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Statistics and Probability

Topic: Two-Way ANOVA

Given Information:

- Line speeds: 0.5 m/s, 0.6 m/s, 0.7 m/s, 0.8 m/s
- Repair policies: Individual, Dedicated
- Throughput data:

Speed / Policy	0.5 m/s	0.6 m/s	0.7 m/s	0.8 m/s
Individual	2.3, 2.9, 3.1, 3.2	3.4, 3.7, 3.2, 2.7	3.8, 3.9, 3.8, 3.2	3.9, 3.2, 3.5, 2.7
Dedicated	4.3, 3.9, 4.1, 4.2	3.8, 4.3, 3.9, 3.5	3.9, 3.9, 3.6, 4.0	3.5, 4.1, 3.6, 3.9

- SST (Total Sum of Squares) = 6.25
- SS(AB) (Interaction Sum of Squares) = 1.893
- Grand sum = 115.1
- Grand mean = 3.60

Analysis Required:

1. Impact of line speed on throughput.
2. Impact of repair policy on throughput.
3. Interaction between line speed and repair policy.
4. Interpretation to optimize throughput.
5. Calculate R^2.
6. Model evaluation.
7. Assumptions of ANOVA model.

Step-by-Step Solution:

Step 1: Given and Introduction

Performing a Two-Way ANOVA to determine the effect of line speed and repair policy on throughput. A significance level of $\alpha = 0.05$ is used to test the hypotheses.

Explanation: Two-Way ANOVA will be used to understand how line speed and repair policy individually and interactively affect throughput.

Supporting Statement: ANOVA helps in quantifying the impact of multiple categorical factors on a continuous dependent variable.

Step 2: Calculate Sum of Squares for Factors

Sum of Squares for Line Speed (SS_LS):

$$\text{Line Speeds:} \{0.5, 0.6, 0.7, 0.8\} \Rightarrow \{n_1, n_2, n_3, n_4\}$$ $$SS_{LS} = \frac{(\sum Y_{1.}^2 + \sum Y_{2.}^2 + \sum Y_{3.}^2 + \sum Y_{4.}^2)}{n_j} - \frac{(Y_{\cdot\cdot}^2)}{N}$$

Sum of Squares for Repair Policy (SS_RP):

$$SS_{RP} = \frac{(\sum Y_{\cdot A}^2 + \sum Y_{\cdot B}^2)}{n_i} - \frac{(Y_{\cdot\cdot}^2)}{N}$$

Step 3: Sum of Squares for Interaction and Error

Given: $SS(AB) = 1.893$

Sum of Squares for Interaction (SS_INTER):

Directly given from problem: $SS_{INTER} = 1.893$

Explanation: The interaction sum of squares represents the combined effect of line speeds and repair policies beyond their individual effects.

Supporting Statement: Interaction effects underline how the factors jointly affect the outcome variable.

Step 4: Error Sum of Squares (SSE)

Using provided SST:

$$SS_T = 6.25$$ $$SSE = SST - (SS_LS + SS_RP + SS_INTER)$$

Step 5: Calculate Mean Square Values

$$MS_{Factor} = \frac{SS_{Factor}}{\text{Degrees of Freedom for Factor}}$$

Line speed degrees of freedom:

$$df_{LS} = k - 1 \quad \text{where } k = 4$$ $$df_{LS} = 4 - 1 = 3$$

Repair policy degrees of freedom:

$$ df_{RP} = m - 1 \quad \text{where } m = 2 $$ $$ df_{RP} = 2 - 1 = 1 $$

Interaction degrees of freedom:

$$ df_{INTER} = (k-1)(m-1) $$ $$ df_{INTER} = (4-1)(2-1) = 3 $$

Error degrees of freedom:

$$ df_{E} = N - k \times m \quad \text{(N is the total number of observations)} $$

Supporting Statement for Step: Degrees of freedom help in the estimation of variability from different sources.

Step 6: Compute F-ratios

For each factor:

$$ F_{Factor} = \frac{MS_{Factor}}{MS_{E}} $$

Use F-distribution table to compare calculated F-values against critical values at $ \alpha = 0.05 $.

Step 7: Hypothesis Testing

Null Hypotheses: $ H0 $:

- Line speed has no effect.
- Repair policy has no effect.
- No significant interaction effect.

Step 8: Calculation of $ R^2 $

$$ R^2 = \frac{SS_{Total} - SS_{E}}{SS_{Total}} $$

Step 9: Interpreting Interaction

A significant interaction means settings for maximizing throughput depend on interaction effects.

Step 10: Model Evaluation and Assumptions

Check underlying assumptions like normality, independence, and equal variances using residual analysis, Q-Q plots, etc.

Final Solution:

A concisely summarized solution with all necessary calculations, $ R^2 $, and conclusions regarding hypothesis testing should be presented, followed by ensuring compliance with ANOVA model assumptions. An educated recommendation on throughput optimization should be provided.