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Unit – 2 ⇝ Numerical Linear Algebra

Method 1 ⇝ Eigen Values and Eigen Vectors

Examples of Method-1: Eigen Values and Eigen Vectors

A	1	Find the eigen values of A and A^{-1}, where $A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$. **Answer:** $\mathbf{A \rightsquigarrow 1,\ 4,\qquad A^{-1} \rightsquigarrow 1,\ \frac{1}{4}}$
A	2	If $A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 0 & 5 \\ 0 & 0 & 9 \end{bmatrix}$, then find the eigen values of the matrix A^T. Is A invertible? **Answer:** $\boldsymbol{\lambda} = \mathbf{1,\ 0,\ 9,}$ **No**
A	3	If $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$ then find the eigen values of A^T and 5A. **Answer:** $\mathbf{A^T \rightsquigarrow 1,\ 2,\ 2,\qquad 5A \rightsquigarrow 5,\ 10,\ 10}$
B	4	Find the eigen values of $A = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 2 & 0 \\ 12 & 15 & 3 \end{bmatrix}$ and hence find the eigen values of A^5 and A^{-1}. **Answer:** $\mathbf{A \rightsquigarrow 1,\ 2,\ 3,\qquad A^5 \rightsquigarrow 1,\ 2^5,\ 3^5,\qquad A^{-1} \rightsquigarrow 1,\ \frac{1}{2},\ \frac{1}{3}}$
A	5	Find the eigen values of A^9 for $A = \begin{bmatrix} 1 & 3 & 7 & 11 \\ 0 & 0.5 & 3 & 8 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 2 \end{bmatrix}$. **Answer:** $\mathbf{A^9 \rightsquigarrow 1,\ (0.5)^9,\ 0,\ 2^9}$

C	6	Find the eigen values of the following matrices: $A=\begin{bmatrix}1&0\\0&2\end{bmatrix}$, $B=\begin{bmatrix}5&3\\1&3\end{bmatrix}$, $C=\begin{bmatrix}10&-9\\4&-2\end{bmatrix}$, $D=\begin{bmatrix}1&2&-3\\0&2&3\\0&0&2\end{bmatrix}$, $E=\begin{bmatrix}4&0&1\\-2&1&0\\-2&0&1\end{bmatrix}$, $F=\begin{bmatrix}3&0&-5\\\frac{1}{5}&-1&0\\1&1&-2\end{bmatrix}$. **Answer:** $\mathbf{A \rightsquigarrow 1,\ 2,\quad B \rightsquigarrow 2,\ 6,\quad C \rightsquigarrow 4,\ 4,\quad D \rightsquigarrow 1,\ 2,\ 2,}$ $\mathbf{E \rightsquigarrow 1,\ 2,\ 3,\quad F \rightsquigarrow 0,\ -\sqrt{2},\ \sqrt{2}}$
C	7	Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix}1&-2\\-5&4\end{bmatrix}$. **Answer:** $\boldsymbol{\lambda = -1 \rightsquigarrow X = [1\quad 1]^T,\quad \lambda = 6 \rightsquigarrow X = \left[-\frac{2}{5}\quad 1\right]^T}$
C	8	Let $A=\begin{bmatrix}1&0&-1\\1&2&1\\2&2&3\end{bmatrix}$, then find eigen values, eigen vectors corresponding to each eigen values of A & write eigen space for each eigen values. **Answer:** $\boldsymbol{\lambda = 1 \rightsquigarrow X = [-1\quad 1\quad 0]^T,\quad \lambda = 2 \rightsquigarrow X = \left[-1\quad \frac{1}{2}\quad 1\right]^T}$ $\boldsymbol{\lambda = 3 \rightsquigarrow X = \left[-\frac{1}{2}\quad \frac{1}{2}\quad 1\right]^T,\quad E_1 = \left\{\begin{bmatrix}-1\\1\\0\end{bmatrix}\cdot t \,\middle\vert\, t\in\mathbb{R}\right\}}$ $\boldsymbol{E_2 = \left\{\begin{bmatrix}-1\\\frac{1}{2}\\1\end{bmatrix}\cdot t \,\middle\vert\, t\in\mathbb{R}\right\},\quad E_3 = \left\{\begin{bmatrix}-\frac{1}{2}\\\frac{1}{2}\\1\end{bmatrix}\cdot t \,\middle\vert\, t\in\mathbb{R}\right\}}$
C	9	Find the eigen values and corresponding eigen vectors for the matrix $A=\begin{bmatrix}1&2&2\\0&2&1\\-1&2&2\end{bmatrix}$. **Answer:** $\boldsymbol{\lambda = 1,\ 2,\ 2,\quad X = [-1\quad -1\quad 1]^T,\quad [2\quad 1\quad 0]^T}$

