Periodic Switched Event-triggered Schemes for Networked Control Systems

Pavagada Mathuranath Karthik
Dept. of Electron. & Comm. Engg.
PES University
Karnataka, IN
pavagadamathuranathkarthik@pesu.pes.edu

K M Amogh
Dept. of Electron. & Comm. Engg.
PES University
Karnataka, IN
pes1ug19ec036@pesu.pes.edu

Adithi Sujay
Dept. of Electron. & Comm. Engg.
PES University
Karnataka, IN
adithisujay@pesu.pes.edu

Nikhil Anil Prakash
Dept. of Electron. & Comm. Engg.
PES University
Karnataka, IN
nikhilanilprakash@pesu.pes.edu

Rajini Makam
Dept. of Electron. & Comm. Engg.
PES University
Karnataka, IN
rajinim@pes.edu

***Abstract*—Event-triggered control is a trending technique used to conserve communication bandwidth in the control of network-controlled systems. At present, there exist many different event-triggered control schemes. One of the parameters that define the performance of such methods is the number of transmissions triggered in a fixed interval of time. The lesser the number of transmissions, the more the bandwidth is conserved. This paper proposes two novel event-triggered schemes called PSET and PCMSET by combining the ideas of periodic sampling and switched event-triggered control. Simulations show the number of transmissions generated by both the novel schemes provided a significant reduction when compared to other existing schemes.**

***Index Terms*—Event-triggered control, Networked Control Systems, Periodic event-triggered control, Switched event-triggered control, Ideal operating period.**

I. Introduction

Networked Control Systems (NCS) are control systems in which the plant of the system is located geographically distant from the controller. The communication between the controller and the plant occurs through a shared communication network. In the present day, the importance of NCS arises from the fact that they enable the performance of various physical tasks from far away. They also reduce wiring complexities of the system and provide ease of system diagnostics and maintenance [1], [2] and [3].

One of the most pertinent challenges faced in NCS is the limited bandwidth of the communication network. This has led to the development of efficient transmission schemes to minimize bandwidth usage while not compromising control performance. Traditionally, the transmission of actuator signals occurs on a periodic basis (time-triggered scheme). But this leads to many redundant transmissions which necessitated the development of a more efficient scheme known as event-triggered control [4]. Event-triggered control is a control scheme for NCS where the transmission of a signal from the actuator to the controller occurs only if the closed-loop performance of the system is not satisfactory [5]. This ensures that there are no unnecessary transmissions and that the communication network is used only when required, thus conserving bandwidth [6].

Surveys were carried out in [7] and [8] on trending event-triggered schemes and modelling, stability, and controller design of event-triggered control (ETC) schemes were presented. Event-triggered sampling systems, self-triggered sampling schemes [9], and discrete event-triggered control schemes were among them.

The first event-triggering condition, known as Continuous Event Triggering (CET), monitors the system state continually and transmits it when the condition is broken. The drawback of CET is that the event-triggering condition has to be monitored continuously. Hence a periodic event-triggering scheme was proposed in [10] by combining periodic sampling with ETC. The triggering condition is checked only at periodic intervals and if the condition is violated, the transmission takes place. The Periodic Event-Triggered Schemes (PET) strategies developed in [10] are applicable to both static state-feedback and dynamical output-based controllers, as well as to both centralized and decentralized ETCs.

Extensive work has been carried out in periodic event-triggered control [11], [12] after its introduction in [10]. A model-based periodic event-triggered control algorithm [13] is proposed in order to conserve the feedback network's bandwidth. Using this approach, longer inter-sampling intervals are achieved. Model-based computations are also exploited in [14] to arrive at an advanced event-triggering mechanism which reduces communication in both the sensor-to-controller channel and the controller-to-actuator channels. Another study on periodic control is presented [15], which proposes a dynamic event-generator for linear systems which requires only periodic sampling of the output. It is hence easy to implement on digital systems. The advantages and benefits of periodic control are vast, hence, motivating research and exploration in the direction of PET.

