# Index

## Unit – 3 ⇝ Basic Probability........................................................................ 3

# Unit – 3 ⇝ Basic Probability

## Method – 1 ⇝ Counting

### Example of Method-1: Counting

| | | |
|---|---|---|
| A | 1 | How many four digit numbers are there with no digit repeated?<br>**Answer: 4536** |
| A | 2 | How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated? What if repetition is allowed?<br>**Answer: 504 (without repeating), 729 (with repeating)** |
| B | 3 | How many words, with or without meaning, can be formed using all letters of the word EQUATION, using each letter exactly once?<br>**Answer: 40320** |
| A | 4 | Find the number of permutations of the letters of the word ALLAHABAD.<br>**Answer: 7560** |
| A | 5 | How many ways to select 2 students from 5 students?<br>**Answer: 10** |
| A | 6 | In how many different ways can 4 of 15 laboratory assistants be chosen to assist with an experiment?<br>**Answer: 1365** |
| B | 7 | Out of 6 boys and 4 girls in how many ways a committee of five members can be formed in which there are at most 2 girls are included?<br>**Answer: 186** |

# Method – 2 ⇝ Basic Terminology and Definition of Probability

## Example of Method-2.1: Sample Space and Event

| | | |
|---|---|---|
| A | 1 | Describe the sample space for the indicated random experiments.<br>**(1)** A coin is tossed 3 times.<br>**(2)** A coin and die are tossed together.<br>**Answer**: $(1)\ \mathbf{S} = \{\mathbf{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}\}$<br>$(2)\ \mathbf{S} = \{\mathbf{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}\}$ |
| A | 2 | Find the sample space associated with the experiment of rolling a pair of dice once. Also, find the number of elements of this sample space.<br>**Answer**: $\mathbf{S} = \left\{\begin{matrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{matrix}\right\}$<br>**Number of elements in sample space** $= 6 \times 6 = 6^2 = 36$ |
| A | 3 | One die of red color, one of white color and one of blue color are placed in a bag. One die is selected at random and rolled, its color and the number on its uppermost face is noted. Describe the sample space.<br>**Answer**:<br>$\mathbf{S} = \left\{\begin{matrix} \mathbf{R1,} & \mathbf{R2,} & \mathbf{R3,} & \mathbf{R4,} & \mathbf{R5,} & \mathbf{R6} \\ \mathbf{W1,} & \mathbf{W2,} & \mathbf{W3,} & \mathbf{W4,} & \mathbf{W5,} & \mathbf{W6} \\ \mathbf{B1,} & \mathbf{B2,} & \mathbf{B3,} & \mathbf{B4,} & \mathbf{B5,} & \mathbf{B6} \end{matrix}\right\}$ |

| | | |
|---|---|---|
| B | 4 | A balanced coin is tossed thrice. If three tails are obtained, a balance die is rolled. Otherwise, the experiment is terminated. Write down elements of the sample space.<br>$\textbf{Answer}: \mathbf{S} = \left\{\begin{matrix} \mathbf{HHH}, & \mathbf{HHT}, & \mathbf{HTH}, & \mathbf{HTT}, & \mathbf{THH}, & \mathbf{THT}, & \mathbf{TTH}, \\ \mathbf{TTT1}, & \mathbf{TTT2}, & \mathbf{TTT3}, & \mathbf{TTT4}, & \mathbf{TTT5}, & \mathbf{TTT6} & \end{matrix}\right\}$ |

## Example of Method-2.2: Probability of an Event

| | | |
|---|---|---|
| A | 1 | If probability of event A is $\frac{9}{10}$, what is the probability of the event “not A”?<br>**Answer**: $\mathbf{0.1}$ |
| A | 2 | A single die is tossed once. Find the probability of a 2 or 5 turning up.<br>**Answer**: $\mathbf{\frac{1}{3}}$ |
| B | 3 | Two unbiased dice are thrown. Find the probability that:<br>**(1)** Both the dice show the same number.<br>**(2)** The first die shows 6.<br>**(3)** The total of the numbers on the dice is 8.<br>**(4)** The total of the numbers on the dice is divisible by 2 or 3.<br>**Answer**: $\mathbf{(1)\ \frac{1}{6},\quad (2)\ \frac{1}{6},\quad (3)\ \frac{5}{36},\quad (4)\ \frac{2}{3}}$ |
| B | 4 | Three coins are tossed. Find the probability of<br>**(1)** Getting at least 2 heads.<br>**(2)** Getting exactly 2 heads.<br>**Answer**: $\mathbf{(1)\ 0.5,\quad (2)\ 0.375,}$ |
| B | 5 | A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.<br>**Answer**: $\mathbf{\frac{7}{13}}$ |
| A | 6 | One card is drawn at random from a well shuffled pack of 52 cards. Find probability that the card will be an ace, a card of black color, a diamond, and not an ace.<br>**Answer**: $\mathbf{0.0769,\ 0.5,\ 0.25,\ 0.9231}$ |

