

ML - Task2

Hypothesis testing

Q1. Suppose a child psychologist claims that the average time working mothers spend talking to their children is at least 11 minutes per day. You conduct a random sample of 1000 working mothers and find they spend an average of 11.5 minutes per day talking with their children. Assume prior research suggests the population standard deviation is 2.3 minutes. Conduct a test with a level of significance of $\alpha = 0.05$.

Null Hypothesis (H_0) = $\mu \geq 11$ minutes

Alternative Hypothesis (H_a) = $\mu < 11$ minutes

Significance level (α) = 0.05

Using the Z test for the mean

$$Z = (\bar{X} - \mu_0) \div (\sigma / \sqrt{n}) \quad \bar{X} = 11.5 \text{ Mins}, \mu_0 = 11 \text{ Mins}, \sigma = 2.3 \text{ Mins},$$

$$n = 1000$$

$$Z = (11.5 - 11) \div (2.3 / \sqrt{1000})$$

$$Z = .5 \div (2.3 \div 31.620)$$

$$Z = 6.87$$

For a one-tailed test with $\alpha = 0.05$ the critical value for Z from the standard normal distribution table is approximately -1.645.

If $Z \geq -1.645$, we fail to reject the null hypothesis.

If $Z < -1.645$, we reject the null hypothesis.

In this case, $Z = 6.87$ is much greater than -1.645.

So we can't deny the null hypothesis, the sample data suggests that working mothers spend, on average, more time than 11 minutes per day talking to their children.

Q2. A coffee shop claims that their average wait time for customers is less than 5 minutes. To test this claim, a sample of 40 customers is taken, and their wait times are recorded. The sample mean wait time is found to be 4.6 minutes with a standard deviation of 0.8 minutes. Perform a hypothesis test at a significance level of 0.05 and determine whether there is enough evidence to support the coffee shop's claim.

Null Hypothesis (H_0) = $\mu < 5$ minutes

Alternative Hypothesis (H_a) = $\mu \geq 5$ minutes

Significance level (α) = 0.05

Using the Z test for the mean

$$Z = (\bar{X} - \mu_0) \div (\sigma / \sqrt{n})$$

$$Z = 4.6 - 5 / (0.8 / \sqrt{40})$$

$$Z = -.4 \div (.8 / 6.3245) = -3.16225$$

If $Z \geq -1.645$, we fail to reject the null hypothesis.

If $Z < -1.645$, we reject the null hypothesis.

So in this case we have Z less than -1.645 so we can deny that the average waiting time in the coffee shop is less than 5 Mins.