CheggSolutions - Thegdp

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## Subject: Differential Equations - Linear Differential Systems

### Topic: Total Response of Linear Differential Systems

**Given:**

$\frac{d^2 y(t)}{dt^2} + 2\frac{d y(t)}{dt} = \frac{d x(t)}{dt} + x(t)$

Initial conditions:

$y(0) = 2, \quad \dot{y}(0) = 1$

Input: $x(t) = u(t)$

**Steps to solve the problem:**

**Step 1: Introduction and Given**

Identify the given system and rewrite the differential equation:

$\frac{d^2 y(t)}{dt^2} + 2\frac{d y(t)}{dt} = \frac{d x(t)}{dt} + x(t)$

Where $x(t)$ is the input function and $y(t)$ is the output function.

**Step 2: Laplace Transform**

Take the Laplace Transform of the differential equation. Convert each term from time domain to the Laplace domain. Assume zero initial conditions for the input $x(t) = u(t)$.

Using:

$L\{ \frac{d^2 y(t)}{dt^2} \} = s^2Y(s) - sy(0) - \dot{y}(0)$
$L\{ \frac{d y(t)}{dt} \} = sY(s) - y(0)$
$L\{ \frac{d x(t)}{dt} \} = sX(s)$
$L\{ x(t) \} = X(s)$

**Step 3: Apply Initial Conditions**

Plug in the initial conditions $y(0) = 2$ and $\dot{y}(0) = 1$:

$\Rightarrow s^2Y(s) - sy(0) - \dot{y}(0) + 2(sY(s) - y(0)) = sX(s) + X(s)$
$\Rightarrow s^2Y(s) - 2s - 1 + 2sY(s) - 4 = sX(s) + X(s)$

**Step 4: Combine Like Terms**

Combine all terms involving $Y(s)$:
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$\Rightarrow (s^2 + 2s)Y(s) - 2s - 1 - 4 = (s + 1)X(s)$
$\Rightarrow (s^2 + 2s)Y(s) - 2s - 5 = (s + 1)X(s)$

Step 5: Solve for $Y(s)$

Solve the equation for $Y(s)$:

$Y(s) = \frac{(s + 1)X(s) + 2s + 5}{s^2 + 2s}$

Step 6: Input $x(t) = u(t)$

$x(t) = u(t) \Rightarrow X(s) = \frac{1}{s}$:

$Y(s) = \frac{(s + 1)\frac{1}{s} + 2s + 5}{s^2 + 2s}$
$\Rightarrow Y(s) = \frac{1 + s + 2s^2 + 5s}{s(s^2 + 2s)}$
$\Rightarrow Y(s) = \frac{2s^2 + 6s + 1}{s(s^2 + 2s)}$
$\Rightarrow Y(s) = \frac{2s + 6 + \frac{1}{s}}{s + 2}$

Step 7: Simplify $Y(s)$

Break down fractions:

$Y(s) = \frac{2s + 6}{s + 2} + \frac{1}{s(s + 2)}$

Step 8: Inverse Laplace Transform

Perform Inverse Laplace Transform to find $y(t)$:

$y(t) = L^{-1}\left\{ \frac{2s + 6}{s + 2} \right\} + L^{-1}\left\{ \frac{1}{s(s + 2)} \right\}$

Use partial fraction decomposition for the second term:

$\frac{1}{s(s + 2)} = \frac{A}{s} + \frac{B}{s + 2}$
Solve for A and B:

$1 = A(s + 2) + Bs$
Set $s = 0$:
$A(0 + 2) = 1 \Rightarrow A = \frac{1}{2}$
Set $s = -2$:
$B(-2) = 1 \Rightarrow B = -\frac{1}{2}$
So,
$\frac{1}{s(s + 2)} = \frac{\frac{1}{2}}{s} - \frac{\frac{1}{2}}{s + 2}$

Step 9: Compute Inverse Laplace Transform

$y(t) = L^{-1} \left\{ 2 + \frac{6}{s + 2} \right\} + L^{-1} \left\{ \frac{1}{2} \left(\frac{1}{s} - \frac{1}{s + 2} \right) \right\}$
$y(t) = 2L^{-1}\{1\} + 6L^{-1}\left\{ \frac{1}{s + 2} \right\} + \frac{1}{2} L^{-1}\{ \frac{1}{s} \} - \frac{1}{2} L^{-1}\left\{ \frac{1}{s + 2} \right\}$

Step 10: Final Solution

$y(t) = 2 + 6e^{-2t} + \frac{1}{2} - \frac{1}{2}e^{-2t}$
Combine similar terms:

$y(t) = 2 + \frac{1}{2} + (6 - \frac{1}{2}) e^{-2t}$
$y(t) = \frac{5}{2} + \frac{11}{2} e^{-2t}$

Therefore, the total response $y(t)$ is:

> **$y(t) = \frac{5}{2} + \frac{11}{2} e^{-2t}$**

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