CheggSolutions - Thegdp

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# Differential Equations

## Total Response for Linear Differential System

### Given Equation:

$$ \frac{d^2 y(t)}{dt^2} + 2 \frac{d y(t)}{dt} = \frac{d x(t)}{dt} + x(t) $$

with initial conditions $y(0) = 2$ and $\dot{y}(0) = 1$, and input $x(t) = u(t)$. The objective is to determine the total response of the system.

### Step 1: Express the Differential Equation

Given differential equation:

$$ \frac{d^2 y(t)}{dt^2} + 2 \frac{d y(t)}{dt} = \frac{d x(t)}{dt} + x(t) $$

and input $x(t) = u(t)$, where $u(t)$ is the unit step function.

### Step 2: Decompose into Homogeneous and Particular Solutions

Split the response into the homogeneous response $y_{h}(t)$ and the particular response $y_{p}(t)$:

$$ y(t) = y_{h}(t) + y_{p}(t) $$

### Step 3: Solve the Homogeneous Equation

The homogeneous differential equation is:

$$ \frac{d^2 y_{h}(t)}{dt^2} + 2 \frac{d y_{h}(t)}{dt} = 0 $$

Solve the characteristic equation:

$$ r^2 + 2r = 0 $$

$$ r(r + 2) = 0 $$

$$ r = 0, -2 $$

The general solution for $y_{h}(t)$ is:

$$ y_{h}(t) = C_1 e^{0 \cdot t} + C_2 e^{-2t} $$

$$ \therefore y_{h}(t) = C_1 + C_2 e^{-2t} $$

### Step 4: Solve for Particular Solution using Input

For $x(t) = u(t)$:
The particular solution $y_{p}(t)$ can be assumed of the form:

$$ y_{p}(t) = A u(t) $$

Substitute $y_{p}(t)$ into the original differential equation:
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\[ 0 + 2 \cdot 0 = \delta(t) + u(t) \]
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\[ 0 = 0 + [A \delta(t) + A u(t)] \]
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Solving for \(A \):

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\[ 0 = A \cdot 1 \cdot \delta(t) + A u(t) \]
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\[ A \cdot \delta(t) + A u(t) = 0 \]
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\[ A = 0 \]
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Thus, the particular solution is:

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\[ y_{p}(t) = 0 \]
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Step 5: Combine General and Particular Solutions

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\[ y(t) = y_{h}(t) + y_{p}(t) \]
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\[ y(t) = (C_1 + C_2 e^{-2t}) + 0 \]
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\[ y(t) = C_1 + C_2 e^{-2t} \]
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Step 6: Apply Initial Conditions

Given \(y(0) = 2 \) and \(\dot{y}(0) = 1 \):

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\[ y(0) = C_1 + C_2 = 2 \]
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\[ \dot{y}(t) = -2 C_2 e^{-2t} \]
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\[ \dot{y}(0) = -2 C_2 = 1 \]
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\[ C_2 = -\frac{1}{2} \]
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So, \(C_1 = 2 - (-\frac{1}{2}) = \frac{5}{2} \)

Step 7: Final Solution

The total solution for \(y(t) \):

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\[ y(t) = \frac{5}{2} - \frac{1}{2} e^{-2t} \]
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Thus, the total response for the given system is:

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\[ \boxed{y(t) = \frac{5}{2} - \frac{1}{2} e^{-2t}} \]
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