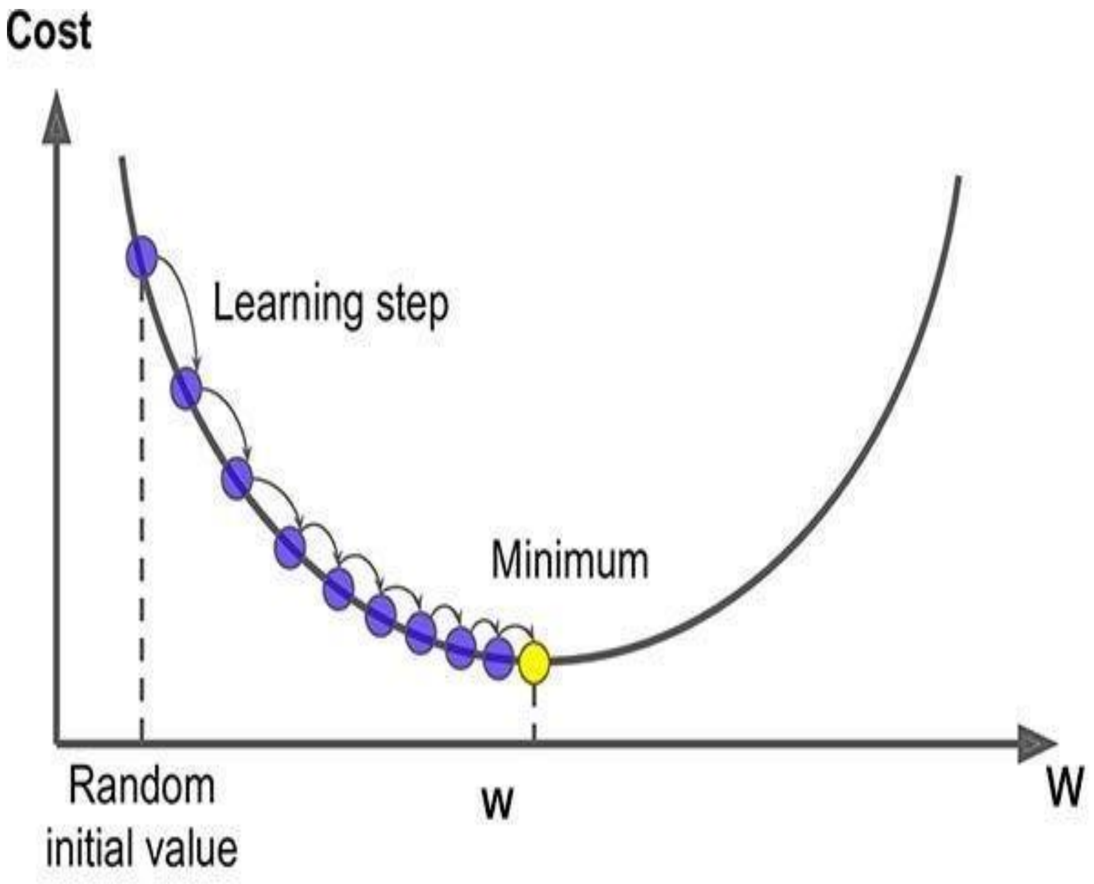
Take a little step towards lowest cost.
1. How little?
2. Which direction?



Step 3. Calculate analytical gradients

$$\mathcal{J}(w,b) = \frac{1}{n}\sum_{i=1}^{n}\left(b + wx^{(i)} - y^{(i)}\right)^2$$

$$\mathcal{J}(w,b) = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{y}^{(i)} - y^{(i)}\right)^2$$



Calculate gradient for m sample steps

$$\frac{\partial \mathcal{J}}{\partial b} = \frac{1}{m}\sum_{i=1}^{m} 2(b + wx^{(i)} - y^{(i)})$$

$$\frac{\partial \mathcal{J}}{\partial w} = \frac{1}{m}\sum_{i=1}^{m} 2x^{(i)}(b + wx^{(i)} - y^{(i)})$$

