CheggSolutions - Thegdp

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Sub-subject: Calculus

Topic: Integration and Trigonometric Simplification

Given the integral function:

$$h(t) = \int_0^t (10 \cos 40t + 40 \sin 40t)\, dt$$

The goal is to simplify this function into the form:

$$A \cos(40t + \varphi)$$

where A is the magnitude of the resultant cosine function, and φ is the phase angle.

Step 1: Integrate the given function

First, integrate each term of the function inside the integral:

$$h(t) = \int_0^t 10 \cos 40t\, dt + \int_0^t 40 \sin 40t\, dt$$

This step divides the integral into two separate integrals, one for $10 \cos 40t$ and one for $40 \sin 40t$.

Step 2: Solve each integral

For the first integral:

$$\int 10 \cos 40t\, dt$$

Use the substitution $u = 40t \Rightarrow du = 40\, dt \Rightarrow dt = du/40$:

$$\int 10 \cos 40t\, dt = \int 10 \cos u\, du/40 = 10/40 \int \cos u\, du = 1/4 \sin u = 1/4 \sin(40t)$$

For the second integral:

$$\int 40 \sin 40t\, dt$$

Again using substitution $u = 40t \Rightarrow du = 40\, dt \Rightarrow dt = du/40$:

$$\int 40 \sin 40t\, dt = \int 40 \sin u\, du/40 = \int \sin u\, du = -\cos u = -\cos (40t)$$

Thus, the simplified solution is:

$$h(t) = 1/4 \sin (40t) - \cos (40t)$$

This step involves evaluating each integral using standard trigonometric integral results and substitution method, simplifying them to basic trigonometric functions.

Step 3: Combine results and express in standard form

Rewrite $h(t)$ in the form: $A \cos(40t + \varphi)$.

Combine $1/4 \sin (40t) - \cos (40t)$ using the formula $R \cos(\theta - \alpha)$.

First, compute the magnitude R:

$$R = \sqrt{((1/4)^2 + (-1)^2)} = \sqrt{(1/16 + 1)} = \sqrt{(17/16)} = \sqrt{17}/4 \approx 1.03$$

Next, determine the phase angle α:

$$\tan \alpha = \text{Coefficient of sin} / \text{Coefficient of cos} = (1/4) / -1 = -1/4 \quad \alpha = \tan^{-1} (-1/4) \approx -14.04^\circ$$

Thus, combining the results, the simplified form is:

$$h(t) = 1.03 \cos(40t - (-14.04^\circ)) = 1.03 \cos(40t + 14.04^\circ)$$

The final expression is obtained using vector (phasor) addition concepts, where the magnitude is calculated and the phase angle is adjusted to bring into the desired range of -180° to 180°.

Final Solution:

The simplified form of the function is:

$$h(t) = 1.03 \cos(40t + 14.04^\circ)$$

This gives the magnitude 1.03 and the phase angle 14.04 degrees.

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