

AC Circuit Analysis

Given:

- a (Resistance) = 10Ω
- b (Inductance) = 0.1 H
- c (Capacitance) = 0.008 F
- d (Inductance) = 0.2 H
- e (Resistance) = 4Ω
- $v_s(t) = 50 \cos(25t) \text{ V}$

a) Find the magnitude of $v_0(t)$

b) Find the phase angle of $v_0(t)$

Step-by-step Solution:

Step 1: Convert $v_s(t)$ to its phasor form

Given: $v_s(t) = 50 \cos(25t)$

Phasor form is: $V_s = 50 \angle 0^\circ$

Step 2: Find impedances of each component at $\omega = 25 \text{ rad/s}$

Resistor (a and e):

$$Z_R = R$$

$$Z_a = 10 \Omega$$

$$Z_e = 4 \Omega$$

Inductor (b and d):

$$Z_L = j\omega L$$

$$Z_b = j \times 25 \times 0.1 = j2.5 \Omega$$

$$Z_d = j \times 25 \times 0.2 = j5 \Omega$$

Capacitor (c):

$$Z_C = 1/j\omega C$$

$$Z_c = 1/(j \times 25 \times 0.008) = -j5 \Omega$$

Step 3: Calculate the total impedance of the series and parallel combinations

• **Series combination across path 1:**

$$Z_{\text{path1}} = Z_b + Z_c + Z_d = j2.5 - j5 + j5 = j2.5 \Omega$$

• **Series combination across path 2:**

$$Z_{\text{path2}} = Z_a + Z_e = 10 + 4 = 14 \Omega$$

• **Parallel combination of Z_{path1} and R:**

$$1/Z_{\text{total}} = 1/Z_{\text{path1}} + 1/Z_{\text{path2}}$$

$$1/Z_{\text{total}} = 1/j2.5 + 1/14$$

$$Z_{\text{total}} = (1/(j2.5) + 1/14)^{-1}$$

$$Z_{\text{total}} = ((14/(j2.5 \times 14)) + (j2.5/(j2.5 \times 14)))^{-1}$$

$$Z_{\text{total}} = ((14 + j2.5)/35)^{-1}$$

$$Z_{\text{total}} = (35/(14 + j2.5))$$

$$Z_{\text{total}} = (35(14 - j2.5)/((14 + j2.5)(14 - j2.5)))$$

$$Z_{\text{total}} = (35(14 - j2.5))/(196 + 6.25)$$

$$Z_{\text{total}} = (490 - j87.5)/202.25$$

$$Z_{\text{total}} = 2.42 - j0.43 \Omega$$

Step 4: Calculate current I using Ohm's Law

$$V_s = I \times Z_{\text{total}}$$

$$50 \angle 0^\circ = I (2.42 - j0.43)$$

$$I = 50 \angle 0^\circ / (2.42 - j0.43)$$

$$I = 50 \times (2.42 + j0.43) / ((2.42 - j0.43)(2.42 + j0.43))$$

$$I = 50(2.42 + j0.43) / (2.9869 + 0.1849)$$

$$I = 50(2.42 + j0.43) / 3.1718$$

$$I = 121 + j21.5 / 3.1718$$

$$I = 38.15 + j6.78$$

- $I \approx 38.76 \text{ A}$, $\angle I \approx 10.4^\circ$

Step 5: Calculate V_0

- $V_0 = I \times Z_c$
- $V_0 = (38.15 + j6.78) \times (-j5)$
- $V_0 = -j(38.15 \times 5 + j6.78 \times 5)$
- $V_0 = -j(190.75 - j33.9)$
- $V_0 = -j190.75 - 33.9$
- $V_0 = -33.9 - j190.75$

Step 6: Find magnitude and phase angle of V_0

- Magnitude: $|V_0| = \sqrt{(-33.9)^2 + (-190.75)^2}$
- $|V_0| \approx \sqrt{(1150.41 + 36391)} \approx \sqrt{37541} \approx 194 \text{ V}$
- Phase angle: $\theta_{V_0} = \tan^{-1}(-190.75 / -33.9)$
- $\theta_{V_0} \approx \tan^{-1}(5.63) \approx -79.94^\circ$

Final Solution:

- (a) The magnitude of $v_0(t) \approx 194 \text{ V}$
- (b) The phase angle of $v_0(t) \approx -79.94^\circ$

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