

ε = 0.01

$$w_1 = 0 \quad w_2 = 0 \quad b = 0$$

$\eta = 1$ learning rate

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 threshold $\theta = 1$

$$y_{in} = w_1 x_1 + w_2 x_2 + b$$

$$0 + 0 + 0 = 0 \neq 1 \Rightarrow -1$$

$$w_1 x_1 + w_2 x_2 : w_{1new} = w_{1old} + \eta t * x_1$$

$$0 + 1 * 1 * 1 = 1$$

$$w_{2new} = w_{2old} + \eta t * x_2$$

$$0 + 1 * 1 * 1 = 1$$

$$b_{new} = b_{old} + \eta t$$

$$0 + 1 * 1 = 1$$

$$w_1 = w_2 = b = 1$$

$$x_1 = 1 \quad x_2 = 1 \quad t = 1$$

$$w_1 = 1 \quad w_2 = 1 \quad b = 1$$

$$y_{in} = x_1 w_1 + x_2 w_2 + b \Rightarrow 1 * 1 + 1 * 1 + 1 = 3 \neq 1 \Rightarrow \checkmark$$

$$y_{in} = t$$

$$x_1 = 1 \quad x_2 = -1 \quad t = 1$$

$$w_1 = 1 \quad w_2 = 1 \quad b = 1$$

$$y_{in} = x_1 w_1 + x_2 w_2 + b \Rightarrow (1 * 1) + (-1 * 1) + 1 = 1 \Rightarrow \checkmark$$

$$y_{in} = t$$

$$x_1 = 1 \quad x_2 = 1 \quad t = 1$$

$$w_1 = 1 \quad w_2 = 1 \quad b = 1$$

$$y_{in} = x_1 w_1 + x_2 w_2 + b \Rightarrow (-1 * 1) + (1 * 1) + 1 = 1 \Rightarrow \checkmark$$

$$y_{in} = t$$

$$x_1 = -1 \quad x_2 = -1 \quad t = 1$$

$$w_1 = 1 \quad w_2 = 1 \quad b = 1$$

$$y_{in} = x_1 w_1 + x_2 w_2 + b \Rightarrow -1 * 1 + -1 * 1 + 1 = -1 \Rightarrow \checkmark$$

$$y_{in} = t$$

$$w_1 + w_2 + b = x_1 + x_2 + 1 \Rightarrow x_1 + x_2 = 1$$

NAND.

$$x_1, x_2 \text{ target } y_{in} = w_1 x_1 + w_2 x_2 + b$$

x_1, x_2 target

$$y_{in} = w_1 x_1 + w_2 x_2 + b$$

$$\begin{array}{ccc} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \end{array} \Rightarrow \begin{array}{l} (1,1) \Rightarrow 1 \cdot 0 + 1 \cdot 0 + 0 = 0 \\ \left\{ \begin{array}{l} -1 y_{in} \leq 1 \\ 1 y_{in} \geq 1 \end{array} \right\} \quad y_{in} \leq 1 \\ -1 = -1 \end{array}$$

$$(1, -1) \Rightarrow 1 \cdot 1 + 1 \cdot -1 + 0 = 0 \quad y(0) = -1 \neq t = -1$$

update weights.

$$w_{1, \text{new}} = w_{1, \text{old}} + \Delta t x_1 \quad w_{2, \text{new}} = w_{2, \text{old}} + \Delta t x_2$$

$$\Rightarrow w_{1, \text{new}} = 0 + 1 \cdot 1 = 1 \quad 0 + 1 \cdot -1 = -1$$

$$b_{\text{new}} = b_{\text{old}} + \Delta t \Rightarrow 0 + 1 \cdot 1 = 1$$

$$(1, 1) \Rightarrow y_{in} = w_1 x_1 + w_2 x_2 + b$$

$$1 \cdot 1 + 1 \cdot 1 + 1 = 3 \quad y_{in} = 3 \quad y(3) = 1 = t$$

$$(1, -1) \Rightarrow y_{in} = w_1 x_1 + w_2 x_2 + b$$

$$1 \cdot 1 + 1 \cdot -1 + 1 = 1 \quad y_{in} = 1 \quad y(1) = -1 \neq t = -1$$

$$y(-1) = -1 \neq t = -1$$

update weights.

