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Unit – 5 ⇝ Applied Statistics

Test for Variables for Large Samples

Method – 2 ⇝ Test of Significance of Mean

Example of Method-2: Test of Significance of Mean

A	1	The mean weight obtained from a random sample of size 100 is 64 gm. The S.D. of the weight distribution of the population is 3 gm. Test the statement that the mean weight of the population is 67 gms. at 5% level of significance. ($\lvert Z_{0.05} \rvert = 1.96$) **Answer: The mean weight of the population is not 67 gm.**
A	2	Sugar is packed in bags by an automation machine with mean contents of bags as 1.000 kg. A random sample of 36 bags is selected and mean mass has been found to be 1.003 kg. If a S.D. of 0.01 kg is acceptable on all the bags being packed, determine on the basis of sample test whether the machine requires adjustment. ($\lvert Z_{0.05} \rvert = 1.96$) **Answer: The machine does not require any adjustment.**
A	3	A random sample of 100 Indians has an average life span of 71.8 years with standard deviation of 8.9 years. Can it be concluded that the average life span of an Indian is 70 years? ($\lvert Z_{0.05} \rvert = 1.96$) **Answer: The average life span of an Indian is not 70 years.**
A	4	A random sample of 50 items gives the mean 6.2 and variance 10.24. Can it be regarded as drawn from a normal population with mean 5.4 at 5% level of significance? ($\lvert Z_{0.05} \rvert = 1.96$) **Answer: The sample is drawn from a normal population with mean 5.4.**

A	5	It is claimed that a random sample of 49 tyre has a mean life of 15200 km. This sample was drawn from a population whose mean is 15150 km and a standard deviation of 1200 km. Test the significance at 0.05 level. ($\lvert Z_{0.05} \rvert = 1.96$) **Answer: The null hypothesis is accepted**
B	6	15.5 % of a random sample of 1600 undergraduate smokers, whereas 20% of a random sample of 900 postgraduate smokers in a state. Can we conclude that less number of undergraduates are smokers than postgraduates? ($Z_{0.05} = -1.645$) **Answer: Yes, less number of undergraduates are smokers than postgraduates.**
A	7	In a random sample of 60 workers, the average time taken by them to get to work is 33.8 minutes with a standard deviation of 6.1 minutes. Can we reject the null hypothesis $\mu > 32.6$ at $\alpha = 0.025$ level of significance? ($Z_{0.05} = 1.645$) **Answer: The null hypothesis is accepted.**
B	8	A tyre company claims that the lives of tyre have mean 42000 km with S.D. of 4000 km. A change in the production process is believed to result in better product. A test sample of 81 new tyre has a mean life of 42500 km. Test at 5% level of significance that the new product is significantly better than the old one. ($Z_{0.05} = 1.645$) **Answer: The new product is not significantly better than the old one.**
C	9	An ambulance service claims that it takes on the average 10 minutes to reach its destination in emergency calls. A sample of 36 calls has a mean of 11 minutes and the variance of 16 minutes. Test the claim at 0.05 level of significance. ($Z_{0.05} = 1.645$) **Answer: The ambulance service takes on the average 10 minutes to reach its destination.**

Method – 3 ⇝ Test of Significance of Difference Between Two Means

Example of Method-3: Test of Significance of Difference Between Two Means

<table>
<tr>
<td>A</td>
<td>1</td>
<td>Random samples drawn from two places gave the following data relating to the heights of children:
<table>
<tr><th></th><th>Mean height in cm</th><th>SD in cm</th><th>No. of samples</th></tr>
<tr><td>Place A</td><td>68.50</td><td>2.5</td><td>1200</td></tr>
<tr><td>Place B</td><td>68.58</td><td>3.0</td><td>1500</td></tr>
</table>
Test at 5% level of significance that the mean height is the same for children at two places. ($|Z_{0.05}| = 1.96$)