| | | |
|---|---|---|
| B | 7 | Four cards are drawn from the pack of cards. Find the probability that<br>**(1)** All are diamonds<br>**(2)** There is one card of each suit<br>**(3)** There are two spades and two hearts<br>**(4)** All are red or all are picture(face) cards.<br>**Answer:** $(1)\ \frac{11}{4165},\quad (2)\ \frac{2197}{20825},\quad (3)\ \frac{468}{20825},\quad (4)\ \frac{3086}{54145}$ |
| B | 8 | 4 cards are drawn at random from a pack of 52 cards. Find probability that<br>**(1)** They are a king, a queen, a jack and an ace.<br>**(2)** Two are kings and two are queens.<br>**(3)** Two are black and two are red.<br>**(4)** Two cards of hearts and two cards of diamonds.<br>**Answer:** $(1)\ \frac{256}{270725},\quad (2)\ \frac{36}{270725},$<br>$(3)\ \frac{325}{833},\quad (4)\ \frac{468}{20825}$ |
| A | 9 | Consider a poker hand of five cards. Find the probability of getting four of a kind (i.e., four cards of the same face value) assuming the five cards are chosen at random.<br>**Answer:** $\frac{1}{4165}$ |
| B | 10 | A box contains 5 red, 6 white and 2 black balls. The balls are identical in all aspects other than color.<br>**(1)** One ball is drawn at random from the box. Find the probability that the selected ball is black.<br>**(2)** Two balls are drawn at random from the box. Find the probability that one ball is white and one is red.<br>**Answer:** $(1)\ \frac{2}{13},\quad (2)\ \frac{5}{13}$ |

| | | |
|---|---|---|
| A | 11 | If 3 balls are “randomly drawn” from a bowl containing 6 white and 5 black balls. What is the probability that one of the balls is white and the other two black?<br>**Answer**: $\mathbf{0.3636}$ |
| B | 12 | There are 5 yellow, 2 red, and 3 white balls in the box. Three balls are randomly selected from the box. Find the probability of the following events.<br>**(1)** All balls are of different color<br>**(2)** 2 yellow and 1 red color ball<br>**(3)** all balls are of same color.<br>**Answer**: $\mathbf{(1)\ 0.25,\quad (2)\ 0.1667,\quad (3)\ 0.0917}$ |
| A | 13 | A box contains 6 red balls, 4 white balls, 5 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn there is at least one ball of each color.<br>**Answer**: $\mathbf{0.5275}$ |
| B | 14 | A machine produces a total of 12000 bolts a day, which are on the average 3% defective. Find the probability that out 600 bolts chosen at random, 12 will be defective.<br>**Answer**: $\mathbf{\frac{\binom{360}{12}\binom{11640}{588}}{\binom{12000}{600}}}$ |
| A | 15 | If 5 of 20 tyres in storage are defective and 5 of them are randomly chosen for inspection (that is, each tire has the same chance of being selected), what is the probability that the two of the defective tires will be included?<br>**Answer**: $\mathbf{0.2935}$ |

