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# Mathematics

## Continuity of Functions

### Given function $f(z) = \frac{z^2 + 1}{z^3 + 9}$:

**Step-by-Step Solution:**

**Step 1: Analyze the Given Function**

The first step is to identify the components of the function $f(z)$:

The function $f(z)$ is given as $f(z) = \frac{z^2 + 1}{z^3 + 9}$.

**Numerator:** $z^2 + 1$

**Denominator:** $z^3 + 9$

**Explanation:** The given function is a rational function, which is in the form $\frac{P(z)}{Q(z)}$. Rational functions are continuous everywhere in their domain, except where their denominator is zero because division by zero is undefined.

**Step 2: Find the Domain of the Function**

The next step is to find the domain of the function by identifying the values of $z$ that make the denominator zero.

Set the denominator equal to zero:

$$z^3 + 9 = 0$$

Solve for $z$:

$$z^3 = -9$$

$$z = \sqrt[3]{-9}$$

**Explanation:** The real cube root of $-9$ is $-\sqrt[3]{9}$. The function $f(z)$ will be undefined at this value because it makes the denominator zero. Therefore, the function is not continuous at $z = -\sqrt[3]{9}$ because division by zero is not allowed.

**Step 3: Establish Points of Discontinuity**

To further consolidate this reasoning, identify the single point at which the function may be discontinuous.

**Explanation:** From the previous step, it is established that the function $f(z)$ is undefined at $z = -\sqrt[3]{9}$. Hence, there is only one discontinuity for the function at this point.

**Step 4: Continuous Everywhere Else**

For all other values of $z$, since the denominator does not become zero, the function $f(z)$ remains continuous.

**Explanation:** Rational functions are continuous wherever they are defined (i.e., wherever their denominator is not zero).

**Final Solution:**

The function $f(z) = \frac{z^2 + 1}{z^3 + 9}$ is continuous everywhere on the complex plane except at \( z = -
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\sqrt[3]{9} \).

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