HW 2 - PHYS 305

Question 1

For small values of Δt (e.g, 0.01 to 2), the solution converges smoothly to a steady state around $x = 1$. But, for larger values of Δt (e.g, 2 to 3), the solutions diverge and become unstable and oscillatory, with increasing divergence from the true steady-state behavior.

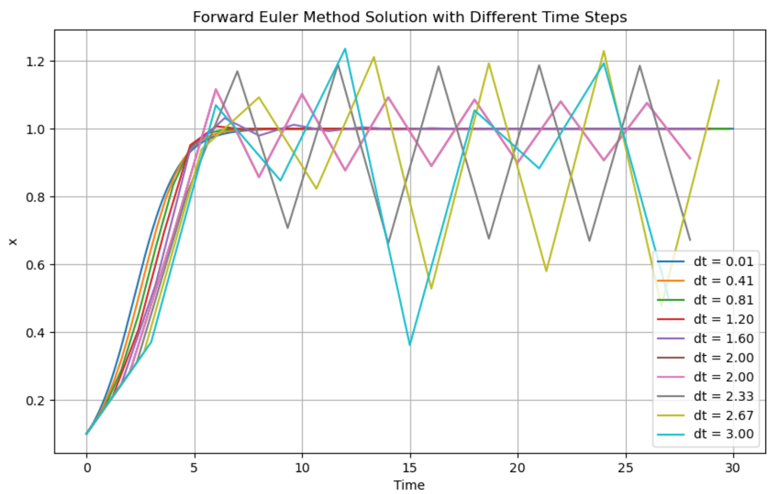


Figure 1: Solution to the equation using forward Euler method for different values of Δt.

Question 2

In order to find the equilibrium solution, we set the derivative of the function to zero and solve for a, b:

$$\frac{dx}{dt} = a - x - x^2 y = 0$$

$$\frac{dy}{dt} = b - x^2 y = 0$$

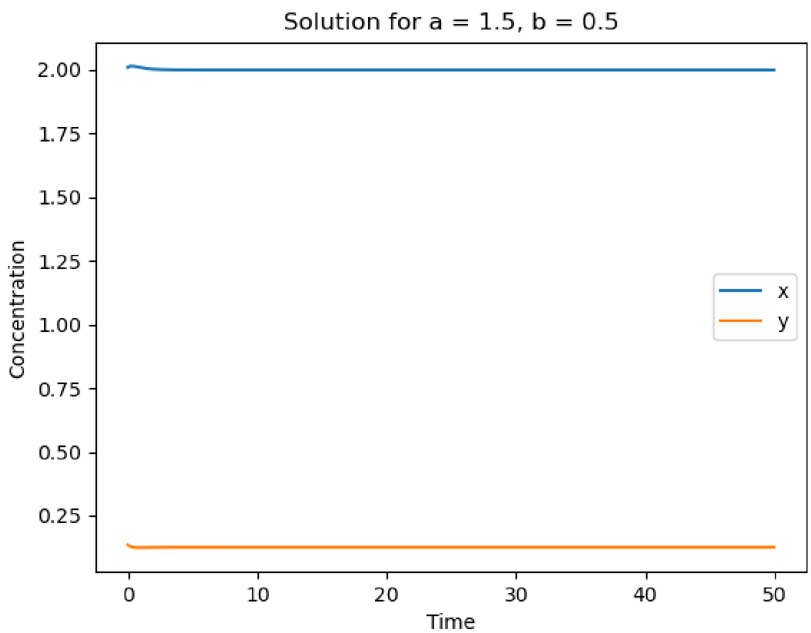
Solving the above equations, we get $x_{eq} = a + b, y_{eq} = b/(a+b)^2$.

In order to plug to the semi-implicit backward Euler method, we need to solve the following equations to get:

$$x_i = \frac{\Delta t(a + x_{i-1}^2 y_{i-1})}{1 + \Delta t}$$

$$y_i = y_{i-1} + \Delta t(b - x_{i-1}^2 y_{i-1})$$

I set a small value of $\epsilon = 0.01$ to check if the solution converge to their equilibrium values or oscillate. The solution converges to the equilibrium values if $a = 1.5, b = 0.5$. But, if $a = 3.5, b = 3.5$, the solution oscillates around the equilibrium values.



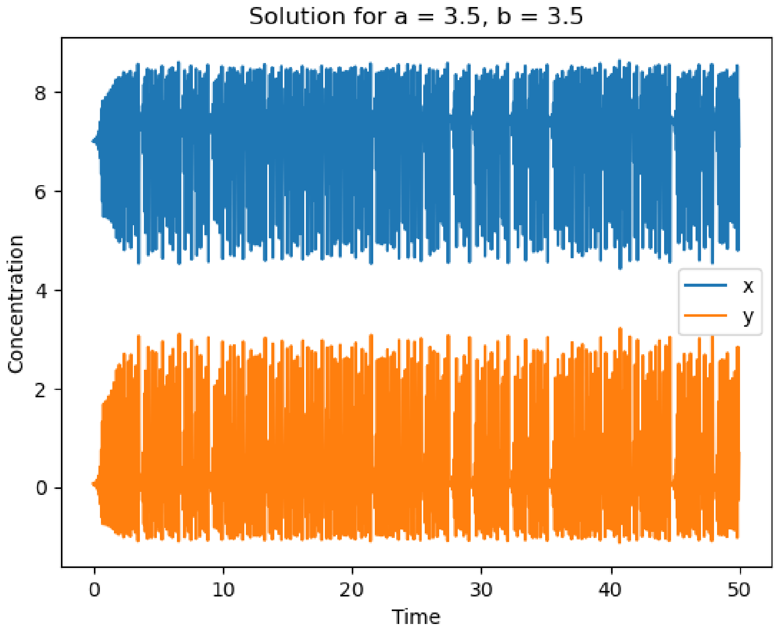


Figure 2: Solution to the equation using semi-implicit backward Euler method for different values of a, b.

Question 3

The equilibrium switches at $\alpha = 6$. The plot showing the equilibrium values of the functions with respect to time is shown below with various alpha is shown below.

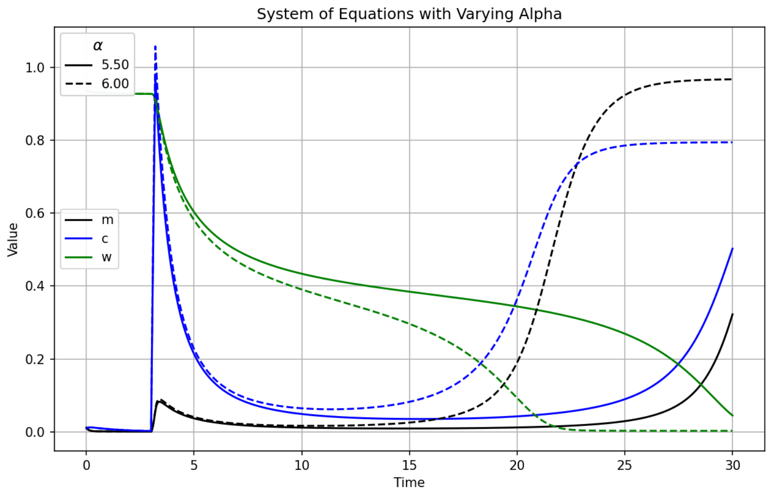


Figure 3: Solution to the system of equations using Runge-Kutta method for different values of α.