A major issue faced during the practical implementation of ETC on a real-world system is the occurrence of Zeno behavior [16]. Zeno behaviour is the phenomenon that occurs when an infinite number of events take place in a finite amount of time. To overcome this, authors in [17] and [18] propose a switching approach to event-triggered control called Switched Event Triggering (SET). SET is a switching approach to ETC where after each transmission a fixed waiting period is introduced. Further, a Mixed Switching Event-Triggered Control scheme was introduced in [19] where the system switches sequentially between time-triggered, self-triggered and Periodic event-triggered schemes. The self-triggered module ensures that the event-triggered condition need not be checked continually at all times.

Although extensive research has been conducted on periodic and switched event-triggered control individually, the combination of the two ideas together has not been explored. In this paper, we propose two novel schemes, based on the combination of the above two ideas; a Periodic Switched Event-Triggered scheme (PSET) and a Periodic Continuous Mixed Switched Event-Triggered scheme (PCMSET). The fusion of both these schemes ensures that we obtain the best of both worlds. The implementation of periodic schemes is much easier as compared to continuous schemes since the system need not be monitored continuously. The switching nature of the schemes adds a waiting time after each transmission. This increases the inter-event time and hence decreases the number of transmissions. This paper also compares the performance of the above two schemes for different periodic intervals. The main contributions of this paper are two novel schemes Periodic Switched Event-Triggered Control (PSET) scheme and Periodic Continuous Mixed Switched Event-Triggered Control (PCMSET) scheme.

The organization of this paper is as follows: Section II reviews different event-triggering conditions. Two novel schemes, PSET and PCMSET are introduced in Section III. Section IV analyses the performance of PSET and PCMSET. Finally, the concluding remarks are given in Section V.

II. Review of Event-triggered conditions

In this section, we review the different event-triggering conditions dealt with in the literature. Let the state space representation of a continuous time system be represented as,

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in R^N$ and $u(t)$ represents the state of the system and the control input respectively. The main concept of CET control scheme is to continuously monitor the system condition, similar to time-triggered control, but to only communicate sampled data as needed. The triggering instant for the continuous event-triggered scheme is computed using:

$$t_{k+1} = \min\{t > t_k | \rho^T(t)\Omega\rho(t) \geq \epsilon x^T(t)\Omega x(t)\} \tag{2}$$

where $\rho(t) := x(t_k) - x(t)$ and $\epsilon > 0$ is a parameter that controls the frequency of the transmissions and $\Omega \geq 0$.

In PET, the state is evaluated periodically rather than continually. Transmission occurs only when the condition is violated. Similarly, the succeeding triggering time instant for the PET scheme is computed using:

$$t_{k+1} = t_k + \min\{ih > 0 | i \in \mathbb{N}, \rho_k^T \Omega \rho_k \geq \epsilon \bar{x_k}^T \Omega \bar{x_k}\} \tag{3}$$

where $\rho_k := \bar{x}_k - x(t_k)$, and $\bar{x}_k := x(t_k + ih)$ with $h > 0$ being the sampling interval. SET is introduced to prevent the occurrence of Zeno behaviour. In SET, the system switches between a sampled-data system with a defined sampling interval (ω) and a system operating under CET. The next triggering instant for the SET scheme is computed using:

$$t_{k+1} = \min\{t > t_k + \omega | \rho^T(t)\Omega\rho(t) \geq \epsilon x^T(t)\Omega x(t)\} \tag{4}$$

Discrete Switched Event-Triggered (DSET) scheme checks the triggering condition at equal intervals beginning after the waiting time according to [20]. In order to prevent Zeno-like behaviour, a waiting period is first established. Once a desired performance dependent condition is met, data transmission is then carried out. The next triggering instant for the DSET scheme is computed using:

$$t_{k+1} = t_k + \omega + \min\{ih \geq 0 | i \in \mathbb{N}, \rho_k^T \Omega \rho_k \geq \epsilon \bar{x_k}^T \Omega \bar{x_k}\} \tag{5}$$

A new scheme is proposed in [19] that switches between three control strategies namely, Time-Triggered control (TTC), Self Triggered control (STC) and Discrete Event-Triggered control (PET) schemes called Mixed Switched Event Triggered scheme (MSET). It combines the schemes in a manner such that the inter-event time is maximized. Each scheme is responsible for contributing a delay component to the overall time between two transmissions. TT scheme hands in a fixed, predefined delay, thereby preventing Zeno behavior. The self-triggered scheme postpones the transmission further by adding a variable delay before switching to PET. The next scheme checks the condition (6) at regular instants and dispatches the state only if it is violated.