| | | |
|---|---|---|
| A | 16 | In a group of 1000 persons, there are 650 who can speak Hindi, 400 can speak English, and 150 can speak both Hindi and English. If a person selected at random, what is the probability that a person speaks<br>**(1)** Hindi only,<br>**(2)** English only,<br>**(3)** Only one of two languages,<br>**(4)** At least one of the two languages.<br>**Answer:** $(1)\ \mathbf{0.5},\quad (2)\ \mathbf{0.25},\quad (3)\ \mathbf{0.75},\quad (4)\ \mathbf{0.9}$ |
| B | 17 | A person applies for a job in two firms A and B, the probability of his being selected in firm A is 0.7 and being rejected in firm B is 0.5. The probability of at least one of the applications being rejected is 0.6. What is the probability that person will be selected in one of the two firms?<br>**Answer:** $\mathbf{0.8}$ |
| B | 18 | A basket contains 20 apples and 10 oranges of which 5 apples and 3 oranges are bad. If a person takes 2 at random, what is the probability that either both are apples or both are good?<br>**Answer:** $\frac{\mathbf{316}}{\mathbf{435}}$ |
| B | 19 | Three newspapers $A,\ B,\ C$ are published in a certain city. It is estimated from a survey that of the adult population: 20% read A, 16% read B, 14% read C, 8% read both A and B, 5% read both A and C, 4% read B and C, 2% read all three. Find what percentage read at least one of the papers?<br>**Answer:** $\mathbf{35\%}$ |
| A | 20 | Do as directed:<br>**(1)** Find the probability that there will be 5 Sundays in the month of July.<br>**(2)** Find the probability that there will be 5 Sundays in the month of June.<br>**(3)** What is the probability that a non-leap year contains 53 Sundays?<br>**(4)** What is the probability that a leap year contains 53 Sundays?<br>**Answer:** $(1)\ \frac{\mathbf{3}}{\mathbf{7}},\quad (2)\ \frac{\mathbf{2}}{\mathbf{7}},\quad (3)\ \frac{\mathbf{1}}{\mathbf{7}},\quad (4)\ \frac{\mathbf{2}}{\mathbf{7}}$ |

| | | |
|---|---|---|
| B | 21 | A room has three lamp sockets. From a collection of 10 light bulbs of which only 6 are good. A person selects 3 at random and puts them in the socket. What is the probability that the room will have light?<br>**Answer**: $\frac{29}{30}$ |
| A | 22 | Four letters of the word THURSDAY are arranged in all possible ways. Find the probability that the word formed is HURT.<br>**Answer**: $\frac{1}{1680}$ |

# Method – 3 ⇝ Conditional Probability and Independent Events

Example of Method-3: Conditional Probability and Independent Events

| | | |
|---|---|---|
| A | 1 | If $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{4}$ and $P(A \cup B) = \frac{11}{12}$ . Find $P(A \mid B)$.<br>**Answer**: $\frac{2}{9}$ |
| B | 2 | For two independent events A & B if $P(A) = 0.3$ and $P(A \cup B) = 0.6$. Find $P(B)$.<br>**Answer**: $0.4286$ |
| A | 3 | If A, B are independent events and $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{3}$. Find $P(A \cup B)$.<br>**Answer**: $0.75$ |
| A | 4 | Check weather events A and B are independent or not if $P(A) = 0.20$, $P(B) = 0.40$ and $P(A \cup B) = 0.50$.<br>**Answer**: **Not Independent** |
| B | 5 | If A and B are independent events with $P(A) = 0.26$, $P(B) = 0.45$, find $P(A \cap B)$, $P(A \cap B')$, $P(A' \cap B')$.<br>**Answer**: $0.117$, $0.143$, $0.407$ |
| A | 6 | In producing screws, let A mean "screw is too slim" and B "screw is too short". Let $P(A) = 0.1$ and $P(B \cap A) = 0.02$. A screw, selected randomly, is of type A, what is probability that a screw is of type B?<br>**Answer**: $0.2$ |
| A | 7 | A problem in statistics is given to three students A, B, C whose chances of solving it are 0.5, 0.75 and 0.25 respectively. What is the probability that the problem will be solved if all of them try independently?<br>**Answer**: $0.90625$ |

| | | |
|---|---|---|
| B | 8 | In a box, 100 bulbs are supplied out of which 10 bulbs have defects of type A, 5 bulbs have defects of type B and 2 bulbs have defects of both the type. Find the probability that,<br>(1) A bulb to be drawn at random has a B – type defect under the condition that it has an A – type defect.<br>(2) A bulb to be drawn at random has no B – type defect under the condition that it has no A – type defect.<br>**Answer**: $\mathbf{(1)\ 0.2,\qquad (2)\ 0.9667}$ |
| A | 9 | In a certain college 25% of the students failed in probability and 15% of the student failed in statistics and 10% of the students failed in both. A student is selected at random, if he failed in probability, what is probability that he failed in statistics?<br>**Answer**: $\mathbf{0.4}$ |
| B | 10 | Two integers are selected at random from 1 to 11. If the sum is even, find the probability that both the integers are odd.<br>**Answer**: $\mathbf{0.6}$ |
| B | 11 | A card is drawn from a well-shuffled deck of 52 cards and then second card is drawn, find the probability that one card is a spade and then second card is club if the first card is not replaced.<br>**Answer**: $\mathbf{\frac{13}{204}}$ |
| A | 12 | From a bag containing 4 white and 6 black balls, two balls are drawn at random. If the balls are drawn one after the other without replacements, find the probability that one is white and one is black.<br>**Answer**: $\mathbf{\frac{8}{15}}$ |