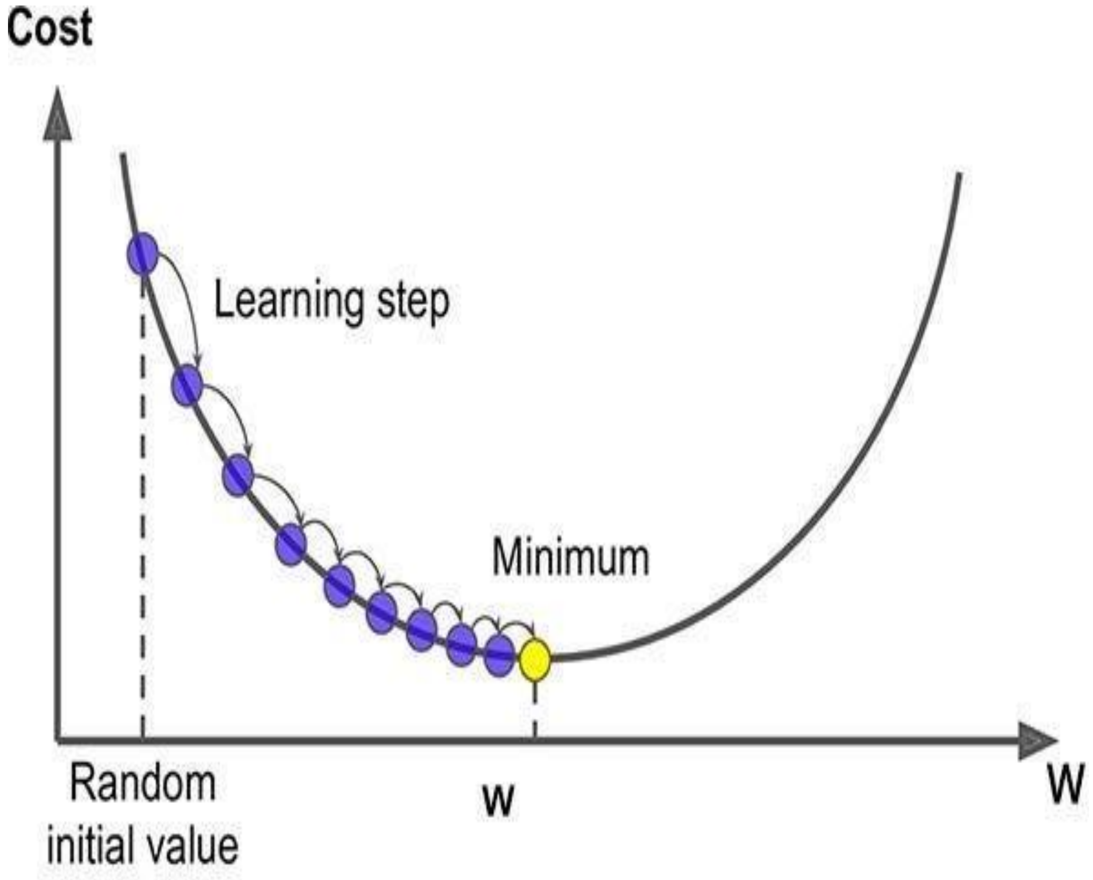
Step 4. Perform numerical gradient descent