Answer: The mean height is same for children at two places.</td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>Samples of students were drawn from two universities and from their weights in kilograms, the mean and standard deviations are calculated. Make a large sample test to test the significance of the difference between the means. ($|Z_{0.05}| = 1.96$)
<table>
<tr><th></th><th>Mean</th><th>SD</th><th>Size of the Sample</th></tr>
<tr><td>University A</td><td>55</td><td>10</td><td>400</td></tr>
<tr><td>University B</td><td>57</td><td>15</td><td>100</td></tr>
</table>
Answer: There is no significant difference between the means.</td>
</tr>
<tr>
<td>A</td>
<td>3</td>
<td>In a certain factory there are two different processes of manufacturing the same item. The average weight in a sample of 250 items produced from one process is found to be 120 gm with a SD of 12 gm; the corresponding figures in a sample of 400 items from the other process are 124 gm and 14 gm. Is this difference between the two sample means significant? ($|Z_{0.05}| = 1.96$)

Answer: There is significant difference between the means.</td>
</tr>
</table>

<table>
<tr><td>A</td><td>4</td><td>The mean life of a sample of 10 electric bulbs was found to be 1456 hours with SD of 423 hours. A second sample of 17 bulbs chosen from a different batch showed a mean life of 1280 with SD of 398 hours. Is there a significant difference between the means of two batches? ($|Z_{0.05}| = 1.96$)

Answer: There is no difference between the mean of two batches.</td></tr>
<tr><td>C</td><td>5</td><td>For sample I, $n_1 = 1000, \sum x = 49{,}000, \sum(x-\bar{x})^2 = 7{,}84{,}000$.
For sample II, $n_2 = 1500, \sum x = 70{,}500, \sum(x-\bar{x})^2 = 24{,}00{,}000$.
Discuss the significance of the difference of the sample means.
($|Z_{0.05}| = 1.96$)

Answer: No significant difference between the sample means.</td></tr>
<tr><td>B</td><td>6</td><td>A company claims that alloying reduces resistance of electric wire by more than 0.050 ohm. To test this claim samples of 32 standard wire and alloyed wire are tested yielding the following results. ($Z_{0.05} = 1.645$).
<table>
<tr><th>Type of wire</th><th>Mean resistance (ohms)</th><th>S.D. (ohms)</th></tr>
<tr><td>Standard</td><td>0.136</td><td>0.004</td></tr>
<tr><td>Alloyed</td><td>0.083</td><td>0.005</td></tr>
</table>
At the 0.05 level of significance, does this support the claim?

Answer: The data supports the claim.</td></tr>
<tr><td>B</td><td>7</td><td>A simple sample of heights of 6400 English men has a mean of 170 cm and a S.D. of 6.4 cm, while a simple sample of heights of 1600 Americans has a mean of 172 cm and a S.D. of 6.3 cm. Do the data indicate that American are, on the average, taller than the English men? ($Z_{0.01} = 2.33$)

Answer: Yes, American are, on the average, taller than the English men.</td></tr>
</table>

Method – 4 ⇝ Test of Significance of Difference Between Two Standard Deviations

Example of Method-4: Test of Significance of Difference Between Two Standard Deviations

<table>
<tr><td>A</td><td>1</td><td>The SD of a random sample of 1000 is found to be 2.6 and the SD of another random sample of 500 is 2.7. Assuming the samples to be independent, find whether the two samples could have come from populations with the same SD. ($|Z_{0.05}| = 1.96$)

Answer: Two samples could have come from populations with the same SD.</td></tr>
<tr><td>A</td><td>2</td><td>Intelligence test of two groups of boys and girls gives the following results:
<table>
<tr><td></td><td>n</td><td>S. D.</td></tr>
<tr><td>Girls</td><td>121</td><td>10</td></tr>
<tr><td>Boys</td><td>81</td><td>12</td></tr>
</table>
Is the difference between the standard deviations significant?
($|Z_{0.05}| = 1.96$)

Answer: There is no significant difference between sample SDs.</td></tr>
<tr><td>A</td><td>3</td><td>The mean yield of two plots and their variability are as given below:
No. of plot $= 40$; SD $= 34$ and No. of plot $= 60$; SD $= 28$.
Check whether the difference in the variability in yields is significant.
($|Z_{0.05}| = 1.96$)

Answer: There is no significant difference between sample SDs.</td></tr>
</table>

A	4	Examine whether the two samples for which the data are given in the following table could have been drawn from populations with the same SD.

	Size	SD
Sample I	100	5
Sample II	200	7

Is the difference between the standard deviation significant?

