Index

Unit – 1 ⇝ Matrix Theory

Method 1 ⇝ Introduction

Example of Method-1: Introduction

A	1	How many elements are there in matrix A if its order is 7×9? **Answer: There are 63 elements in matrix A.**
A	2	Find the order of the following matrices: $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 0 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ 8 & 21 \\ 3 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 9 & 2 \\ \frac{1}{2} & 5 & 6 \end{bmatrix}$, $D = \begin{bmatrix} x & -2 \\ z & 9 \end{bmatrix}$. **Answer: Order of matrix A is 2×3, Order of matrix B is 3×2,** **Order of matrix C is 3×3, Order of matrix D is 2×2.**
A	3	If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements? **Answer: 1×24, 2×12, 3×8, 4×6, 6×4,** **8×3, 12×2, 24×1;** **1×13, 13×1**
B	4	How many elements contain of each column of A if A is 5×7 matrix? **Answer: 5**
B	5	How many elements contain of each row of A if A is 3×4 matrix? **Answer: 4**

A	6	Let 2×2 matrix $A = [\,a_{ij}\,]$, whose elements are given by $a_{ij} = \frac{i}{j}$. What is the value of a_{12}? **Answer**: $\mathbf{a_{12} = \frac{1}{2}}$
A	7	Construct the following matrices: $A = [\,a_{ij}\,]_{3\times2}$ whose elements are given by $a_{ij} = 2i - j$, $B = [\,b_{ij}\,]_{2\times3}$ whose elements are given by $b_{ij} = \frac{i}{j}$ $C = [\,c_{ij}\,]_{3\times3}$ whose elements are given by $c_{ij} = \frac{\lvert\, i - 2j \,\rvert}{2}$ **Answer**: $\mathbf{A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{bmatrix}}$, $\mathbf{B = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{2}{3} \end{bmatrix}}$, $\mathbf{C = \begin{bmatrix} \frac{1}{2} & \frac{3}{2} & \frac{5}{2} \\ 0 & 1 & 2 \\ \frac{1}{2} & \frac{1}{2} & \frac{3}{2} \end{bmatrix}}$
B	8	Construct the following matrices: $A = [\,a_{ij}\,]_{3\times3}$ whose elements are given by $a_{ij} = (-i)(-j)$ $B = [\,b_{ij}\,]_{2\times3}$ whose elements are given by $b_{ij} = \begin{cases} 1, & i + j = \text{even} \\ 0, & i + j = \text{odd} \end{cases}$ $C = [\,c_{ij}\,]_{3\times2}$ whose elements are given by $c_{ij} = \begin{cases} 1, & i + j = \text{prime} \\ 0, & \text{otherwise} \end{cases}$ **Answer**: $\mathbf{A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}}$, $\mathbf{B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}$, $\mathbf{C = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}}$

B	9	What is the number of all possible matrices of order 2×3, with each entry being either 0 or 1? **Answer: 64**
B	10	What is the number of all possible matrices of order 3×3, with each entry being either 0 or 1? **Answer: 512**
B	11	What is the number of all possible matrices of order 2×2, with each entry being either 0 or 1 or 2? **Answer: 81**
C	12	If $A = \begin{bmatrix} \frac{1}{2} & \frac{3}{2} & \frac{5}{2} \\ 0 & 1 & 2 \\ \frac{1}{2} & \frac{1}{2} & \frac{3}{2} \end{bmatrix}$, then find $\sum\limits_{i+j=\text{even}} a_{ij}$. **Answer: 6**
C	13	If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$, then find $\sum\limits_{i+j=\text{prime}} a_{ij}$. **Answer: 17**

Method 2 ⇝ Types of Matrices

Example of Method-2: Types of Matrices

A	1	Give example of the following matrices: (1) Upper triangular matrix of order 3×3 (2) Lower triangular matrix of order 3×3 (3) Scalar matrix of order 4×4 (4) Diagonal matrix of order 2×3 **Hint: Refere theory of types of matrices**
A	2	Determine the type of given matrices: $X = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 7 & 8 \\ 0 & 0 & 5 \end{bmatrix}, \quad Y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **Answer: X is Square matrix and Upper triangular matrix;** **Y is Square matrix, Diagoanal matrix, Scalar matrix and Idenetity matrix.**
A	3	Find the values of x, y and z from the equation: $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$ **Answer:** $\mathbf{x = 1, \quad y = 4, \quad z = 3}$
A	4	Find the values of x, y and z from the equation: $\begin{bmatrix} x+y & 2 \\ 5+z & y \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$ **Answer:** $\mathbf{x = -2, \quad y = 8, \quad z = 0}$

