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# Retirement Savings Calculation

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## Part (a): Calculate the amount at age 65

**Given:**

- Current age = 33 years
- Annual savings = $8,000
- Interest rate = 10%
- Number of years saving = 32 years (65 - 33)

**Future Value of an Annuity Formula:**

$$FV = P \times [(1 + r)^n - 1] / r$$

**Step 1:** Calculate the future value

$$FV = 8000 \times [(1 + 0.10)^{32} - 1] / 0.10$$

$$(1 + 0.10)^{32} \approx 17.4494$$

$$FV = 8000 \times (17.4494 - 1) / 0.10$$

$$FV = 8000 \times 164.494$$

**FV ≈ $1,315,952.00**

**Explanation:** Using the Future Value of an Annuity formula, the future value of the savings is calculated by substituting the given values of annual payment, interest rate, and number of periods.

**Supporting Statement:** The calculation shows that by saving $8,000 annually at an interest rate of 10%, the client will have approximately $1,315,952.00 at age 65.

## Part (b): Calculate the amount at age 70

**Given:**

- Age to retire = 70
- Number of years saving = 37 years (70 - 33)
- Annual savings = $8,000
- Interest rate = 10% annually

**Step 2:** Calculate the future value

$$FV = 8000 \times [(1 + 0.10)^{37} - 1] / 0.10$$

$$(1 + 0.10)^{37} \approx 28.1024$$

$$FV = 8000 \times (28.1024 - 1) / 0.10$$

$$FV = 8000 \times 271.024$$

**FV ≈ $2,168,192.00**

**Explanation:** Using the same formula with updated values for the number of periods (37 years), the future value at age 70 is calculated.

**Supporting Statement:** The calculation shows that by saving $8,000 annually at an interest rate of 10%, the client will have approximately $2,168,192.00 at age 70.

## Part (c): Calculate the annual withdrawals at each retirement age

**Given:**

- Expected life after retirement at 65 = 20 years
- Expected life after retirement at 70 = 15 years
- Interest rate = 10%

**Withdrawal Amount Calculation Using the Present Value of Annuities Formula:**

$$PV = PMT \times [1 - (1 + r)^{-n}] / r$$

**Retirement at 65:**

**Step 3:** Calculate the annual withdrawal amount

Given PV = 1,315,952.00:

1,315,952.00 = PMT × [1 - (1 + 0.10)^-20] / 0.10

PMT = 1,315,952.00 × 0.10 / [1 - 0.1486]

PMT ≈ **$154,554.06**

**Explanation:** Using the Present Value of Annuities formula, the annual withdrawal amount if the client retires at 65 and withdraws evenly over 20 years is calculated.

**Supporting Statement:** The client will be able to withdraw approximately $154,554.06 annually for 20 years if retiring at 65.

## Retirement at 70:

**Step 4:** Calculate the annual withdrawal amount

Given PV = 2,168,192.00:

2,168,192.00 = PMT × [1 - (1 + 0.10)^-15] / 0.10

PMT = 2,168,192.00 × 0.10 / [1 - 0.2394]

PMT ≈ **$284,918.92**

**Explanation:** Using the Present Value of Annuities formula, the annual withdrawal amount if the client retires at 70 and withdraws evenly over 15 years is calculated.

**Supporting Statement:** The client will be able to withdraw approximately $284,918.92 annually for 15 years if retiring at 70.

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# Final Solution:

- At age 65, the future value of savings is approximately **$1,315,952.00**.
- At age 70, the future value of savings is approximately **$2,168,192.00**.
- Annual withdrawals:
  - **$154,554.06** if retiring at 65.
  - **$284,918.92** if retiring at 70.

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