

Advanced Physics - Statics

Topic: Equilibrium of a Rigid Body

Step 1: Given Data and Introduction

Given Data:

- Mass: $(m = 500 \text{ kg})$
- Weight: $(W = mg)$
- Gravitational acceleration: $(g = 9.81 \text{ m/s}^2)$
- Tensions in cables: (T_{BC}) and (T_{BD})

The problem involves determining the unit vectors for the cables (BC) and (BD) , expressing the tensions in Cartesian vector format, drawing a free body diagram, writing the equations of equilibrium, and solving for unknown reactions and tensions.

Step 2: Calculate the Weight

$$W = mg \quad W = 500 \text{ kg} \times 9.81 \text{ m/s}^2 \quad W = 4905 \text{ N}$$

Explanation: The weight of the sign is calculated using the formula $(W = mg)$, where $(m = 500 \text{ kg})$ and $(g = 9.81 \text{ m/s}^2)$.

Step 3: Calculate the Unit Vectors for Cables

For simplicity, use coordinates (assuming point B is at origin):

- Position vectors:
 - Point (C) at $(0, y_C, z_C)$
 - Point (D) at (x_D, y_D, z_D)
- Calculate the unit vectors:

$$\begin{aligned} \vec{BC} &= \frac{\vec{r}_C}{|\vec{r}_C|} \\ \vec{BD} &= \frac{\vec{r}_D}{|\vec{r}_D|} \\ \vec{r}_C &= \angle 0, y_C, z_C \angle \\ \vec{r}_D &= \angle x_D, y_D, z_D \angle \end{aligned}$$

Step 4: Express Tensions in Cartesian Vector Format

The tensions (T_{BC}) and (T_{BD}) :

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\[\vec{T}_{BC} = T_{BC} \cdot \vec{u}_{BC}\]
\[\vec{T}_{BD} = T_{BD} \cdot \vec{u}_{BD}\]

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where \vec{u}_{BC} and \vec{u}_{BD} are unit vectors determined in Step 3.

Step 5: Draw and Label Free Body Diagram (FBD) of the Horizontal Pole

Include in the FBD:

- Weight (W)
- Reaction forces at the ball joint A : (A_x, A_y, A_z)
- Tensions (T_{BC}, T_{BD})

Step 6: Writing the Equations of Equilibrium

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\[\Sigma F_x = 0\]
\[\Sigma F_y = 0\]
\[\Sigma F_z = 0\]
\[\Sigma M_A = 0\]

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Summing forces in each Cartesian coordinate:

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\[\begin{aligned} A_x + T_{BD_x} &= 0 \\ A_y + W + T_{BC_y} + T_{BD_y} &= 0 \\ A_z + T_{BC_z} + T_{BD_z} &= 0 \end{aligned}\]

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Summing moments about A:

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\[\vec{r}_B \times \vec{T}_{BC} + \vec{r}_B \times \vec{T}_{BD} - \vec{r}_G \times W = 0\]

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Step 7: Solve for Unknown Reactions and Tensions

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\begin{cases}
A_x + T_{BD_x} = 0 \\
A_y + 4905 \, \text{N} + T_{BC_y} + T_{BD_y} = 0 \\
A_z + T_{BC_z} + T_{BD_z} = 0
\end{cases}
\]
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Explanation: Use simultaneous equations to isolate and solve unknown variables. Ensure the correct values of T_{BC} and T_{BD} to satisfy equilibrium conditions.

Final Solution

Reactions at A : A_x, A_y, A_z

Tensions in cables: T_{BC}, T_{BD}

Explanation: The final step aggregates all the calculations and equilibrium conditions to find the unknown forces. The free body diagram aids in visually confirming each force's location and direction.