DIGITAL CONTROL SYSTEMS

MAE 444/544

Project 1

Professor- Dr. Aaron Estes

By- Nikhil Arora

UB Person No- 50320282

$$G(z) = \frac{Y(z)}{U(z)} = \frac{1.4432}{z - 0.2299}$$

given
(i) Settling time $0.4 < t_s < 0.8s$
(ii) Overshoot $M_p < 0.2$
(iii) Rise time $0.1 < t_r < 0.4s$
(iv) $K_p = \infty$

We know $M_p = e^{-\pi\zeta / \sqrt{1-\zeta^2}}$

$\Rightarrow e^{-\pi\zeta/\sqrt{1-\zeta^2}} < 0.2$

Taking ln on both sides

$$\cancel{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}} < 0.2}$$

$$\frac{-\pi\zeta}{\sqrt{1-\zeta^2}} < \ln(0.2)$$

$$\frac{+\pi\zeta}{\sqrt{1-\zeta^2}} < +1.609$$

$$\frac{\pi \zeta}{\sqrt{1-\zeta^2}} < 1.609$$

$$\frac{\zeta}{\sqrt{1-\zeta^2}} < \frac{1.609}{\pi}$$

$$\zeta^2 < \frac{(1.609)^2}{\pi^2}(1-\zeta^2)$$

$$\zeta^2 < \left(\frac{1.609}{\pi}\right)^2 - \zeta^2\left(\frac{1.609}{\pi}\right)^2$$

$$\zeta^2\left[1+\left(\frac{1.609}{\pi}\right)^2\right] < \left(\frac{1.609}{\pi}\right)^2$$

