AML5103 | Applied Probability and Statistics | Problem Set-4

1. We have a box with 10 balls (4 white and 6 black). We randomly pick 5 balls. What is the probability that we get exactly 3 white balls if the sampling is

   (i) *with replacement* (ii) *without replacement*?

   The sampling *with replacement* scenario is called the *binomial experiment* and the sampling *without replacement* scenario is called the *hypergeometric experiment*.

   To calculate the probabilities, start with the sampling space $s = \{w_1, w_2, w_3, w_4, b_1, b_2, b_3, b_4, b_5, b_6\}$ and complete the steps below:

   - build the sample space S;
   - check if the outcomes are equally likely;
   - assign probability measure to the outcomes;
   - build the event set E;
   - calculate the probability of event E.

2. The *binomial experiment* explained above can be abstracted as picking from a box containing a white and black ball where the probability of drawing a white ball is 4/10 and probability of drawing a black ball is 6/10, and we pick one ball *with replacement* from the box and repeat this process 5 times. Now calculate the probability that we get exactly 3 white balls using simulation and confirm that you get the same answer as in the previous question. To this end, use the code template below:

```
## Sampling space
s = c('w', 'b')

## Probability for sampling an element from the sampling space
p = c(?, ?)

## Simulate the random experiment of drawing 4 balls from 10 balls with
## replacement
nsimulations = 1e5
samplesize = ?
simulatedData = replicate(nsimulations, sample(s, size = ?, replace = TRUE, prob = ?))

# Check event if 3 of the 5 drawn balls are white
checkEvent = function(data){
  if (sum(data == ?) == ?){
    return(?)
  }
  else{
    return(?)
  }
}

# Calculate probability that 3 of the 5 drawn balls are white
?
```