CheggSolutions - Thegdp

Operations Research

Linear Programming - Cost Minimization Problem

Given:

Chicken:

- Protein: 10 grams/ounce
- Fat: 5 grams/ounce
- Cost: 11¢/ounce

Grain:

- Protein: 2 grams/ounce
- Fat: 2 grams/ounce
- Cost: 1¢/ounce

Requirements:

- Total protein: at least 246 grams
- Total fat: at least 156 grams

Step 1: Define variables

Let:

- x be the number of ounces of chicken.
- y be the number of ounces of grain.

Variable x represents the amount of chicken, and y represents the amount of grain. Both variables must be non-negative.

Variables are defined to translate the requirements into mathematical expressions.

Step 2: Formulate the objective function

The objective is to minimize cost. The cost function can be represented as:

Cost = 11x + 1y

The total cost is determined by the cost per ounce of chicken and grain multiplied by their respective quantities.

The objective function captures the economics of utilizing chicken and grain.

Step 3: Formulate the constraints

Constraints are derived from the nutritional requirements:

- Protein requirement: $10x + 2y \geq 246$
- Fat requirement: $5x + 2y \geq 156$
- Non-negativity constraints: $x \geq 0$
- $y \geq 0$

The constraints ensure that the nutritional limits for protein and fat are met and that the variables remain within a feasible range.

Constraints are essential to ensure that the final solution meets the minimum nutrition requirements.

Step 4: Graph the feasible region

Graphing the inequalities will help identify the feasible region where the solution exists.

- $10x + 2y \geq 246$
- $5x + 2y \geq 156$
- $x \geq 0$
- $y \geq 0$

Plotting these constraints on a graph helps visualize the feasible region, where all conditions are satisfied simultaneously.

The graphical method is used to find the region where all constraints overlap.

Step 5: Find corner points of the feasible region

Solve the equations to find the intersection points:

- $10x + 2y = 246$
- $5x + 2y = 156$

Solving these simultaneously:

- Multiply the second equation by 2: $10x + 4y = 312$ $10x + 2y = 246$
- Subtract the first from the second: $2y = 66 \implies y = 33$
- Substitute y into $10x + 2y = 246$: $10x + 2(33) = 246$ $10x + 66 = 246$ $10x = 180 \implies x = 18$

Corner Points: (18, 33)

The intersection provides the potential optimal solution due to satisfying both constraints.

Corner points of the feasible region are candidates for the optimal solution in linear programming problems.

Step 6: Calculate the cost at feasible point

Plug in $x = 18$ and $y = 33$ into the cost function:

Cost = 11(18) + 1(33) = 231

Evaluating the cost at the identified corner point gives the total expenditure on ingredients.

The cost calculation confirms the minimum expenditure required to meet the nutritional requirements.

Final Solution

Ruff, Inc. should use 18 ounces of chicken and 33 ounces of grain to minimize costs while satisfying the nutritional requirements. The minimum cost is **231¢**.

The identified quantities for chicken and grain minimize the cost while meeting all given nutritional constraints.

The final solution effectively balances both cost and nutritional requirements, ensuring optimal resource utilization.