<u>PHYS 305 – Homework #6</u> **Due: Nov. 18, 2024**

1. Determine the steady state profile of the following equation:

$$\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial x^2} - \frac{\partial}{\partial x}\left(\sin(x) C\right)$$

   on a domain that goes from $x = 0$ to $x = 4\pi$ using periodic boundary conditions. Make sure to use the appropriate upwind derivatives.

2. Solve the following system of equations numerically using Gauss-Jordan elimination:

$$\begin{aligned} x_2 - x_1 &= -1 \\ x_3 - 3x_2 + x_1 &= 0 \\ x_4 - 3x_3 + x_2 &= 0 \\ x_5 - 3x_4 + x_3 &= 0 \\ x_5 - x_4 &= 1 \end{aligned}$$

3. Consider the Fisher equation that we examined on the last homework:

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + u(1-u)$$

   on the domain with $0 < x < L$, with boundary conditions $u(0,t) = 1$ and $u(L,t) = 0$ for all t.

   A. Write out a semi-implicit Backward Euler discretization of this equation.
   B. Using a domain with only 5 nodes, write out explicitly the 5 equations that represent this discretization.
   C. Write this system of 5 equations as a matrix equation.