

Electrical Engineering

Topic: Power Electronics - Controlled Rectifiers

Given Data:

- Resistance, $(R = 2 \text{ } \Omega)$ ohms
- Inductance, $(L = 75 \text{ } \text{mH})$ (0.075 H)
- DC Voltage Source, $(V_{dc} = 48 \text{ } \text{V})$ V
- Source Voltage, $(V_s = 120 \text{ } \text{V})$ V (RMS)
- Frequency, $(f = 60 \text{ } \text{Hz})$ Hz
- Delay Angle, $(\alpha = 50^\circ)$

(a) Expression for Load Current

Step 1: Convert RMS Voltage to Peak Voltage

$$V_m = V_s \cdot \sqrt{2}$$

$$V_m = 120 \cdot \sqrt{2} \approx 169.71 \text{ } \text{V}$$

Explanation: The peak voltage (V_m) is found by multiplying the RMS value by $(\sqrt{2})$.

Step 2: Calculate the Angular Frequency

$$\omega = 2 \pi f$$

$$\omega = 2 \pi \cdot 60 = 120 \pi \text{ } \text{rad/s}$$

Explanation: The angular frequency is calculated by multiplying (2π) by the frequency.

Step 3: Derive the Differential Equation for Load Current

The controlled half-wave rectifier with a resistive (R), inductive (L), and DC voltage source (V_{dc}) is considered:

$$V_m \sin(\omega t - \alpha) = L \frac{di(t)}{dt} + Ri(t) + V_{dc}$$

Explanation: This equation relates all voltage drops across the resistance, inductance, and DC source.

Step 4: Solve the Differential Equation

Using the standard solution for first-order linear differential equations considering initial conditions and integrating factor:

$$i(t) = \frac{V_m \sin(\omega t - \alpha) - V_{dc}}{R} + \left(i(0) - \frac{V_m \sin(\omega t - \alpha) - V_{dc}}{R} \right) e^{-\frac{R}{L}t}$$

Explanation: This represents the current in the RL circuit with time taking into account the voltage source drops along with resistor and inductor.

(b) Power Absorbed by the DC Voltage Source

Step 5: Calculate Average Load Current (I_{avg})

The load current over one cycle for a controlled rectifier with an inductor:

$$I_{avg} = \frac{V_m \cos(\alpha) - V_{dc}}{R}$$

Where,

$$V_m \approx 169.71 \text{ } \text{V}$$

$$\alpha = 50^\circ$$

$$\cos(50^\circ) \approx 0.6428$$

$$I_{avg} = \frac{169.71 \cdot 0.6428 - 48}{2} \approx \frac{109.04 - 48}{2} \approx 30.52 \text{ } \text{A}$$

Explanation: This is the average current derived by considering the cosine of the delay angle and circuit parameters.

Step 6: Calculate Power Absorbed by DC Voltage Source

$$P_{dc} = V_{dc} \cdot I_{avg}$$

$$P_{dc} = 48 \cdot 30.52 \approx 1465 \text{ } \text{W}$$

Explanation: Power absorbed by the DC source is the product of its voltage and the average current.

(c) Power Absorbed by Resistor

Step 7: Calculate the RMS Value of Load Current

Given the half-wave rectifier:

$$I_{\text{RMS}} = I_{\text{avg}} \sqrt{\frac{1}{2}}$$

$$I_{\text{RMS}} = 30.52 \cdot \sqrt{0.5} \approx 21.58 \text{ A}$$

Explanation: The RMS value for half-wave rectified current is the factor by root half of average current.

Step 8: Calculate Power Absorbed by the Resistor

$$P_{\text{R}} = I_{\text{RMS}}^2 \cdot R$$

$$P_{\text{R}} = (21.58)^2 \cdot 2 \approx 931 \text{ W}$$

Explanation: Power absorbed by the resistor is calculated using the square of RMS current times the resistance.

Final Solution

- (a) The expression for load current:

$$i(t) = \frac{169.71}{\sqrt{2}} \sin(\omega t - 50^\circ) - 48 + \left(i(0) - \frac{169.71}{\sqrt{2}} \sin(\omega t - 50^\circ) - 48 \right) e^{-\frac{2}{0.075}t}$$

- (b) Power absorbed by the DC voltage source:

$$P_{\text{dc}} \approx 1465 \text{ W}$$

- (c) Power absorbed by the resistor:

$$P_{\text{R}} \approx 931 \text{ W}$$