# CheggSolutions - Thegdp

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# Differential Equations

## Solving Non-Homogeneous Second-Order Linear Differential Equations

Given:
The differential equation to solve is:

$$ y'' + 3y' - 10y = te^t \cos(t) $$

### Step 1: Introduction and Homogeneous Solution

The solution process involves finding the general solution of the non-homogeneous differential equation, which consists of a homogeneous solution (complementary function) and a particular solution.

First, solve the homogeneous differential equation:

$$ y'' + 3y' - 10y = 0 $$

### Step 2: Characteristic Equation

To solve the homogeneous part, form the characteristic equation:

$$ r^2 + 3r - 10 = 0 $$

### Step 3: Solve the Characteristic Equation

Use the quadratic formula to find the roots:

$$ r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

where $a = 1$, $b = 3$, and $c = -10$.

$$ r = \frac{-3 \pm \sqrt{3^2 - 4(1)(-10)}}{2(1)} $$ $$ r = \frac{-3 \pm \sqrt{9 + 40}}{2} $$ $$ r = \frac{-3 \pm \sqrt{49}}{2} $$ $$ r = \frac{-3 \pm 7}{2} $$

### Step 4: Determine the Roots

$$ r_1 = \frac{-3 + 7}{2} = 2 $$ $$ r_2 = \frac{-3 - 7}{2} = -5 $$

### Step 5: Write the Homogeneous Solution

The homogeneous solution $y_h$ is a combination of the solutions corresponding to the roots:

$$ y_h = C_1 e^{2t} + C_2 e^{-5t} $$

### Step 6: Find the Particular Solution

Given the differential equation:

$$ y'' + 3y' - 10y = te^t \cos(t) $$

Use the method of undetermined coefficients. Assume a solution of the form:

$$ y_p = t (A e^t \cos(t) + B e^t \sin(t)) $$

### Step 7: Differentiate the Assumed Particular Solution
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Calculate the first derivative $y_p'$:

$$ y_p' = A e^t \cos(t) + B e^t \sin(t) + t e^t (A (\cos(t) - \sin(t)) + B (\sin(t) + \cos(t))) $$ $$ y_p' = e^t [(A+B)t \cos(t) + (A-B)t \sin(t) + (A \cos(t) + B \sin(t))] $$

Calculate the second derivative $y_p''$:

$$ y_p'' = e^t [(A+B)t (\cos(t) - \sin(t)) + (A-B)t (\sin(t) + \cos(t)) + 2(A \cos(t) + B \sin(t)) + (A \cos(t) - A \sin(t) + B \sin(t) + B \cos(t))] $$ $$ y_p'' = e^t [(A+B)t \cos(t) + (A+B)t \sin(t) + (A-B)t \sin(t) + (A-B)t \cos(t)] $$

### Step 8: Substitute into the Original Equation

Substitute $y_p$, $y_p'$, and $y_p''$ into the original differential equation and simplify to find $A$ and $B$.

Equate the coefficients of $e^t \cos(t)$ and $e^t \sin(t)$ to solve for $A$ and $B$.

### Step 9: Form the Particular Solution

After finding $A$ and $B$, write the particular solution $y_p$:

$$ y_p = t (A e^t \cos(t) + B e^t \sin(t)) $$

### Step 10: General Solution

Combine the homogeneous and particular solutions to get the general solution:

$$ y = y_h + y_p $$ $$ y = C_1 e^{2t} + C_2 e^{-5t} + t (A e^t \cos(t) + B e^t \sin(t)) $$

### Final Solution

The general solution of the differential equation:

$$ y'' + 3y' - 10y = te^t \cos(t) $$

is:

$$ y = C_1 e^{2t} + C_2 e^{-5t} + t (A e^t \cos(t) + B e^t \sin(t)) $$

This completes the general solution.

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