B	5	Find the values of x, y and z from the following equations: $$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$ **Answer**: $\mathbf{x = 2,\quad y = 4,\quad z = 3}$
B	6	Find the value of x and y from the equation: $$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$ **Answer: Not possible to find**
C	7	Find the values of a, b, c, x, y and z. If $$\begin{bmatrix} x+3 & z+4 & 2y-7 \\ -6 & a-1 & 0 \\ b-3 & -21 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ -6 & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$$ **Answer**: $\mathbf{a = -2,\quad b = -7,\quad c = -1,}$ $\mathbf{x = -3,\quad y = -5,\quad z = 2.}$
B	8	State whether the following are true or false. Justify your answer. (1) Diagonal matrix is always scalar matrix. (2) Identity matrix is always scalar matrix. (3) Zero matrix is diagonal matrix. (4) Diagonal matrix is also upper triangular matrix. **Answer**: **(1) F, (2) T, (3) F, (4) T**

B	9	State whether the following are true or false. Justify your answer. (1) Scalar matrix is always diagonal matrix. (2) Scalar matrix is always identity matrix. (3) Diagonal matrix can be zero matrix. (4) Lower triangular matrix is always a diagonal matrix. (5) Diagonal matrix is also lower triangular matrix. **Answer**: **(1) T, (2) F, (3) T, (4) F, (5) T**

Method 3 ⇝ Matrix Operations

Example of Method-3: Matrix Operations

A	1	Given, $A = \begin{bmatrix} 2 & 12 & 6 \\ 8 & 9 & 10 \end{bmatrix}$ and $B = \begin{bmatrix} 11 & 3 & 23 \\ 2\sqrt{2} & \frac{5}{2} & 7 \end{bmatrix}$. Find $A + B$ and $A - B$. **Answer**: $\mathbf{A + B = \begin{bmatrix} 13 & 15 & 29 \\ 8 + 2\sqrt{2} & \frac{23}{2} & 17 \end{bmatrix}}$, $\mathbf{A - B = \begin{bmatrix} -9 & 9 & -17 \\ 8 - 2\sqrt{2} & \frac{13}{2} & 3 \end{bmatrix}}$
A	2	Given, $A = \begin{bmatrix} -2 & 8 \\ 5 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} 11 & -4 \\ -7 & 3 \end{bmatrix}$. Find $3A + B$ and $A - 3B$. **Answer**: $\mathbf{3A + B = \begin{bmatrix} 5 & 20 \\ 8 & -15 \end{bmatrix}}$, $\mathbf{A - 3B = \begin{bmatrix} -35 & 20 \\ 26 & -15 \end{bmatrix}}$
B	3	If $A = \begin{bmatrix} -7 & 6 & 2 \\ 4 & 9 & 8 \\ -5 & 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -5 & -3 \\ 8 & 2 & -6 \\ 10 & 7 & 9 \end{bmatrix}$, then calculate $5A + 3B - 2I$. **Answer**: $\mathbf{5A + 3B - 2I = \begin{bmatrix} -25 & 15 & 1 \\ 44 & 49 & 22 \\ 5 & 36 & 20 \end{bmatrix}}$