$(|Z_{0.05}| = 1.96)$

Answer: The sample standard deviations do not differ significantly.

Tests of Significance for Small Samples

Method – 5 ⇝ Test of Significance of Mean

Example of Method-5: t - Test for Single Mean

A	1	A machine is designed to produce insulting washers for electrical devices of average thickness of 0.025 cm. A random sample of 10 washers was found to have an average thickness of 0.024 cm wih S.D. of 0.002 cm. Test the significance of the deviation. $(\lvert t_{0.05,9} \rvert = 2.2622)$ **Answer: There is no significant difference between population mean and sample mean.**
A	2	A random sample of six steel beams has a mean compressive strength of 58392 psi (pounds per square inch) with a SD of 648 psi. Use this information and level of significance $\alpha = 0.05$ to test whether the true average compressive strength of the steel from which this sample came is 58000 psi. Assume normality. $(\lvert t_{0.05,5} \rvert = 2.5706)$ **Answer: The average compressive strength of the steel beam is not equal to 58000 psi.**
A	3	A random sample of size 16 from a normal population showed a mean of 103.75 cm and sum of squares of deviations from the mean 843.75 cm^2 can we say that the population has a mean of 108.75 cm? $(\lvert t_{0.05,15} \rvert = 2.1314)$ **Answer: No, We cannot say that the population mean is 108.75 cm.**
B	4	A manufacturer of external hard drives claims that only 10% of his drives require repairs within the warranty period of 12 months. If 5 of 20 of his drives required repairs within the first year, does this tend to support or refute the claim? $(\lvert t_{0.05,19} \rvert = 1.7291)$ **Answer: The claim should be refuted.**

A	5	A random sample of 10 boys had the following IQs: 70, 120, 110, 101, 88, 83, 95, 98, 107 and 100. Do these data support the assumption of a population mean IQ of 100? Find 95% confidence limits for the mean IQ. $(\lvert t_{0.05,9} \rvert = 2.2622)$ **Answer: 86.9892 and 107.4108**
A	6	The 9 items of a sample have the values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these values differ significantly from assumed mean 47.5? $(\lvert t_{0.05,9} \rvert = 2.2622)$ **Answer: The mean of given values does not differ significantly from assumed mean 47.5.**
B	7	Producer of gutkha claims that the nicotine content in his gutkha on the average is 1.83 mg. Can this claim be accepted if a random sample of 8 gutkha of this type have the nicotine contents of 2, 1.7, 2.1, 1.9, 2.2, 2.1, 2, 1.6 mg? Use a 0.05 level of significance. $(\lvert t_{0.05,7} \rvert = 2.3646)$ **Answer: Yes, The claim is accepted.**
C	8	The mean weekly sales of soap bars in departmental stores was 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was the advertising campaign successful? $(t_{0.05,21} = 1.7207)$ **Answer: The advertisement campaign is successful.**
B	9	A random sample from a company's very extensive files shows that the orders for a certain kind of machinery were filled respectively in 10, 12, 19, 14, 15, 18, 11 and 13 days. Use the level of significance $\alpha = 0.01$ to test the claim that on the average such orders are filled in 10.5 days. Choose and Test the alternative hypothesis so that rejection of null hypothesis $\mu = 10.5$ days implies that it takes longer than indicated. $(t_{0.01,7} = 2.9980)$ **Answer: The orders on average are filled in more than 10.5 days.**

Method – 6 ⇝ Test of Significance of Difference Between Two Means

Example of Method-6: Test of Significance of Difference Between Two Means

<table>
<tr>
<td>A</td>
<td>1</td>
<td>Two sample of 6 and 5 items, respectively, gave the following data.
<table>
<tr><td></td><td>1st sample</td><td>2nd sample</td></tr>
<tr><td>Mean</td><td>40</td><td>50</td></tr>
<tr><td>S.D.</td><td>8</td><td>10</td></tr>
</table>
Is the difference of the means significant? (Test at 5% level of significance)
$(| t_{0.05,9} | = 2.2622)$

