CheggSolutions - Thegdp

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## Subject: Electromagnetism

### Topic: Magnetic Field at the Center of Current Loops

### Given

**Steady Current (I)**

**Distance from center to side (R)**

### (a) Find the magnetic field at the center of a square loop

**Step 1: Introduction**

A square loop carries a steady current I. The goal is to find the magnetic field at the center of the loop. For a square loop, symmetry can be utilized to simplify the problem.

**Step 2: Magnetic field contribution from one side of the loop**

The Biot-Savart law gives the magnetic field contribution from a current element as:

$$ d\mathbf{B} = \frac{\mu_0 I}{4\pi} \frac{d\mathbf{l} \times \mathbf{\hat{r}}}{r^2} $$

Here:

- $\mu_0$ is the permeability of free space,
- $I$ is the current,
- $d\mathbf{l}$ is the length element of the current,
- $\mathbf{\hat{r}}$ is the unit vector from the element to the point of interest,
- $r$ is the distance from the element to the point of interest.

**Explanation:** The Biot-Savart law is essential to calculate the magnetic field generated by a current-carrying conductor.

**Step 3: Analyze a single segment**

Each side of the square loop will contribute equally to the magnetic field at the center. Since $R$ is the distance from the center to the middle of a side, the distance from any segment to the center is:

$$ a = \sqrt{2}R $$

The length of the side of the square loop $a = 2R$

**Explanation:** The distance is calculated using the fact that the side length is equivalent to $2R$.

**Step 4: Integrating over one side**

The magnetic field produced by each segment will all point in the same direction by symmetry. For each segment, the length element $d\mathbf{l}$ and distance $r$ remain constant, so it simplifies the integration:

$$ B_{\text{one\ side}} = \frac{\mu_0 I}{4\pi} \int_{0}^{a} \frac{dl \sin(\theta)}{(\sqrt{2}R)^2} $$

**Explanation:** The integration simplifies due to symmetric properties and direct distances.

**Step 5: Summation of contributions**

There are four sides, and each contributes equally:

$$ B_{total} = 4 \times B_{\text{one\ side}} = 4 \times \left( \frac{\mu_0 I}{4\pi} \cdot \frac{\sqrt{2}}{(2R)^2} \cdot a / \sqrt{2} \right) $$

$$ B_{total} = \frac{4 \mu_0 I}{4\pi} \cdot \frac{1}{2R^2} \cdot R $$

$$ B_{total} = \frac{\mu_0 I}{\pi R} $$

**Explanation:** Each side contributes $1/4$th part, and by summing up all segments' contributions, the total magnetic field at the center is derived.

### (b) Find the field at the center of a regular n-sided polygon

**Step 1: Analysis parameter continuation**
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For a regular n-sided polygon, similar symmetry can be utilized. Each side's contribution needs to be evaluated, and the distance from center is consistently R.

Explanation: Continue with more general (n)-sided) geometry properties without losing the symmetry.

Step 2: Integration over each side and polygon

Using the same logic but expressing side length in polygon terms, continue integrating each segment:

$$B_{\text{side}} = \frac{\mu_0 I}{4\pi} \int_{0}^{a_n} \frac{dl \sin(\theta)}{R^2}$$

Where $a_n = 2R \sin(\pi/n)$, distance from center remains R.

Explanation: Polygon's central distance and diverse n-sided property must be tightly held to continue with integration.

Step 3: Final expression with total contributions

Sum of magnetic fields:

$$B_n = n B_{\text{one\ side}} = n \times \frac{\mu_0 I}{4\pi} \cdot \frac{\sin(\pi/n)}{R}$$

$$B_n = \frac{\mu_0 I}{2 \pi R} \cdot n \sin(\pi/n)$$

Explanation: Final summation takes into account each part's consistency to evenly contributing sum.

(c) Check formula for $n \to \infty$

Step 1: Limit analysis

In the limit of $n \to \infty$, a polygon approaches a circle. Thus, R and number of sides render:

$$\lim_{n \to \infty} n \sin(\pi/n) = \pi$$

This comes to:

$$B_{\text{circle}} = \frac{\mu_0 I}{2R}$$

Explanation: Cleanly checking earlier polygon formula ensures circle equivalent magnetic formula derivation for limit cases $n \to \infty$.

Final solutions

(a) Magnetic field at the center of the square loop is: $\boxed{\frac{\mu_0 I}{\pi R}}$

(b) Magnetic field at the center of a regular n-sided polygon is: $\boxed{\frac{\mu_0 I}{2 \pi R} \cdot n \sin(\pi/n)}$

(c) Field reduces to center of circular loop in the limit $n \to \infty$: $\boxed{\frac{\mu_0 I}{2R}}$