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# Total Response of Linear Differential System

## Given and Introduction

Given the linear differential equation:

$$ \frac{d^2}{dt^2}y(t) + 2\frac{d}{dt}y(t) = \frac{d}{dt}x(t) + x(t) $$

Initial conditions:

$$ y(0) = 2, \quad \dot{y}(0) = 1 $$

Input:

$$ x(t) = u(t) $$

## Step-by-Step Solution

### 1. Homogeneous Solution

First, solve the homogeneous part of the differential equation:

$$ \frac{d^2}{dt^2}y(t) + 2\frac{d}{dt}y(t) = 0 $$

Characteristic equation:

$$ r^2 + 2r = 0 $$

Explanation: The characteristic equation is formed by substituting $y(t) = e^{rt}$ into the homogeneous differential equation.

Solving the characteristic equation:

$$ r(r + 2) = 0 $$$$ r_1 = 0, \quad r_2 = -2 $$

Explanation: The roots of the characteristic equation indicate the form of the homogeneous solution.

Homogeneous solution:

$$ y_h(t) = C_1 e^{0t} + C_2 e^{-2t} $$$$ y_h(t) = C_1 + C_2 e^{-2t} $$

Explanation: The general solution of the homogeneous differential equation involves a linear combination of terms with the roots of the characteristic equation.

### 2. Particular Solution

Next, find a particular solution to the non-homogeneous equation $x(t) = u(t)$. Since $u(t)$ is the unit step function, assume:

$$ y_p(t) = A $$

Explanation: Assume a simple form for $y_p(t)$ since $u(t)$ is a step function.

Substitute $y_p(t) = A$ into the differential equation:

$$ 0 + 2 \cdot 0 = \frac{d}{dt}u(t) + u(t) $$$$ 0 = \delta(t) + u(t) $$

Explanation: The differentiation of the step function $u(t)$ gives an impulse function $\delta(t)$.

To eliminate the impulse function's effect, assume $y_p(t) = B \cdot u(t)$:

$$ \frac{d}{dt}y_p(t) = B \cdot \delta(t) \Rightarrow \frac{d^2}{dt^2}y_p(t) = B \cdot \delta'(t) $$$$ B \cdot \delta'(t) + 2 \cdot B \cdot \delta(t) $$

Therefore, the particular solution:

$$ y_p(t) = u(t) $$

Explanation: By balancing the coefficient, the particular solution is identified as $u(t)$.

### 3. Total Solution

The total solution is:

$$ y(t) = y_h(t) + y_p(t) = C_1 + C_2 e^{-2t} + u(t) $$

Explanation: The total solution is the sum of the homogeneous solution and the particular solution.

### 4. Applying Initial Conditions

Use the initial conditions $y(0) = 2$ and $\dot{y}(0) = 1$ to find $C_1$ and $C_2$.
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At $t = 0$:

$$y(0) = C_1 + C_2 + u(0)$$
$$2 = C_1 + C_2 \cdot 1 + 0$$
$$2 = C_1 + C_2$$

Take the derivative of $y(t)$:

$$\dot{y}(t) = -2 C_2 e^{-2t} + u'(t)$$
$$1 = -2 C_2 + 0$$
$$C_2 = -\frac{1}{2}$$
$$C_1 = 2 + \frac{1}{2} = \frac{5}{2}$$

Explanation: The initial conditions help determine the constants in the total solution.

Final Solution

Thus, the total response for the given linear differential system is:

$$y(t) = \frac{5}{2} - \frac{1}{2} e^{-2t} + u(t)$$

Explanation: This solution includes both the transient (homogeneous) and steady-state (particular) responses, satisfying the differential equation and initial conditions.

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