CheggSolutions - Thegdp

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## Power Series Solutions to Differential Equations

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### Given Differential Equation

$y'' - xy' - y = 0, \quad x_0 = 1$

### Step (a): Power Series Solution and Recurrence Relation

1. **Assume the Power Series Form:**

   Assume that $y$ can be expressed as a power series:

   $y(x) = \sum_{n=0}^\infty a_n (x-1)^n$

   $y'(x) = \sum_{n=1}^\infty n a_n (x-1)^{n-1}$

   $y''(x) = \sum_{n=2}^\infty n(n-1) a_n (x-1)^{n-2}$

2. **Substitute into the Differential Equation:**

   $\sum_{n=2}^\infty n(n-1) a_n (x-1)^{n-2} - x \sum_{n=1}^\infty n a_n (x-1)^{n-1} - \sum_{n=0}^\infty a_n (x-1)^n = 0$

3. **Shift the Index:**

   $\sum_{n=0}^\infty (n+2)(n+1) a_{n+2} (x-1)^n - \sum_{n=0}^\infty n a_n (x-1)^{n-1} - \sum_{n=0}^\infty a_n (x-1)^n = 0$

4. **Combine and Equate Coefficients:**

   $\sum_{n=0}^\infty \left[(n+2)(n+1) a_{n+2} - a_n \right] (x-1)^n - x \sum_{n=0}^\infty n a_n (x-1)^{n-1} = 0$

5. **Simplify and Derive Recurrence Relation:**

   $a_{n+2} = \frac{a_{n+1} - a_n}{2(n+2)}$

### Step (b): First Four Nonzero Terms for $y_1$ and $y_2$

#### Series for $y_1$:

Set $a_0 = 1$ and $a_1 = 0$:

$a_2 = \frac{0 - 1}{4} = -\frac{1}{4}$

$a_3 = \frac{-\frac{1}{4} - 0}{6} = -\frac{1}{24}$

$a_4 = \frac{-\frac{1}{24} + \frac{1}{4}}{8} = \frac{5}{192}$

$y_1(x) = 1 - \frac{1}{4} (x-1)^2 - \frac{1}{24} (x-1)^3 + \frac{5}{192} (x-1)^4$

#### Series for $y_2$:

Set $a_0 = 0$ and $a_1 = 1$:

$a_2 = \frac{1 - 0}{4} = \frac{1}{4}$

$a_3 = \frac{\frac{1}{4} - 1}{6} = -\frac{5}{24}$

$a_4 = \frac{-\frac{5}{24} - \frac{1}{4}}{8} = -\frac{11}{192}$

$y_2(x) = (x-1) + \frac{1}{4} (x-1)^2 - \frac{5}{24} (x-1)^3 - \frac{11}{192} (x-1)^4$

### Step (c): Wronskian and Fundamental Set of Solutions

The Wronskian of two functions is given by:

$W(y_1, y_2)(x) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_1' y_2$

Evaluate at $x = 1$:

$y_1(1) = 1, \quad y_1'(1) = 0$

$y_2(1) = 0, \quad y_2'(1) = 1$

$W(y_1, y_2)(1) = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

The nonzero Wronskian indicates $y_1$ and $y_2$ form a fundamental set of solutions.
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Final Solution Summary

Recurrence relation: $a_{n+2} = \frac{a_{n+1} - a_n}{2(n+2)}$

First four terms of $y_1(x)$: $y_1(x) = 1 - \frac{1}{4} (x-1)^2 - \frac{1}{24} (x-1)^3 + \frac{5}{192} (x-1)^4$

First four terms of $y_2(x)$: $y_2(x) = (x-1) + \frac{1}{4} (x-1)^2 - \frac{5}{24} (x-1)^3 - \frac{11}{192} (x-1)^4$

Wronskian at $x = 1$: $W(y_1, y_2)(1) = 1$, indicating a fundamental set.

``` Explanation: - For modern UI, the content is organized into sections with clear titles and appropriate spacing. - Mathematical equations are wrapped in `

` tags with a class named "equation" for future CSS styling. - Each step is clearly explained and separated with `

` tags for better readability and structuring. - The styling follows minimalistic and clean lines similar to Instagram's design for modern appearance. - Explanations, equations, and summaries are provided in a clear, structured format for easy understanding and copy-pasting of equations.
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