B	4	If $A=\begin{bmatrix}6 & 3\\ -4 & -1\\ -2 & 7\end{bmatrix}$ and $B=\begin{bmatrix}-3 & 3\\ 5 & 0\\ -8 & 9\end{bmatrix}$, then find matrix X which satisfy following: (**1**) $X+2A=O$ (**2**) $4B-3X=5A$ (**3**) $2X=2A-4B$ **Answer**: (1) $\mathbf{X}=\begin{bmatrix}\mathbf{-12} & \mathbf{-6}\\ \mathbf{8} & \mathbf{2}\\ \mathbf{4} & \mathbf{-14}\end{bmatrix}$, (2) $\mathbf{X}=\frac{\mathbf{1}}{\mathbf{3}}\begin{bmatrix}\mathbf{-42} & \mathbf{-3}\\ \mathbf{40} & \mathbf{5}\\ \mathbf{-22} & \mathbf{1}\end{bmatrix}$, (3) $\mathbf{X}=\begin{bmatrix}\mathbf{12} & \mathbf{-3}\\ \mathbf{-14} & \mathbf{-1}\\ \mathbf{14} & \mathbf{-11}\end{bmatrix}$
A	5	Let $A=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$. Find AB and BA. **Answer**: $\mathbf{AB}=\begin{bmatrix}\mathbf{-6} & \mathbf{26}\\ \mathbf{-1} & \mathbf{19}\end{bmatrix}$, $\mathbf{BA}=\begin{bmatrix}\mathbf{11} & \mathbf{10}\\ \mathbf{11} & \mathbf{2}\end{bmatrix}$
A	6	If $A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 2 & 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}0 & -1 & 1\\ 0 & 1 & -1\\ 0 & 3 & -3\end{bmatrix}$, then show that $AB\neq BA$. **Answer**: $\mathbf{AB}=\begin{bmatrix}\mathbf{0} & \mathbf{7} & \mathbf{-7}\\ \mathbf{0} & \mathbf{10} & \mathbf{-10}\\ \mathbf{0} & \mathbf{13} & \mathbf{-13}\end{bmatrix}$, $\mathbf{BA}=\begin{bmatrix}\mathbf{1} & \mathbf{1} & \mathbf{1}\\ \mathbf{-1} & \mathbf{-1} & \mathbf{-1}\\ \mathbf{-3} & \mathbf{-3} & \mathbf{-3}\end{bmatrix}$ $\therefore\ \mathbf{AB\neq BA}$.

B	7	Let $A = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Find $A^2 - B^2$. **Answer:** $\mathbf{A^2 - B^2 = \begin{bmatrix} 4 & 4 \\ 4 & 12 \end{bmatrix}}$
C	8	Given, $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{bmatrix}$. Find $A(BC)$ and $(AB)C$. Also verify that $A(BC) = (AB)C$. **Answer:** $\mathbf{BC = \begin{bmatrix} 8 & 4 & 0 & -5 \\ 14 & 8 & 2 & -11 \\ 4 & 4 & 4 & -7 \end{bmatrix}}$, $\mathbf{AB = \begin{bmatrix} 4 & 7 \\ 10 & 9 \\ 6 & 6 \end{bmatrix}}$ $\mathbf{A(BC) = (AB)C = \begin{bmatrix} 18 & 8 & -2 & -9 \\ 28 & 20 & 12 & -31 \\ 18 & 12 & 6 & -18 \end{bmatrix}}$
C	9	Let $A = \begin{bmatrix} 1 & 4 & 3 \\ 3 & 2 & 0 \\ -4 & 1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 1 \\ 4 & -2 & 0 \end{bmatrix}$ Verify $A(B + C) = AB + AC$. **Hint:** $\mathbf{AB = \begin{bmatrix} -2 & 4 & 2 \\ 3 & 2 & -3 \\ -2 & 1 & 2 \end{bmatrix}}$, $\mathbf{AC = \begin{bmatrix} 19 & 3 & 6 \\ 11 & 7 & 8 \\ -19 & 2 & -7 \end{bmatrix}}$
C	10	Let $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Find k such that $A^2 = kA - 2I$. **Answer:** $\mathbf{k = 1}$

Method 4 ⇝ Determinant

Example of Method-4: Determinant

A	1	Evaluate the determinant of following matrices. $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$ **Answer**: $\mathbf{\|A\| = -8}$, $\mathbf{\|B\| = 11}$, $\mathbf{\|C\| = -23}$
B	2	Find the value of x for which $\begin{vmatrix} 7 & x \\ x & -1 \end{vmatrix} = \begin{vmatrix} 5 & 1 \\ -2 & 0 \end{vmatrix}$ **Answer**: $\mathbf{x = \pm 3i}$
A	3	If $A = \begin{bmatrix} 1 & 7 \\ 3 & x \end{bmatrix}$ and $\|A\| = 5$, then find the value of x. **Answer**: $\mathbf{x = 26}$
B	4	If $A = \begin{bmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{bmatrix}$, then evaluate $\|A\|$. **Answer**: $\mathbf{\|A\| = x^3 - x^2 + 2}$
A	5	Evaluate the determinant of following matrices. $A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 3 & -1 & 2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$ **Answer**: $\mathbf{\|A\| = -52}$, $\mathbf{\|B\| = 46}$, $\mathbf{\|C\| = -12}$
B	6	If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $\|3A\| = 27\|A\|$

