

Discrete Mathematics and Algorithms

Topic: Integer Pair Transformation

Given:

Consider a pair of integers (a, b) . The following operations can be performed on (a, b) in any order, zero or more times:

1. $(a, b) \rightarrow (a + b, b)$
2. $(a, b) \rightarrow (a, a + b)$

The task is to return a string that denotes whether or not (a, b) can be converted to (c, d) by performing the operations zero or more times.

Example:

- Input: $(a, b) = (1, 1)$
- Output: $(c, d) = (5, 2)$

Solution Outline:

1. Given and Initial Analysis:

Given (a, b) and (c, d) , it is possible to transform (a, b) into (c, d) if and only if (c, d) can be derived using the allowed operations.

2. Reverse Transformations:

To check if (c, d) can be reached from (a, b) , consider reversing the operations:

- If $c > d$, consider reversing (c, d) as $(c - d, d)$
- If $d > c$, consider reversing (c, d) as $(c, d - c)$

3. Recursion Base Case:

The recursion can stop if:

- If $c < a$ or $d < b$, it's impossible to transform.
- If $c = a$ and $d = b$, the transformation is achieved.

4. Implementation Strategy:

Write a recursive function that checks the possibility of transforming (c, d) back to (a, b) using the reverse operations.

Step-by-Step Solution:

1. Initialization and Input:

```
def isConvertible(a, b, c, d):  
    if c < a or d < b:  
        return "No"  
  
    if a == c and b == d:  
        return "Yes"  
  
    if c > d:  
        return isConvertible(a, b, c - d, d)  
    else:  
        return isConvertible(a, b, c, d - c)
```

```
# Define the pairs
```

```
a, b = 1, 1  
c, d = 5, 2
```

```
# Run the check and print the result  
print(isConvertible(a, b, c, d))
```

2. Explanation of the Code:

- The code defines the function **isConvertible** that checks if (a, b) can be transformed into (c, d).
- **Base case 1:** Checks if (c < a or d < b), returning "No" because reverse transformation can't lower values below originals.
- **Base case 2:** Checks if initial and target pairs are equal, returning check successful "Yes".
- **Recursive Cases:** Applies reverse transformations until one of the base cases is met.

3. Detailed Explanation:

- **Initial Check:** If either component of (c, d) is smaller than (a, b), print "No" indicating an invalid transformation pathway.
- **Exact Match:** If both pairs match, transformation possible.
- **Recursive Reverse Transform:** Depending on whether c > d or vice versa, recursively adjusting c or d, ensuring each step considers both paths to potentially reach the target pair.

4. Final Solution:

Execute **isConvertible** function with arguments a=1, b=1, c=5, d=2. The expected print result is "Yes".

Conclusion:

This approach ensures that the proposed transformations and logic detect the validity of transforming any given pair of integers (a, b) to another pair (c, d) using allowed operations effectively.

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