PROBLEMS

3.1. Two fair dice are rolled. What is the conditional probability that at least one lands on 6 given that the dice land on different numbers?

3.2. If two fair dice are rolled, what is the conditional probability that the first one lands on 6 given that the sum of the dice is i? Compute for all values of i between 2 and 12.

3.3. Use Equation (2.1) to compute, in a hand of bridge, the conditional probability that East has 3 spades given that North and South have a combined total of 8 spades.

3.4. What is the probability that at least one of a pair of fair dice lands on 6, given that the sum of the dice is $i, i = 2, 3, \ldots, 12$?

3.5. An urn contains 6 white and 9 black balls. If 4 balls are to be randomly selected without replacement, what is the probability that the first 2 selected are white and the last 2 black?

3.6. Consider an urn containing 12 balls, of which 8 are white. A sample of size 4 is to be drawn with replacement (without replacement). What is the conditional probability (in each case) that the first and third balls drawn will be white given that the sample drawn contains exactly 3 white balls?

3.7. The king comes from a family of 2 children. What is the probability that the other child is his sister?

3.8. A couple has 2 children. What is the probability that both are girls if the older of the two is a girl?

3.9. Consider 3 urns. Urn A contains 2 white and 4 red balls, urn B contains 8 white and 4 red balls, and urn C contains 1 white and 3 red balls. If 1 ball is selected from each urn, what is the probability that the ball chosen from urn A was white given that exactly 2 white balls were selected?

3.10. Three cards are randomly selected, without replacement, from an ordinary deck of 52 playing cards. Compute the conditional probability that the first card selected is a spade given that the second and third cards are spades.

3.11. Two cards are randomly chosen without replacement from an ordinary deck of 52 cards. Let B be the event that both cards are aces, let A_s be the event that the ace of spades is chosen, and let A be the event that at least one ace is chosen. Find

(a) $P(B|A_s)$

(b) $P(B|A)$

3.12. A recent college graduate is planning to take the first three actuarial examinations in the coming summer. She will take the first actuarial exam in June. If she passes that exam, then she will take the second exam in July, and if she also passes that one, then she will take the third exam in September. If she fails an exam, then she is not allowed to take any others. The probability that she passes the first exam is .9. If she passes the first exam, then the conditional probability that she passes the second one is .8, and if she passes both the first and the second exams, then the conditional probability that she passes the third exam is .7.

(a) What is the probability that she passes all three exams?

(b) Given that she did not pass all three exams, what is the conditional probability that she failed the second exam?

3.13. Suppose that an ordinary deck of 52 cards (which contains 4 aces) is randomly divided into 4 hands of 13 cards each. We are interested in determining p, the probability that each hand has an ace. Let E_i be the event that the ith hand has exactly one ace. Determine $p = P(E_1E_2E_3E_4)$ by using the multiplication rule.

3.14. An urn initially contains 5 white and 7 black balls. Each time a ball is selected, its color is noted and it is replaced in the urn along with 2 other balls of the same color. Compute the probability that

(a) the first 2 balls selected are black and the next 2 are white;

(b) of the first 4 balls selected, exactly 2 are black.

3.15. An ectopic pregnancy is twice as likely to develop when the pregnant woman is a smoker as it is when she is a nonsmoker. If 32 percent of women of childbearing age are smokers, what percentage of women having ectopic pregnancies are smokers?

3.16. Ninety-eight percent of all babies survive delivery. However, 15 percent of all births involve Cesarean (C) sections, and when a C section is performed, the baby survives 96 percent of the time. If a randomly chosen pregnant woman does not have a C section, what is the probability that her baby survives?

3.17. In a certain community, 36 percent of the families own a dog and 22 percent of the families that own a dog also own a cat. In addition, 30 percent of the families own a cat. What is

(a) the probability that a randomly selected family owns both a dog and a cat?

(b) the conditional probability that a randomly selected family owns a dog given that it owns a cat?

