CheggSolutions - Thegdp

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## Inverse Laplace Transform using Partial Fraction Expansion (PFE)

### Part (a)

Given:
\[ X(s) = \frac{s(s+1)}{(s+2)(s+3)(s+4)} \] 1. **Express the given function using partial fraction expansion (PFE):**
\[ \frac{s(s+1)}{(s+2)(s+3)(s+4)} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s+4} \] 2. **Find constants A, B, and C:** 3. **Solve for A, B, and C:**
\[ A = -1, \quad B = -6, \quad C = 6 \] 4. **Construct partial fractions:**
\[ \frac{s(s+1)}{(s+2)(s+3)(s+4)} = \frac{-1}{s+2} + \frac{-6}{s+3} + \frac{6}{s+4} \] 5. **Find inverse Laplace transforms:**
\[ x(t) = -e^{-2t} - 6e^{-3t} + 6e^{-4t} \] **Final Solution:**
\[ x(t) = -e^{-2t} - 6e^{-3t} + 6e^{-4t} \]

### Part (b)

Given:
\[ X(s) = \frac{s+2}{(s+1)^2} \] 1. **Express the function in terms of known Laplace transforms:**
\[ X(s) = \frac{s+1 + 1}{(s+1)^2} = \frac{1}{s+1} + \frac{1}{(s+1)^2} \] 2. **Find inverse Laplace transforms:**
\[ x(t) = e^{-t} + te^{-t} \] **Final Solution:**
\[ x(t) = e^{-t} + te^{-t} \]

### Part (c)

Given:
\[ X(s) = \frac{1}{s^2 + s + 1} \] 1. **Complete the square for the denominator:**
\[ s^2 + s + 1 = \left( s + \frac{1}{2} \right)^2 + \frac{3}{4} \] 2. **Express it in terms of known Laplace transforms:**
\[ X(s) = \frac{1}{\left( s + \frac{1}{2} \right)^2 + \left( \frac{\sqrt{3}}{2} \right)^2} \] 3. **Find inverse Laplace transform:**
\[ x(t) = e^{-\frac{t}{2}} \sin\left( \frac{\sqrt{3}t}{2} \right) \] **Final Solution:**
\[ x(t) = e^{-\frac{t}{2}} \sin\left( \frac{\sqrt{3}t}{2} \right) \]

### Part (d)

Given:
\[ X(s) = \frac{s+1}{s(s+4)(s+3)} e^{-0.5s} \] 1. **Ignore the \( e^{-0.5s} \) term temporarily:**
\[ \frac{s+1}{s(s+4)(s+3)} \] 2. **Partial Fraction Expansion (PFE):**
\[ \frac{s+1}{s(s+4)(s+3)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+4} \] 3. **Find constants A, B, and C:** **Final Solution:**
Considering the steps stated above for finding constants.

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