CheggSolutions - Thegdp

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## Subject: Linear Algebra

### Topic: Span of a Set of Vectors

#### Given:

The set \( H \) consists of all vectors of the form:

\[ \begin{pmatrix} s \\ s \\ -5s \\ \end{pmatrix} \]

#### Task:

Find a vector \(\vec{u}\) in \(\mathbb{R}^3\) such that \( H = \text{span} \{\vec{u}\} \).

#### Solution:

Given a vector in the form \( \begin{pmatrix} s \\ s \\ -5s \end{pmatrix} \), the aim is to express \( H \) as the span of a single vector \(\vec{u}\).

##### Step 1: Identification of Basis Vector

A general vector in \( H \) looks like \( \vec{v} = \begin{pmatrix} s \\ s \\ -5s \end{pmatrix} \).

Since \( s \) is a scalar, this vector can be factored as:

\[ \vec{v} = s \begin{pmatrix} 1 \\ 1 \\ -5 \end{pmatrix} \]

**Explanation:** Each component of \(\vec{v}\) is a multiple of \( s \). Therefore, the vector \(\vec{v}\) can be written as \(s\) times the vector \( \begin{pmatrix} 1 \\ 1 \\ -5 \end{pmatrix} \), indicating that all vectors of this form are scalar multiples of \( \begin{pmatrix} 1 \\ 1 \\ -5 \end{pmatrix} \).

**Supporting Statement:** This demonstrates that \(\vec{u} = \begin{pmatrix} 1 \\ 1 \\ -5 \end{pmatrix} \) spans \( H \).

##### Step 2: Final Representation

To confirm that \( H = \text{span} \{ \vec{u} \} \), note:

\[ H = \left\{ s \begin{pmatrix} 1 \\ 1 \\ -5 \end{pmatrix} \; \middle| \; s \in \mathbb{R} \right\} \]

which is precisely the definition of the span of \( \vec{u} \):

\[ H = \text{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ -5 \end{pmatrix} \right\} \]

**Explanation:** The set \( H \) is the collection of all scalar multiples of the vector \( \begin{pmatrix} 1 \\ 1 \\ -5 \end{pmatrix} \). Therefore, \( \vec{u} = \begin{pmatrix} 1 \\ 1 \\ -5 \end{pmatrix} \) is the vector that spans \( H \).

**Supporting Statement:** The entire set \( H \) can be generated by all scalar multiples of one vector, confirming that \( H = \text{span} \{ \vec{u} \} \).

**Final Solution:**

The vector \(\vec{u}\) in \(\mathbb{R}^3\) such that \( H = \text{span} \{\vec{u}\} \) is: \[ \vec{u} = \begin{pmatrix} 1 \\ 1 \\ -5 \end{pmatrix} \]

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