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Big Data Analytics

Applied probability and statistics
Assignment problem set-1

1) i) There are no restrictions.

$n=10 \quad r=7$

$^{10}C_7 \quad \binom{10}{7} \Rightarrow \frac{10!}{7!(3!)}$

ii) Answer exactly 2 of the last 4 : total to answer = 7

from last 4 ques. $^4C_2 \times {}^6C_5 \rightarrow$ from first 6 ori

$\binom{4}{2} \cdot \binom{6}{5}$

iii) Answer exactly 2 of the first 6 :-

$^6C_2 \times {}^4C_5 \rightarrow$ not possible selecting 5 out of 4 question so zero.

$\binom{6}{2}\binom{4}{5} \quad \frac{6!}{2!(4!)} \times 0$

iv) Answer atleast 3 of the first 5 question.

$({}^5C_3 \times {}^5C_4) + ({}^5C_4 \times {}^5C_3) + ({}^5C_5 \times {}^5C_2) \Rightarrow \binom{5}{3}\binom{5}{4} + \binom{5}{4}\binom{5}{3} + \binom{5}{5}\binom{5}{2}$

2) $r = 8$ (Identical) $n = 4$.

i) no of divisions are (for formula)

$$\binom{n+r-1}{r} = \binom{4+8-1}{8} = \binom{11}{8} \quad {}^{11}C_8$$

ii) If at least one blackboard to each school.

$$\binom{r-1}{n-1} = \binom{8-1}{4-1} = \binom{7}{3} \quad {}^7C_3$$

3) 9 computer - - - - - - - - - - - 3 → PC 4 → MAC 2 → Lin.

i) In how many distinguishable ways (n distinct object into k unlabelled groups)

$\frac{9!}{3!4!2!}$ or ${}^{9}C_{3} \times {}^{6}C_{4} \times {}^{2}C_{2}$ ⇒ $\frac{n!}{r_1!.r_2!..r_k!}$

ii) If the first 5 machines include all 4 MACs.

$\frac{5!}{4! \times 1!} \times \frac{4!}{2! \times 1!}$ or ${}^{5}C_{4} \times {}^{5}C_{3} \times {}^{2}C_{2}$

MAC PC Lin ↑PC ↑Linux

iii) If 2 PCs must be in the first three and 1 PC must be in last three.

$\underset{\text{first 3}}{{}^{3}C_{2}} \times \underset{\text{last 3}}{{}^{3}C_{1}} \times \underbrace{{}^{6}C_{4} \times {}^{2}C_{2}}_{\text{rest}}$

4) 60 → students → n · 3 members, lit, cod, docum → r

i) no restrictions

${}^{n}C_{r}$ ⇒ ${}^{60}C_{3}$ ⇒ $\binom{60}{3}$

ii) Two students will not work together.

Tot no of selection = no. of selections in which 2 students are together + no of selutions in which 2 students are not together

⇒ $\left({}^{58}C_{1} \times 3!\right) + \left({}^{58}C_{3} \times 3!\right)$ 3 ways to assign the roles in team.

two already selected two eliminated

iii) Two students will work together or not at all.

$\left({}^{58}C_{1} \times 3!\right)$ → 2 are together.

iv) one student must be in the team.

$\left({}^{59}C_{2} \times 3!\right)$ → select 1 already and select from rest and 3! ways of assign roles.

v) One student can only do coding.

$\left({}^{59}C_{2} \times 2!\right)$ → one position set for coding for 1 student.

5) 100 → weights A → 10 B → 12 C → 12 D & E combined (4)

total be 38 assign 38 from 100 D (0, 1, 2, 3, 4) E (0, 1, 2, 3, 4) } 5 ways.

$100 - 38 = r^*$, $n = 5$

r^* where D & E values changes.

$$\binom{5 + \sum_{j=1}^{5} 62_j - 1}{62_j}$$

where it changes for values of D & E

6) 11 → soccer players

11 → 3F 3M 4d 1 goalie.

2 → can play goalie or any.

4 → only for or midfield.

5 → only def or midfield.

no of possible ways to divide the team.

from hint.

$$^{2}C_{1} * \left(^{6}C_{4} * {}^{5}C_{3} * {}^{3}C_{3} \right)$$

7) r → Identitical, souvlees requests among n → server. $r \geq (m_1 + m_2 + \dots + m_n)$

if suppose 10 request 2 request given among 3 servers. then $10 - 6 = 4$ rest for his new r^* and how to divide among the servers.

Here we have m_n request to be ~~reserved~~ served for n servers.

so the formula. updated r values for this it is

$$\binom{n + r - 1}{r} \Rightarrow \binom{n + r^* - 1}{r^*} \qquad \binom{n + \sum_{i=1}^{r} m_i - 1}{\sum_{i=1}^{7} m_i}$$

8) only one up or down event values. by considering

n_u -> no of up steps n_D -> no of down steps.

① -> $n_u + n_D = n$ -> total no of steps.

② -> $n_u - n_D = k$ -> position of the points.

from eq ① & ② upon adding them we get.

$$2n_u = n+k \Rightarrow n_u = \frac{n+k}{2}$$

from eq ① & ② upon substracting them we get

$$2n_D = n-k \Rightarrow n_D = \frac{n-k}{2}$$

for the given graph we leave from origin to origin.