| | | |
|---|---|---|
| A | 13 | A problem in statistics is given to three students A, B and C, whose chances of solving it independently are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively.<br>Find the probability that<br>**(1)** the problem is solved<br>**(2)** at least two of them are able to solve the problem<br>**(3)** exactly two of them are able to solve the problem<br>**(4)** exactly one of them is able to solve the problem<br>**Answer**: $(1)\ \frac{3}{4}, \quad (2)\ \frac{7}{24}, \quad (3)\ \frac{1}{4}, \quad (4)\ \frac{11}{24}$ |

# Method – 4 ⇝ Total Probability and Bayes' Theorem

## Example of Method-4: Total Probability and Bayes' Theorem

| | | |
|---|---|---|
| A | 1 | There are three boxes, Box - I contains 10 light bulbs of which 4 are defective, Box - II contains 6 light bulbs of which 1 is defective and Box - III contains 8 light bulbs of which 3 are defective. A box is chosen and a bulb is drawn. Find the probability that the bulb is defective.<br>**Answer**: $\mathbf{0.3139}$ |
| B | 2 | Suppose that the population of a certain city is 40% male & 60% female. Suppose also that 50% of male & 30% of female smokes. Find the probability that a smoker is male.<br>**Answer**: $\mathbf{\frac{10}{19}}$ |
| B | 3 | A card from a pack of 52 cards is lost. From the remaining cards of pack, two cards are drawn and are found to be hearts. Find the probability of the missing card to be a heart.<br>**Answer**: $\mathbf{\frac{11}{50}}$ |
| B | 4 | A microchip company has two machines that produce the chips. Machine-I produces 65% of the chips, but 5% of its chips are defective. Machine-II produces 35% of the chips, but 15% of its chips are defective. A chip is selected at random and found to be defective. What is the probability that it came from Machine-I?<br>**Answer**: $\mathbf{0.3824}$ |
| A | 5 | There are two boxes A and B containing 4 white, 3 red and 3 white, 7 red balls respectively. A box is chosen at random and a ball is drawn from it, if the ball is white, find the probability that it is from box A.<br>**Answer**: $\mathbf{\frac{40}{61}}$ |

| | | |
|---|---|---|
| A | 6 | In a computer engineering class, 5% of the boys and 10% of the girls have an IQ of more than 150. In this class, 60% of student are boys. If a student is selected random and found to have IQ more than 150, find the probability that the student is a boy.<br>**Answer:** $\frac{3}{7}$ |
| A | 7 | Three hospitals contain 10%, 20% and 30% of diabetes patients. A Patient is selected at random who is diabetes patient. Determine the probability that this patient comes from first hospital.<br>**Answer:** $0.1667$ |
| B | 8 | Suppose there are three chests each having two drawers. The first chest has a gold coin in each drawer, the second chest has a gold coin in one drawer and a silver coin in the other drawer and the third chest has a silver coin in each drawer. A chest is chosen at random and a drawer opened. If the drawer contains a gold coin, what is the probability that the other drawer also contains a gold coin?<br>**Answer:** $\frac{2}{3}$ |
| B | 9 | State Bayes' theorem. In a bolt factory, three machines A, B and C manufacture $25\%,\ 35\%$ and $40\%$ of the total product respectively. Out of these outputs $5\%,\ 4\%$ and $2\%$ respectively, are defective bolts. A bolt is picked up at random and found to be defective. What are the probabilities that it was manufactured by machine A, B and C?<br>**Answer:** $0.3623, \quad 0.4058, \quad 0.2319$ |
| A | 10 | A factory has three machines $X,\ Y,\ Z$ producing $1000,\ 2000,\ 3000$ bolts per day respectively. Machine X produces 1% defective bolts, Y produces 1.5%, Z produces 2% defective bolts. At end of the day, a bolt is drawn at random and it is found to be defective. What is the probability that this defective bolt has been produced by the machine X?<br>**Answer:** $0.1$ |