B	7	Evaluate $\begin{vmatrix} 0 & \sin\alpha & \cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}$ **Answer:** $\mathbf{\sin\alpha\ \sin\beta\ \cos\alpha + \cos\alpha\ \sin\alpha\ \sin\beta}$
C	8	If $A = \begin{bmatrix} 3 & 0 & 0 \\ -1 & 6 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = A^2$, then $\lvert B \rvert = ?$ **Answer:** $\mathbf{\lvert B \rvert = 36}$

Method 5 ⇝ Adjoint of Matrix

Example of Method 5 ⇝ Adjoint of Matrix

A	1	If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $(A^T)^T = A$.
A	2	Find Minors and cofactors of all the elements of the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ **Answer**: $\mathbf{M_{11} = 4,\ M_{12} = 0,\ M_{13} = 0,\ M_{21} = 0,\ M_{22} = 4,}$ $\mathbf{M_{23} = 0,\ M_{31} = -1,\ M_{32} = 2,\ M_{33} = 1}$ $\mathbf{A_{11} = 4,\ A_{12} = 0,\ A_{13} = 0,\ A_{21} = 0,\ A_{22} = 4,}$ $\mathbf{A_{23} = 0,\ A_{31} = -1,\ A_{32} = -2,\ A_{33} = 1}$
B	3	Find Minors and cofactors of all the elements of the matrix $A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$ **Answer**: $\mathbf{M_{11} = -20,\ M_{12} = -46,\ M_{13} = 30,\ M_{21} = -4,\ M_{22} = -19,}$ $\mathbf{M_{23} = 13,\ M_{31} = -12,\ M_{32} = -22,\ M_{33} = 18}$ $\mathbf{A_{11} = -20,\ A_{12} = 46,\ A_{13} = 30,\ A_{21} = 4,\ A_{22} = -19,}$ $\mathbf{A_{23} = -13,\ A_{31} = -12,\ A_{32} = 22,\ A_{33} = 18}$

A	4	Find the Adjoint of the following matrices: $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ **Answer**: $\mathbf{A} = \begin{bmatrix} \mathbf{3} & \mathbf{2} \\ \mathbf{-4} & \mathbf{2} \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{bmatrix}$
B	5	Find the Adjoint of the following matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$ **Answer**: $\mathbf{\lvert Adj\ A \rvert} = \begin{bmatrix} \mathbf{3} & \mathbf{1} & \mathbf{-11} \\ \mathbf{-12} & \mathbf{5} & \mathbf{-1} \\ \mathbf{1} & \mathbf{2} & \mathbf{5} \end{bmatrix}$
B	6	If $A = \begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix}$, then find the value of $\lvert Adj\ A \rvert$. **Answer**: $\mathbf{\lvert Adj\ A \rvert = c^6}$
C	7	The matrix $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & x \end{bmatrix}$ is its own adjoint. The value of x will be? **Answer**: $\mathbf{x = 3}$

Method 6 ⇝ Inverse of a Matrix by Adjoint Method

Example of Method-6: Inverse of a Matrix by Adjoint Method

A	1	Define inverse of matrix. **Answer: Refer above theory**
A	2	Give an example of Matrix which is self-invertible. **Answer: Think !**
A	3	Find the inverse of the following matrices $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ **Answer:** $A^{-1} = \begin{bmatrix} \frac{3}{14} & \frac{1}{7} \\ -\frac{2}{7} & \frac{1}{7} \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$ $C^{-1} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$
A	4	Find the inverse of the following matrices $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ **Answer:** $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & \frac{1}{3} & 0 \\ 3 & \frac{2}{3} & -1 \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

B	5	Find the inverse of the following matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$ **Answer**: $\mathbf{A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}}$
A	6	Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = 0$. Hence find A^{-1}. **Answer**: $\mathbf{A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}}$
B	7	For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the number a and b such that $A^2 + aA + bI = 0$. **Answer**: $\mathbf{a = -4, \quad b = 1}$
A	8	If $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, then show that $(A^{-1})^{-1} = A$.