$$\zeta^2 < \frac{(1.609/\pi)^2}{\left[1+(1.609/\pi)^2\right]}$$

$$\boxed{\zeta < 0.45}$$

From $0.4 < t_s < 0.8$

$$t_s = \frac{4}{\zeta \omega_n}$$

$$0.4 < \frac{4}{\zeta\omega_n} < 0.8$$

$\frac{4}{\zeta \omega_n} < 0.8$

$\zeta \omega_n > 5$

$\frac{4}{\zeta \omega_n} > 0.4$

$\zeta \omega_n < 10$

Also, $\lambda_d = e^{-\zeta \omega_n T}$

$\lambda_d = e^{-10 \times 0.05}$

$\lambda_d = 0.606$

$\lambda_d = e^{-5 \times 0.05}$

$\lambda_d = 0.778$

We know, $t_r = \frac{1.8}{\omega_n}$

$0.1 < t_r < 0.4$

$0.1 < \frac{1.8}{\omega_n} < 0.4$

$\frac{1.8}{\omega_n} > 0.1$

$\omega_n < 18$

$\frac{1.8}{\omega_n} < 0.4$

$\omega_n > 1.8/0.4$

$\omega_n > 4.5$

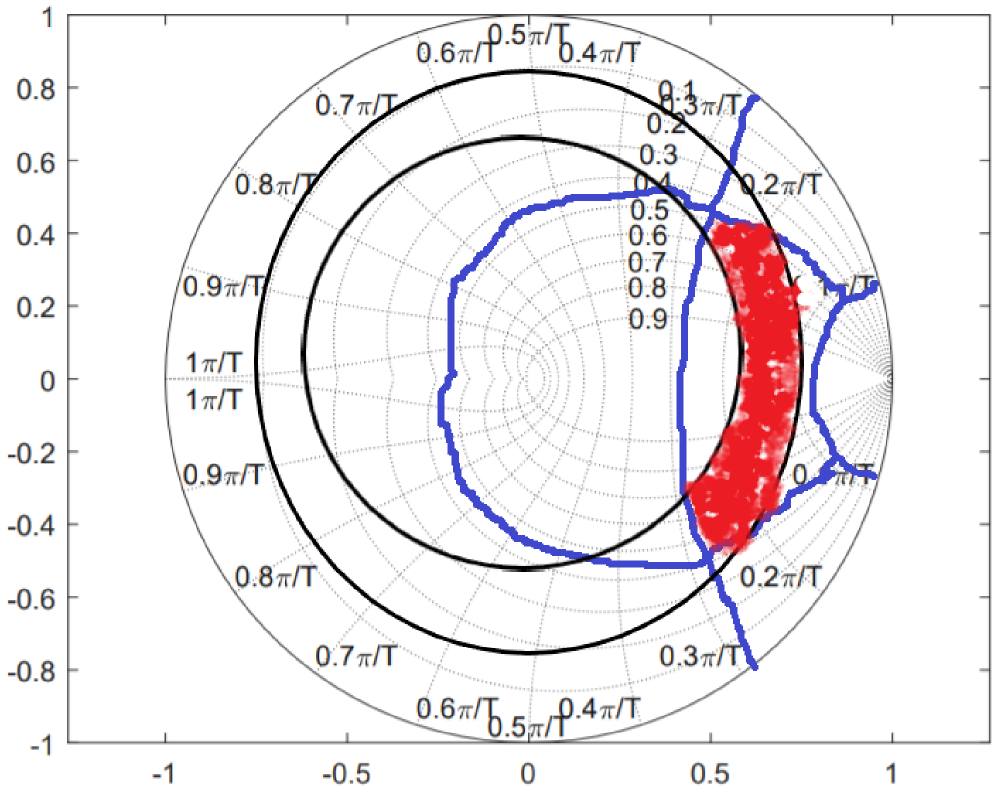
$\omega_n = \frac{x\pi}{T}$

$18 = \frac{x\pi}{T}$ | $4.5 = \frac{x\pi}{T}$

$x = \frac{18 \times 0.05}{3.14}$ | $x = \frac{4.5 \times 0.05}{3.14}$

$x = 0.2866$ | $x = 0.0716$

$\omega_n < \frac{0.2866\pi}{T}$ | $\omega_n > \frac{0.0716\pi}{T}$

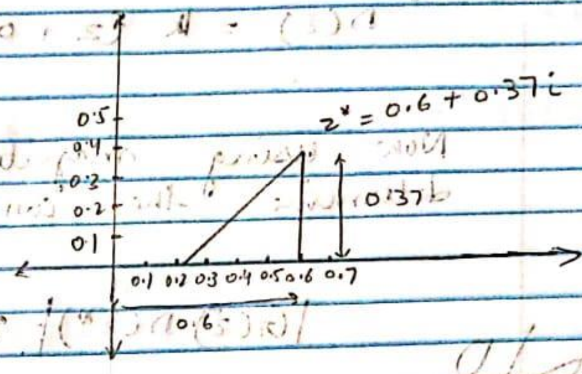


Red part is the accepted region

```
clear all; clc;
N = 100;
z_star = 0.6+0.37i;
sampling_freq = 20;
T = 1/sampling_freq;
G=tf([1.4432],[1 -0.2299],T);
%D_ang =pi+atan(imag(z_star)/(real(z_star)-0.229));
G_ang =angle(evalfr(G,z_star));
d_ang = pi - G_ang;
d_ang2 = d_ang - pi;
alpha_ang=linspace(d_ang2+10*pi/180, pi-10*pi/180,N);
beta1 = -1;
d_ang1 = angle(z_star+beta1);
alpha =zeros(N,1);
beta =zeros(N,1);
beta_ang =zeros(N,1);
myPoles =[];
for k = 1:N
    alpha(k) = imag(z_star)/tan(alpha_ang(k))-real(z_star);
    beta_ang(k) = alpha_ang(k)-d_ang2-d_ang1;
    beta1(k) = imag(z_star)/tan(beta_ang(k))-real(z_star);
    %beta2(k)=imag(z_star)/tan(alpha_ang(k))-real(z_star);
    D_z(k) = tf([1 alpha(k)], [1 beta1(k)-1 -beta1(k)], T);
    K(k) = 1/abs(evalfr(G*D_z(k), z_star));
    D(k) = K(k)*D_z(k);
    %controller_G = minreal(G*D_z/(1+G*D_z));
    controller_G2 = feedback(G*D(k),1);
    myPoles(k,:) = pole(controller_G2);
    L = tf([1 -1],[1 0],T);
    Kv = evalfr(minreal(L*G*D),1)/T
end

figure
plot(alpha, real(myPoles), 'kx', 'markersize',3, 'linewi',2)
grid on

Tfinal = 2;
t = (0:T:Tfinal);
stepReference = ones(length(t),1)*2*pi;
rampReference = t;
M = 5;
alpha_restrict = linspace(-0.8,-0.1,M);
mystepResponse = zeros(length(t),M);
myrampResponse = zeros(length(t),M);
mystepControl = zeros(length(t),M);
myrampcontrol = zeros(length(t),M);
clear alpha_ang beta_ang beta K D myPoles Kv D_z Kv
for i = 1:M
    alpha_ang(i) = angle(alpha_restrict(i)+z_star);
    beta_ang = alpha_ang(i) - d_ang2 - d_ang1;
```

```
    beta1(i) = imag(z_star)./tan(beta_ang)-real(z_star);
    D_z = tf([1 alpha_restrict(i)], [1 beta1(i)-1 -beta1(i)], T);
    K(i) = 1/abs(evalfr(G*D_z, z_star));
    D = K(i)*D_z;
    controller_G2 = feedback(G*D,1);
    myPoles(k,:) = pole(controller_G2);
    L = tf([1 -1],[1 0],T);
    Kv(i) = evalfr(minreal(L*G*D),1)/T;
    mystepResponse(:,i) = lsim(controller_G2,stepReference,t);
    myrampResponse(:,i) = lsim(controller_G2,rampReference,t);
    G_u = minreal(D/(1+G*D));
    myStepControl(:,i) = lsim(G_u, stepReference, t);
    myrampControl(:,i) = lsim(G_u, rampReference, t);
end

%Kv = rampReference-myrampResponse;
figure
plot(t,mystepResponse)
grid on
hold on
plot(t,stepReference)
%stepinfo(t,mystepReference)
grid on
hold off
title('Step Response')
figure
plot(t,myrampResponse)
grid on
hold on
plot(t,rampReference)
grid on
hold off

  title('Ramp Response')
  figure
  stairs(t,myrampControl)
  grid on
  title('Ramp Control')
  figure
  stairs(t,myStepControl)
  grid on
  title('Step Control')
```

