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## Electrical Engineering

**Topic: Induction Motor Torque**

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### Given and Introduction Step

Given:

• Per-unit constants of a three-phase 50-Hz induction motor:
- $R_s = R_r = 0.02 \ \Omega$
- $X_s = X_r = 0.1 \ \Omega$

• The motor operates with constant volts per hertz control when its supply frequency and magnitude of the voltage are made half (i.e., new frequency $f_n = \frac{f_r}{2}$ and new voltage $V_n = \frac{V_r}{2}$).

### Objective

Calculate the ratio of:

1. Maximum value of torque at new values as compared to that at rated values.
2. Starting torque at new values as compared to that at rated values.

### Maximum Value of Torque Ratio Calculation

**1. Rated Maximum Torque:**

$$T_{max,r} = \frac{{k_r V_r^2}}{{2 (R_r + R_s) X_r}}$$

**2. New Maximum Torque:**

When the frequency and voltage are halved, the new synchronous reactance $X_r'$ becomes:

$$X_r' = \frac{X_r f_n}{f_r} = \frac{0.1 \cdot 25}{50} \ = 0.05 \ \Omega$$ $$T_{max,n} = \frac{{k_n V_n^2}}{{2 (R_r + R_s) X_r'}}$$

Considering that the constants $k_r$ and $k_n$ remain the same (assuming the same structure and design characteristics):

$$T_{max,n} = \frac{{k_r \left(\frac{V_r}{2}\right)^2}}{{2(0.02 + 0.02) \cdot 0.05}}$$ $$T_{max,n} = \frac{{k_r \frac{V_r^2}{4}}}{{2(0.02 + 0.02) \cdot 0.05}} = \frac{{k_r V_r^2}}{{8 \cdot 0.04 \cdot 0.05}}$$ $$T_{max,n} = \frac{{k_r V_r^2}}{{0.016}}$$

**3. Torque Ratio Calculation:**

$$\text{Torque ratio} = \frac{{T_{max,n}}}{{T_{max,r}}} = \frac{\frac{k_r V_r^2}{0.016}}{\frac{k_r V_r^2}{0.008}}$$ $$\text{Torque ratio} = \frac{\frac{1}{0.016}}{\frac{1}{0.008}} = \frac{0.008}{0.016} = 0.5$$

**4. Supporting Statement and Explanation:**

The maximum torque at the new conditions (with reduced voltage and frequency) is half of the torque at the rated conditions.

### Starting Torque Ratio Calculation

**1. Rated Starting Torque:**

$$T_{start,r} = \frac{{3V_r^2 R_r}}{{(\omega_r (R_r + R_s)^2 + (\omega_r X_r^2)^2)}}$$

**2. New Starting Torque:**

$$T_{start,n} = \frac{{3 \left(\frac{V_r}{2}\right)^2 R_r}}{{(\omega_n (R_r + R_s)^2 + (\omega_n X_r')^2)}}$$ Where $\omega_n = 2\pi f_n = \pi f_r$ (since the frequency is halved): $$T_{start,n} = \frac{{3 \frac{V_r^2}{4} \cdot 0.02}}{{(\pi f_r \cdot (0.04)^2 + (\pi f_r \cdot 0.05)^2)}}$$

**3. Torque Ratio Calculation:**

$$\text{Starting torque ratio} = \frac{{T_{start,n}}}{{T_{start,r}}} = \frac{\frac{3 \cdot \frac{V_r^2}{4} \cdot 0.02}{(\pi f_r (0.04)^2 + (\pi (f_r 0.05)^2)}}{\frac{3 V_r^2 \cdot 0.02}{(2(2\pi f_r) (0.04)^2 + (2\pi f_r(0.1)^2)}}$$ $$\text{Starting torque ratio} = \frac{\frac{0.02 V_r^2}{4 (\pi f_r (0.04)^2 + \pi (f_r 0.05)^2)}}{\frac{0.02 V_r^2}{2 (2\pi f_r) (0.04)^2 + (2\pi (f_r 0.1)^2)}}$$ Simplifying terms: $$\text{Starting torque ratio} = \frac{1}{4} \cdot \left(\frac{2(2\pi f_r) 0.04^2 + 2 (\pi f_r 0.1^2)}{(\pi f_r (0.04)^2 + \pi f_r 0.05^2)}\right)$$ $$\text{Starting torque ratio} = \frac{1}{2}$$
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Final Solution:

1. The maximum torque at the new values is half of the maximum torque at rated values.
2. The starting torque at the new values is half of the starting torque at the rated values.

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