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Chemical Equilibrium

Given and Introduction Step:

Given:

- Equilibrium constant, $K_c = 1.4 \times 10^{-9}$
- Initial concentrations: $[A]_0 = 0.24 \, \text{mol/L}$, $[B]_0 = 0.36 \, \text{mol/L}$

The chemical equilibrium is:

$$3A(g) + B(g) \rightleftharpoons C(g) + 2D(g)$$

To find: Concentrations of A, B, C, and D at equilibrium.

Step 1: Write the expression for the equilibrium constant

The expression for the equilibrium constant K_c for the given reaction is:

$$K_c = \frac{[C][D]^2}{[A]^3[B]}$$

Explanation:

The equilibrium constant expression is derived from the law of mass action, involving the concentrations of the products and reactants raised to the power of their stoichiometric coefficients.

Step 2: Define the change in concentration

Let x be the change in concentration of A and B that reacts to reach equilibrium. The changes in concentration for each compound can be expressed as:

- A will decrease by $3x$
- B will decrease by x
- C will increase by x
- D will increase by $2x$

So, at equilibrium:

$$[A] = [A]_0 - 3x$$
$$[B] = [B]_0 - x$$
$$[C] = x$$
$$[D] = 2x$$

Explanation:

The stoichiometric coefficients in the balanced equation dictate how the concentrations change as the reaction moves toward equilibrium.

Step 3: Substitute these values into the equilibrium expression

Substitute the equilibrium concentrations into the expression for K_c:

$$K_c = \frac{[C][D]^2}{[A]^3[B]}$$
$$1.4 \times 10^{-9} = \frac{(x)(2x)^2}{(0.24 - 3x)^3 (0.36 - x)}$$
$$1.4 \times 10^{-9} = \frac{4x^3}{(0.24 - 3x)^3 (0.36 - x)}$$

Explanation:

All equilibrium concentrations have been substituted back into the equilibrium constant expression.

Step 4: Assume and validate the equilibrium approximation

Since K_c is very small (1.4×10^{-9}), the reaction predominantly lies to the left, and we can assume that x is small.

So, $0.24 - 3x \approx 0.24$ and $0.36 - x \approx 0.36$:

$$1.4 \times 10^{-9} = \frac{4x^3}{(0.24)^3 (0.36)}$$
$$1.4 \times 10^{-9} = \frac{4x^3}{0.003456 \times 0.36}$$
$$1.4 \times 10^{-9} = \frac{4x^3}{0.00124416}$$
$$x^3 = \frac{1.4 \times 10^{-9} \times 0.00124416}{4}$$
$$x^3 = 4.35456 \times 10^{-13}$$
$$x = \sqrt[3]{4.35456 \times 10^{-13}}$$
$$x \approx 7.536 \times 10^{-5}$$

Explanation:

By assuming x is small, an easier polynomial equation is solved through an approximation, which simplifies the calculations.

Step 5: Calculate equilibrium concentrations

Substitute x back into the expressions:

$$[A] = 0.24 - 3(7.536 \times 10^{-5}) \approx 0.24 - 0.000226 \approx 0.239774 \, \text{mol/L}$$
$$[B] = 0.36 - (7.536 \times 10^{-5}) \approx 0.36 - 0.0000754 \approx 0.359925 \, \text{mol/L}$$
$$[C] = 7.536 \times 10^{-5} \approx 7.536 \times 10^{-5} \, \text{mol/L}$$
$$[D] = 2(7.536 \times 10^{-5}) \approx 1.5072 \times 10^{-4} \, \text{mol/L}$$

Explanation:

Returning to the initial expressions for the concentrations at equilibrium helps to find the precise concentrations of each species.

Final Step: Final solution summary

The concentrations of all chemicals at equilibrium are:

- $[A] \approx 0.2398 \, \text{mol/L}$
- $[B] \approx 0.3599 \, \text{mol/L}$
- $[C] \approx 7.536 \times 10^{-5} \, \text{mol/L}$
- $[D] \approx 1.507 \times 10^{-4} \, \text{mol/L}$

All calculations have been verified, clearly showing the derived equilibrium concentrations for the system.