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Kinetics of Second-Order Reactions

Given and Introduction:

The reaction $A \rightarrow \text{product}$ is a second-order reaction with a rate constant $k = 0.540 \, M^{-1} \, s^{-1}$. The objective is to find the time at which the concentration of species A becomes $\frac{1}{3}$ of its original concentration.

Second-Order Reaction Formula:

For a second-order reaction, the integrated rate law is given by:

$$\frac{1}{[A]} = kt + \frac{1}{[A]_0}$$

Where:
$[A]$ = concentration of A at time t
k = rate constant
t = time
$[A]_0$ = initial concentration of A

Step-by-Step Solution:

1. **Identify given values and what needs to be found:**
   - $[A]_t = \frac{1}{3} [A]_0$ (concentration of A at time t is one-third of its initial concentration)
   - $k = 0.540 \, M^{-1} \, s^{-1}$
   - Initial concentration $[A]_0$

   Explanation: The problem provides the rate constant and the fraction of the initial concentration. Based on these values, the goal is to find the time needed for the concentration of A to drop to $\frac{1}{3}$ of its original value.
2. **Substitute the known values into the integrated rate law equation:**

   $$\frac{1}{\frac{1}{3} [A]_0} = kt + \frac{1}{[A]_0}$$

   Explanation: By substituting $\frac{1}{3} [A]_0$ for $[A]$, the integrated rate law equation can be rearranged to solve for time t.
3. **Simplify the equation:**

   $$3 \cdot \frac{1}{[A]_0} = kt + \frac{1}{[A]_0}$$

   Explanation: Multiply the left side by 3 since $\frac{1}{\frac{1}{3} [A]_0}$ simplifies to $3 \cdot \frac{1}{[A]_0}$.
4. **Rearrange to solve for t:**

   $$3 \cdot \frac{1}{[A]_0} - \frac{1}{[A]_0} = kt$$
   $$\frac{3 - 1}{[A]_0} = kt$$
   $$\frac{2}{[A]_0} = kt$$

   Explanation: Separate the terms involving $[A]_0$ and isolate t.
5. **Solve for t:**

   $$t = \frac{2}{k[A]_0}$$

   Explanation: Rearrange the equation to solve for t by dividing both sides by k and $[A]_0$.
6. **Substitute the value of k:**

   $$t = \frac{2}{0.540 \, M^{-1} \, s^{-1} \times [A]_0}$$

   Explanation: Substitute the given rate constant into the equation.
7. **Calculate the value of t:** Since $[A]_0$ cancels out, the equation becomes:

   $$t = \frac{2}{0.540}$$
   $$t \approx 3.70 \, \text{s}$$

Explanation: Perform the division to find the value of t.

Final Solution:

The time required for the concentration of species A to become $\frac{1}{3}$ of its original concentration is approximately $\boxed{3.70 \, \text{s}}$.