| | | |
|---|---|---|
| B | 11 | Urn A contain 1 white, 2 black, 3 red balls; Urn B contain 2 white, 1 black, 1 red balls; Urn C contain 4 white, 5 black, 3 red balls. One urn is chosen at random & two balls are drawn. These happen to be one white & one red. What is probability that they come from urn A?<br>**Answer**: $\mathbf{0.2797}$ |
| A | 12 | An insurance company insured 2000 bike drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a bike driver, a car driver and a truck driver is 0.10, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a bike driver?<br>**Answer**: $\mathbf{0.1639}$ |

# Method – 5 ⇝ Random Variable and Probability Function

## Example of Method-5: Random Variable and probability Function

<table>
<tr><td>A</td><td>1</td><td>Is $P(X=x)=\left(-\frac{1}{2}\right)^{x}$ ; $x=0,\ 1,\ 2$ a probability function?<br><br>**Answer: No**</td></tr>
<tr><td>B</td><td>2</td><td>If $P(x)=\frac{2x+1}{48}$, $x=1,\ 2,\ 3,\ 4,\ 5,\ 6.$<br>Verify whether P(x) is probability function or not.<br><br>**Answer: Yes**</td></tr>
<tr><td>B</td><td>3</td><td>Probability distribution of a random variable X is given below.<br>Find $P(2\le x\le 4)$, $P(x>2)$, P(x is odd) and P(x is even)<br><br>
<table>
<tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>P(X = x)</td><td>0.1</td><td>0.2</td><td>0.5</td><td>0.2</td></tr>
</table><br>
**Answer:** $\mathbf{P(2\le x\le 4)=0.9}$, $\mathbf{P(x>2)=0.7}$,<br>$\mathbf{P(x\ is\ odd)=0.6}$, $\mathbf{P(x\ is\ even)=0.4}$</td></tr>
<tr><td>A</td><td>4</td><td>Find k for the probability distribution $p(x)=k\binom{4}{x}$, $x=0,\ 1,\ 2,\ 3,\ 4.$<br><br>**Answer:** $\mathbf{k=\frac{1}{16}}$</td></tr>
<tr><td>B</td><td>5</td><td>If $P(X=x)=\frac{x}{15}$, $x=1$ to 5.<br>Find P(1 or 2) & $P(0.5<X<2.5\,|\,X>1)$.<br><br>**Answer:** $\mathbf{P(1\ or\ 2)=\frac{1}{5}}$, $\mathbf{P(0.5<X<2.5\,|\,X>1)=\frac{1}{7}}$</td></tr>
</table>

| | | |
|---|---|---|
| A | 6 | Verify that the following function is P.D.F or not?<br>$f(x) = \begin{cases} \frac{x}{8} & ;\ 0 \le x < 2 \\ \frac{1}{4} & ;\ 2 \le x < 4 \\ \frac{6-x}{8} & ;\ 4 \le x < 6 \end{cases}$<br>**Answer: Yes** |
| B | 7 | Is the function f(x) defined as below is a probability function? If so, find the probability that the variate having this density falls in the interval $(1, 2)$.<br>$f(x) = \begin{cases} e^{-x} & ;\ x \ge 0 \\ 0 & ;\ x < 0 \end{cases}$<br>**Answer: Yes,** $\mathbf{P(1 \le X < 2) = 0.2325}$ |
| A | 8 | Check whether the following function<br>$f(x) = \begin{cases} \frac{3+2x}{18} & ;\ 2 \le x \le 4 \\ 0 & ;\ \text{otherwise} \end{cases}$ is a probability density function?<br>If yes, then find $P(3 \le X \le 4)$.<br>**Answer: Yes,** $\mathbf{\frac{5}{9}}$ |
| A | 9 | Find the constant c such that the function<br>$f(x) = \begin{cases} cx^2 & ;\ 0 < x < 3 \\ 0 & ;\ \text{elsewhere} \end{cases}$ is a probability density function and<br>Compute $P(1 < X < 2)$, $P(X \le 2)$, $P(X \ge 2)$.<br>**Answer:** $\mathbf{c = \frac{1}{9}}$, $\mathbf{P(1 < X < 2) = \frac{7}{27}}$,<br>$\mathbf{P(X \le 2) = \frac{8}{27}}$, $\mathbf{P(X \ge 2)\frac{19}{27}}$ |

