DC Circuit Analysis

Subject: Electrical Engineering

Topic: DC Circuit Analysis

Given and Introduction:

The circuit in question consists of a 12V DC source, a 1Ω resistor, a 5Ω resistor, an inductor of 2H, a 4Ω resistor, and a capacitor of 1F.

Solution Steps:

(a) To find the current i_L through the inductor:

1. **Steady-State Current through Resistor Circuit (No Transient Components):**
   For a DC steady-state condition, the inductor will behave as a short circuit because the voltage across the inductor is zero in a steady state ($V_L = L \frac{dI}{dt} = 0$)
2. **Combining the Resistors:**
   At steady state, combine resistors to simplify the circuit:
   - Combining 4Ω resistor in parallel with the short-circuited branch: Since the inductor acts like a short, the parallel combination of 4Ω and 0Ω results in 0Ω (effective resistance zero).
3. **Effective Series Resistance:**
   Combine remaining resistances in series:

   $R_{\text{total}} = 1Ω + 5Ω = 6Ω$
4. **Applying Ohm's Law:**
   Calculate the current from the source I through the effective resistor:

   $I = \frac{V}{R_{\text{total}}} = \frac{12V}{6Ω} = 2A$

   Because of steady-state, the same current flows through the inductor since the inductor is treated as a short circuit in steady state.

Thus, the current i_L through the inductor is $2A$.

(b) To find the current i through the 1Ω resistor:

As determined from the previous step, the same current I flows through the 1Ω resistor in series.

Thus, the current i through the 1Ω resistor is also $2A$.

(c) To find the voltage v_C across the capacitor:

1. **Applying Kirchhoff's Voltage Law:**
   At steady state, the voltage across the capacitor will be the same as across the 4Ω resistor since they are in parallel. Given that 4Ω is effectively bypassed by the short-circuited inductor.
2. **Effective Voltage across Combined Resistor:**
   Calculate the potential difference across 5Ω resistor:

   $V_{5Ω} = I \times 5Ω = 2A \times 5Ω = 10V$

   So, the remaining voltage drops across 1Ω resistor and battery terminations should add to 12V. Thus there is no drop across capacitor.

Thus, the voltage v_C across the capacitor is $0V$.

(d) To find the energy stored in the inductor:

1. **Using the formula for energy stored in the inductor:**

   $W_L = \frac{1}{2} L i_L^2$

   Where $L = 2H$ and $i_L = 2A$,
2. **Substituting the values:**

   $W_L = \frac{1}{2} \times 2H \times (2A)^2 = \frac{1}{2} \times 2 \times 4 = 4 \, \text{J}$

Thus, the energy stored in the inductor is $4J$.

(e) To find the energy stored in the capacitor:

1. **Using the formula for the energy stored in the capacitor:**

   $W_C = \frac{1}{2} C v_C^2$

   Where $C = 1F$ and $v_C = \text{0V}$,
2. **Substituting the values:**

   $W_C = \frac{1}{2} \times 1F \times (0V)^2 = 0 \, J$

Thus, the energy stored in the capacitor is $0J$.

Final Solutions:

(a) The current i_L through the inductor $= 2A$

(b) The current i through the 1Ω resistor $= 2A$

(c) The voltage v_C across the capacitor $= 0V$

(d) The energy stored in the inductor $= 4J$

(e) The energy stored in the capacitor $= 0J$