$$z^* = 0.6 \pm 0.37i$$



$$\angle D(z^*) = \pi - \angle G(z^*)$$

$$= \pi - (-0.2299)$$
$$= 3.3699$$

$$\angle D(z^*) = 3.3699 - \pi$$
$$= 0.2299 \text{ rad}$$
$$= 13.17 \text{ degrees}$$

$$\tan(13.17) = \frac{0.37}{0.6 - z_D}$$

$$0.2339\,(0.6 - z_D) = 0.37$$

$$0.14 - 0.2339 z_D = 0.37$$

$$z_D = \frac{0.14 - 0.37}{-0.2339}$$

$$Z_D = 0.9833$$

Therefore,

$$D(z) = K(z + 0.9833)$$

We know $|G(z^*) D(z^*)| = 1$

From the Matlab program

$$K = 0.2087$$

Hence

$$D(z) = 0.2087 (z + 0.9833)$$

$$U(z) = 0.2087 (z + 0.9833) E(z)$$

multiply by z^{-1}

$$z^{-1} U(z) = 0.2087 (1 + 0.9833 z^{-1}) E(z)$$

$$u_{k-1} = 0.2087 (e_k + 0.9833 e_{k-1})$$

$$u_{k-1} = 0.2087 e_k + 0.205 e_{k-1}$$

$$u_k = 0.2087 e_{k+1} + 0.205 e_k$$

from the Matlab Step response.

