Operating Systems: Page Replacement Algorithms

Part a) Defining the Page-Replacement Algorithm

i. Initial Value of Counters

Each page frame will start with a counter value of 0 since no page has been loaded into the frames at the beginning.

Explanation: At the beginning, all page frames are empty, so the counters are initialized to zero.

ii. When Counters are Increased

The counter associated with a page frame is increased every time a page in that frame is accessed or loaded.

Explanation: This increment ensures that frequently accessed pages have a high counter value, reflecting their heavy usage.

iii. When Counters are Decreased

The counter associated with a page frame is decreased when the page in that frame is not accessed for a while, using a specified interval or condition.

Explanation: This decrease can help in tracking pages that become less frequently used over time.

iv. How the Page to be Replaced is Selected

The page to be replaced is the one in the frame with the smallest counter value. If there are multiple frames with the same smallest counter value, use the Least Recently Used (LRU) strategy among them.

Explanation: This ensures the replacement of the least used or older pages, thereby minimizing page faults.

Part b) Calculating the Number of Page Faults with the Algorithm (Not Random)

Reference String with Four Page Frames:

```
1, 2, 3, 4, 5, 3, 4, 1, 6, 7, 8, 7, 8, 9, 7, 8, 9, 5, 4, 5, 4, 2
```

1. **1**: [1, -, -, -] Counters: [1, 0, 0, 0] — Page Fault 2. **2**: [1, 2, -, -] Counters: [1, 1, 0, 0] — Page Fault 3. **3**: [1, 2, 3, -] Counters: [1, 1, 1, 0] — Page Fault

Process:

```
4. 4: [1, 2, 3, 4] Counters: [1, 1, 1, 1] — Page Fault
5. 5: [5, 2, 3, 4] Counters: [1, 1, 1, 1] — Page Fault (1 replaced, lowest counter)
6. 3: [5, 2, 3, 4] Counters: [1, 1, 2, 1] — Hit
7. 4: [5, 2, 3, 4] Counters: [1, 1, 2, 2] — Hit
8. 1: [5, 2, 1, 4] Counters: [1, 1, 1, 1] — Page Fault (3 replaced, lowest nonzero counter)
9. 6: [5, 2, 1, 6] Counters: [1, 0, 1, 1] — Page Fault (4 replaced)
10. 7: [5, 7, 1, 6] Counters: [1, 1, 1, 1] — Page Fault (2 replaced)
11. 8: [8, 7, 1, 6] Counters: [1, 1, 1, 1] — Page Fault (5 replaced)
12. 7: [8, 7, 1, 6] Counters: [1, 2, 1, 1] — Hit
13. 8: [8, 7, 1, 6] Counters: [2, 2, 1, 1] — Hit
14. 9: [8, 7, 9, 6] Counters: [2, 2, 1, 1] — Hit
16. 8: [8, 7, 9, 6] Counters: [3, 3, 1, 1] — Hit
17. 9: [8, 7, 9, 6] Counters: [3, 3, 2, 1] — Hit
18. 5: [8, 7, 9, 5] Counters: [3, 3, 2, 1] — Page Fault (6 replaced)
```

19. 4: [4, 7, 9, 5] Counters: [1, 3, 2, 1] — Page Fault (8 replaced, using LRU for same lowest counter)

```
20. 5: [4, 7, 9, 5] Counters: [1, 3, 2, 2] — Hit
21. 4: [4, 7, 9, 5] Counters: [2, 3, 2, 2] — Hit
22. 2: [4, 7, 9, 2] Counters: [2, 3, 2, 1] — Page Fault (5 replaced, using LRU for same lowest counter)
Total Page Faults:
14
Part c) Minimum Number of Page Faults for an Optimal Page-Replacement Strategy
Algorithm: Optimal Page Replacement
Reference String with Four Page Frames:
1, 2, 3, 4, 5, 3, 4, 1, 6, 7, 8, 7, 8, 9, 7, 8, 9, 5, 4, 5, 4, 2
Process:
1. 1: [1, -, -, -] — Page Fault
2. 2: [1, 2, -, -] — Page Fault
3. 3: [1, 2, 3, -] — Page Fault
4. 4: [1, 2, 3, 4] — Page Fault
5. 5: [1, 2, 3, 5] — Page Fault (4 replaced, 4 accessed latest in sequence)
6. 3: [1, 2, 3, 5] — Hit
7. 4: [4, 2, 3, 5] — Page Fault (1 replaced, 2 accessed latest in sequence)
8. 1: [4, 1, 3, 5] — Page Fault (2 replaced)
9. 6: [4, 6, 3, 5] — Page Fault (1 replaced)
10. 7: [4, 6, 7, 5] — Page Fault (3 replaced)
11. 8: [4, 6, 7, 8] — Page Fault (5 replaced)
12. 7: [4, 6, 7, 8] — Hit
13. 8: [4, 6, 7, 8] — Hit
14. 9: [9, 6, 7, 8] — Page Fault (4 replaced, no longer needed)
15. 7: [9, 6, 7, 8] — Hit
16. 8: [9, 6, 7, 8] — Hit
17. 9: [9, 6, 7, 8] — Hit
18. 5: [9, 6, 7, 5] — Page Fault (8 replaced, no longer needed)
19. 4: [9, 4, 7, 5] — Page Fault (6 replaced, 7 accessed latest)
20. 5: [9, 4, 7, 5] — Hit
21. 4: [9, 4, 7, 5] — Hit
22. 2: [9, 4, 2, 5] — Page Fault (7 replaced)
```

Total Page Faults:

13

Final Solution

- Defined the page-replacement algorithm using counters for each page frame to keep track of usage.
- Calculated the number of page faults using the defined algorithm: 14 page faults.
- Derived the minimum number of page faults using the optimal page-replacement strategy: 13 page faults.

...