which can be generalized to give

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum\sum_{i<j} P(A_i A_j) + \sum\sum\sum_{i<j<k} P(A_i A_j A_k)$$
$$+ \cdots + (-1)^{n+1} P(A_1 \cdots A_n)$$

If S is finite and each one point set is assumed to have equal probability, then

$$P(A) = \frac{|A|}{|S|}$$

where $|E|$ denotes the number of outcomes in the event E.

$P(A)$ can be interpreted either as a long-run relative frequency or as a measure of one's degree of belief.

PROBLEMS

1. A box contains 3 marbles: 1 red, 1 green, and 1 blue. Consider an experiment that consists of taking 1 marble from the box and then replacing it in the box and drawing a second marble from the box. Describe the sample space. Repeat when the second marble is drawn without replacing the first marble.

2. In an experiment, die is rolled continually until a 6 appears, at which point the experiment stops. What is the sample space of this experiment? Let E_n denote the event that n rolls are necessary to complete the experiment. What points of the sample space are contained in E_n? What is $\left(\bigcup_1^{\infty} E_n\right)^c$?

3. Two dice are thrown. Let E be the event that the sum of the dice is odd, let F be the event that at least one of the dice lands on 1, and let G be the event that the sum is 5. Describe the events $EF, E \cup F, FG, EF^c$, and EFG.

4. A, B, and C take turns flipping a coin. The first one to get a head wins. The sample space of this experiment can be defined by

$$S = \left\{ \begin{array}{l} 1, 01, 001, 0001, \ldots, \\ 0000 \cdots \end{array} \right.$$

(a) Interpret the sample space.
(b) Define the following events in terms of S:
   (i) A wins $= A$.
   (ii) B wins $= B$.
   (iii) $(A \cup B)^c$.
   Assume that A flips first, then B, then C, then A, and so on.

5. A system is comprised of 5 components, each of which is either working or failed. Consider an experiment that consists of observing the status of

each component, and let the outcome of the experiment be given by the vector $(x_1, x_2, x_3, x_4, x_5)$, where x_i is equal to 1 if component i is working and is equal to 0 if component i is failed.

(a) How many outcomes are in the sample space of this experiment?
(b) Suppose that the system will work if components 1 and 2 are both working, or if components 3 and 4 are both working, or if components 1, 3, and 5 are all working. Let W be the event that the system will work. Specify all the outcomes in W.
(c) Let A be the event that components 4 and 5 are both failed. How many outcomes are contained in the event A?
(d) Write out all the outcomes in the event AW.

6. A hospital administrator codes incoming patients suffering gunshot wounds according to whether they have insurance (coding 1 if they do and 0 if they do not) and according to their condition, which is rated as good (g), fair (f), or serious (s). Consider an experiment that consists of the coding of such a patient.

(a) Give the sample space of this experiment.
(b) Let A be the event that the patient is in serious condition. Specify the outcomes in A.
(c) Let B be the event that the patient is uninsured. Specify the outcomes in B.
(d) Give all the outcomes in the event $B^c \cup A$.

7. Consider an experiment that consists of determining the type of job—either blue-collar or white-collar—and the political affiliation—Republican, Democratic, or Independent—of the 15 members of an adult soccer team. How many outcomes are

(a) in the sample space?
(b) in the event that at least one of the team members is a blue-collar worker?