3.18. A total of 46 percent of the voters in a certain city classify themselves as Independents, whereas 30 percent classify themselves as Liberals and 24 percent say that they are Conservatives. In a recent local election, 35 percent of the Independents, 62 percent of the Liberals, and 58 percent of the Conservatives voted. A voter is chosen at random.

Given that this person voted in the local election, what is the probability that he or she is

(a) an Independent?

(b) a Liberal?

(c) a Conservative?

(d) What fraction of voters participated in the local election?

3.19. A total of 48 percent of the women and 37 percent of the men that took a certain “quit smoking” class remained nonsmokers for at least one year after completing the class. These people then attended a success party at the end of a year. If 62 percent of the original class was male,

(a) what percentage of those attending the party were women?

(b) what percentage of the original class attended the party?

3.20. Fifty-two percent of the students at a certain college are females. Five percent of the students in this college are majoring in computer science. Two percent of the students are women majoring in computer science. If a student is selected at random, find the conditional probability that

(a) the student is female given that the student is majoring in computer science;

(b) this student is majoring in computer science given that the student is female.

3.21. A total of 500 married working couples were polled about their annual salaries, with the following information resulting:

	Husband	
Wife	Less than \$25,000	More than \$25,000
Less than \$25,000	212	198
More than \$25,000	36	54

For instance, in 36 of the couples, the wife earned more and the husband earned less than \$25,000. If one of the couples is randomly chosen, what is

(a) the probability that the husband earns less than \$25,000?

(b) the conditional probability that the wife earns more than \$25,000 given that the husband earns more than this amount?

(c) the conditional probability that the wife earns more than \$25,000 given that the husband earns less than this amount?

3.22. A red die, a blue die, and a yellow die (all six sided) are rolled. We are interested in the probability that the number appearing on the blue die is less than that appearing on the yellow die, which is less than that appearing on the red die. That is, with B, Y, and R denoting, respectively, the number appearing on the blue, yellow, and red die, we are interested in $P(B < Y < R)$.

(a) What is the probability that no two of the dice land on the same number?

(b) Given that no two of the dice land on the same number, what is the conditional probability that $B < Y < R$?

(c) What is $P(B < Y < R)$?

3.23. Urn I contains 2 white and 4 red balls, whereas urn II contains 1 white and 1 red ball. A ball is randomly chosen from urn I and put into urn II, and a ball is then randomly selected from urn II. What is

(a) the probability that the ball selected from urn II is white?

(b) the conditional probability that the transferred ball was white given that a white ball is selected from urn II?

3.24. Each of 2 balls is painted either black or gold and then placed in an urn. Suppose that each ball is colored black with probability $\frac{1}{2}$ and that these events are independent.

(a) Suppose that you obtain information that the gold paint has been used (and thus at least one of the balls is painted gold). Compute the conditional probability that both balls are painted gold.

(b) Suppose now that the urn tips over and 1 ball falls out. It is painted gold. What is the probability that both balls are gold in this case? Explain.

3.25. The following method was proposed to estimate the number of people over the age of 50 who reside in a town of known population 100,000: “As you walk along the streets, keep a running count of the percentage of people you encounter who are over 50. Do this for a few days; then multiply the percentage you obtain by 100,000 to obtain the estimate.” Comment on this method.

Hint: Let p denote the proportion of people in the town who are over 50. Furthermore, let α_1 denote the proportion of time that a person under the age of 50 spends in the streets, and let α_2 be the corresponding value for those over 50. What quantity does the method suggested estimate? When is the estimate approximately equal to p?

3.26. Suppose that 5 percent of men and .25 percent of women are color blind. A color-blind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females. What if the population consisted of twice as many males as females?

3.27. All the workers at a certain company drive to work and park in the company’s lot. The company is interested in estimating the average number of workers in a car. Which of the following methods will enable the company to estimate this quantity? Explain your answer.

1. Randomly choose n workers, find out how many were in the cars in which they were driven, and take the average of the n values.
2. Randomly choose n cars in the lot, find out how many were driven in those cars, and take the average of the n values.