Answer: There is no significant difference between two population means.</td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>Two sample of 10 and 14 items, respectively, gave the following data.
<table>
<tr><td></td><td>1st sample</td><td>2nd sample</td></tr>
<tr><td>Mean</td><td>20.3</td><td>18.6</td></tr>
<tr><td>S.D.</td><td>3.5</td><td>5.2</td></tr>
</table>
Is the difference of the means significant? $(| t_{0.05,22} | = 2.0739)$

Answer: There is no significant difference between two population means.</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>A mechanist is making engine parts with axle diameter of 0.7 cm. A random sample of 10 parts shows a mean diameter of 0.742 cm with a standard deviation of 0.04 cm. Compute the statistic you would use to test whether work is meeting the specification at 0.05 level of significance.
$(| t_{0.05,9} | = 2.2622)$

Answer: The sample are not drawn from the same population.</td>
</tr>
</table>

A	4	The following figures refer to observations in live independent samples: Sample I: 25, 30, 28, 34, 24, 20, 13, 32, 22, 38 Sample II: 40, 34, 22, 20, 31, 40, 30, 23, 36, 17 Analyze whether the samples have been drawn from the population of equal means. Test whether the means of two populations are same at 5% level. $(\lvert t_{0.05,18} \rvert = 2.1009)$ **Answer: Samples have been drawn from population with equal mean.**
A	5	Random samples of specimens of coal from two mines A & B are drawn and their heat producing capacity (in millions of calories per ton) were measured yielding the following result: Mine A: 8260, 8130, 8350, 8070, 8340, – Mine B: 7950, 7890, 7900, 8140, 7920, 7840 Test whether the difference between the means of these two samples is significant. $(\lvert t_{0.05,9} \rvert = 2.2622)$ **Answer: There is significant difference in average heat producing capacity of coal from mines.**
B	6	A group of 5 patients treated with medicine A weight 42, 39, 48, 60 and 41 kg. Second group of 7 patients from the same hospitals treated with medicine B weigh 38, 42, 56, 64, 68, 69 and 62 kg. do you agree with the claim that medicine B increases the weight significantly? $(t_{0.05,10} = -1.8125)$ **Answer: The medicine B does not increase in weight.**
B	7	A large group of teachers are trained, where some are trained by institution A and some are trained by institution B. In a random sample of 10 teachers taken from a large group; the following marks are obtained in an appropriate achievement test. Institution A: 65, 69, 73, 71, 75, 66, 71, 68, 68, 74 Institution B: 78, 69, 72, 77, 84, 70, 73, 77, 75, 65 Test the claim that institute B is more effective. $(t_{0.05,18} = 1.7341)$ **Answer: The claim is valid.**

Method – 7 ⇝ F-test for Equality of Two Population Variances

Example of Method-7: F-test for Equality of Two Population Variances

<table>
<tr><td>A</td><td>1</td><td>In two independent samples of sizes 8 and 10 the sum of squares of derivations of the sample's values from the respective sample means were 84.4 and 102.6. Test whether the difference of variances of the populations is significant or not. ($F_{0.05}(7,9) = 3.29$)

Answer: There is no significant difference between the variances of two populations.</td></tr>
<tr><td>A</td><td>2</td><td>Two samples of size 9 and 8 give the sum of squares of deviations from their respective means equal 160 inches and 91 inches respectively. Can they be regarded as drawn from two normal populations with the same variance? ($F_{0.05}(8,7) = 3.73$)

Answer: The samples can be regarded as drawn from normal population with same SD.</td></tr>
<tr><td>B</td><td>3</td><td>Two independent sample of size 7 and 6 had the following values:
<table>
<tr><td>A</td><td>28</td><td>30</td><td>32</td><td>33</td><td>31</td><td>29</td><td>34</td></tr>
<tr><td>B</td><td>29</td><td>30</td><td>30</td><td>24</td><td>27</td><td>28</td><td>-</td></tr>
</table>
Examine whether the samples have been drawn from normal populations having the same variance. ($F_{0.05}(5,6) = 4.39$)

Answer: Samples have been drawn from the normal populations with same variance.</td></tr>
</table>

