CheggSolutions - Thegdp

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## Discrete Mathematics: Solving Congruences

**(a) 20x ≡ 4 (mod 30)**

**Step 1: Simplifying the Congruence**

20x ≡ 4 (mod 30)

Divide both sides by the gcd(20, 30) = 10:

2x ≡ 0.4 (mod 3)

Explanation: Reducing the coefficients by the greatest common divisor simplifies the equation.

**Step 2: Removing the Fraction**

Convert to integer by multiplying by 5:

10x ≡ 2 (mod 15)

Explanation: Multiplying by 5 ensures the equation has integer coefficients.

**Step 3: Solving the Simplified Equation**

10x ≡ 2 (mod 15)

Multiply both sides by the inverse of 10 mod 15, which is 3(10 * 3 = 1 mod 15):

x ≡ 6 (mod 15)

Explanation: Multiplying by the modular inverse isolates \(x\).

**Final Solution: x ≡ 6 (mod 15).**

**(b) 20x ≡ 30 (mod 4)**

**Step 1: Simplifying the Congruence**

Reduce modulo 4:

0x ≡ 2 (mod 4)

Explanation: Since 20 ≡ 0 mod 4 and 30 ≡ 2 mod 4, the equation simplifies.

**Step 2: Analyzing Simplification**

0 cannot equal 2, so:

No solutions

Explanation: If a coefficient's modulo result is 0, it cannot equal a non-zero remainder.

**Final Solution: No solutions.**

**(c) 353x ≡ 254 (mod 400)**

**Step 1: Simplifying the Congruence**

Check coefficients:

gcd(353, 400) = 1

Explanation: The gcd should be 1 to ensure invertibility.

**Step 2: Solving with Multiplicative Inverse**

Determine the modular inverse of 353 mod 400:

$353^{-1}$ ≡ 233 (mod 400)

Explanation: Using the Extended Euclidean Algorithm to compute the inverse.

**Step 3: Multiplying Both Sides**

x ≡ 254 * 233 (mod 400)
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$x \equiv 59182 \pmod{400}$

$x \equiv 182 \pmod{400}$

Explanation: Calculating the result to satisfy the congruence.

Final Solution: $x \equiv 182 \pmod{400}$.

(d) $57x \equiv 87 \pmod{105}$

Step 1: Simplify the Congruence

Check gcd:

$\gcd(57, 105) = 3$

Divide congruence by 3:

$19x \equiv 29 \pmod{35}$

Explanation: Simplifying with the gcd.

Step 2: Solving the Simplified Equation

Find the inverse of 19 mod 35:

$19^{-1} \equiv 16 \pmod{35}$

Multiply both sides:

$x \equiv 464 \pmod{35}$

$x \equiv 9 \pmod{35}$

Explanation: Calculating using the modular inverse.

Final Solution: $x \equiv 9 + 35k$.

(e) $64x \equiv 83 \pmod{105}$

Step 1: Simplify the Congruence

Check gcd:

$\gcd(64, 105) = 1$

Explanation: gcd = 1 confirms invertibility.

Step 2: Solving with Multiplicative Inverse

Find inverse:

$64^{-1} \equiv 1 \pmod{105}$

Multiply both sides:

$x \equiv 83 \pmod{105}$

Final Solution: $x \equiv 83 \pmod{105}$.

(f) $589x \equiv 209 \pmod{817}$

Step 1: Simplify the Congruence

Check gcd:

$\gcd(589, 817) = 1$

Explanation: gcd = 1 confirms invertibility.

Step 2: Solving with Multiplicative Inverse

Find inverse:

$589^{-1} \equiv 1 \pmod{817}$

Multiply both sides:

$x \equiv 209 \pmod{817}$

Final Solution: $x \equiv 209 \pmod{817}$.

(g) $49x \equiv 5000 \pmod{999}$

Step 1: Simplify the Congruence

Check gcd:

gcd(49, 999) = 1

Explanation: gcd = 1 confirms invertibility.

Step 2: Solving with Multiplicative Inverse

Find inverse:

$49^{-1} \equiv 1 \pmod{999}$

Multiply both sides:

$x \equiv 5000 \pmod{999}$

$x \equiv 4 \pmod{999}$

Explanation: Inverting and solving gives the solution directly.

Final Solution: $x \equiv 4 \pmod{999}$.

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