CheggSolutions - Thegdp

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# Solow Growth Model

## Given problem is based on the Solow Growth Model with labor-augmenting technological progress.

### Given:

**1. Production function:**

$$ Y = K^\alpha (EL)^{1-\alpha} $$

**2. Growth rate of technology:**

$$ \frac{\Delta E}{E} = g $$

**3. Labor growth:**

$$ \frac{\Delta L}{L} = 0 $$

**4. Capital accumulation equation:**

$$ \Delta K = I - \delta K $$

**5. Investment defined as a fraction of output:**

$$ I = sY $$

### Define the variables in efficiency units:

$$ \hat{y} = \frac{Y}{EL} $$

$$ \hat{k} = \frac{K}{EL} $$

$$ \hat{i} = \frac{I}{EL} $$

#### Part (a): Derive the production function in efficiency units

First, rewrite the production function using the specified efficiency units.

Given:

$$ Y = K^\alpha (EL)^{1-\alpha} $$

Dividing both sides by $EL$:

$$ \frac{Y}{EL} = \left( \frac{K}{EL} \right)^\alpha $$

This becomes:

$$ \hat{y} = \hat{k}^\alpha $$

**Explanation:** The production function in efficiency units ($\hat{y}$) is derived by normalizing the original production function ($Y$) by dividing both sides by $EL$, yielding $\hat{y} = \hat{k}^\alpha$.

**Supporting Statement:** Production in efficiency units uses capital per efficiency unit labor ($\hat{k}$) as its primary factor, leading to $\hat{y} = \hat{k}^\alpha$.

#### Part (b): Derive the fundamental equation of capital in efficiency units

Starting from the capital accumulation equation:

$$ \Delta K = I - \delta K $$

Convert this to per efficiency unit terms:

$$ \Delta \hat{k} = \frac{\Delta K}{EL} - \delta \frac{K}{EL} - \frac{K \Delta(EL)}{(EL)^2} $$

Using:

$$ \frac{\Delta K}{EL} = \hat{i} - \delta \hat{k} - (\hat{k} (\frac{\Delta E}{E} + \frac{\Delta L}{L})) $$

Given:

$$ \hat{i} = \frac{sY}{EL} = s \hat{y} = s \hat{k}^\alpha $$
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$$ \frac{\Delta E}{E} = g $$

$$ \frac{\Delta L}{L} = 0 $$

Therefore:

$$ \Delta \hat{k} = s \hat{k}^\alpha - \delta \hat{k} - g \hat{k} $$

This simplifies to:

$$ \Delta \hat{k} = s \hat{k}^\alpha - (\delta + g) \hat{k} $$

So, the evolution of capital in efficiency units is:

$$ \Delta \hat{k} = s \hat{k}^\alpha - (\delta + g) \hat{k} $$

Explanation: The above equation shows the growth rate of capital per unit of effective labor ($\hat{k}$) as a function of savings rate, the depreciation rate, and the rate of technological progress.

Supporting Statement: The fundamental equation of capital in efficiency units reflects how capital accumulation changes over time, factoring in investments, depreciation, and technological growth.

Final Solution:

1. $\hat{y} = \hat{k}^\alpha$

2. $\Delta \hat{k} = s \hat{k}^\alpha - (\delta + g) \hat{k}$

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