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Inductive Proof for Discrete Mathematics

Given: The problem involves using inductive proofs to prove statements of the form $\forall\ n \in D,\ n \geq a,\ P(n)$, where $P(n)$ is a predicate. The task is to select all valid domains, D, to which inductive proof can be applied.

Step-by-Step Solution:

Step 1: Understanding Inductive Proof

Inductive proof is a method used to prove statements that are supposed to be true for all integers n greater than or equal to some initial value a. The process involves:

1. Proving the base case, which verifies the statement for the initial value $n = a$.
2. Proving the inductive step, which shows that if the statement holds for $n = k$, then it also holds for $n = k + 1$.

Supporting Statement: Inductive proofs are primarily used for sets of numbers that have a clear successor for any element, which is essential for the inductive step.

Step 2: Validating Domains

Considering the properties of the domains:

- **\(D = \mathbb{N}\) (Natural Numbers):**
  - Natural numbers start from 0 or 1 and have a clear successor (next integer).
  - *Valid for Inductive Proof*
- **\(D = \mathbb{Z}\) (Integers):**
  - Includes all whole numbers (positive, negative, and zero).
  - Inductive proofs generally start from a specific integer value and can proceed to \(n+1\).
  - *Valid for Inductive Proof*
- **\(D = \mathbb{R}\) (Real Numbers):**
  - Includes all rational and irrational numbers within the continuum.
  - Inductive steps are not meaningful as there is no "next" real number.
  - *Invalid for Inductive Proof*
- **\(D = \mathbb{C}\) (Complex Numbers):**
  - Includes numbers in the form \(a + bi\).
  - No natural order or "next" complex number making inductive proofs inapplicable.
  - *Invalid for Inductive Proof*

Supporting Statement: Only those sets that have a well-defined and discrete successor for each element are suitable for inductive proofs.

Step 3: Conclusion

Based on the properties and structure of the respective sets, induction can be applied to the domains `&mathbb{N}` and `&mathbb{Z}`, but not to `&mathbb{R}` or `&mathbb{C}`.

Final Solution:

Select all valid domains, D, to which inductive proof can be applied:

- $D = \mathbb{N}$
- $D = \mathbb{Z}$