

Sub-Subject: Propositional Logic

Topic: Conditional Statement and its Inverse

Given:

Formula $((p \rightarrow q))$, where (p) and (q) are propositional variables.

Objective:

Find specific propositions for (p) and (q) which make $((p \rightarrow q))$ true but the inverse $((\neg p \rightarrow \neg q))$ false.

Step-by-Step Solution:

Step 1: Understand the formula $((p \rightarrow q))$

The conditional statement $((p \rightarrow q))$ (if (p) then (q)) is true in all cases except when (p) is true and (q) is false.

Supporting statement: It is essential to know when a conditional statement is true or false to determine the specific propositions.

Step 2: Identify the inverse of $((p \rightarrow q))$

The inverse of $((p \rightarrow q))$ is $((\neg p \rightarrow \neg q))$, which is interpreted as "if not (p) then not (q) ."

Supporting statement: The inverse is important in this situation because we need the inverse to be false.

Step 3: Determine truth values for $((p \rightarrow q))$

To satisfy $((p \rightarrow q))$ being true:

- If (p) is true, (q) must be true.
- If (p) is false, (q) can be either true or false.

Supporting statement: Setting specific truth values for (p) and (q) will help in achieving the desired condition.

Step 4: Determine truth values for $((\neg p \rightarrow \neg q))$

To make $((\neg p \rightarrow \neg q))$ false, $((\neg p))$ must be true, and $((\neg q))$ must be false. Hence, (p) must be false and (q) must be true.

Supporting statement: It is critical to have $((\neg p))$ true and $((\neg q))$ false to make the inverse false.

Step 5: Choose specific propositions for (p) and (q)

Let:

- $[p = \text{false}]$
- $[q = \text{true}]$

Supporting statement: These propositions ensure that $((p \rightarrow q))$ is true and $((\neg p \rightarrow \neg q))$ is false.

Verification:

For $(p = \text{false})$:

- $((\neg p = \text{true}))$
- $((p \rightarrow q) = (\text{false} \rightarrow \text{true}) = \text{true})$
- Inverse: $((\neg p \rightarrow \neg q) = (\text{true} \rightarrow \text{false}) = \text{false})$

Supporting statement: These propositions align with the required conditions.

Final Solution:

To make $(p \rightarrow q)$ true but $(\neg p \rightarrow \neg q)$ false, choose the propositions:

- $p = \text{false}$
- $q = \text{true}$