PHYS 305 – Homework #4 **Due: Oct. 7, 2024**

1. A self-propelled object of mass m moves with velocity $\mathbf{v} = v_0\mathbf{d}$, where v_0 is the constant "swimming" speed of the object and $\mathbf{d}$ is a unit vector that points along the object's direction of motion. When two of these objects are near each other they interact in such a way that the velocities and orientations try to entrain with each other. The equations of motion for these objects is governed by

$$m\frac{d\mathbf{v}_1}{dt} = \zeta(v_0\mathbf{d}_1 - \mathbf{v}_1) + \frac{\alpha}{r}\left(\mathbf{v}_2 - \mathbf{v}_1\right)$$

$$\frac{d\mathbf{d}_1}{dt} = \frac{\beta}{r^2}(\mathbf{d}_1 \times \mathbf{d}_2) \times \mathbf{d}_1$$

$$m\frac{d\mathbf{v}_2}{dt} = \zeta(v_0\mathbf{d}_2 - \mathbf{v}_2) + \frac{\alpha}{r}(\mathbf{v}_1 - \mathbf{v}_2)$$

$$\frac{d\mathbf{d}_2}{dt} = \frac{\beta}{r^2}(\mathbf{d}_2 \times \mathbf{d}_1) \times \mathbf{d}_2$$

   where ζ, α, and β are constants, and $r = \left|\mathbf{r}_1 - \mathbf{r}_2\right|$. Write a function that will solve these equations given values for the constants.

2. Simulate a 1000 step random walk in two dimensions using a fixed step size. What is the average displacement over the entire walk? What is the squared displacement as a function of the number of steps? Does your results agree with what we predicted in class?
3. Simulate the 2D motion of a 2 μm diameter, neutrally-buoyant Brownian particle moving through water at room temperature using Langevin dynamics with a time step of 10^{-9} s and for a total time of 0.001 s. Use the midpoint rule for handling the integration and compute the random forcing at the half time step. Show that the MSD as a function of time is initially quadratic. At what time does it become linear? Does this agree with what we discussed in class?