CheggSolutions - Thegdp

Subject: Linear Algebra

Topic: Linear (In)dependence of Vectors

(1) Linear Independence of Vectors

Given:

```
\[ \mathbf{v_1} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{v_2} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \]
```

Introduction

To determine if the vectors $\mathbf{v_1}$ and $\mathbf{v_2}$ are linearly independent, check if the only solution to the equation $c_1\mathbf{v_1} + c_2\mathbf{v_2} = \mathbf{0}$ is $c_1 = 0$ and $c_2 = 0$.

Step 1: Set Up the Equation

```
\[ c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \]
```

Explanation: This equation must be solved to find out if $c_1 = 0$ and $c_2 = 0$ are the only solutions.

Step 2: Formulate System of Equations

```
\[ c_1 (1) + c_2 (1) = 0 \]
\[ c_1 (-1) + c_2 (1) = 0 \]

This transforms into:

\[ c_1 + c_2 = 0 \]
\[ -c_1 + c_2 = 0 \]
```

Step 3: Solve System of Equations

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Add both equations:
\[ (c_1 + c_2) + (-c_1 + c_2) = 0 + 0 \]
\[ 2c_2 = 0 \]
\[ c_2 = 0 \]

Substitute \(c_2\) back into the first equation:
\[ c_1 + 0 = 0 \]
\[ c_1 = 0 \]
```

Explanation: Both c_1 and c_2 are zero, indicating linear independence.

Final Solution

$$ \{ \mathbf{v_1}, \mathbf{v_2} \} \text{ is linearly independent.} $$

(2) Linear Independence of Vectors

Given:

```
\[ \mathbf{v_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{v_2} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \mathbf{v_3} = \begin{pmatr
```

Introduction

To determine if $\mathbf{v_1}$, $\mathbf{v_2}$, and $\mathbf{v_3}$ are linearly independent, check if the only solution to $c_1\mathbf{v_1} + c_2\mathbf{v_2} + c_3\mathbf{v_3} = \mathbf{0}$ is $c_1 = c_2 = c_3 = 0$.

Step 1: Set Up the Equation

```
\[ c_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + c_3 \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}
```

Explanation: Solve this system to find c_1, c_2, c_3.

Step 2: Formulate System of Equations

```
\[ c_1 + c_2 - c_3 = 0 \]
```

```
\[ c_1 + c_2 + 2c_3 = 0 \]
\[ c_1 - c_2 + 0 = 0 \]
```

Step 3: Solve System of Equations

```
Write in matrix form and row reduce:
\[ \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 2 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\
Perform row operations to apply row reduction:
1. \( R2 = R2 - R1 \)
2. \( R3 = R3 - R1 \)

New matrix:
\[ \begin{pmatrix} 1 & 1 & -1 \\ 0 & 0 & 3 \\ 0 & -2 & 1 \end{pmatrix} \]
Notice inconsistency:
- \(0 = 0\)
- \( -2c_2 + c_3 = 0\)

Explanation: System implies non-trivial solution exists.
```

Final Solution

\[\{ \mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3} \} \text{ is linearly dependent}.\]

(3) Linear Independence of Vectors

Given:

```
\[ \mathbf{v_1} = \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix}, \mathbf{v_2} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \mathbf{v_3} = \begin{pmatr
```

Introduction

To determine if \(\mathbf{v_1}\), \(\mathbf{v_2}\), and \(\mathbf{v_3}\) are linearly independent, check if the only solution to \(c_1\mathbf{v_1} + c_2\mathbf{v_2} + c_3\mathbf{v_3} = \mathbf{0}\) is \(c_1 = c_2 = c_3 = 0\).

Step 1: Set Up the Equation

```
\[ c_1 \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + c_3 \begin{pmatrix} 8 \\ 12 \\ 4 \end{pmatrix}
```

Explanation: Solve this system to find \(c_1, c_2, c_3\).

Step 2: Formulate System of Equations

```
\[ 4c_1 + c_2 + 8c_3 = 0 \]
\[ 4c_1 - c_2 + 12c_3 = 0 \]
\[ 2c_1 + 2c_2 + 4c_3 = 0 \]
```

Step 3: Solve System of Equations

```
Write in matrix form and row reduce:
\[ \begin{pmatrix} 4 & 1 & 8 \\ 4 & -1 & 12 \\ 2 & 2 & 4 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\
Perform row operations to apply row reduction:
1. \( R2 = R2 - R1 \)
2. \( R3 = R3 - (1/2)R1 \)

New matrix:
\[ \begin{pmatrix} 4 & 1 & 8 \\ 0 & -2 & 4 \\ 0 & 1/2 & 0 \end{pmatrix} \]
Perform:
- \( R3 = 2R3 \)
- \( R2 = R2 + 4R3 \)

Explanation: System implies \(c_1 = 0, c_2 = 0, c_3 = 0 \).
```

Final Solution

\[\{ \mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3} \} \text{ is linearly independent}.\]