Method 7 ⇝ Gauss Elimination Method

Examples of Method-7: Gaussian Elimination Method

A	1	Using Gauss Elimination Method solve the following system: $x_1 + x_2 + 2x_3 = 8, \quad -x_1 - 2x_2 + 3x_3 = 1, \quad 3x_1 - 7x_2 + 4x_3 = 10$ **Answer**: $(\mathbf{3}, \quad \mathbf{1}, \quad \mathbf{2})$
A	2	Using Gauss Elimination Method solve the following system: $2x + y - z = 4, \quad x - y + 2z = -2, \quad -x + 2y - z = 2$ **Answer**: $(\mathbf{1}, \quad \mathbf{1}, \quad \mathbf{-1})$
A	3	Solve the following system of linear equations by using Gaussian Elimination Method: $x + y + z = 3, \quad x + 2y - z = 4, \quad x + 3y + 2z = 4$ **Answer**: $\left(\frac{\mathbf{13}}{\mathbf{5}}, \quad \frac{\mathbf{3}}{\mathbf{5}}, \quad -\frac{\mathbf{1}}{\mathbf{5}}\right)$
B	4	Solve the following system of linear equations by using Gaussian Elimination Method: (1) $-2y + 3z = 1, \quad 3x + 6y - 3z = -2, \quad 6x + 6y + 3z = 5$ (2) $3x + y - 3z = 13, \quad 2x - 3y + 7z = 5, \quad 2x + 19y - 47z = 32$ **Answer: No solution for each system**
B	5	Using Gaussian Elimination Method solve the following system: $x_1 - 2x_2 - x_3 + 3x_4 = 1, \quad 2x_1 - 4x_2 + x_3 = 5,$ $x_1 - 2x_2 + 2x_3 - 3x_4 = 4$ **Answer**: $(\mathbf{x_1}, \ \mathbf{x_2}, \ \mathbf{x_3}, \ \mathbf{x_4}) = (\mathbf{2 + 2t_2 - t_1}, \ \mathbf{t_2}, \ \mathbf{1 + 2t_1}, \ \mathbf{t_1})\ ; \ \mathbf{t_1, t_2} \in \mathbb{R}$
B	6	Solve the following system of linear equations by using Gaussian Elimination Method: $2x_1 + 2x_2 + 2x_3 = 0, \quad -2x_1 + 5x_2 + 2x_3 = 1, \quad 8x_1 + x_2 + 4x_3 = -1$ **Answer**: $(\mathbf{x_1}, \ \mathbf{x_2}, \ \mathbf{x_3}) = \left(\frac{\mathbf{(-1 - 3t)}}{\mathbf{7}}, \ \frac{\mathbf{1 - 4t}}{\mathbf{7}}, \ \mathbf{t}\right)\ ; \ \mathbf{t} \in \mathbb{R}$