<table>
<tr>
<td>B</td>
<td>4</td>
<td>Two independent samples of 8 and 7 items respectively had the following values of the variable (weight in kg):
<table>
<tr><td>Sample I</td><td>9</td><td>11</td><td>13</td><td>11</td><td>15</td><td>9</td><td>12</td><td>14</td></tr>
<tr><td>Sample II</td><td>10</td><td>12</td><td>10</td><td>14</td><td>9</td><td>8</td><td>10</td><td>-</td></tr>
</table>
Do the two estimates of population variance differ significantly?
$(F_{0.05}(7,6) = 4.21)$

Answer: There is no significant difference between two estimates of population variances.</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>Two samples are drawn from two normal populations. From the following data test whether the two samples have the same variance at 5 % level?
$(F_{0.05}(9,7) = 3.68)$
<table>
<tr><td>Sample I</td><td>60</td><td>65</td><td>71</td><td>74</td><td>76</td><td>82</td><td>85</td><td>87</td><td>-</td><td>-</td></tr>
<tr><td>Sample II</td><td>61</td><td>66</td><td>67</td><td>85</td><td>78</td><td>63</td><td>85</td><td>86</td><td>88</td><td>91</td></tr>
</table>
Answer: Two samples have the same variances.</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>The standard deviations calculated from two random samples of sizes 9 and 13 are 2.1 and 1.8 respectively. Can the samples be regarded as drawn from normal populations with the same SD?
$(F_{0.05}(8,12) = 2.85 \; ; \; |t_{0.05,20}| = 2.0860)$

Answer: The samples can be regarded as drawn from normal population with same SD.</td>
</tr>
<tr>
<td>C</td>
<td>7</td>
<td>Two random samples drawn from 2 normal populations are as follows:
<table>
<tr><td>A</td><td>17</td><td>27</td><td>18</td><td>25</td><td>27</td><td>29</td><td>13</td><td>17</td></tr>
<tr><td>B</td><td>16</td><td>16</td><td>20</td><td>27</td><td>26</td><td>25</td><td>21</td><td>-</td></tr>
</table>
Test whether the samples are drawn from the same normal population.
$(F_{0.05}(7,6) = 4.21 \; ; |t_{0.05,13}| = 2.1604)$

Answer: The samples are drawn from same normal population.</td>
</tr>
</table>

Method – 8 ⇝ Chi-square Test: for Goodness of Fit

Example of Method-8: Chi-square Test: for Goodness of Fit

A 1

A die is thrown 276 times and the results of these throws are given below:

Number appeared on the die	1	2	3	4	5	6
Frequency	40	32	29	59	57	59

Test whether the die is biased or not. $\left(\chi^2_{0.05,5} = 11.070\right)$

Answer: The die is biased.

A 2

The following table gives the number of accidents that took place in an industry during various days of the week. Test if accidents are uniformly distributed over the week.

Day	Mon	Tue	Wed	Thus	Fri	Sat
No. of accidents	14	18	12	11	15	14

$\left(\chi^2_{0.05,5} = 11.070\right)$

Answer: The accidents are uniformly distributed over the week.

C 3

A sample analysis of examination results of 200 Computer Engineer was made. It was found that 46 students had failed, 68 secured a third division, 62 secured a second division and the rest were placed in first division. Are these figures commensurate with the general examination result which is in the ratio of $4 : 3 : 2 : 1$ for various categories respectively?

$\left(\chi^2_{0.05,3} = 7.815\right)$

Answer: The data are not commensurate with the general examination result.

<table>
<tr>
<td>C</td>
<td>4</td>
<td>A set of five similar coins is tossed 320 times and result is obtained as follows:
<table>
<tr><td>No. of male heads</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>Frequency</td><td>6</td><td>27</td><td>72</td><td>112</td><td>71</td><td>32</td></tr>
</table>
Test the hypothesis that the data follow a binomial distribution.
$(\chi^2_{0.05,5} = 11.070)$

Answer: The data don’t follow binomial distribution.</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>The following mistakes per page were observed in a book:
<table>
<tr><td>No. of mistakes per page</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>No. of pages</td><td>211</td><td>90</td><td>19</td><td>5</td><td>0</td></tr>
</table>
Fit a Poisson distribution and test the goodness of fit. $(\chi^2_{0.05,4} = 9.488)$

Answer: Mistakes follow Poisson's distribution.</td>
</tr>
</table>

*** * * * * End of the Unit * * * * ***