$$\begin{aligned} t_{k+1} \quad = \quad & t_k + \omega + s_k + \min\{ih \geq 0 | i \in \mathbb{N}, \\ & \varrho_k^T(t)\Omega\varrho_k(t) \geq \epsilon(t_k)(x_k)^T(t_k)\Omega x_k(t_k)\} \end{aligned} \tag{6}$$

III. Periodic Switched Event-triggered Control Schemes

In this section, we propose periodic schemes that reduce the number of transmissions by a significant percentage compared to the schemes reviewed in Section II. If we associate periodic sampling to (2), we obtain the periodic event-triggered scheme proposed in [10]. On interpolating this idea to SET and MSET, we approach two new schemes, PSET and PCMSET as described in (7) and (8) respectively.

A. Periodic Switched Event-Triggered Control Scheme

Switching control causes a waiting period after each transfer, which helps prevent Zeno behaviour. This concept is combined with periodic sampling to give rise to periodic switched event-triggered control (PSET). The line diagram in Fig. 2 depicts the PSET scheme in a pictorial manner. The system switches from a time-triggered scheme to a periodic event-triggered scheme. In the periodic scheme, the condition is checked at all time instants which are integral multiples of the fixed period p. The instants at which the condition is violated, the transmissions take place. In other words, the condition is checked only at instants of time which are integral multiples of the selected period. 1 presents the flow of the PSET scheme. The next transmitting instance for PSET is computed using,

$$\begin{aligned} t_{k+1} \quad &= \quad \min\{(t > t_k + \omega) \ \& \ (t\%p == 0)| \\ &\qquad \rho^T(t)\Omega\rho(t) \geq \epsilon x^T(t)\Omega x(t)\} \end{aligned} \tag{7}$$

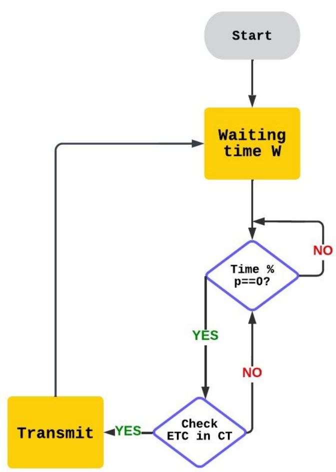

Figure 1. Flow Diagram for PSET

Remark. ***(Difference between DSET and PSET):*** *The main difference between DSET and PSET is that the PSET scheme starts measuring the intervals from the beginning of the simulation, whereas, DSET starts measuring the time intervals after the waiting period of each transmission. PSET uses a modulo operator and considers time as a global factor independent of transmissions. PSET and DSET can be visually distinguished from the line diagrams presented in Fig. 2 and Fig. 3 respectively.*

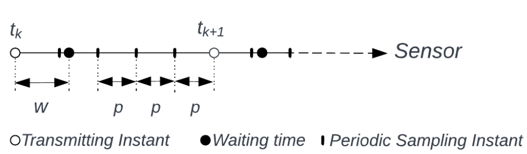

Figure 2. Line Diagram for PSET

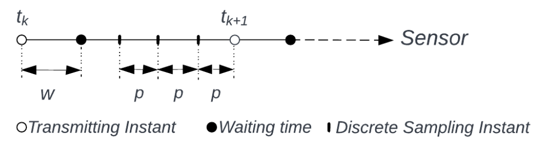

Figure 3. Line Diagram for DSET

B. Periodic Continuous Mixed Switched Event-Triggered Scheme

MSET is proposed in [19], in which the sensor switches between three control strategies namely, Time-Triggered control (TTC), Self-Triggered control (STC) and Periodic Event-Triggered (PET). The algorithm reduces the number of transmissions to 25 in a duration of 30 seconds. We combine this scheme with periodic sampling. This leads to the Periodic Continuous Mixed Switched Event-Triggered Control scheme (PCMSET). The line diagram of PCMSET is presented in Fig. 4. Here, the sensor switches to periodic monitoring post the ST scheme. From this point in time, the condition is checked continuously and the state is transmitted only if the time instant is a multiple of a predefined period p. In other words, the condition is evaluated at periodic intervals p and the state is transmitted only if the condition is not satisfied. The flow of the scheme is depicted in Fig. 5. The event-triggering condition is computed using,

$$\begin{aligned} t_{k+1} \quad &= \quad \min\{(t > t_k + \omega + s_k) \ \& \ (t\%p == 0)| \\ &\qquad \varrho_k^T(t)\Omega\varrho_k(t) \geq \epsilon(t_k)(x_k)^T(t_k)\Omega x_k(t_k)\} \end{aligned} \tag{8}$$