6 steps up up down up down down. by this it reach origin here we have 3 up 3 down from eq ① & ② we

① -> $\underset{n_u + n_D}{3+3} = \underset{= n}{6} \Rightarrow n$ ② -> $\underset{n_U\ n_D}{3-3} = \underset{k}{0}$.

from $\binom{n}{\frac{n+k}{2}} \Rightarrow \binom{6}{\frac{6+0}{2}} \Rightarrow \binom{6}{3} \Rightarrow {}^6C_3 \Rightarrow 20$ ways

for this steps in 20 ways the partical can move.

9) a) here the sample space is n^r where n is either 0 or 1 and r is 5 judges where in we have $n^r \Rightarrow 2^5 = 32$ outcomes in the sample spaces.

where we consider two examples $D_1 = (1,1,1,0,0)$ where 3 out of 5 judges have given the correct decision hence court give the court decision, $D_2 = (0,0,1,0,1)$ where 3 outs have give the court decision. so court have given incorrect decision

5 judges gave incorrect decision.

b) $n^r = n=2\ r=5\ n^r \Rightarrow 2^5 = 32$ outcomes in samplespace

c) $({}^5C_3 + {}^5C_4 + {}^5C_5)$ where in the $P(E)$ is court incorrect decision where in atleast 3 judges go wrong.

d) The events of interest are not equally likely and the $P(E)$ or $n(E)$ is calculated as $n(E)/n$ because each judge give independent decisions.

10) a) i) Sample spaces : S = {(A1, A2).. (A1, B1) (A1, B10)
{(A2 A3).. (A1, B1) (A1, B10) }

Event of interest E = {(A1, B1) (A1 B10), (A2 .. B1) (A2 B10)}

Total no of ways = $^2C_1 * {}^{18}C_1$ => likelihood of one occuring.

$\frac{1}{^2C_1 \times {}^{18}C_1}$ select 1 from bottle & 1 from pile where it should be from bottle b => $P(E) = \frac{1}{^2C_1 * {}^{18}C_1} * {}^2C_1 * {}^{10}C_1 \doteq 0.5$

ii) selecting both from the pile where omit the two tablets from bottle A is $\frac{1}{^{18}C_2}$ => likelihood and select 2 out 8 from type A and 2 out of 10 from type B = $P(E) = \frac{1}{^{18}C_2} * {}^8C_1 * {}^{10}C_1$

b) Selecting all the tablets from the pile so 20 tablets in pile. where likelihood one selection is $\frac{1}{^{20}C_2}$ and for select 1 from A and selecting 1 from B it is $\frac{1}{^{20}C_2} * {}^{10}C_1 * {}^{10}C_1 = 0.53$.

is the probability.

11) 28% => cigarettes => 0.28 => P(A)

7% => cigares => 0.07 => P(B)

5% => both C & C => 0.05 => $P(A \cap B)$

i) $(A \cup B)^c$ => the chosen person will not smoke cigarettes or cigar.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ => 0.28 + 0.07 - 0.05

= 0.3.

for $P(A \cup B)^c$ => not/compliment

$P(A \cup B)^c = 1 - P(A \cup B)$

1 - 0.3 = 0.7. 70% do not smoke.

ii) $(B \cap A^c)$ => person smoke cigar but not cigarette.

P(B) - P(A and B) = 0.07 - 0.05 = 0.02.

$P(B \cap A^c) = 0.02$..

12) i) single die → 6 outcomes. $n = 6.(1,2,3,4,5,6)$
4 times roll single die $r = 4$.
with replacement order matter. $.n^r = 6^4$.
the posibility of getting 6 out 6 outcomes is $(1/6)$
$n = (1/6) \cdot n^r = (1/6)^4$

ii) two die → 36 outcomes $((1,1) \quad (6,6))$ $n = 36$.
24 times rolls $\therefore r = 24$.
sum of the dies is 12 only ones that $\left(\frac{1}{36}\right) = n.\ r = 24$

$$\left(\frac{1}{36}\right)^{24}.$$

13) total no of residents
$952 + 1050 + 563 + 456 + 2055 + 1570 + 54 + 952 + 1008$
$= 8150$

i) age less than 25 from table.
$952 + 1050 + 53 \Rightarrow 2055/8150 = 0.2521\%$.
$= 25\%$.

ii) age greater than 25 from table (from above result)
$1 - P(25) \Rightarrow 1 - 0.25\cancel{\%} \Rightarrow 0.75 \Rightarrow 75\%$.

iii) earn less than 70,000\$
$952 + 1050 + 456 + 2055 + 54 + 952. \Rightarrow 5519$.
$\frac{5519}{8150} = 0.6771 = 67\%$

iv) age less than 25 and earns more than \$70,000
$\frac{53}{8150} = 0.0065$

v) earns less than 25000 and between 25-45
earns less 25,000
$= 952 + 456 + 54 = 1,462$
age between 25-45 = 456
$\frac{456}{1462} = 0.311$

vi) ager 45 ; probability earns less than \$ 70,000.
people age >45 = $54 + 952 + 1008 \Rightarrow 2014$
earns less than 70000 = $54 + 952 = 1006 \Rightarrow \frac{1006}{2014} = 0.499$