$$w_{1, \text{new}} = w_{1, \text{old}} + \Delta t x_1 \quad w_{2, \text{new}} = w_{2, \text{old}} + \Delta t x_2$$

$$2 + 1 \cdot 1 = 3 \quad 0 + 1 \cdot -1 = -1$$

$$b_{\text{new}} = b_{\text{old}} + \Delta t \Rightarrow 1 + 1 = 2$$

$$w_1 = 3 \quad w_2 = -1 \quad b = 2$$

$$(1, 1) \Rightarrow y_{in} = w_1 x_1 + w_2 x_2 + b$$

$$y(3) = 1$$

$$1 \cdot 1 + 1 \cdot 1 + 2 = 4 = t = 1$$

$$(-1, -1) \Rightarrow y_{in} = w_1 x_1 + w_2 x_2 + b$$

$$(-1) + (-1) + 2 = 0$$

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$$y_{in}(0) = -1 \neq t = 1$$

$$\text{update weights } w_{1, \text{new}} = w_{1, \text{old}} + \Delta t x_1 \quad w_{2, \text{new}} = w_{2, \text{old}} + \Delta t x_2 \quad b_{\text{new}} = b_{\text{old}} + \Delta t$$

$$\Rightarrow w_{1, \text{new}} = 1 + 1 \cdot 1 = 2 \quad w_{2, \text{new}} = 1 + (-1) \cdot (-1) = 2$$

$$b = 2 + 1 \cdot 1 = 3$$

$$w_1 = 2 \quad w_2 = 2 \quad b = 3$$

$$(-1, -1) \Rightarrow y_{in} = w_1 x_1 + w_2 x_2 + b \Rightarrow 0 - 1 + 0 - 1 + 3 = 1$$

$$\begin{array}{ccc} x_1 \rightarrow 0 & & w_1 = w_2 = b = 0. \quad \Delta t = 1 \quad t = 1 \\ x_2 \rightarrow 0 & \text{AND} & y_{in} = w_1 x_1 + w_2 x_2 + b \end{array}$$

$$0 \cdot 1 + 0 \cdot 1 + 0 = y_{in} = 0 \neq t = 1$$

Input x_1, x_2, t_1, t_2 update weights.

$$\begin{array}{ccc} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \end{array} \quad \begin{array}{l} w_{1, \text{new}} = w_{1, \text{old}} + \Delta t x_1 \quad w_{2, \text{new}} = w_{2, \text{old}} + \Delta t x_2 \\ = 0 + 1 \cdot 1 = 1 \quad = 0 + 1 \cdot 1 = 1 \\ b_{\text{new}} = b_{\text{old}} + \Delta t \Rightarrow 0 + 1 = 1 \end{array}$$

$$y_{in} = w_1 x_1 + w_2 x_2 + b \Rightarrow w_1 = w_2 = b = 1$$

$$y_{in} = 1 \cdot 1 + 1 \cdot 1 + 1 = 3 \quad y(3) = 1$$

$$x_1 = 1 \quad x_2 = -1 \quad t = -1$$

$$y_{in} = 1 \cdot 1 + (-1) \cdot 1 + 1 = 1$$

$$y(1) = 1 \quad t = -1$$

update weights.