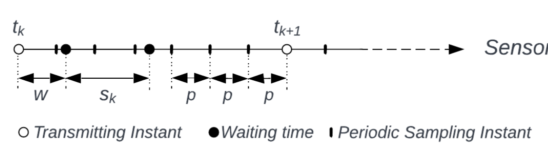

Figure 4. Line Diagram for PCMSET

IV. Simulation Results

In this section, we present the comparison between the schemes proposed in section III with other methods.

A. Example 1

To show the efficacy of the proposed schemes, we consider the system (1) with A = $\begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix}$, BK = $\begin{bmatrix} 0 & 0 \\ -3 & 0 \end{bmatrix}$. This is the benchmark system provided in [17].

Using the triggering matrices given in Table I, simulation of CET and PET for a duration of 30 seconds is shown in Fig. 6 and 7 respectively. Simulations reveal that the

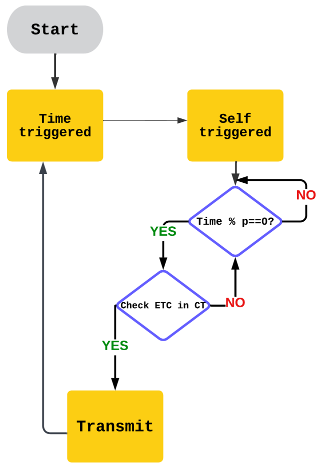

Figure 5. Flow Diagram for PCMSET

Table I
TRIGGERING MATRICES Ω FOR VARIOUS SCHEMES [19]

Scheme	Ω
CET	$\begin{bmatrix} 11.8130 & -0.0431 \\ -0.0431 & 8.9075 \end{bmatrix}$
PET	$\begin{bmatrix} 133.9546 & -0.2205 \\ -0.2205 & 17.6298 \end{bmatrix}$
SET and PSET	$\begin{bmatrix} 157.8802 & 0.1372 \\ 0.1372 & 19.5053 \end{bmatrix}$
MSET and PCMSET	$\begin{bmatrix} 192.8534 & 0.7626 \\ 0.7626 & 21.2071 \end{bmatrix}$

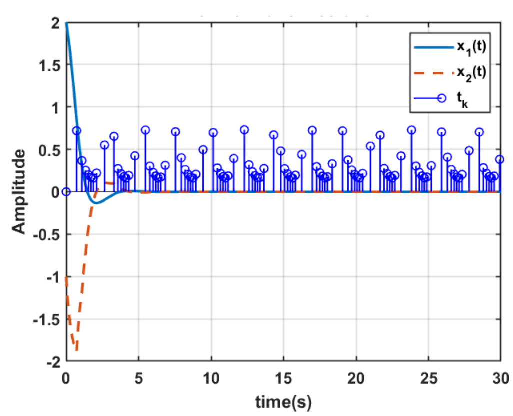

Figure 6. Transmissions for Continuous Event-Triggered Scheme

number of events occurring in PET and CET are 38 and 97 respectively. We see that the number of transmissions is significantly reduced in PET. This simulation shows that the periodic scheme in general can reduce the number of

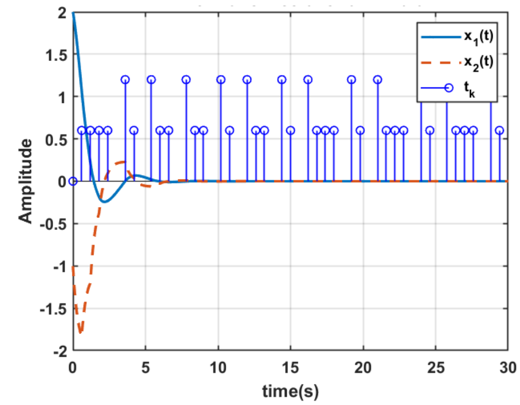

Figure 7. Transmissions for Periodic Event-Triggered Scheme

transmissions. The states of the system $x_1(t)$ and $x_2(t)$ are shown in solid and dashed lines respectively. Both the states converge to zero eventually.

