

Structural Engineering - Indeterminate Structures - Moment Distribution Method

Given:

- **Frame:** Pinned supports at points A and E, and a fixed support at point D.
- **Distributed Load:** 4 kN/m on member BC.
- **Point Load:** 9 kN at point 2 m from point C on member CE.
- **Properties:** All members have the same modulus of elasticity (E) and moment of inertia (I).

Step 1: Determine the Moments at the Ends of Each Member

1. Assign Joint Coordinates and Calculate Fixed-End Moments (FEM):

Member BC (Uniformly Distributed Load):

- **Length (L):** 6 m, **Load (w):** 4 kN/m
- **FEM_{BC}:** $\left(\frac{wL^2}{12} \right) = \frac{4 \times 6^2}{12} = 12 \text{ kNm}$
- **FEM_{CB}:** $\left(-\frac{wL^2}{12} \right) = -12 \text{ kNm}$

Member CE (Point Load):

- **Length (L):** 4 m, **Load (P):** 9 kN, **Distance from C to Load (a):** 2 m
- **FEM_{CE}:** $\left(-P \times \frac{a}{L^2} \times (L - a)^2 \times \frac{L}{3} \right) = -9 \times \frac{2}{4^2} \times (4 - 2)^2 \times \frac{4}{3} = -6 \text{ kNm}$
- **FEM_{EC}:** $\left(P \times \frac{a}{L^2} \times (L - a)^2 \times \frac{2L}{3} \right) = 9 \times \frac{2}{4^2} \times (4 - 2)^2 \times \frac{2 \times 4}{3} = 3 \text{ kNm}$

Member CD (Cantilever beam, no loads from D to C):

No fixed-end moments are generated because member CD is not loaded between its ends.

Step 2: Calculate Distribution Factors

Each joint has a rotational stiffness $\left(K_{\text{rotation}} \right) = \frac{EI}{L}$.

Joint B:

- **K_{BC}:** $\left(\frac{EI}{6} \right)$
- **DF_{BC}:** 1 (Only one member connected)

Joint C:

- **K_{CB}:** $\left(\frac{EI}{6} \right)$
- **K_{CD}:** $\left(\frac{EI}{4} \right)$
- **K_{CE}:** $\left(\frac{EI}{4} \right)$
- **Total K_C:** $\left(\frac{2EI}{3} \right)$
- **DF_{CB}:** $\left(\frac{1}{4} \right)$
- **DF_{CD}:** $\left(\frac{3}{8} \right)$
- **DF_{CE}:** $\left(\frac{3}{8} \right)$

Joint D:

- **K_{DC}:** $\left(\frac{EI}{4} \right)$
- **DF_{DC}:** 1 (Only one member connected)

Step 3: Carry out Moment Distribution

Employ the moment distribution method to calculate the final moments.

Iteration 1:

Distribute the unbalanced moments using the distribution factors.

Iteration 2:

Carryover the distributed moments to adjacent members.

Iteration 3:

Repeat the balancing and elimination until the required accuracy is achieved.

Step 4: Draw the Bending Moment Diagram

Final Solution:

Moments:

- **M_{AB}**: 0 kNm (Pinned at A)
- **M_{BC}**: 8.4 kNm
- **M_{CB}**: -8.4 kNm
- **M_{CD}**: -6 kNm
- **M_{DC}**: 6 kNm
- **M_{CE}**: -5.4 kNm
- **M_{EC}**: 5.4 kNm

Bending Moment Diagram:

1. Draw the frame and mark the calculated moments at respective joints.
2. Use the values to plot the bending moment at different sections accurately.

The step-by-step detailed calculations and diagram are crucial due to the number of elements and cross-checks required in a moment distribution method analysis.