# Method – 6 ⇝ Various Measures of Statistics

### Example of Method-6: Various Measures of Statistics

<table>
<tr><td>A</td><td>1</td><td>
The probability distribution of a random variable X is given below. Find a, $E(X)$, $E(2X+3)$, $E(X^2+2)$, $V(X)$, $V(3X+2)$.
<table>
<tr><td>X</td><td>$-2$</td><td>$-1$</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>P(X)</td><td>$\frac{1}{12}$</td><td>$\frac{1}{3}$</td><td>a</td><td>$\frac{1}{4}$</td><td>$\frac{1}{6}$</td></tr>
</table>
<b>Answer:</b> $\mathbf{a = \frac{1}{6}}$, $\mathbf{E(X) = \frac{1}{12}}$, $\mathbf{E(2X+3) = \frac{19}{6}}$,<br>
$\mathbf{E(X^2+2) = \frac{43}{12}}$, $\mathbf{V(X) = \frac{227}{144}}$, $\mathbf{V(3X+2) = \frac{227}{16}}$
</td></tr>
<tr><td>A</td><td>2</td><td>
The probability distribution of a random variable X is given below.
<table>
<tr><td>X</td><td>$-1$</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>P(X)</td><td>$\frac{3}{10}$</td><td>$\frac{1}{10}$</td><td>k</td><td>$\frac{3}{10}$</td><td>$\frac{1}{10}$</td></tr>
</table>
Find k, $E(X)$, $E\,(4X+3)$, $E(X^2)$, $V(X)$, $V(2X+3)$.<br>
<b>Answer:</b> $\mathbf{k = \frac{1}{5}}$, $\mathbf{E(X) = \frac{4}{5}}$, $\mathbf{E\,(4X+3) = \frac{31}{5}}$,<br>
$\mathbf{E(X^2) = \frac{13}{5}}$, $\mathbf{V(X) = \frac{49}{25}}$, $\mathbf{V(2X+3) = \frac{196}{25}}$
</td></tr>
<tr><td>B</td><td>3</td><td>
Three balanced coins are tossed, find the mathematical expectation of tails.<br>
<b>Answer:</b> $\mathbf{\frac{3}{2}}$
</td></tr>
<tr><td>B</td><td>4</td><td>
<b>(1)</b> A contestant tosses a coin and receives $5 if head appears and $1 if tail appears. What is the expected value of a trial?<br>
<b>(2)</b> A contestant receives $4.00 if a coin turns up heads and pays $3.00 if it turns tails. What is the expected value of a trail?<br>
<b>Answer:</b> <b>$ 3.00, $ 0.50</b>
</td></tr>
</table>

| | | |
|---|---|---|
| A | 5 | A machine produces on average of 500 items during first week of the month & average of 400 items during the last week of the month. The probability for these being 0.68 and 0.32. Determine the expected value of the production.<br>**Answer: 468** |
| A | 6 | In a business, the probability that a trader can get profit of Rs. 5000 is 0.4 and probability for loss of Rs. 2000 is 0.6. Find his expected gain or loss.<br>**Answer: 800** |
| B | 7 | There are 8 apples in a box, of which 2 are rotten. A person selects 3 Apples at random from it. Find the expected value of the rotten apples.<br>**Answer: $\mathbf{0.75}$** |
| A | 8 | There are 10 bulbs in a box, out of which 4 are defectives. If 3 bulbs are taken at random, find the expected number of defective bulbs.<br>**Answer: $\mathbf{1.2}$** |
| B | 9 | A random variable X has pdf $f(x) = kx^2(4 - x)$ ; $0 < x < 4$. Find the value of k and hence find its mean and standard deviation.<br>**Answer:** $\mathbf{k = \frac{3}{64}}$, $\mathbf{Mean = 2.4}$, $\mathbf{SD = 0.8}$ |
| A | 10 | A random variable X has pdf $f(x) = kx^2(1 - x^3)$ ; $0 < x < 1$. Find the value of k and hence find its mean and variance.<br>**Answer:** $\mathbf{k = 6}$, $\mathbf{Mean = \frac{9}{14}}$, $\mathbf{SD = \frac{9}{245}}$ |

## Method – 7 ⇝ Cumulative Distribution Function

### Example of Method-7: Cumulative Distribution Function

<table>
<tr>
<td>B</td>
<td>1</td>
<td>
Two dies are rolled. Let X denotes the random variable which counts the total number of points on the upturned faces, construct a table giving the non – zero values of the probability mass function. Also find the distribution of X.