Now, we give a comparison between our scheme PSET with SET. The simulations were carried out for a duration of 30 seconds. Table II provides the number of transmissions for SET and PSET. The states and transmission events for SET and PSET are shown in Fig. 8 and Fig. 9 respectively. It can be clearly observed that the number of transmissions for the PSET scheme is reduced by 34% compared to SET. Further, the states of the system converge to zero.

Table II
SET V/S PSET

Number of transmissions of SET	Number of transmissions of PSET
56	37

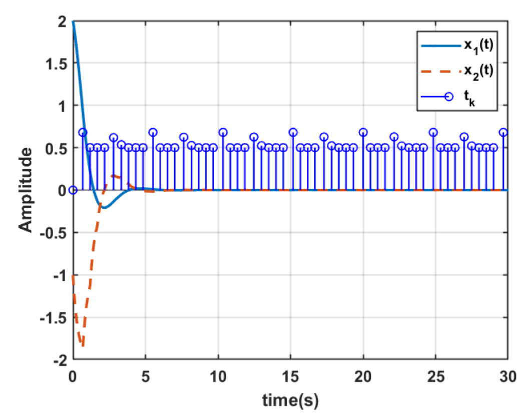

Figure 8. Switched Event-Triggered Scheme

Further, we compare another proposed scheme PCMSET with the counter scheme MSET. The number of transmissions in MSET is 24 while PCMSET transmits only 17 states for $p = 0.5$, which is 32% better than MSET as tabulated in Table III. The details of the choice p is given in Section IV-C. Fig. 10 and Fig. 11 respectively show the states and transmission events

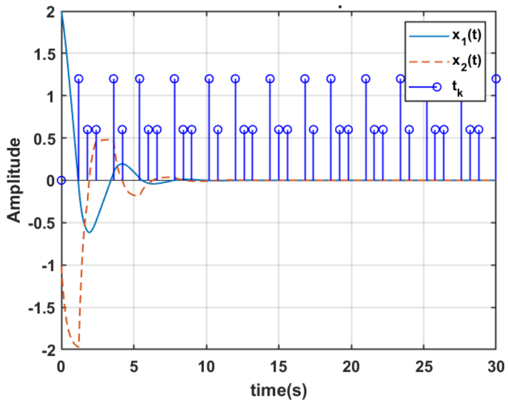

Figure 9. Periodic Switched Event Triggered Scheme

for MSET and PCMSET. The average inter-sampling period of PCMSET is 1.7647 whereas the average inter-sampling period of MSET is 1.25, with a reduction of 41%.

Table III
MSET V/s PCMSET

Number of transmissions of MSET	Number of transmissions of PCSET
25	17

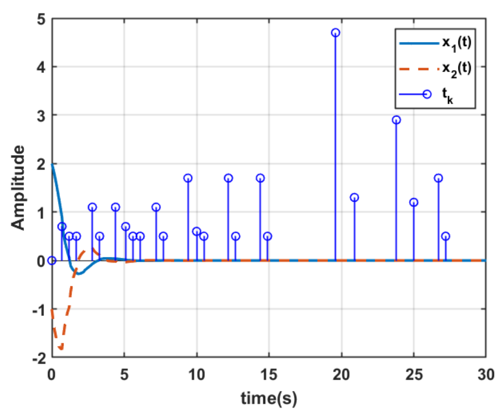

Figure 10. Mixed Switched Event-Triggered Scheme

B. Example 2

To further show the efficacy of the PCMSET, a fourth-order system is considered. This is an inverted pendulum system presented in [21]. The system matrix and input matrix are respectively,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{g}{l} & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ -\frac{1}{Ml} \end{bmatrix}$$