- Backward pass – update b and w

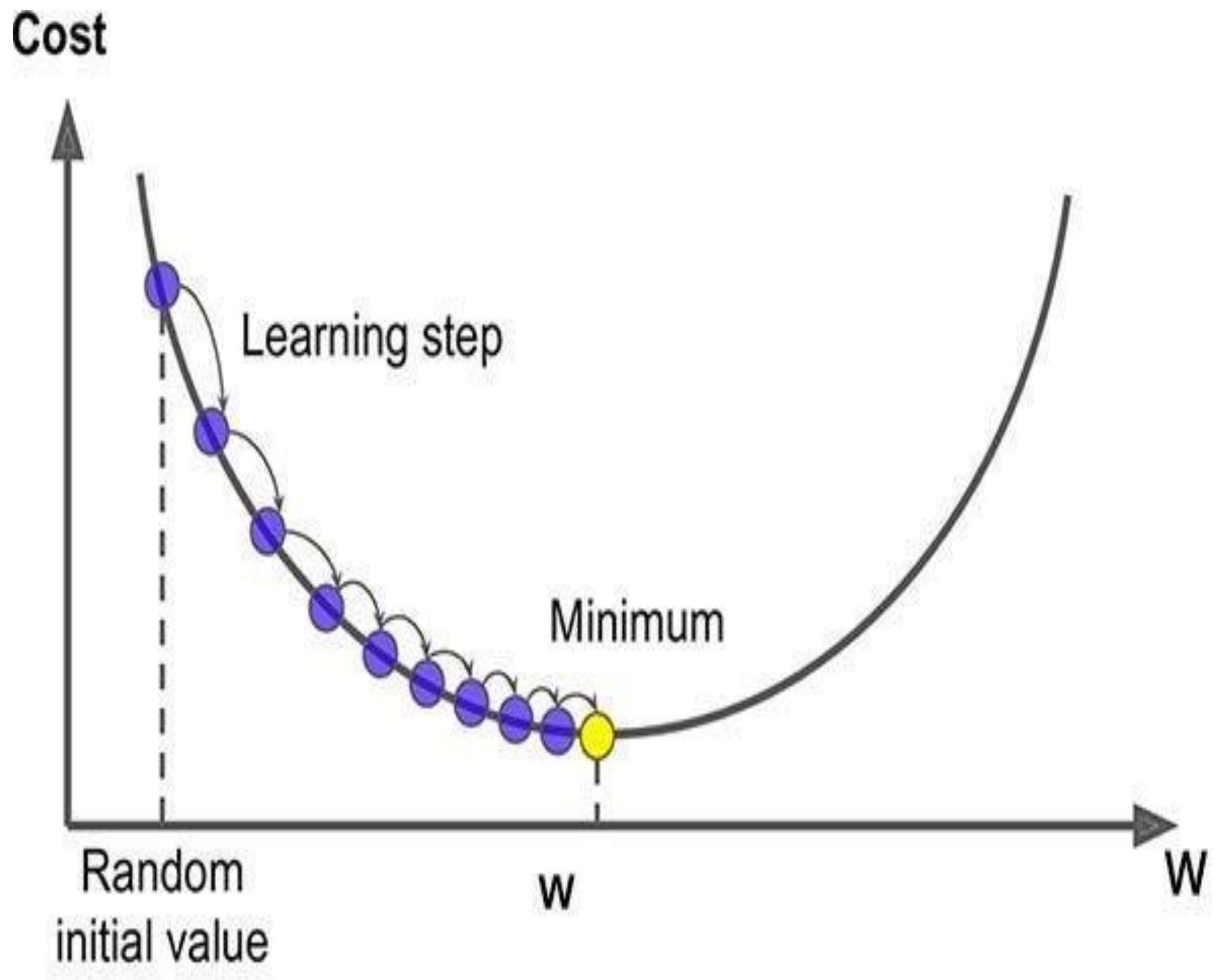
$$b = b - \eta \frac{\partial \mathcal{J}}{\partial b}$$