(i) $t_s = 0.7$ sec

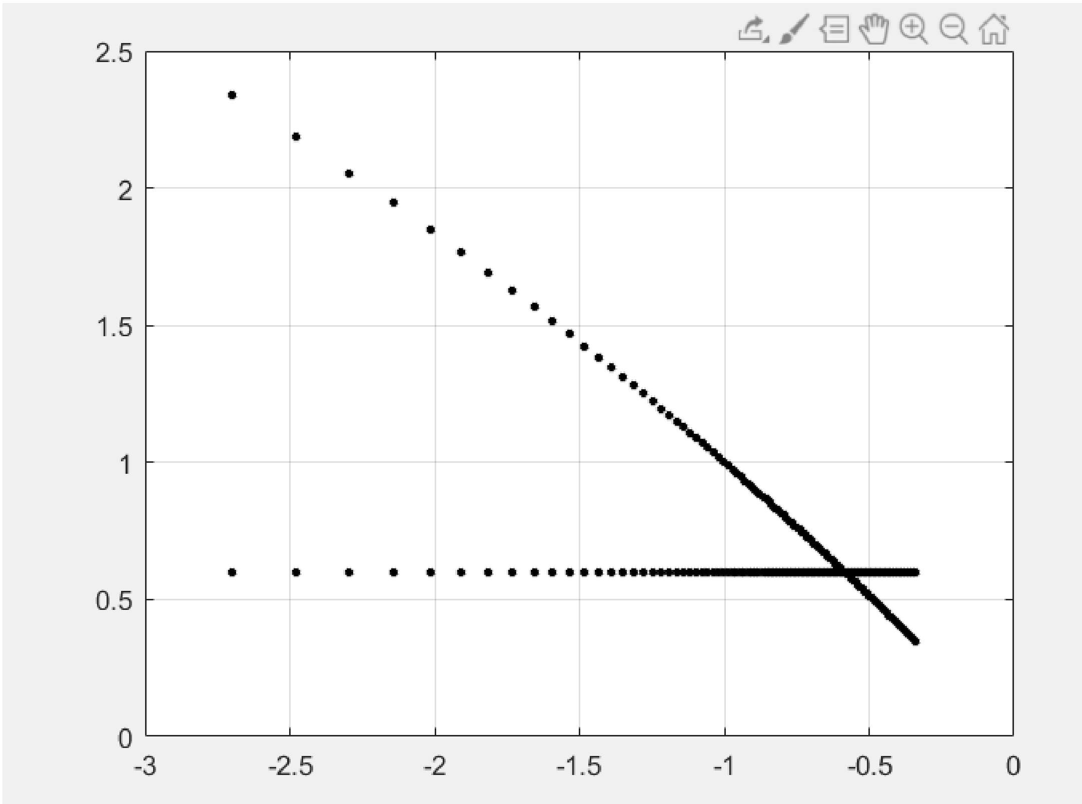
(ii) $M_p = 0.2$

(iii) $t_r = 0.25$ sec

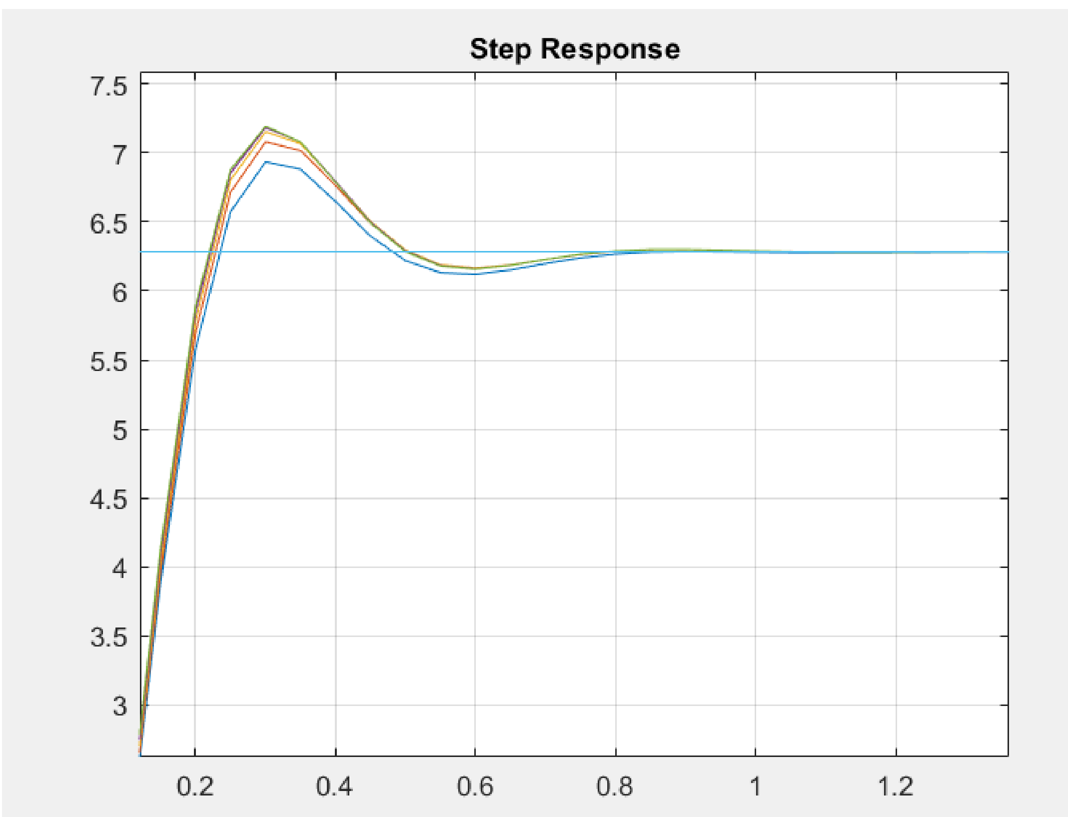
