CheggSolutions - Thegdp

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## Subject: Probability Theory

### Topic: Conditional Probability

**Given:**

$X, Y, Z$ be discrete random variables.

**Statements:**

1. $p_{X,Y,Z}(x, y, z) = p_Y(y)p_{Z|Y}(z | y)p_{X|Y,Z}(x | y, z)$
2. $p_{X,Y|Z}(x, y | z) = p_X(x)p_{Y|Z}(y | z)$
3. $p_{X,Y|Z}(x, y | z) = p_{X|Z}(x | z)p_{Y|X,Z}(y | x, z)$
4. $\sum_{x} p_{X|Y,Z}(x | y, z) = 1$
5. $\sum_{y}\sum_{x} p_{X,Y|Z}(x, y | z) = 1$
6. $p_{X,Y|Z}(x, y | z) = \frac{p_{X,Y,Z}(x, y, z)}{p_Z(z)}$
7. $p_{X|Y,Z}(x | y, z) = \frac{p_{X,Y,Z}(x, y, z)}{p_{Y,Z}(y, z)}$

**Analysis:**

**1. Statement:**

$p_{X,Y,Z}(x, y, z) = p_Y(y)p_{Z|Y}(z | y)p_{X|Y,Z}(x | y, z)$

**True**

*Explanation:* This is the chain rule for joint probability distribution, which breaks down a joint probability into the product of conditional probabilities.

**2. Statement:**

$p_{X,Y|Z}(x, y | z) = p_X(x)p_{Y|Z}(y | z)$

**False**

*Explanation:* This statement would be true if $X$ and $Y$ were independent given $Z$, which is not generally the case unless specified.

**3. Statement:**

$p_{X,Y|Z}(x, y | z) = p_{X|Z}(x | z)p_{Y|X,Z}(y | x, z)$

**True**

*Explanation:* This is another form of the chain rule for conditional probabilities, expressing a joint conditional in terms of nested conditionals.

**4. Statement:**

$\sum_{x} p_{X|Y,Z}(x | y, z) = 1$

**True**

*Explanation:* The sum of the conditional probabilities over all possible values of $X$ given $y$ and $z$ must equal 1 by the law of total probability.

**5. Statement:**

$\sum_{y}\sum_{x} p_{X,Y|Z}(x, y | z) = 1$

**True**

*Explanation:* The sum of the joint conditional probabilities over all possible values of $X$ and $Y$ given $z$ must be 1 by the law of total probability.
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6. Statement:

$$p_{X,Y|Z}(x, y \mid z) = \frac{p_{X,Y,Z}(x, y, z)}{p_Z(z)}$$

True

Explanation: This is the definition of conditional probability, decomposing the joint distribution by dividing it over the marginal distribution of Z.

7. Statement:

$$p_{X|Y,Z}(x \mid y, z) = \frac{p_{X,Y,Z}(x, y, z)}{p_{Y,Z}(y, z)}$$

True

Explanation: This is the definition of conditional probability for the variable X given Y and Z, dividing the joint distribution over the marginal distribution of Y and Z.

Final Solution:

1. True
2. False
3. True
4. True
5. True
6. True
7. True

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