CheggSolutions - Thegdp

Subject: Mathematics

Topic: Exponential Decay

Given:

Initial mass of radioactive substance, \(M_0 \): 100 mg
Mass after 6 hours (\(M_6 \)): decreased by 4%

Introduction:

The decay of the radioactive substance follows the exponential decay model, which is represented by the formula:
\[M(t) = M_0 e^{-kt} \]

where:

- \(M(t)\) is the amount of substance at time \(t\),
- \(M_0\) is the initial amount of substance,
- \(k\) is the decay constant,
- \(t\) is the time.

Step-by-Step Solution:

Step 1: Determine the mass after 6 hours.

\[M_6 = M_0 \times (1 - 0.04) \]

\[M_6 = 100 \times 0.96 \]

\[M_6 = 96 \text{ mg} \]

Explanation: After 6 hours, the mass has decreased by 4%, so the remaining mass is 96% of the initial mass.

Supporting Statement: 96% of 100 mg is calculated to obtain the mass after 6 hours.

Step 2: Determine the decay constant \(k\).

Using the exponential decay formula:

\[M(t) = M_0 e^{-kt} \]

Set \(t = 6 \text{ hours,} M(t) = 96 \text{ mg and} M_0 = 100 \text{ mg} \).

\[96 = 100 e^{-6k} \]

Solving for \(k\):

\[\frac{96}{100} = e^{-6k} \]

\[0.96 = e^{-6k} \]

Take natural logarithm on both sides:

\[\ln(0.96) = -6k \]

\[k = -\frac{\ln(0.96)}{6} \]

\[k \approx 0.006939 \]

Explanation: The decay constant is computed using the logarithmic properties to isolate \(k\).

Supporting Statement: The decay constant is derived from the relationship between the remaining mass and the initial mass over a known time period.

Step 3: Calculate the amount of substance remaining after 24 hours.

Using the exponential decay model:

$$ M(24) = 100 e^{-0.006939 \times 24} $$

Simplify the exponent:

$$ M(24) = 100 e^{-0.166536} $$

Evaluate the exponential function:

$$ M(24) \approx 100 \times e^{-0.166536} $$

$$ M(24) \approx 100 \times 0.846088 $$

$$ M(24) \approx 84.6 \text{ mg} $$

Explanation: The formula is used with the determined decay constant to find the remaining mass after 24 hours.

Supporting Statement: The exponential decay formula is used after substituting $t = 24$ hours to get the remaining mass.

Final Solution:

The amount remaining after 24 hours is approximately 84.6 milligrams.

Explanation: The calculation has shown that the substance decays exponentially, leaving around 84.6 mg after 24 hours.

Supporting Statement: The decay model accurately predicts the remaining mass after a specified time, taking into account the proportional decrease observed initially.