(iv) $K_p = \lim_{z \to 1} G(z) D(z)$

from Matlab

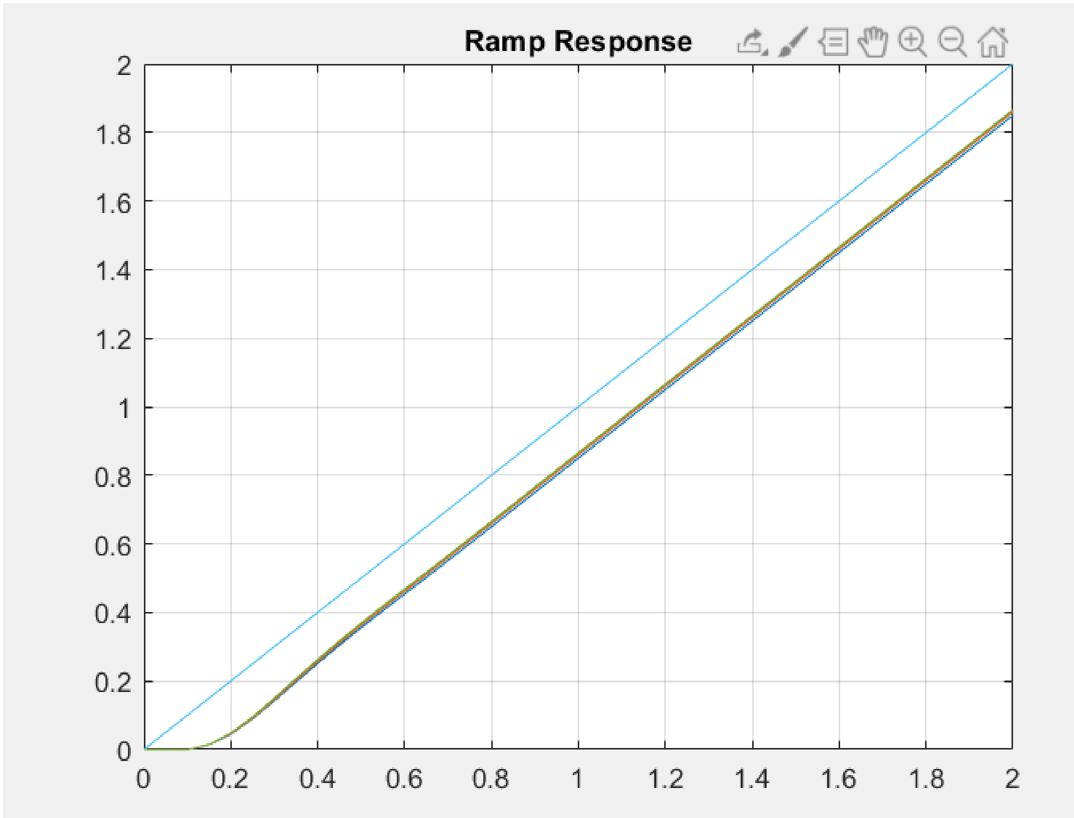
$K_p = 1.38 \times 10^{16} \approx \infty$