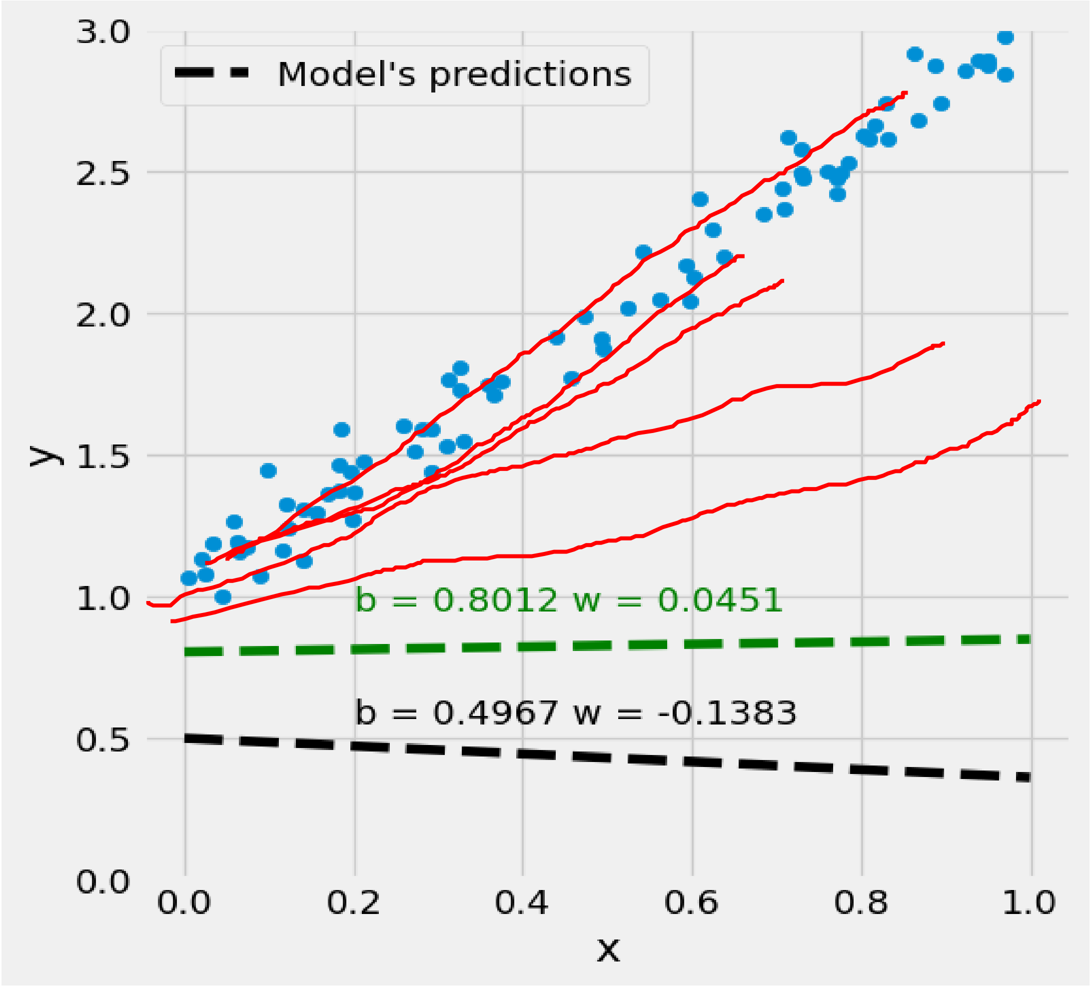
$$w = w - \eta \frac{\partial \mathcal{J}}{\partial w}$$

