

Sub-Subject: Microeconomics Topic: General Equilibrium and Welfare Theorems

Given Data and Introduction

Individuals A and B have utility functions and endowments as follows:

- Utility functions:
 $U^A = x^A y^A$
 $U^B = x^B y^B - 0.5x^A$
- Endowments:
 $e^A = (e_x^A, e_y^A) = (15, 5)$
 $e^B = (e_x^B, e_y^B) = (10, 15)$

Given resources in the economy:

$$x^A + x^B = e_x^A + e_x^B = 15 + 10 = 25$$
$$y^A + y^B = e_y^A + e_y^B = 5 + 15 = 20$$

(a) Utility Impact

Step 0: Analyzing Utility Functions

- For Individual A:
 $U^A = x^A y^A$
- For Individual B:
 $U^B = x^B y^B - 0.5 x^A$

Supporting Statement: The form of individual A's utility function indicates it solely depends on their own consumption of goods x and y .

Explanation for Step 0

Explanation: Individual A's utility is functionally dependent only on their own consumption levels of goods x and y and does not incorporate the consumption levels of individual B.

Supporting Statement: Therefore, individual B's consumption does not affect individual A's utility.

Supporting Statement: Conversely, individual B's utility is a function of their consumption of goods x and y , with an additional term negatively related to individual A's consumption of good x .

Explanation: Thus, individual B's utility is indeed affected by individual A's consumption since individual A's consumption of good x detracts from individual B's utility.

Conclusion for (a)

Individual A's utility is not affected by individual B's consumption. Individual B's utility is negatively affected by individual A's consumption of x .

(b) Finding the Equilibrium Allocation

Step 1: Setting Up the Budget Constraints

For both individuals, the market value of their initial endowment must equal the market value of their consumption bundle. Let p_x and p_y be the prices of goods x and y respectively.

- Individual A:
 $p_x x^A + p_y y^A = p_x e_x^A + p_y e_y^A$
 $p_x x^A + p_y y^A = p_x \times 15 + p_y \times 5$
- Individual B:
 $p_x x^B + p_y y^B = p_x e_x^B + p_y e_y^B$
 $p_x x^B + p_y y^B = p_x \times 10 + p_y \times 15$

Supporting Statement: The equilibrium condition requires the sum of consumption to equal the total endowments, $x^A + x^B = 25$ and $y^A + y^B = 20$.

Explanation for Step 1

Explanation: The budget constraints ensure that each individual's consumption set is limited to their means as determined by their initial endowments and the prices.

Step 2: Marginal Rate of Substitution (MRS)

For optimal consumption, the ratio of marginal utilities (Marginal Rate of Substitution - MRS) must equal the ratio of prices:

- For Individual A:
 $MRS^A = \frac{MU_x^A}{MU_y^A} = \frac{y^A}{x^A}$
- For Individual B:
 $MRS^B = \frac{MU_x^B}{MU_y^B} = \frac{y^B}{x^B}$

At equilibrium, $MRS^A = \frac{p_x}{p_y}$ and $MRS^B = \frac{p_x}{p_y}$.

However, individual B's utility function complicates the process, so we solve for general equilibrium:

$$\frac{y^A}{x^A} = \frac{y^B}{x^B}$$

Explanation for Step 2

Explanation: At equilibrium, both individuals must value the marginal utility per unit price of goods x and y equivalently, ensuring no arbitrage opportunity.

Step 3: Solving System of Equations

The equations to solve are:

$$\begin{aligned} x^A + x^B &= 25 \\ y^A + y^B &= 20 \end{aligned}$$

From earlier constraints and optimization, substitute to solve for allocated amounts.

- Individual A:
 $\frac{y^A}{x^A} = \frac{p_x}{p_y} \Rightarrow y^A = \left(\frac{p_x}{p_y} \right) x^A$
- Individual B:
 $\frac{y^B}{x^B} = \frac{p_x}{p_y} \Rightarrow y^B = \left(\frac{p_x}{p_y} \right) x^B - 0.5 x^A$

Explanation for Step 3

Explanation: Solve the system while maintaining the constraints defined by the total amounts of goods x and y available equally to both individuals' valuations.

Final Equilibrium Allocation

$$\begin{aligned} x^A + x^B &= 25 \\ y^A + y^B &= 20 \end{aligned}$$

(c) Social Efficiency Analysis

Step 4: Pareto Efficiency

Efficiency requires that no reallocation can make one person better off without making the other worse off.

Given Individual A's consumption doesn't affect B except through overall constraint impacts, reallocation complexities show outcomes cannot meet both individuals' marginal rate subsidiaries.

Explanation for Step 4

Explanation: The equilibrium achieved doesn't ensure all fit expectations, verifying that individual B's utility spoilage from component deviation exactly equalizes.

Conclusion for (c)

Thus, the equilibrium allocation is not socially efficient because individual B's utility function inherently punishes marginal increase effects typical in the defined given goods practice.

(d) First Welfare Theorem Observation

Step 5: Checking First Welfare Theorem

The First Welfare Theorem asserts that competitive equilibria results in Pareto optimal allocations of goods according to agents.

Explanation for Step 5

Explanation: Given equilibrium constraints expected narrowly between rivalries, deviations ascertain mutual effects connector norms.

Conclusion for (d)

Given disutility from individual A's consumption affecting B's overall utility kabab condition:

- Market forces aren't homogeneous supporting Pareto proof per ideal. Hence, while informative, results don't fit theorem stances directly.