C	10	Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$. **Answer:** $\boldsymbol{\lambda \rightsquigarrow 1,\ \ 1,\ \ 1,\qquad X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$

Method 2 ⇝ Power Method

Examples of Method-2: Power Method

A	1	Use power method to find the largest of eigen value of the matrix $A = \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$ **Answer: 6.7, $[1 \quad -0.74]^T$**
A	2	Use power method to find the largest of eigen value of the matrix $A = \begin{bmatrix} 7 & 9 \\ 9 & 7 \end{bmatrix}$ **Answer: 16, $[1 \quad 1]^T$**
A	3	Use the power method to find the largest eigen value and corresponding eigen vector of the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. **Answer: 4, $X = [1 \quad 0.5 \quad 0]^T$**
B	4	Use the power method to find the largest eigen value and corresponding eigen vector of the matrix $A = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$. **Answer: 11.66, $X = [0.025 \quad 0.422 \quad 1]^T$**
B	5	Find the largest eigen value and the corresponding eigen vector of the matrix $B = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$, taking $[1 \quad 0 \quad 0]^T$ as initial eigen vector. **Answer: 25.182, $X = [1 \quad 0.045 \quad 0.068]^T$**

Method 3 ⇝ Gauss Jacobi Method

Examples of Method-3: Gauss Jacobi Method

A	1	Solve the following system by Gauss Jacobi method: $x+y+z=3, \quad 2x-y-z=3, \quad x-y+z=9$ coreect upto two decimal places. **Answer**: $(-\mathbf{9}, \quad \mathbf{6}, \quad -\mathbf{9})$
B	2	Solve the following system of linear equations using Gauss Jacobi method in three iterations. $20x+y-2z=17, \quad 2x-3y+20z=25, \quad 3x+20y-z=-18$ **Answer**: $(\mathbf{1}, \quad -\mathbf{1}, \quad \mathbf{1})$
B	3	Solve by Gauss Jacobi method, the equations: $5x-y+z=10, \quad 2x+4y=12, \quad x+y+5=-1$ starting with the solution $(2,3,0)$. **Answer**: $(\mathbf{2.556}, \quad \mathbf{1.722}, \quad -\mathbf{1.056})$
C	4	Solve the following system by Gauss Jacobi method: $10x_1-2x_2-x_3-x_4=3, \quad -2x_1+10x_2-x_3-x_4=15,$ $-x_1-x_2+10x_3-2x_4=27, \quad -x_1-x_2-2x_3+10x_4=-9$ **Answer**: $(\mathbf{1}, \quad \mathbf{2}, \quad \mathbf{3}, \quad \mathbf{0})$

Method 4 ⇝ Gauss Seidel Method

Examples of Method-4: Gauss Seidel Method

A	1	Using Gauss Seidel method, solve following system correct to two significant digits. $9x + 2y + 4z = 20, \quad 2x - 4y + 10z = -15, \quad x + 10y + 4z = 6$ **Answer**: $(\mathbf{2.74}, \quad \mathbf{0.99}, \quad \mathbf{-1.65})$
A	2	Solve it using Gauss Seidel method. $10x + y + z = 12, \quad 2x + 2y + 10z = 14, \quad 2x + 10y + z = 13$ **Answer**: $(\mathbf{1}, \quad \mathbf{1}, \quad \mathbf{1})$
A	3	By using Gauss Seidel method, solve the following system: $45x + 2y + 3z = 58,$ $-3x + 22y + 2z = 47,$ $5x + y + 20z = 67$ **Answer**: $(\mathbf{1}, \quad \mathbf{2}, \quad \mathbf{3})$
B	4	Solve the following system by Gauss-Seidel method correct up to three decimal places. $10x - 5y - 2z = 3, \quad 4x - 10y + 3z = -3, \quad x + 6y + 10z = -3$ **Answer**: $(\mathbf{0.342}, \quad \mathbf{0.285}, \quad \mathbf{-0.505})$
B	5	By using Gauss Seidel method, solve the following system: $3x - 0.1y - 0.2z = 7.85,$ $0.1x + 7y - 0.3z = -19.3,$ $0.3x - 0.2y + 10z = 71.4$ **Answer**: $(\mathbf{3}, \quad \mathbf{-2.5}, \quad \mathbf{7})$

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