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## Physics - Electromagnetism

### Topic: Electric Field Due to an Infinite Line Charge

**Question:**

The electric field in the $xy$-plane due to an infinite line of charge along the $z$-axis is the gradient of a field potential $\rho_{o} \ln(r)$, where $\rho_{o}$ is a constant and $r = \sqrt{x^{2} + y^{2}}$. Determine the electric field $\mathbf{E}$ in the $xy$-plane.

**Solution:**

**Step 1: Introduction and Given Data**

Given the electric potential $V$ in the $xy$-plane due to an infinite line charge along the $z$-axis:

$$ V(r) = \rho_{o} \ln(r) $$

where:

- $\rho_{o}$ is a constant.
- $r$ is the radial distance from the $z$-axis, expressed as $r = \sqrt{x^{2} + y^{2}}$.

Objective: To determine the electric field $\mathbf{E}$ in the $xy$-plane using the given potential function.

**Explanation**

To find the electric field $\mathbf{E}$, the gradient of the potential $V(r)$ should be calculated.

**Step 2: Formula for Electric Field from Potential**

The electric field $\mathbf{E}$ is related to the electric potential $V$ by:

$$ \mathbf{E} = -\nabla V $$

Where $\nabla$ is the gradient operator. In Cartesian coordinates:

$$ \nabla V = \left( \frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z} \right) $$

Since $V$ is independent of $z$, $\frac{\partial V}{\partial z} = 0$.

**Explanation**

The gradient operator in Cartesian coordinates is used for calculating the partial derivatives of $V$ with respect to $x, y,$ and $z$.

**Step 3: Calculation of $\frac{\partial V}{\partial x}$ and $\frac{\partial V}{\partial y}$**

First, express $V$ in terms of $x$ and $y$:

$$ V(x, y) = \rho_{o} \ln(\sqrt{x^2 + y^2}) $$

Using the chain rule:

$$ V(x, y) = \rho_{o} \ln(r) $$

$$ \frac{\partial V}{\partial x} = \rho_{o} \frac{1}{r} \frac{\partial r}{\partial x} $$

$$ \frac{\partial V}{\partial y} = \rho_{o} \frac{1}{r} \frac{\partial r}{\partial y} $$

The derivatives of $r$ are:

$$ \frac{\partial r}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{r} $$

$$ \frac{\partial r}{\partial y} = \frac{y}{\sqrt{x^2 + y^2}} = \frac{y}{r} $$

Hence:

$$ \frac{\partial V}{\partial x} = \rho_{o} \frac{1}{r} \frac{x}{r} = \rho_{o} \frac{x}{r^2} $$

$$ \frac{\partial V}{\partial y} = \rho_{o} \frac{1}{r} \frac{y}{r} = \rho_{o} \frac{y}{r^2} $$

**Explanation**

The partial derivatives of $V$ have been calculated using the chain rule and the relationship between $r, x,$ and $y$.
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Step 4: Determining the Components of Electric Field $\mathbf{E}$

Apply the negative gradient:

$$ \mathbf{E}_x = -\frac{\partial V}{\partial x} = -\rho_{o} \frac{x}{r^2} $$

$$ \mathbf{E}_y = -\frac{\partial V}{\partial y} = -\rho_{o} \frac{y}{r^2} $$

Combining the components:

$$ \mathbf{E} = \left(-\rho_{o} \frac{x}{r^2}, -\rho_{o} \frac{y}{r^2} \right) $$

Given that $r = \sqrt{x^2 + y^2}$:

$$ \mathbf{E} = -\rho_{o} \frac{(x, y)}{x^2 + y^2} $$

Explanation

The components of the electric field $\mathbf{E}$ are derived by applying the negative gradient to each partial derivative of the electric potential.

Step 5: Final Solution

Thus, the electric field $\mathbf{E}$ in the xy-plane due to an infinite line charge along the z-axis is:

$$ \mathbf{E} = -\rho_{o} \frac{(x, y)}{x^2 + y^2} $$

Explanation

The final result provides the expression for the electric field in vector form, giving the field's magnitude and direction dependent on the position coordinates x and y.

Final Solution:

$$ \mathbf{E} = -\rho_{o} \frac{(x, y)}{x^2 + y^2} $$

This is the comprehensive and accurate solution for the electric field $\mathbf{E}$ in the xy-plane due to an infinite line charge along the z-axis.

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