PHYS 305 – Homework #2 **Due: Sep. 16, 2024**

1. Use the forward Euler method to solve

$$\frac{dx}{dt} = x - x^2$$

with initial condition, $x(0) = 0.1$. Investigate how the solution depends on time step, using a range of values between $\Delta t = 0.01$ and 3. Pay special attention to the range of Δt between 2 and 3. Provide graphs to show all the behaviors you find. Try to come up with an explanation for what happens between $\Delta t = 2$ and 3.

2. Oscillations are common in biological systems. A simple chemical system that leads to oscillations is given by the following chemical reactions:

$$X \underset{k_+}{\overset{k_-}{\rightleftarrows}} A$$

$$B \xrightarrow{k_2} Y$$

$$2X + Y \xrightarrow{k_3} 3X$$

If the concentrations of A and B are held fixed by external processes, the Law of Mass Action leads to the following form for the dynamics of x and y:

$$\frac{dx}{dt} = a - x + x^2 y$$
$$\frac{dy}{dt} = b - x^2 y$$

where a and b are constants. Write a function that uses a semi-implicit Backward Euler to solve this system of equations and find values for a and b such that x and y oscillate in time. For time-stepping using the semi-implicit method, evaluate the linear term in x at time $t + \Delta t$, but evaluate the term x^2y at time t. Hint for finding the oscillations: The initial concentrations of x and y need to be close to but not equal to the equilibrium concentrations. Determine the equilibrium values and use values that are close to this for the initial conditions.

Make a plot of x and y as functions of time showing a situation where you get oscillations, and one where the concentrations go to their equilibrium values by the end of the simulation. Make sure to tell what values you used for the initial conditions and for a and b for each case.

3. Write code that uses 4th order Runge-Kutta to solve

$$\frac{dm}{dt} = c\left(1-m\right) - 10wm$$
$$\frac{dc}{dt} = 5m\left(1-c\right) - 1.25c + S\left(t\right)$$
$$\frac{dw}{dt} = 0.1\left(1-w\right) - 4mw$$

where $S\left(t\right) = \alpha$ when t is between 3 and 3.2, and is zero otherwise. Use initial conditions, $m(0) = 0.0114$, $c(0) = 0.0090$, and $w(0) = 0.9374$. Vary α to find a value for which the equilibrium switches to a new location. You should use a $\Delta t = 0.002$ and run for a total time of 30. Make a plot showing m, c, and w as functions of time when the equilibrium position switches.