where the mass of the pendulum bob $m = 1\ kg$, mass of the cart is $M = 10\ kg$, length of the pendulum arm $l = 3\ m$ and $g = 10\ m/s^2$. Feedback gain and initial state are

$$K = \begin{bmatrix} 2 & 12 & 378 & 210 \end{bmatrix}$$

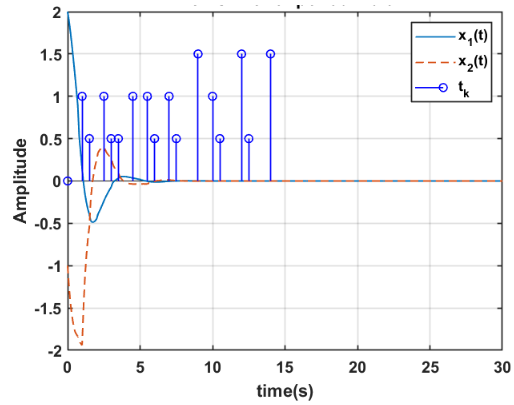

Figure 11. Periodic Continuous Mixed Switched Event-Triggered Scheme

$$x(0) = \begin{bmatrix} 0.98 & 0 & 0.2 & 0 \end{bmatrix}^T$$

respectively. As provided in [19], we plug in the values of ω, ϵ and Ω as 0.212, 0.35 and $\begin{bmatrix} 1 & 2 & 76 & 39 \\ 2 & 18 & 423 & 262 \\ 76 & 423 & 13637 & 7282 \\ 39 & 262 & 7282 & 4361 \end{bmatrix}$ respectively. The value of p is chosen as 0.21 for the best performance. The simulation is shown in Fig.12. All the states converge within 20 secs. The number of transmissions required is 46.

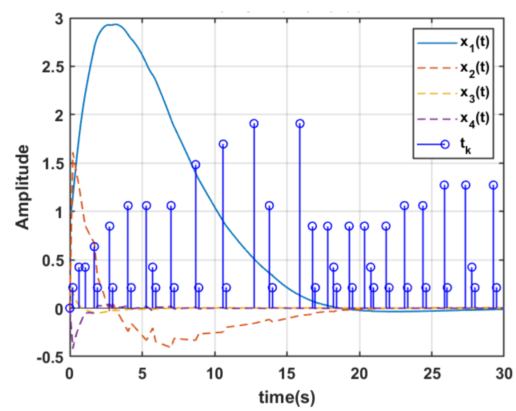

Figure 12. PCMSET - 4th order system

C. Ideal Operating Period (IOP)

Ideal operating period is defined as the period p which yields the least number of transmissions while maintaining the stability of states. IOP is found out by noting the number of transmissions corresponding to different periods p. The period p which gives a stable graph and the least amount of transmissions is chosen as the IOP. This is given for the second-order system considered in Section IV-A

1) IOP for PSET: The periods p and the corresponding number of transmissions are presented in Table IV. Since $p = 0.6$ provides the least number of transmissions, it is chosen as the IOP for PSET.

Table IV
PERIOD VS NUMBER OF TRANSMISSIONS PSET

Period (s)	Number of Transmissions
0.1	55
0.2	47
0.3	44
0.4	38
0.5	47
0.6	**37**
0.7	43
0.8	38

2) IOP for PCMSET: The periods p and corresponding number of transmissions which give stable graphs are presented in Table V. Since $p = 0.5$ provides the least number of transmissions, it is chosen as the IOP for PCMSET.

Table V
PERIOD VS NUMBER OF TRANSMISSIONS FOR PCMSET

Period (s)	Number of Transmissions
0.1	39
0.2	35
0.3	25
0.4	31
0.5	**17**
0.6	25
0.7	31
0.8	23
0.9	22
1.0	23

V. CONCLUSION

Two novel event-triggering schemes were proposed namely PSET and PCMSET that significantly reduced the number of transmissions for linear continuous-time network-controlled systems. Both these schemes were based on a combination of the ideas of periodic sampling and switched event-triggered control. These schemes reduced the number of transmissions by a minimum of 30% compared to other existing schemes. This paper has effectively demonstrated the potential of the periodic switched event triggering schemes as a novel solution for reducing the number of transmissions in networked control systems and provides a promising avenue for future research.

