CheggSolutions - Thegdp

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## AC Circuit Analysis - Unbalanced Wye-Connected Load

### Given Data and Introduction

- A balanced wye-connected source supplies power to an unbalanced wye-connected load.
- Source voltage $V_{an} = 208 \angle 40^\circ$ volts (rms).
- Phase impedances: $Z_a = 10 + j20$ ohms, $Z_b = 8 + j12$ ohms, $Z_c = 6 + j22$ ohms.

### Objective

To find the load currents and the current in the neutral wire.

### Step-by-Step Solution

**Step 1: Calculate the line-to-neutral voltages for other phases.**

The source is balanced, so:

$V_{bn} = 208 \angle(-80^\circ)$

$V_{cn} = 208 \angle(160^\circ)$

**Explanation:**

The phase voltages in a balanced system are separated by $120^\circ$.

**Supporting Statement:**

This helps in calculating the individual phase currents for the unbalanced loads.

**Step 2: Calculate the phase currents $I_a$, $I_b$, and $I_c$.**

Using Ohm's Law, $I = \frac{V}{Z}$:

$$I_a = \frac{V_{an}}{Z_a}$$

$$I_a = \frac{208 \angle 40^\circ}{10 + j20}$$

$$Z_a = 10 + j20$$

$$|Z_a| = \sqrt{10^2 + 20^2} = \sqrt{100 + 400} = \sqrt{500} = 22.36 \, \text{ohms}$$

$$\theta_a = \tan^{-1}\left(\frac{20}{10}\right) = \tan^{-1}\left(2\right) = 63.435^\circ$$

$$Z_a = 22.36 \angle 63.435^\circ$$

$$I_a = \frac{208 \angle 40^\circ}{22.36 \angle 63.435^\circ}$$

$$I_a = \frac{208}{22.36} \angle (40^\circ - 63.435^\circ)$$

$$I_a = 9.3 \angle -23.435^\circ \, \text{A (Amps)}$$

**Explanation:**

The magnitude of $I_a$ is obtained by dividing the magnitudes of $V_{an}$ and $Z_a$, and the angle is the difference between the angles of $V_{an}$ and $Z_a$.
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Supporting Statement:

This gives the phase current I_a.

Step 3: Calculate Phase Currents I_b and I_c.

For I_b:

$$ I_b = \frac{V_{bn}}{Z_b} $$

$$ I_b = \frac{208 \angle -80^\circ}{8 + j12} $$

$$ Z_b = 8 + j12 $$

$$ |Z_b| = \sqrt{8^2 + 12^2} = \sqrt{64 + 144} = \sqrt{208} = 14.42 \, \text{ohms} $$

$$ \theta_b = \tan^{-1}\left(\frac{12}{8}\right) = \tan^{-1}(1.5) = 56.31^\circ $$

$$ Z_b = 14.42 \angle 56.31^\circ $$

$$ I_b = \frac{208 \angle -80^\circ}{14.42 \angle 56.31^\circ} $$

$$ I_b = \frac{208}{14.42} \angle (-80^\circ - 56.31^\circ) $$

$$ I_b = 14.42 \angle -136.31^\circ \, \text{A} $$

For I_c:

$$ I_c = \frac{V_{cn}}{Z_c} $$

$$ I_c = \frac{208 \angle 160^\circ}{6 + j22} $$

$$ Z_c = 6 + j22 $$

$$ |Z_c| = \sqrt{6^2 + 22^2} = \sqrt{36 + 484} = \sqrt{520} = 22.8 \, \text{ohms} $$

$$ \theta_c = \tan^{-1}\left(\frac{22}{6}\right) = \tan^{-1}\left(3.67\right) = 74.05^\circ $$

$$ Z_c = 22.8 \angle 74.05^\circ $$

$$ I_c = \frac{208 \angle 160^\circ}{22.8 \angle 74.05^\circ} $$

$$ I_c = \frac{208}{22.8} \angle (160^\circ - 74.05^\circ) $$

$$ I_c = 9.12 \angle 85.95^\circ \, \text{A} $$

Explanation:

Proceed similarly to the calculation for I_a to find the phase currents I_b and I_c.

Supporting Statement:

This helps in calculating the individual phase currents for the unbalanced loads.

Step 4: Calculate the neutral current I_n.

$$ I_n = I_a + I_b + I_c $$

Convert currents to rectangular form:

$$ I_a = 9.3 \angle -23.435^\circ = 9.3 (\cos(-23.435^\circ) + j \sin(-23.435^\circ)) $$

$$ I_a = 9.3 \times 0.919 + j \times 9.3 \times -0.398 $$

$$ I_a \approx 8.55 - j 3.7 \, \text{A} $$

$$ I_b = 14.42 \angle -136.31^\circ = 14.42 (\cos(-136.31^\circ) + j \sin(-136.31^\circ)) $$

$$ I_b = 14.42 \times -0.719 + j \times 14.42 \times -0.695 $$

$$ I_b \approx -10.36 - j 10.02 \, \text{A} $$

$$ I_c = 9.12 \angle 85.95^\circ = 9.12 (\cos(85.95^\circ) + j \sin(85.95^\circ)) $$

$$ I_c = 9.12 \times 0.06 + j \times 9.12 \times 0.998 $$

$$ I_c \approx 0.547 + j 9.10 \, \text{A} $$

Sum the rectangular components:

$$ I_n = (8.55 - 10.36 + 0.547) + j (-3.7 - 10.02 + 9.10) $$

$$ I_n = -1.263 - j 4.62 \, \text{A} $$

Convert back to polar form:

$$ |I_n| = \sqrt{(-1.263)^2 + (-4.62)^2} = \sqrt{1.595 + 21.344} = \sqrt{22.94} \approx 4.79 \, \text{A} $$

$$ \theta_n = \tan^{-1}\left(\frac{-4.62}{-1.263}\right) = \tan^{-1}(3.66) = -74.83^\circ $$

$$ I_n = 4.79 \angle -74.83^\circ \, \text{A} $$

Explanation:

The neutral current is the vector sum of the individual phase currents. Converting the phase currents to rectangular form and summing was necessary to get the result accurately.

Supporting Statement:

The resulting current in the neutral wire gives a complete understanding of how the sum of the unbalanced currents impacts the system.

Final Solution

- $I_a = 9.3 \angle -23.435^\circ$ A
- $I_b = 14.42 \angle -136.31^\circ$ A
- $I_c = 9.12 \angle 85.95^\circ$ A
- $I_n = 4.79 \angle -74.83^\circ$ A

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