- Repeat Step 2, 3, 4

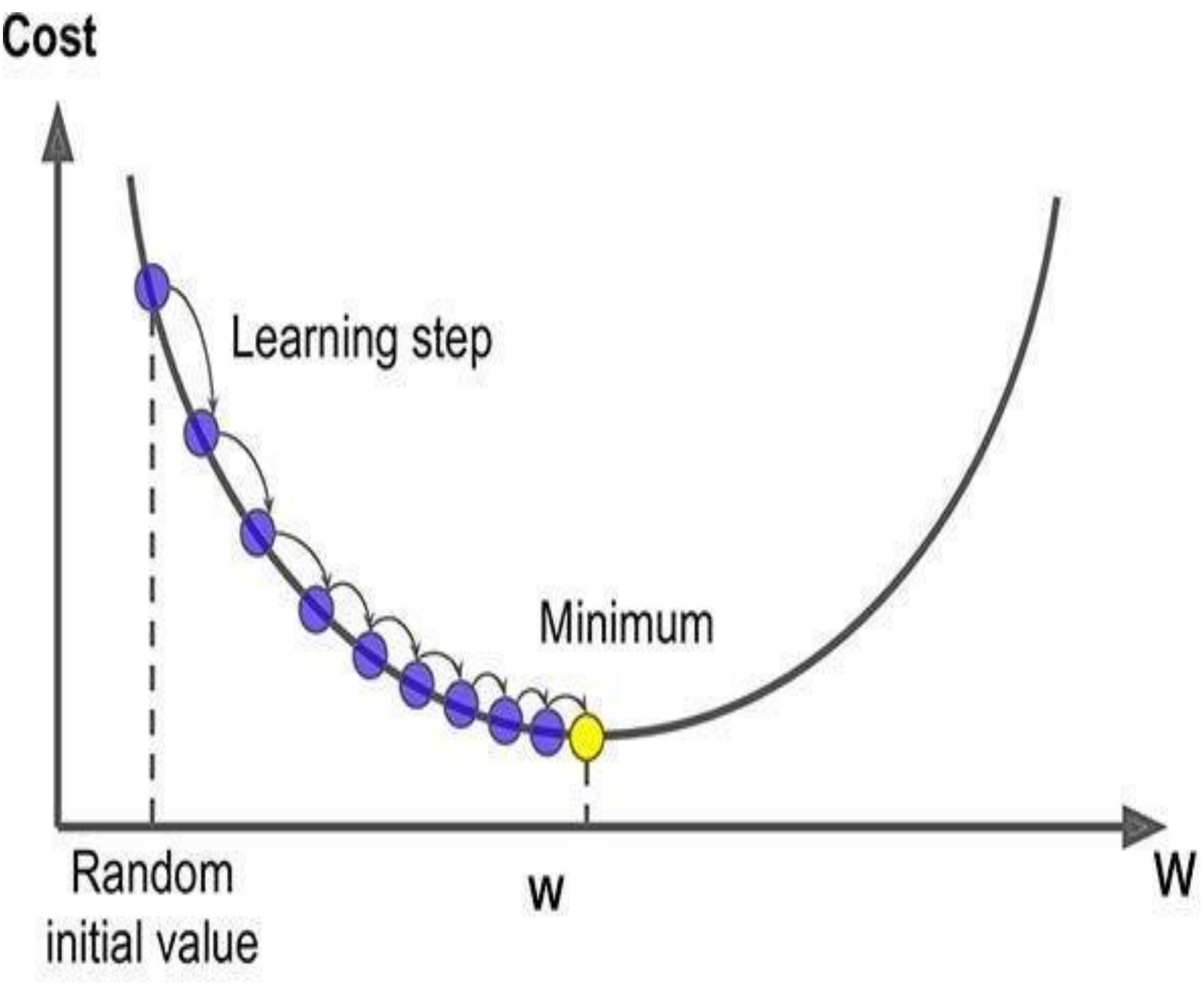


Change in w and b with gradient descent





Gradient descent summary



Iteration	w	b	Cost	dJ/dw	dJ/db
0	0.0110	0.0195	2.0443	-1.1077	-1.9538
1000	1.4309	1.2985	0.0178	-0.0407	0.02078
2000	1.7162	1.1527	0.0071	-0.0191	0.00976
3000	1.8502	1.0842	0.0047	-0.0090	0.00459
4000	1.9132	1.0520	0.0042	-0.0042	0.00215
5000	1.9430	1.0369	0.0041	-0.0020	0.00101
6000	1.9567	1.0298	0.0040	-0.0009	0.00047
7000	1.9632	1.0265	0.0040	-0.0004	0.00022
8000	1.9663	1.0249	0.0040	-0.0002	0.00010
9000	1.9677	1.0242	0.0040	-9.637e-05	4.925e-05

Example-1

Number of hours studied (x) IDV	**Marks scored (y) DV**	$\boldsymbol{X^2}$	$\boldsymbol{x.y}$	$(\hat{y} - y)^2$
2	**75**			
3	**82**			
4	**93**			
5	**89**			
6	**98**			

Example-1

$$\hat{y} = b_0 + b_1 * x$$

$$b_1 = \frac{\overline{xy} - \overline{x} * \overline{y}}{\overline{x^2} - (\overline{x})^2}$$

$$b_0 = \overline{y} - b_1 * \overline{x}$$

$$standard\ error = \sqrt{\frac{\sum(\hat{y} - y)^2}{n - 2}}$$

Assignment: Obtain a line that best fit the sample data given in the table. Evaluate the model by finding the standard error.

(x) IDV	Marks scored (y) DV		
1	2		
2	4		
3	5		
4	4		
5	5		

$$standard\ error = \sqrt{\frac{\sum(\hat{y} - y)^2}{n - 2}}$$

Thank you