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Engineering Mechanics

Centroid of Composite Areas

Given:

- A semi-circle with a radius of $10 \, \text{in}$.
- A triangle at the middle of the base of the semi-circle with dimensions $6 \, \text{in}$ height and $12 \, \text{in}$ base.

To find the centroid of a composite area, decompose the shape into simpler components (a semi-circle and a triangle), find their individual centroids, and then compute the overall centroid using the method of composite areas.

Step 1: Find centroids of individual shapes

Semi-Circle:

- Radius, $R = 10 \, \text{in}$
- Centroid coordinates for a semi-circle:
- $$ \overline{x}_1 = 0 \, \text{in} \quad (\text{Symmetry about the y-axis}) $$
- $$ \overline{y}_1 = \frac{4R}{3\pi} = \frac{4 \times 10 \, \text{in}}{3\pi} \approx 4.24 \, \text{in} $$

The centroid of a semi-circle of radius R on the y-axis is at $\frac{4R}{3\pi}$ from the flat edge.

Triangle:

- Base, $b = 12 \, \text{in}$
- Height, $h = 6 \, \text{in}$
- Centroid coordinates:
- $$ \overline{x}_2 = 0 \, \text{in} \quad (\text{Symmetry about the y-axis}) $$
- $$ \overline{y}_2 = h/3 = 6 \, \text{in} / 3 = 2 \, \text{in} $$

The centroid lies $\frac{1}{3}$ from the base.

Step 2: Compute the areas A_1 and A_2 of the individual shapes

Semi-Circle:

$$ A_1 = \frac{1}{2} \pi R^2 = \frac{1}{2} \pi (10 \, \text{in})^2 = 50\pi \, \text{in}^2 \approx 157.08 \, \text{in}^2 $$

Triangle:

$$ A_2 = \frac{1}{2} b \times h = \frac{1}{2} \times 12 \, \text{in} \times 6 \, \text{in} = 36 \, \text{in}^2 $$

The area of a semi-circle is half the area of a full circle, and the area of a triangle is half the product of its base and height.

Step 3: Calculate the composite centroid $\overline{x}$ and $\overline{y}$

Since both centroids lie on the y-axis, $\overline{x}$ is 0.

For $\overline{y}$:

$$ \overline{y} = \frac{\sum \overline{y}_i A_i}{\sum A_i} $$

Numerator:

$$ \sum \overline{y}_i A_i = \overline{y}_1 A_1 + \overline{y}_2 A_2 = (4.24 \, \text{in} \times 157.08 \, \text{in}^2) + (2 \, \text{in} \times 36 \, \text{in}^2) $$ $$ = 666.04 \, \text{in}^3 + 72 \, \text{in}^3 = 738.04 \, \text{in}^3 $$

Denominator:

$$ \sum A_i = A_1 + A_2 = 157.08 \, \text{in}^2 + 36 \, \text{in}^2 = 193.08 \, \text{in}^2 $$

Calculate $\overline{y}$:

$$ \overline{y} = \frac{738.04 \, \text{in}^3}{193.08 \, \text{in}^2} \approx 3.82 \, \text{in} $$

The composite centroid involves finding the weighted average of the individual centroids with respect to their areas.

Final Step: State the composite centroid

The centroid of the shaded area is approximately:

$$\left(\overline{x}, \overline{y} \right) = \left(0 \, \text{in}, 3.82 \, \text{in} \right)$$

Thus, the centroid of the shaded area is $(0 \, \text{in}, 3.82 \, \text{in})$.