

Sub-subject: Electromagnetism

Topic: Magnetic Force on a Current-Carrying Loop

Given and Introduction

- Distance of the inner leg of the loop from the wire: $d_1 = 0.9 \text{ m}$
- Distance of the outer leg of the loop from the wire: $d_2 = 1.5 \text{ m}$
- Other dimension of the rectangle (height): $L = 0.8 \text{ m}$
- Current in the straight conductor: $I_1 = 7 \times 10^6 \text{ A}$
- Current in the loop: $I_2 = 1.4 \times 10^5 \text{ A}$

The task is to calculate the net force on the rectangular loop due to the magnetic field created by the straight conductor. The force on each leg is calculated and summed up to find the total force.

Step-by-Step Solution

1. Magnetic Field due to a Long Straight Conductor

The magnetic field at a distance d from a very long straight conductor carrying current I_1 is given by Ampere's law:

$$B = \mu_0 * I_1 / (2 * \pi * d)$$

Where:

$$\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$$

Explanation: This formula gives the magnetic field produced by a long straight current-carrying conductor at a perpendicular distance d .

2. Force on a Current-Carrying Conductor in a Magnetic Field

The force on a segment of length L of a conductor carrying current I_2 in a magnetic field B is given by:

$$F = I_2 * L * B$$

3. Calculate the Force on the Inner Leg

Magnetic field at $d_1 = 0.9 \text{ m}$:

$$\begin{aligned} B_1 &= \mu_0 * I_1 / (2 * \pi * d_1) \\ B_1 &= (4\pi \times 10^{-7}) (7 \times 10^6) / (2\pi * 0.9) \\ B_1 &\approx 3.111 \times 10^{-1} \text{ T} \end{aligned}$$

Force on the inner leg:

$$\begin{aligned} F_1 &= I_2 * L * B_1 \\ F_1 &= (1.4 \times 10^5) (0.8) (3.111 \times 10^{-1}) \\ F_1 &\approx 34840 \text{ N} \end{aligned}$$

Explanation: The magnetic force is calculated for the inner segment of the loop in the presence of the magnetic field B_1 .

4. Calculate the Force on the Outer Leg

Magnetic field at $d_2 = 1.5 \text{ m}$:

$$\begin{aligned} B_2 &= \mu_0 * I_1 / (2 * \pi * d_2) \\ B_2 &= (4\pi \times 10^{-7}) (7 \times 10^6) / (2\pi * 1.5) \\ B_2 &\approx 1.867 \times 10^{-1} \text{ T} \end{aligned}$$

Force on the outer leg:

$$F_2 = I_2 * L * B_2$$

$$F_2 = (1.4 \times 10^5) (0.8) (1.867 \times 10^{-1})$$

$$F_2 \approx 20856 \text{ N}$$

Explanation: Similarly, the magnetic force on the outer segment is calculated.

5. Net Force on the Loop

The direction of the magnetic force on the inner leg is towards the conductor (attraction), and on the outer leg is away from the conductor (repulsion).

The net force is:

$$F_{\text{net}} = F_1 - F_2$$

$$F_{\text{net}} = 34840 \text{ N} - 20856 \text{ N}$$

$$F_{\text{net}} = 13984 \text{ N}$$

Explanation: The forces in opposite directions result in a net force given by their difference.

6. Conversion to Pounds-Force

$$1 \text{ Newton} = 0.224809 \text{ lb}$$

$$F_{\text{net}} (\text{in pounds}) = 13984 * 0.224809$$

$$F_{\text{net}} \approx 3141.6 \text{ lb}$$

Explanation: Conversion from the SI unit of force (Newton) to pounds-force.

Final Solution

The net force on the rectangular loop of current is:

13984 N directed towards the long straight conductor.

In pounds-force, the net force is approximately:

3141.6 lb.