CheggSolutions - Thegdp

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## Sub-Subject: Macroeconomics

### Topic: Differentiables in an Open Economy Macroeconomic Model

#### Given Data:

Two equations representing the equilibrium conditions for a hypothetical open economy, one for the combined goods market and another for the money market:

**1) Combined Goods Market:**

$$F^1(Y, r; M_{0}, X_{0}) = I(r) + X_{0} - S(Y, r) - Z(Y, r) = 0$$

**2) Money Market:**

$$F^2(Y, r; M_{0}, X_{0}) = L(Y, r) - M_{0} = 0$$

where:

- $Y$: Income
- $r$: Interest rate
- $M_{0}$: Money supply
- $X_{0}$: Autonomous exports

Additional information about derivatives:

- $I_{r} < 0$
- $0 < S_{Y} < 1$
- $S_{r} > 0$
- $0 < Z_{Y} < 1$
- $Z_{r} < 0$
- $L_{Y} > 0$
- $L_{r} < 0$

#### Part (i): Total Differentials of the General Implicit Form

The total differential of a function $F^i(Y, r; M_{0}, X_{0}) = 0$ with respect to $Y$, $r$, $M_{0}$, and $X_{0}$ is given by:

$$dF^i = \frac{\partial F^i}{\partial Y} dY + \frac{\partial F^i}{\partial r} dr + \frac{\partial F^i}{\partial M_{0}} dM_{0} + \frac{\partial F^i}{\partial X_{0}} dX_{0} = 0$$

**For $F^1$:**

$$F^1(Y, r; M_{0}, X_{0}) = I(r) + X_{0} - S(Y, r) - Z(Y, r)$$

Taking the total differential:

$$dF^1 = \frac{\partial I}{\partial r} dr + dX_{0} - \left( \frac{\partial S}{\partial Y} dY + \frac{\partial S}{\partial r} dr \right) - \left( \frac{\partial Z}{\partial Y} dY + \frac{\partial Z}{\partial r} dr \right) = 0$$

Simplifying:

$$\left( \frac{\partial I}{\partial r} - \frac{\partial S}{\partial r} - \frac{\partial Z}{\partial r} \right) dr + \left( -\frac{\partial S}{\partial Y} - \frac{\partial Z}{\partial Y} \right) dY + dX_{0} = 0$$

Supporting Statement: This equation represents the change in the equilibrium of the goods market in response to changes in the variables.

**For $F^2$:**

$$F^2(Y, r; M_{0}, X_{0}) = L(Y, r) - M_{0}$$

Taking the total differential:

$$dF^2 = \frac{\partial L}{\partial Y} dY + \frac{\partial L}{\partial r} dr - dM_{0} = 0$$

Supporting Statement: This equation represents the change in the equilibrium of the money market in response to changes in the variables.
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Part (ii): Total Differentials of the Specific Implicit Forms

Specify partial derivatives:

- $\frac{\partial I}{\partial r} = I_r$
- $\frac{\partial S}{\partial Y} = S_Y$
- $\frac{\partial S}{\partial r} = S_r$
- $\frac{\partial Z}{\partial Y} = Z_Y$
- $\frac{\partial Z}{\partial r} = Z_r$
- $\frac{\partial L}{\partial Y} = L_Y$
- $\frac{\partial L}{\partial r} = L_r$

For F^1:

$$\left(I_r - S_r - Z_r \right) dr + \left(-S_Y - Z_Y \right) dY + dX_{0} = 0$$

Supporting Statement: The equation is transformed by plugging specific partial derivative terms into the general implicit form.

For F^2:

$$L_Y dY + L_r dr - dM_{0} = 0$$

Supporting Statement: The equation is transformed by plugging specific partial derivative terms into the general implicit form.

Part (iii): Matrix Form $Jx = b$

The total derivatives with respect to X_{0}:

Define the vector of differentials:

$$dx = \begin{pmatrix} dY \\ dr \end{pmatrix}$$

Define the Jacobian matrix J:

$$J = \begin{pmatrix} -S_Y - Z_Y & I_r - S_r - Z_r \\ L_Y & L_r \end{pmatrix}$$

Now, the vector b representing the right-hand side for changes in X_{0} and M_{0}:

For X_0:

$$db = \begin{pmatrix} -dX_{0} \\ 0 \end{pmatrix}$$

Thus, the total derivatives in matrix form $Jx = b$:

$$\begin{pmatrix} -S_Y - Z_Y & I_r - S_r - Z_r \\ L_Y & L_r \end{pmatrix} \begin{pmatrix} dY \\ dr \end{pmatrix} = \begin{pmatrix} -dX_{0} \\ 0 \end{pmatrix}$$

Supporting Statement: By representing the total derivatives in matrix form, the systemic nature of the changes in the economic variables is captured.

Final Solution:

The matrix form of the total derivatives is:

$$\begin{pmatrix} -S_Y - Z_Y & I_r - S_r - Z_r \\ L_Y & L_r \end{pmatrix} \begin{pmatrix} dY \\ dr \end{pmatrix} = \begin{pmatrix} -dX_{0} \\ 0 \end{pmatrix}$$

This concludes the comprehensive and cohesive solution to finding the total differentials and their matrix form for the given macroeconomic equilibrium conditions in an open economy.

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