# CheggSolutions - Thegdp

## Subject: Calculus

### Topic: Convergence Tests and Estimation via Integrals

### Problem (a)

**Given Statement:**

For each integer $k \ge 1$, define $a_k = \int_k^\infty \frac{k}{x^3} \, dx$. By virtue of the Integral Test, we may conclude that the infinite series $\sum_{k=1}^\infty \frac{k}{k^3}$ is convergent.

**Solution:**

1. **Define the integrand and perform the integration.**
   $$a_k = \int_k^\infty \frac{k}{x^3} \, dx$$ $$\int \frac{k}{x^3} \, dx = k \int x^{-3} \, dx$$ The indefinite integral of $x^{-3}$ is: $$\int x^{-3} \, dx = \frac{x^{-2}}{-2} = -\frac{1}{2x^2}$$
2. **Evaluate the definite integral:**
   $$a_k = \left[ -\frac{k}{2x^2} \right]_k^\infty = 0 - \left( -\frac{k}{2k^2} \right) = \frac{1}{2k}$$ Thus, $a_k = \frac{1}{2k}$.

Explanation: The given integral has been evaluated using standard integration techniques. The bounds are applied to find the final value of $a_k$.

**Conclusion:**

By the Integral Test, the original statement about the series:

$$\sum_{k=1}^\infty \frac{k}{k^3} = \sum_{k=1}^\infty \frac{1}{k^2}$$ True, since $\sum_{k=1}^\infty \frac{1}{k^2}$ is known to be convergent.

**Answer:** True

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### Problem (b)

**Question:**

Does the Remainder Estimate Theorem for the Integral Test apply to $\sum_{k=7}^\infty \frac{1}{k^3}$ for $n = 7$ and $N \to \infty$?

**Solution:**

1. **Consider the given series:**
   $$\sum_{k=7}^\infty \frac{1}{k^3}$$
2. **Check if the conditions for the Integral Test apply:**
   The function $f(x) = \frac{1}{x^3}$ is positive, continuous, and decreasing for $x \ge 7$. Therefore, the remainder estimate theorem can be applied.

Explanation: The check ensures the function satisfies the conditions required for the Integral Test and Remainder Estimate Theorem.

**Answer:** Yes

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### Problem (c)

**Given:**

Compute $R_7$ such that the Remainder Estimate Theorem guarantees: $$\int_{7}^\infty \frac{1}{x^3} \, dx < R_7 < \int_{6}^\infty \frac{1}{x^3} \, dx$$

**Solution:**

1. **Evaluate the integrals:**
   $$\int_{7}^\infty \frac{1}{x^3} \, dx = \left[ -\frac{1}{2x^2} \right]_7^\infty = 0 - \left( -\frac{1}{2 \cdot 7^2} \right) = \frac{1}{98}$$ $$\int_{6}^\infty \frac{1}{x^3} \, dx = \left[ -\frac{1}{2x^2} \right]_6^\infty = 0 - \left( -\frac{1}{2 \cdot 6^2} \right) = \frac{1}{72}$$

Explanation: The integrals are evaluated similarly as in part (a), considering the new bounds.

Therefore, the remainder $R_7$ is bounded as:

$$\frac{1}{98} < R_7 < \frac{1}{72}$$

Explanation: The inequality form of the Remainder Estimate Theorem is used to establish the bounds for $R_7$.

**Answer:** $\frac{1}{98}$ and $\frac{1}{72}$

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## Final Solution Summary

- (a) True
- (b) Yes
- (c) $\frac{1}{98}$ and $\frac{1}{72}$