$$x_1 = -1 \quad x_2 = 1 \quad t = -1$$

$$y_{in} = 1 \cdot (-1) + 1 \cdot 1 + 1 = 1$$

$$w_{1, \text{new}} = 1 + 1 \cdot 1 = 2$$

$$w_{2, \text{new}} = 1 + 1 \cdot (-1) = 0$$

$$b_{\text{new}} = 1 + 1 \cdot 1 = 2$$

$$y_{in} = 2 \cdot 1 + 0 \cdot 1 + 2 = 4$$

$$x_1 = -1 \quad x_2 = -1 \quad t = -1$$

$$y_{in} = 1 \cdot (-1) + 1 \cdot (-1) + 2 = 0$$

$$y_1 = 1 \quad w_{11} = 1 \quad b_{11} = 1$$

$$y_2 = -1$$

$$y_3 = -1$$

$$y_4 = -1$$

$$y_{2in} = w_{11} y_1 + b_{11} = 1 \cdot 1 + 1 = 2$$

$$y_{2in} = w_{11} y_1 + b_{11} = 1 \cdot -1 + 1 = 0$$

$$y_{3in} = 1 \cdot -1 + 1 = 0 \quad y_4 = 1 \cdot -1 + 1 = 0$$

$$w_1 = w_2 = b = 1$$

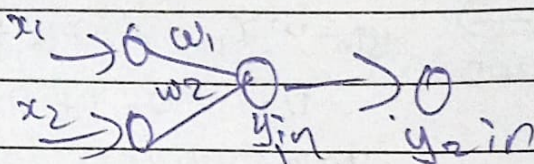
$$w_{11} = b_{11} = 1$$

$$\begin{cases} 1 & y_{in} > 1 \\ -1 & y_{in} \leq 1 \end{cases}$$

$$\begin{cases} 1 & y_{in} \geq 1 \\ -1 & y_{in} < 1 \end{cases}$$

NOR.

x_1	x_2	t_1	t_2
1	1	1	-1
1	-1	-1	-1
-1	1	1	-1
-1	-1	-1	1



$$\alpha = 1 \quad \theta = 1$$

$$\begin{cases} 1 & y_{in} \geq 1 \\ -1 & y_{in} < 1 \end{cases}$$

$$u_{in} = w_1 x_1 + w_2 x_2 + b$$

$$0 + 1 + 0 + 1 + 0 = 0$$

$$w_1 = 0 + 1 \cdot 1 = 1$$

update weight.

$$w_2 = 0 + 1 \cdot 1 = 1$$

$$w_{new} = w_{old} + \alpha t x_1$$

$$b = 0 + 1 \cdot 1 = 1$$

$$w_{2new} = w_{2old} + \alpha t x_2$$

$$w_1 = w_2 = b = 1$$

$$b_{new} = b_{old} + \alpha t$$

$$y_{in} = w_1 x_1 + w_2 x_2 + b \Rightarrow 1 \cdot 1 + 1 \cdot 1 + 1 = 3$$

$$y_{in}(3) = 1 \leq t = 1 \quad \checkmark$$

$$x_1 = 1 \quad x_2 = -1 \quad t = 1 \Rightarrow 1 \cdot 1 + (1 \cdot (-1)) + 1 = 1$$

$$w_1 = 1 \quad w_2 = 1 \quad b = 1$$

$$y_{in}(1) = 1 = t = 1$$

$$y_{in} = x_1 w_1 + x_2 w_2 + b$$

$$x_1 = 1 \quad x_2 = 1 \quad t = 1$$

$$w_1 = 1 \quad w_2 = 1 \quad b = 1$$

$$y_{in} = x_1 w_1 + x_2 w_2 + b$$

$$x_1 = -1 \quad x_2 = -1 \quad t = -1$$

$$w_1 = 1 \quad w_2 = 1$$

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$$1 \cdot 1 + 1 \cdot -1 + 1 = 1 - 1 + 1 = 1$$

$$1 \cdot 1 + 1 \cdot 1 + 1 = 1$$

$$y_{in}(1) = t = 1$$

$$y(-2) = t = -1 \checkmark$$

second gate is not. $y_{in} = w_{11} x_{y1} + b$ $w_{11} = b = 1$

$$\begin{matrix} 1 & -1 \\ -1 & 1 \end{matrix}$$

$$w_1 = w_2 = b = 1 = w_{11} = b_{11} = 1 \checkmark$$

$$\begin{cases} 1 & y_{in} \geq 1 \\ -1 & y_{in} \leq -1 \end{cases}$$

NOT t

$$\begin{matrix} 1 & -1 \\ -1 & 1 \end{matrix}$$

$$y_{in} = w_1 x_1 + b \quad w_1 = b = 1$$

$$1 \cdot 1 + 1 = 2$$

$$y_{in}(2) = 1 = t = 1$$

$$1 \cdot -1 + 1 = 0$$

$$y_{in}(0) = 1 = t = 1$$

$$\begin{cases} 1 & y_{in} \geq 1 \\ -1 & y_{in} \leq -1 \end{cases}$$

... in the artificial