2.1.2 Flame model

The fraction of heat release rate perturbation per volume, $q_1(x,t)$ and mean heat release rate, q_{tot} can be defined as using local n-τ model;

$$\frac{q_1(\mathbf{x},t)}{q_{tot}} = \frac{nh(\mathbf{x})\int_\Omega w(\chi)\mathbf{u}_1(\chi,t-\tau(\mathbf{x}))\cdot\mathbf{n}_{ref}d\chi}{U_{bulk}} \tag{2.20}$$

where n is a constant with a unit of J.m^{-1}, $h(\mathbf{x})$ is heat release rate distribution, $\tau(\mathbf{x})$ is a time delay distribution and $(\mathbf{x})$ is a distribution of measurement region.

Moving q_{tot} to the right hand side and converting into frequency space leads;

$$q_1(\mathbf{x},t) = \frac{q_{tot}}{U_{bulk}}nh(\mathbf{x})\int_\Omega w(\chi)\mathbf{u}_1(\chi,t-\tau(\mathbf{x}))\cdot\mathbf{n}_{ref}d\chi \tag{2.21a}$$

$$\hat{q}_1 = \frac{q_{tot}}{U_{bulk}}nh(\mathbf{x})\int_\Omega w(\chi)\hat{u}(\chi)e^{i\omega\tau(\mathbf{x})}\cdot\mathbf{n}_{ref}d\chi \tag{2.21b}$$

$$\hat{q}_1 = \frac{q_{tot}}{U_{bulk}}nh(\mathbf{x})e^{i\omega\tau(\mathbf{x})}\int_\Omega w(\chi)\hat{u}(\chi)\cdot\mathbf{n}_{ref}d\chi \tag{2.21c}$$

Since $\hat{u} = \nabla\hat{p}/(-i\omega\rho_0)$ from momentum equation (2.3b);

$$\hat{q}_1 = \frac{q_{tot}}{-i\omega U_{bulk}}nh(\mathbf{x})e^{i\omega\tau(\mathbf{x})}\int_\Omega \frac{w(\chi)}{\rho_0}\nabla\hat{p}\cdot\mathbf{n}_{ref}d\chi \tag{2.22}$$

The thermoacoustic Helmholtz equation with unhomogeneous flame part becomes;

$$\nabla\cdot\left(c^2\nabla\hat{p}\right)+\omega^2\hat{p} = (\gamma-1)\frac{q_{tot}}{U_{bulk}}nh(\mathbf{x})e^{i\omega\tau(\mathbf{x})}\int_\Omega \frac{w(\chi)}{\rho_0}\nabla\hat{p}\cdot\mathbf{n}_{ref}d\chi \tag{2.23}$$

In Helmholtz solver, the components of unhomogeneous part are sub-labelled after finite element discretization;

$$\underbrace{\overbrace{(\gamma-1)\frac{q_{tot}}{U_{bulk}}}^{\text{coefficient}}\int_\Omega \phi_i nh(\mathbf{x})e^{i\omega\tau(\mathbf{x})}d\mathbf{x}}_{\text{left vector}}\underbrace{\int_\Omega \frac{w(\chi)}{\rho_0(\chi)}\nabla\phi_j\cdot\mathbf{n}_{ref}d\chi}_{\text{right vector}} \tag{2.24}$$

where ϕ_i and ϕ_j are trial and test functions.