REFERENCES

[1] X.-M. Zhang, Q.-L. Han, X. Ge, D. Ding, L. Ding, D. Yue, and C. Peng, "Networked control systems: a survey of trends and techniques," *IEEE/CAA Journal of Automatica Sinica*, vol. 7, no. 1, pp. 1–17, 2020.

[2] Y.-T. C, "Networked control system: a brief survey," *IEE Proceedings-Control Theory and Applications*, vol. 153, no. 4, pp. 403–412, 2006.

[3] Y. Tipsuwan and M.-Y. Chow, "Control methodologies in networked control systems," *Control Engineering Practice*, vol. 11, no. 10, pp. 1099–1111, 2003.

[4] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks," *IEEE Transactions on Automatic Control*, vol. 52, no. 9, pp. 1680–1685, 2007.

[5] W. Heemels, M. Donkers, and A. Teel, "Periodic event-triggered control based on state feedback," in *2011 50th IEEE Conference on Decision and Control and European Control Conference*, 2011, pp. 2571–2576.

[6] W. Heemels, K. Johansson, and P. Tabuada, "An introduction to event-triggered and self-triggered control," in *2012 IEEE 51st IEEE Conference on Decision and Control (CDC)*, 2012, pp. 3270–3285.

[7] C. Peng and F. Li, "A survey on recent advances in event-triggered communication and control," *Information Sciences*, vol. 457-458, pp. 113–125, 2018.

[8] Z. Heng, P. Chen, Z. Jin, and Z. Chu, "Event-triggered control in networked control systems: A survey," in *The 27th Chinese Control and Decision Conference (2015 CCDC)*, 2015, pp. 3092–3097.

[9] M. Mazo, A. Anta, and P. Tabuada, "An iss self-triggered implementation of linear controllers," *Automatica*, vol. 46, no. 8, pp. 1310–1314, 2010.

[10] W. Heemels, M. Donkers, and A. Teel, "Periodic event-triggered control for linear systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 4, pp. 847–861, 2013.

[11] H. Yu, T. Chen, and F. Hao, "Output-based periodic event-triggered control for nonlinear plants: An approximate-model method," *IEEE Transactions on Control of Network Systems*, vol. 7, no. 3, pp. 1342–1354, 2020.

[12] H. Yu and T. Chen, "Periodic event-triggered networked control systems subject to large transmission delays," *IEEE Transactions on Automatic Control*, vol. 68, no. 1, pp. 63–79, 2023.

[13] R. Dou and Q. Ling, "Model-based periodic event-triggered control strategy to stabilize a scalar nonlinear system," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 9, pp. 5322–5335, 2021.

[14] W. Heemels and M. Donkers, "Model-based periodic event-triggered control for linear systems," *Automatica*, vol. 49, no. 3, pp. 698–711, 2013.

[15] D. Borgers, V. Dolk, and W. H. (Maurice), "Dynamic periodic event-triggered control for linear systems," 04 2017, pp. 179–186.

[16] H. Yu and T. Chen, "On zeno behavior in event-triggered finite-time consensus of multiagent systems," *IEEE Transactions on Automatic Control*, vol. 66, no. 10, pp. 4700–4714, 2021.

[17] A. Selivanov and E. Fridman, "Event-triggered h_∞ control: A switching approach," *IEEE Transactions on Automatic Control*, vol. 61, no. 10, pp. 3221–3226, 2016.

[18] Y. Huang and Y. Liu, "Switching event-triggered control for a class of uncertain nonlinear systems," *Automatica*, vol. 108, p. 108471, 10 2019.

[19] X. Wang, J. Sun, G. Wang, and L. Dou, "A mixed switching event-triggered transmission scheme for networked control systems," *IEEE Transactions on Control of Network Systems*, vol. 9, no. 1, pp. 390–402, 2022.

[20] Y. Qi, X. Xu, S. Lu, and Y. Yu, "A waiting time based discrete event-triggered control for networked switched systems with actuator saturation," *Nonlinear Analysis: Hybrid Systems*, vol. 37, p. 100904, 08 2020.

[21] D. Yue, E. Tian, and Q.-L. Han, "A delay system method for designing event-triggered controllers of networked control systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 2, pp. 475–481, 2013.