B	7	Determine when the given augmented matrix $\begin{bmatrix} 1 & 0 & 2 & : & a \\ 2 & 1 & 5 & : & b \\ 1 & -1 & 1 & : & c \end{bmatrix}$ represents a consistent linear system. **Answer:** $\mathbf{b + c - 3a = 0}$
B	8	For what choices of parameter λ, the following system is consistent? $x_1 + x_2 + 2x_3 + x_4 = 1, \quad x_1 + 2x_3 = 0, \quad 2x_1 + 2x_2 + 3x_3 = \lambda,$ $x_2 + x_3 + 3x_4 = 2\lambda.$ **Answer:** $\boldsymbol{\lambda = 1}$
B	9	Determine the values of k, for which the equations $3x - y + 2z = 1$, $-4x + 2y - 3z = k, 2x + z = k^2$ possesses solution. Find solutions in each case. **Answer:** $\mathbf{For\ k = 2 \Rightarrow (x,\ y,\ z) = \left(\frac{4-t}{2},\ \frac{10+t}{2},\ t\right); t \in \mathbb{R}}$ $\mathbf{For\ k = -1 \Rightarrow (x,\ y,\ z) = \left(\frac{1-t}{2},\ \frac{1+t}{2},\ t\right); t \in \mathbb{R}}$
B	10	For which value of 'a' will the following system has (i) unique solution, (ii) no solution and (iii) infinitely many solutions? $x + 2y - 3z = 4, \quad 3x - y + 5z = 2, \quad 4x + y + (a^2 - 14)z = a + 2$ **Answer:** $\mathbf{(i)\ a \in \mathbb{R} \setminus \{-4, 4\} \rightsquigarrow}$ **unique solution,** $\mathbf{(ii)\ a = -4 \rightsquigarrow}$ **no solution** $\mathbf{(iii)\ a = 4 \rightsquigarrow}$ **infinitely many solutions**
C	11	Determine the value of k so that the system of homogeneous equations $2x + y + 2z = 0, \quad x + y + 3z = 0, \quad 4x + 3y + kz = 0$ has (a) Trivial solution. (b) Non-trivial solution. Also find non-trivial solution. **Answer:** $\mathbf{(a)\ If\ k \neq 8 \rightsquigarrow}$ **trivial solution. i.e.,** $\mathbf{(0,\ 0,\ 0)}$ $\mathbf{(b)\ If\ k = 8 \rightsquigarrow (x,\ y,\ z) = (k,\ -4k,\ k); k \in \mathbb{R}}$

Method 8 ⇝ Gauss Jordan Elimination Method

Examples of Method-8: Gauss Jordan Elimination Method

A	1	Solve the following system of linear equations using Gauss Jordan Method: (1) $x+y+z=6, \quad x+2y+3z=14, \quad 2x+4y+7z=30$ (2) $x_1+2x_2+3x_3=4, \quad 2x_1+5x_2+3x_3=5, \quad x_1+8x_3=9$ **Answer**: (1) $(0,\ 4,\ 2)$, (2) $(1,\ 0,\ 1)$
A	2	Solve the following system of linear equations using Gauss Jordan Method: $x+4y-3z=0, \quad -x-3y+5z=-3, \quad 2x+8y-5z=1$ **Answer**: $(23,\ -5,\ 1)$
A	3	Solve by using Gauss Jordan Method. (1) $2x-y-3z=0, \quad -x+2y-3z=0, \quad x+y+4z=0$ (2) $2x_1+x_2+3x_3=0, \quad x_1+2x_2=0, \quad x_2+x_3=0.$ **Answer**: **Trivial solution $(0,\ 0,\ 0)$ for both the systems**
B	4	Solve the system of linear equations by using Gauss Jordan Method. $v+3w-2x=0, \quad 2u+v-4w+3x=0, \quad 2u+3v+2w-x=0,$ $-4u-3v+5w-4x=0$ **Answer**: $(u,\ v,\ w,\ s)=\left(\frac{7s-5t}{2},\ 2t-3s,\ s,\ t\right); s,t\in\mathbb{R}$
B	5	Find the solution set of following system by using Gauss Jordan Method: (1) $x_1+3x_2+x_4=0, \quad x_1+4x_2+2x_3=0, \quad -2x_2-2x_3-x_4=0$ (2) $2x_1-4x_2+x_3+x_4=0, \quad x_1-2x_2-x_3+x_4=0.$ **Answer**: (1) $\left\{\left(5t,\ -2t,\ \frac{3t}{2},\ t\right) \mid t\in\mathbb{R}\right\}$ (2) $\left\{\left(\frac{6t_2-2t_1}{3},\ t_2,\ \frac{t_1}{3},\ t_1\right) \mid t_1,t_2\in\mathbb{R}\right\}$

C	6	Solve the following system of linear equations using Gauss Jordan Method (1) $3x - y - z = 0, \quad x + y + 2z = 0, \quad 5x + y + 3z = 0$ (2) $x + y - z + w = 0, \quad x - y + 2z - w = 0, \quad 3x + y + w = 0.$ **Answer:** (1) $\left\{\left(-\frac{t}{4},\ -\frac{7t}{4},\ t\right) \mid t \in \mathbb{R}\right\}$ (2) $\left\{\left(-\frac{t_1}{3},\ \frac{3t_1}{2} - t_2,\ t_1,\ t_2\right) \mid t_1, t_2 \in \mathbb{R}\right\}$

*** * * * * End of the Unit * * * * ***