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## Chemistry - Acid-Base Equilibria

### Problem Statement

Calculate the concentrations of $\text{CO}_3^{2-}$, $\text{H}_3\text{O}^+$, and $\text{OH}^-$ in a 0.180 M solution of $\text{H}_2\text{CO}_3$ given that $K_{a1} = 4.3 \times 10^{-7}$ and $K_{a2} = 5.6 \times 10^{-11}$.

**Given:**

- Concentration of $\text{H}_2\text{CO}_3$ (C) = 0.180 M
- $K_{a1} = 4.3 \times 10^{-7}$
- $K_{a2} = 5.6 \times 10^{-11}$

### Step-by-Step Solution

**Step 1: Introduction and Given Data**

This problem involves calculating the concentrations of various species in a diprotic acid solution. Here, the diprotic acid is $\text{H}_2\text{CO}_3$ which dissociates in two steps:

$$\text{H}_2\text{CO}_3 \rightleftharpoons \text{H}^+ + \text{HCO}_3^-$$

$$\text{HCO}_3^- \rightleftharpoons \text{H}^+ + \text{CO}_3^{2-}$$

**Step 2: Calculate the concentration of $\text{H}_3\text{O}^+$ from the first ionization**

$$\text{H}_2\text{CO}_3 \rightleftharpoons \text{H}^+ + \text{HCO}_3^-$$

$$K_{a1} = 4.3 \times 10^{-7}$$

Using the approximation method:

- Let's assume $x$ is the concentration of $\text{H}^+$ formed in the first dissociation.
- $\text{H}^+$ and $\text{HCO}_3^-$ will both have concentration $x$.
- $[\text{H}_2\text{CO}_3] \approx 0.180 \text{M} - x \approx 0.180 \text{M}$

$$K_{a1} = \frac{[\text{H}^+][\text{HCO}_3^-]}{[\text{H}_2\text{CO}_3]}$$

$$4.3 \times 10^{-7} = \frac{x^2}{0.180}$$

**Step 3: Solve for $x$**

$$x^2 = 4.3 \times 10^{-7} \times 0.180$$

$$x^2 = 7.74 \times 10^{-8}$$

$$x = \sqrt{7.74 \times 10^{-8}}$$

$$x \approx 8.80 \times 10^{-4} \text{M}$$

Thus, $\left[\text{H}_3\text{O}^+\right] \approx 8.80 \times 10^{-4} \text{M}$

**Step 4: Calculate $\text{CO}_3^{2-}$ concentration from the second ionization**

$$\text{HCO}_3^- \rightleftharpoons \text{H}^+ + \text{CO}_3^{2-}$$

$$K_{a2} = 5.6 \times 10^{-11}$$

Using approximation:

- Let's assume $y$ is the concentration of $\text{CO}_3^{2-}$ formed from the second dissociation.
- $\text{H}^+$ concentration from the second dissociation will be small compared to the first.
- $y = [\text{CO}_3^{2-}]$
- $[\text{HCO}_3^-] \approx x$

$$K_{a2} = \frac{y[\text{H}^+]}{[\text{HCO}_3^-]}$$

$$5.6 \times 10^{-11} = \frac{y \cdot 8.80 \times 10^{-4}}{8.80 \times 10^{-4}}$$

$$y = 5.6 \times 10^{-11}$$

Thus, $[\text{CO}_3^{2-}] \approx 5.6 \times 10^{-11} \text{M}$

**Step 5: Calculate $\text{OH}^-$ concentration using $K_w$**
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$$ K_w = [\text{H}_3\text{O}^+] [\text{OH}^-] = 1.0 \times 10^{-14} $$

$$ 1.0 \times 10^{-14} = 8.80 \times 10^{-4} [\text{OH}^-] $$

$$ [\text{OH}^-] = \frac{1.0 \times 10^{-14}}{8.80 \times 10^{-4}} $$

$$ [\text{OH}^-] \approx 1.14 \times 10^{-11} \text{M} $$

Final Concentrations

$$ [\text{CO}_3^{2-}] \approx 5.6 \times 10^{-11} \text{M} $$

$$ [\text{H}_3\text{O}^+] \approx 8.80 \times 10^{-4} \text{M} $$

$$ [\text{OH}^-] \approx 1.14 \times 10^{-11} \text{M} $$

Final Answer

$$ \left[\text{CO}_3^{2-}, \text{H}_3\text{O}^+, \text{OH}^- \right] = 5.6 \times 10^{-11} \text{M}, 8.80 \times 10^{-4} \text{M}, 1.14 \times 10^{-11} \text{M} $$

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