Mini - Project

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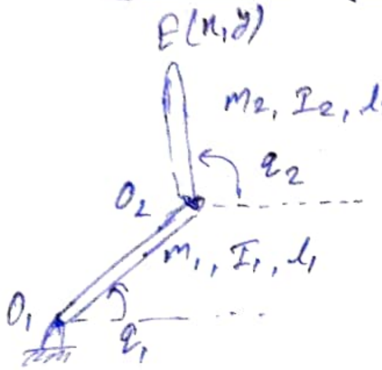
Task. 0. 2R Elbow Manipulator



E = end effector

(q_1, q_2) - joint angles

Motor is connected at O_1, O_2.

$$x = l_1 \cos q_1 + l_2 \cos q_2$$
$$y = l_1 \sin q_1 + l_2 \sin q_2$$

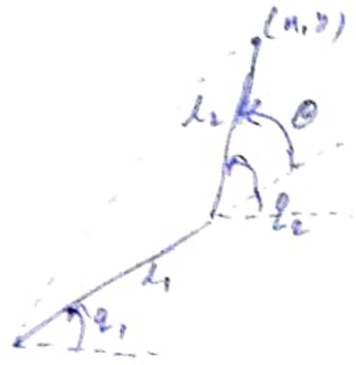
$\sin \rightarrow s$, $\cos \rightarrow c$

then
$$\left.\begin{aligned} x &= l_1 c q_1 + l_2 c q_2 \\ y &= l_1 s q_1 + l_2 s q_2 \end{aligned}\right\} \text{—} (1)$$

differentating (1), we get.
$$\dot{x} = -l_1 s q_1 \cdot \dot{q}_1 - l_2 s q_2 \cdot \dot{q}_2$$
$$\dot{y} = l_1 c q_1 \cdot \dot{q}_1 + l_2 c q_2 \cdot \dot{q}_2$$

End effector velocity:
$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & -l_2 s q_2 \\ l_1 c q_1 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \text{ — } (2)$$

To determine q_1, q_2 from given (x, y)



Cosine rule in triangle :

$$\theta = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2 l_1 l_2}\right)$$

$$q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2\cos\theta}\right)$$

$$q_2 = \theta + q_1$$

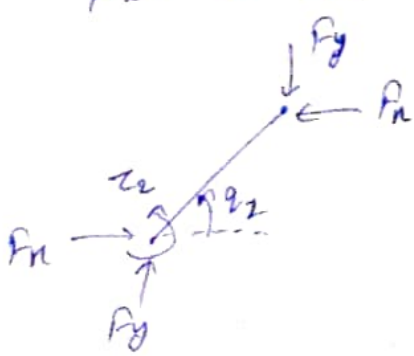
— (3)

$\Rightarrow$ Force on the wall :

static Equilibrium :

$\Sigma M_{O_1} = 0$, $\Sigma M_{O_2} = 0$

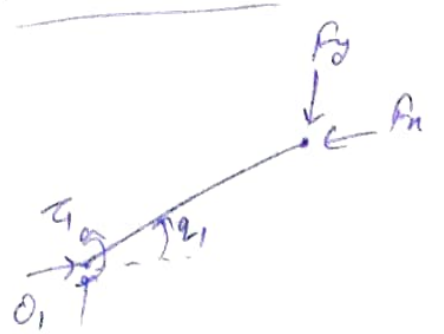
FBD of link 2. (Ignoring gravity)



$\Sigma M_{O_2} = 0$

$\Rightarrow F_y l_2 c q_2 - F_x l_2 s q_2 = \tau_2$

FBD of link 2 :



$$M_{O_1} = 0$$

$$F_y l_1 cq_1 - F_x l_1 sq_1 = \tau_1$$

$$\Rightarrow \quad \left.\begin{aligned} F_y l_1 cq_1 - F_x l_1 sq_1 &= \tau_1 \\ F_y l_2 cq_2 - F_x l_2 sq_2 &= \tau_2 \end{aligned}\right\} \text{ — (4)}$$

Lagrangian Equation :

$$\mathcal{L} = \underbrace{K}_{K.E} - \underbrace{V}_{P.E}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i' \quad \text{— (5)}$$

Q_i' are generalized forces derived using principal of virtual work.

$$K = \underbrace{\frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right) \dot{q}_1^2}_{\text{Pure rotation about } O_1} + \underbrace{\frac{1}{2} m_2 V_{c_2}^2}_{\text{translation of } L_2} + \underbrace{\frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right) \dot{q}_2^2}_{\text{rotation of } L_2 \text{ about C.G.}}$$

$$V_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2} \dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_2 \cos(q_2 - q_1)$$

now considering gravity :

$$V = mg \frac{l_1}{2} sq_2 + m_2 g \left(l_1 sq_1 + \frac{l_2}{2} sq_2\right)$$

So the :

$$\tau_1 = \frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1)$$

$$- m_2 \frac{l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_1 g \frac{l_1}{2} cq_2 + m_2 g l_1 \, cq_1$$

$$\tau_2 = \frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{2} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1)$$

$$- m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_2 g \frac{l_2}{2} sq_2$$

— (6)

$\Rightarrow$ f_x & f_y

$f_x = kx$ — more generally $f_x = k_x (x - x_0)$

$f_y = ky$ $\quad$ $f_y = k_y (y - y_0)$

for now:

$f_x = kx$, $f_y = ky$

from (1) $\Rightarrow$

$$f_x = k(l_1 cq_1 + l_2 cq_2)$$

$$f_y = k(l_1 sq_1 + l_2 sq_2)$$

from (4) $\Rightarrow$

$$k(l_1 sq_1 + l_2 sq_2) l_2 cq_2 - k(l_1 cq_1 + l_2 cq_2) l_2 sq_2 = \tau_{2s}$$

$$k(l_1 sq_1 + l_2 sq_2) l_1 cq_1 - k(l_1 cq_1 + l_2 cq_2) l_1 sq_1 = \tau_{1s}$$

— (7)