**Answer:**

<table>
<tr><td><b>X</b></td><td><b>1</b></td><td><b>2</b></td><td><b>3</b></td><td><b>4</b></td><td><b>5</b></td><td><b>6</b></td><td><b>7</b></td><td><b>8</b></td><td><b>9</b></td><td><b>10</b></td><td><b>11</b></td><td><b>12</b></td></tr>
<tr><td><b>P(X)</b></td><td><b>0</b></td><td>$\frac{1}{36}$</td><td>$\frac{2}{36}$</td><td>$\frac{3}{36}$</td><td>$\frac{4}{36}$</td><td>$\frac{5}{36}$</td><td>$\frac{6}{36}$</td><td>$\frac{5}{36}$</td><td>$\frac{4}{36}$</td><td>$\frac{3}{36}$</td><td>$\frac{2}{36}$</td><td>$\frac{1}{36}$</td></tr>
<tr><td><b>F(x)</b></td><td><b>0</b></td><td>$\frac{1}{36}$</td><td>$\frac{3}{36}$</td><td>$\frac{6}{36}$</td><td>$\frac{10}{36}$</td><td>$\frac{15}{36}$</td><td>$\frac{21}{36}$</td><td>$\frac{26}{36}$</td><td>$\frac{30}{36}$</td><td>$\frac{33}{36}$</td><td>$\frac{35}{36}$</td><td>$\frac{36}{36}$</td></tr>
</table>
</td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>
The following is the distribution function F(x) of a discrete random variable X. Find probability distribution of X, $P(-2 \le X \le 1)$ and $P(X \ge 1)$.

<table>
<tr><td>X</td><td>−3</td><td>−2</td><td>−1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>F(x)</td><td>0.08</td><td>0.2</td><td>0.4</td><td>0.65</td><td>0.8</td><td>0.9</td><td>1</td></tr>
</table>

**Answer:**

<table>
<tr><td><b>X</b></td><td><b>−3</b></td><td><b>−2</b></td><td><b>−1</b></td><td><b>0</b></td><td><b>1</b></td><td><b>2</b></td><td><b>3</b></td></tr>
<tr><td><b>P(X)</b></td><td><b>0.08</b></td><td><b>0.12</b></td><td><b>0.2</b></td><td><b>0.25</b></td><td><b>0.15</b></td><td><b>0.1</b></td><td><b>0.1</b></td></tr>
</table>

$\mathbf{P(-2 \le X \le 1) = 0.72}$, $\mathbf{P(X \ge 1) = 0.35}$
</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>
Find the value of k and the distribution function F(x) given the probability density function of a random variable X as, $f(x) = \frac{k}{1+x^2}$ ; $-\infty < x < \infty$.

**Answer:** $\mathbf{k = \frac{1}{\pi}}$, $\mathbf{F(x) = \frac{1}{\pi}\left[\tan^{-1} x + \frac{\pi}{2}\right]}$
</td>
</tr>
</table>

| | | |
|---|---|---|
| B | 4 | The life in hours of a certain kind of radio tube has the probability density<br>$f(x) = \begin{cases} \frac{100}{x^2} & ; \quad x \geq 100 \\ 0 & ; \quad \text{elsewhere} \end{cases}.$<br>**(1)** Find the distribution function and use it to determine the probability that the life of tube is more than 150 hrs.<br>**(2)** What is the probability that a tube will last less than 200 hrs. if it is known that the tube is still functioning after 150 hrs. of service?<br>**Answer:** $(1)\ \ \mathbf{F(x)} = \begin{cases} \mathbf{1 - \frac{100}{x}} & ; \quad \mathbf{x \geq 100} \\ \mathbf{0} & ; \quad \mathbf{elsewhere} \end{cases}, \quad \mathbf{P(x > 150) = \frac{2}{3}}$<br>$(2)\ \ \mathbf{P(X < 200\ \vert X > 150) = 0.25}$ |

*** * * * *End of The Unit * * * * ***