CheggSolutions - Thegdp

CheggMasterBot Solutions

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Electromagnetism

Electric Field and Potential

Problem Statement:

Determine the gradient of the field potential given an infinite line charge along the z-axis.

Step-by-Step Solution:

Step 1: Understanding the Problem and Given Data

Given:
- An infinite line charge along the z-axis.
- Goal: Find the gradient of the field potential V.

Explanation: The problem revolves around finding the electric field due to an infinite line charge which involves both the potential V and its gradient.

Step 2: Defining the Potential Due to an Infinite Line Charge

The electric potential V at a distance ρ from an infinite line charge with linear charge density λ is given by:

$$V(\rho) = -\frac{\lambda}{2 \pi \epsilon_0} \ln\left(\frac{\rho}{\rho_0}\right)$$

where:
- λ is the linear charge density,
- ϵ_0 is the vacuum permittivity,
- ρ is the radial distance from the z-axis,
- ρ_0 is a reference distance.

Explanation: $V(\rho)$ is the potential due to an infinite line charge derived from using Gauss's law and considering the symmetry of the problem.

Step 3: Computing the Gradient of the Potential

The electric field $\mathbf{E}$ is related to the potential V by:

$$\mathbf{E} = -\nabla V$$

Considering cylindrical coordinates (ρ, ϕ, z), the potential V primarily depends on ρ:

$$\nabla V = \frac{\partial V}{\partial \rho} \hat{\rho} + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \hat{\phi} + \frac{\partial V}{\partial z} \hat{z}$$

Since V is a function of ρ only:

$$\mathbf{E} = -\frac{dV}{d\rho} \hat{\rho}$$

Explanation: The partial derivatives with respect to ϕ and z are zero because V does not depend on ϕ or z.

Step 4: Calculating the Gradient

Calculate $-\frac{dV}{d\rho}$:

$$V(\rho) = -\frac{\lambda}{2 \pi \epsilon_0} \ln\left(\frac{\rho}{\rho_0}\right)$$

$$\frac{dV}{d\rho} = -\frac{\lambda}{2 \pi \epsilon_0} \cdot \frac{1}{\rho}$$

Therefore:

$$\mathbf{E} (\rho) = -\left(-\frac{\lambda}{2 \pi \epsilon_0} \cdot \frac{1}{\rho}\right) \hat{\rho}$$

$$\mathbf{E} (\rho) = \frac{\lambda}{2 \pi \epsilon_0 \rho} \hat{\rho}$$

Explanation: Derivation involves differentiating the potential function with respect to ρ.

Step 5: Final Solution and Supporting Statement

The gradient of the field potential V resulting in the electric field $\mathbf{E}$:

$$\mathbf{E} = \frac{\lambda}{2 \pi \epsilon_0 \rho} \hat{\rho}$$

This matches the expected result considering an infinite line charge generates an electric field that diminishes with $1/\rho$.

Final Solution:

The gradient of the electric potential V due to an infinite line charge along the z-axis is:

$$\mathbf{E} (\rho) = \frac{\lambda}{2 \pi \epsilon_0 \rho} \hat{\rho}$$

This electric field points radially outward with magnitude inversely proportional to the distance ρ from the z-axis.