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Mechanical Engineering

Topic: Torsional Shear Stress

Given Data:

Radius of the shaft, $r = 20 \, \text{mm} = 0.02 \, \text{m}$

Power transmitted, $P = 2.5 \, \text{kW} = 2500 \, \text{W}$

Frequency of rotation, $f = 50 \, \text{Hz}$

Introduction:

The problem involves calculating the maximum shear stress in a solid shaft that is transmitting a given power at a specified frequency. The maximum shear stress in a rotating shaft can be determined using the relationship between power, angular velocity, and torque. The formula for shear stress, in terms of torque and the polar moment of inertia, will also be required.

Step-by-Step Solution:

Step 1: Calculate the Angular Velocity

The angular velocity (ω) in radians per second can be found from the frequency:

$$\omega = 2 \pi f$$

Explanation: The angular velocity is the rate of change of the angular position of the shaft, which is determined by the formula $\omega = 2 \pi f$.

Calculation:

$$\omega = 2 \pi \times 50$$

$$\omega = 100 \pi \, \text{rad/s}$$

Supporting Statement: The frequency is given as 50 Hz, and the angular velocity is calculated using the standard formula $\omega = 2 \pi f$.

Step 2: Calculate the Torque

The relationship between power (P) and torque (T) is given by the equation:

$$P = T \omega$$

Supporting Statement: Power, torque, and angular velocity are related; hence, the torque can be computed by rearranging the power formula.

Calculation:

$$T = \frac{P}{\omega}$$

$$T = \frac{2500 \, \text{W}}{100 \pi \, \text{rad/s}}$$

$$T = \frac{2500}{100 \pi}$$

$$T \approx 7.96 \, \text{Nm}$$

Step 3: Calculate the Polar Moment of Inertia

For a solid cylindrical shaft, the polar moment of inertia (J) is given by:

$$J = \frac{\pi r^4}{2}$$

Explanation: The polar moment of inertia for a circular cross-section is crucial for determining the distribution of shear stress in the shaft.

Calculation:

$$J = \frac{\pi (0.02 \, \text{m})^4}{2}$$

$$ J = \frac{\pi (1.6 \times 10^{-7} \, \text{m}^4)}{2} $$

$$ J = 2.513 \times 10^{-8} \, \text{m}^4 $$

Supporting Statement: The Polar moment of inertia for the shaft with a given radius is calculated using the given formula.

Step 4: Determine Maximum Shear Stress

The maximum shear stress (τ_{max}) can be calculated using:

$$ \tau_{\text{max}} = \frac{T r}{J} $$

Explanation: The shear stress in the shaft due to torsional loading is given by the formula $\tau_{\text{max}} = \frac{T r}{J}$, where T is the torque, r is the radius, and J is the polar moment of inertia.

Calculation:

$$ \tau_{\text{max}} = \frac{7.96 \, \text{Nm} \times 0.02 \, \text{m}}{2.513 \times 10^{-8} \, \text{m}^4} $$

$$ \tau_{\text{max}} = \frac{0.1592}{2.513 \times 10^{-8}} $$

$$ \tau_{\text{max}} \approx 6.33 \times 10^6 \, \text{Pa} $$

$$ \tau_{\text{max}} \approx 6.33 \, \text{MPa} $$

Supporting Statement: The maximum shear stress is calculated using the relationship between torque, radius, and the polar moment of inertia.

Final Solution:

The maximum shear stress in the shaft is **$\tau_{\text{max}} = 6.33 \, \text{MPa}$**