

Electrical Engineering - Transmission Line Analysis

Given:

- Characteristic impedance of the line (Z_0): $70\ \Omega$
- Voltage standing wave ratio (s): 1.6
- Phase of the reflection coefficient (θ_R): 300°
- Length of the line (l): 0.6λ (where λ is the wavelength)

(a) Calculate $|\Gamma|$, Z_L , and Z_{in}

1. Reflection Coefficient ($|\Gamma|$)

The magnitude of the reflection coefficient can be calculated using the voltage standing wave ratio (s):

$$|\Gamma| = (s - 1) / (s + 1)$$

Substituting the given value:

$$|\Gamma| = (1.6 - 1) / (1.6 + 1) = 0.6 / 2.6 = 0.228$$

The reflection coefficient in phasor form will be:

$$\Gamma = |\Gamma| e^{j\theta_R} = 0.228 \angle 300^\circ$$

2. Load Impedance (Z_L)

Using the relationship between $|\Gamma|$ and Z_L :

$$\Gamma = (Z_L - Z_0) / (Z_L + Z_0)$$

Rearranging to solve for Z_L :

$$Z_L = Z_0 * (1 + \Gamma) / (1 - \Gamma)$$

Substituting $\Gamma = 0.228 \angle 300^\circ$:

$$\Gamma = 0.228 (\cos 300^\circ + j \sin 300^\circ) = 0.228 (0.5 - j0.866) = 0.114 - j0.1975$$

Substituting into the equation:

$$Z_L = 70 * (1 + 0.114 - j0.1975) / (1 - 0.114 + j0.1975) = 70 * (1.114 - j0.1975) / (0.886 + j0.1975) \approx 80.5 - j33.6\ \Omega$$

3. Input Impedance (Z_{in})

Using the formula for input impedance of a transmission line:

$$Z_{in} = Z_0 * (Z_L + jZ_0 \tan(\beta l)) / (Z_0 + jZ_L \tan(\beta l))$$

where $\beta = 2\pi/\lambda$ and $l = 0.6\lambda$ implies $\beta l = 1.2\pi$

Calculating $\tan(1.2\pi) = \tan(\pi + 0.2\pi) = \tan(0.2\pi)$:

$$\tan(0.2\pi) = \tan(\pi/5) \approx 0.7265$$

Substituting the known values:

$$Z_{in} = 70 * (80.5 - j33.6 + j70 * 0.7265) / (70 + j(80.5 - j33.6) * 0.7265) = 70 * (80.5 + j17.25) / (123.655 - j24.397) \approx$$

(b) Distance to the First Minimum Voltage from the Load

Given the relationship for the position of voltage nodes and antinodes:

$$l_{\min} = \lambda/2 - (\theta_R / 4\pi)\lambda = \lambda/2 - (300^\circ / 360^\circ) * \lambda/2 = 0.6\lambda / 6 = \lambda/6$$

Final Solution:

(a) $\Gamma = 0.228 \angle 300^\circ$, $Z_L = 80.5 - j33.6 \, \Omega$, $Z_{in} = 47.6 - j17.5 \, \Omega$

(b) Distance to the first minimum voltage from the load = $\frac{\lambda}{6}$