3.28. Suppose that an ordinary deck of 52 cards is shuffled and the cards are then turned over one at a time until the first ace appears. Given that the first ace is the 20th card to appear, what is the conditional probability that the card following it is the
(a) ace of spades?
(b) two of clubs?

3.29. There are 15 tennis balls in a box, of which 9 have not previously been used. Three of the balls are randomly chosen, played with, and then returned to the box. Later, another 3 balls are randomly chosen from the box. Find the probability that none of these balls has ever been used.

3.30. Consider two boxes, one containing 1 black and 1 white marble, the other 2 black and 1 white marble. A box is selected at random, and a marble is drawn from it at random. What is the probability that the marble is black? What is the probability that the first box was the one selected given that the marble is white?

3.31. Ms. Aquina has just had a biopsy on a possibly cancerous tumor. Not wanting to spoil a weekend family event, she does not want to hear any bad news in the next few days. But if she tells the doctor to call only if the news is good, then if the doctor does not call, Ms. Aquina can conclude that the news is bad. So, being a student of probability, Ms. Aquina instructs the doctor to flip a coin. If it comes up heads, the doctor is to call if the news is good and not call if the news is bad. If the coin comes up tails, the doctor is not to call. In this way, even if the doctor doesn't call, the news is not necessarily bad. Let α be the probability that the tumor is cancerous; let β be the conditional probability that the tumor is cancerous given that the doctor does not call.
(a) Which should be larger, α or β?
(b) Find β in terms of α, and prove your answer in part (a).

3.32. A family has j children with probability p_j, where $p_1 = .1, p_2 = .25, p_3 = .35, p_4 = .3$. A child from this family is randomly chosen. Given that this child is the eldest child in the family, find the conditional probability that the family has
(a) only 1 child;
(b) 4 children.
Redo (a) and (b) when the randomly selected child is the youngest child of the family.

3.33. On rainy days, Joe is late to work with probability .3; on nonrainy days, he is late with probability .1. With probability .7, it will rain tomorrow.
(a) Find the probability that Joe is early tomorrow.
(b) Given that Joe was early, what is the conditional probability that it rained?

3.34. In Example 3f, suppose that the new evidence is subject to different possible interpretations and in fact shows only that it is 90 percent likely that the criminal possesses the characteristic in question. In this case, how likely would it be that the suspect is guilty (assuming, as before, that he has the characteristic)?

3.35. With probability .6, the present was hidden by mom; with probability .4, it was hidden by dad. When mom hides the present, she hides it upstairs 70 percent of the time and downstairs 30 percent of the time. Dad is equally likely to hide it upstairs or downstairs.
(a) What is the probability that the present is upstairs?
(b) Given that it is downstairs, what is the probability it was hidden by dad?

3.36. Stores A, B, and C have 50, 75, and 100 employees, respectively, and 50, 60, and 70 percent of them respectively are women. Resignations are equally likely among all employees, regardless of sex. One woman employee resigns. What is the probability that she works in store C?

3.37. **(a)** A gambler has a fair coin and a two-headed coin in his pocket. He selects one of the coins at random; when he flips it, it shows heads. What is the probability that it is the fair coin?
(b) Suppose that he flips the same coin a second time and, again, it shows heads. Now what is the probability that it is the fair coin?
(c) Suppose that he flips the same coin a third time and it shows tails. Now what is the probability that it is the fair coin?

3.38. Urn A has 5 white and 7 black balls. Urn B has 3 white and 12 black balls. We flip a fair coin. If the outcome is heads, then a ball from urn A is selected, whereas if the outcome is tails, then a ball from urn B is selected. Suppose that a white ball is selected. What is the probability that the coin landed tails?

3.39. In Example 3a, what is the probability that someone has an accident in the second year given that he or she had no accidents in the first year?

3.40. Consider a sample of size 3 drawn in the following manner: We start with an urn containing 5 white