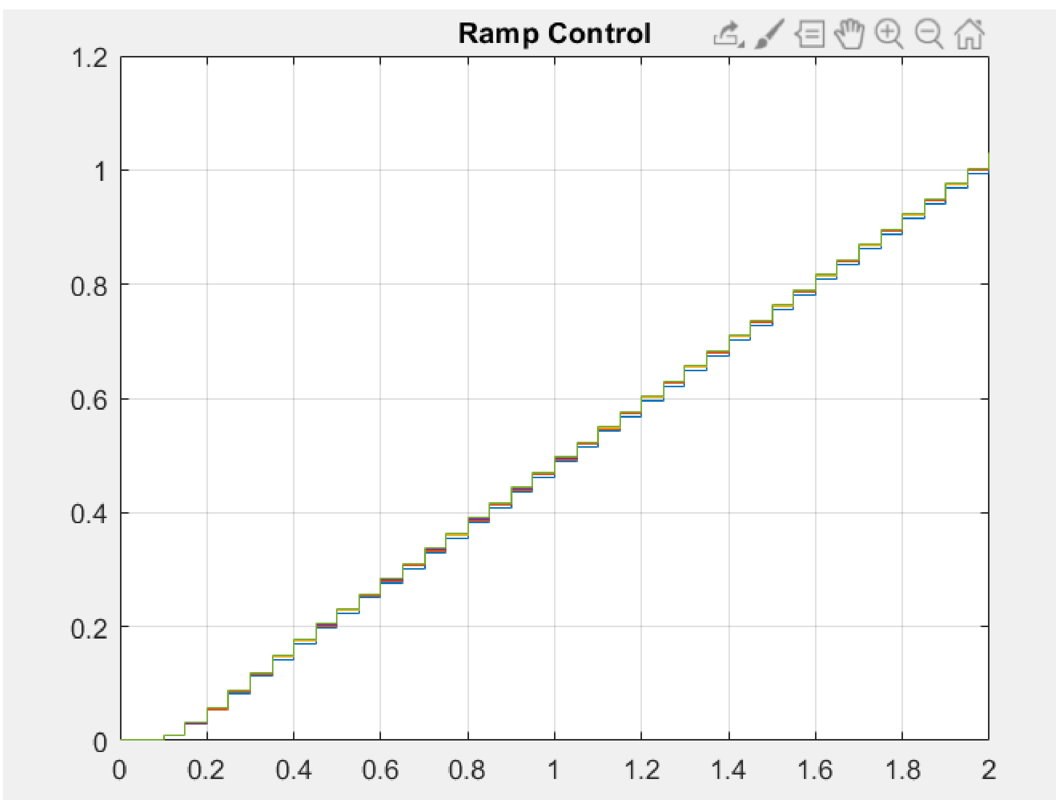


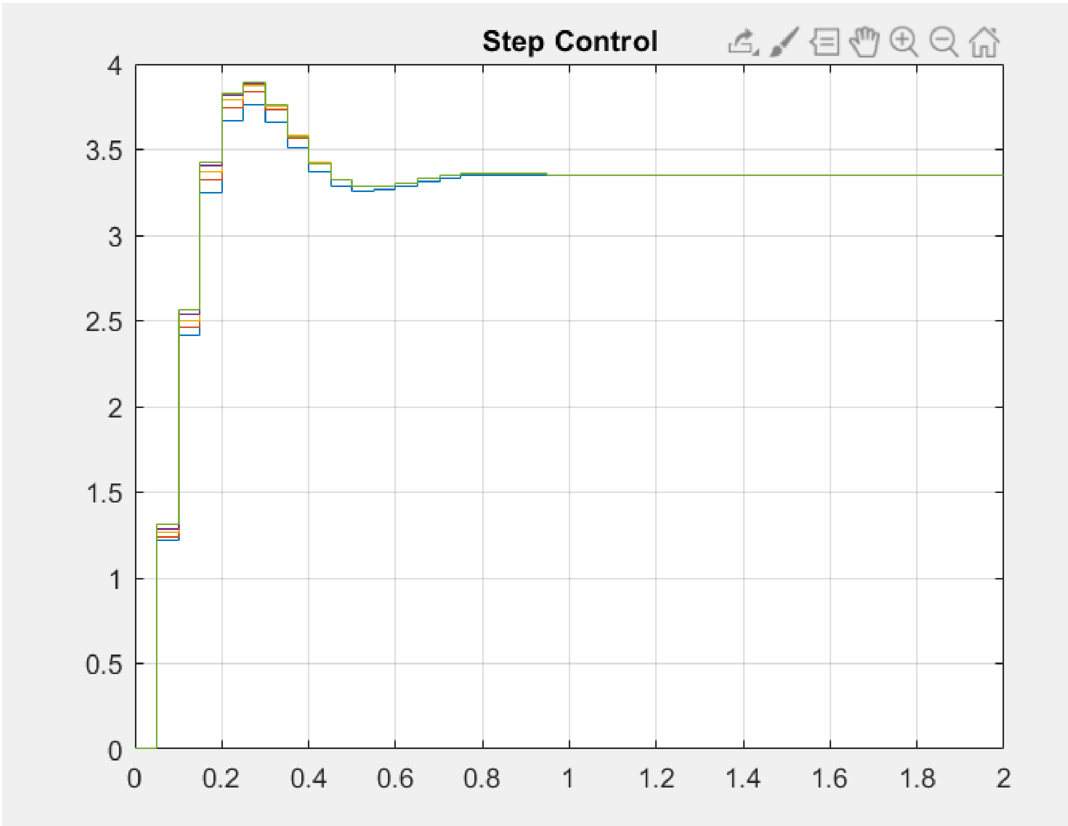
2.5
2
1.5
1
0.5
0
-3
-2.5
-2
-1.5
-1
-0.5
0



Step Response
7.5
7
6.5
6
5.5
5
4.5
4
3.5
3
0.2
0.4
0.6
0.8
1
1.2


Ramp Response
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Ramp Control
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Step Control
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2