# CheggSolutions - Thegdp

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## Control Systems Engineering

### Inverse Laplace Transformation and Partial Fraction Expansion (PFE)

#### Given:

**Problem:** Find *x(t)* for the given *X(s)* using Partial Fraction Expansion (PFE) where required.

## (a)

**Given:**

```
X(s) = s(s+1) / [(s+2)(s+3)(s+4)]
```

**Step 1:**

Express *X(s)* in partial fractions:

```
X(s) = A/(s+2) + B/(s+3) + C/(s+4)
```

**Step 2:**

Find constants *A*, *B*, and *C*:

```
Multiplying both sides by (s+2)(s+3)(s+4):
s(s+1) = A(s+3)(s+4) + B(s+2)(s+4) + C(s+2)(s+3)

Set s = -2 :
(-2)(-1) = A(1)(2) => A = 1

Set s = -3 :
(-3)(-2) = B(-1)(1) => B = -6

Set s = -4 :
(-4)(-3) = C(-2)(-1) => C = 6
```

So, the partial fraction decomposition is:

```
X(s) = 1/(s+2) - 6/(s+3) + 6/(s+4)
```

**Step 3:**

Find the inverse Laplace Transform:

```
x(t) = e^(-2t) - 6e^(-3t) + 6e^(-4t)
```

**Final Solution:**

```
x(t) = e^(-2t) - 6e^(-3t) + 6e^(-4t)
```

## (b)

**Given:**

```
X(s) = (s+2) / (s+1)^2
```

**Step 1:**
```

Express *X(s)* in a form suitable for inverse transformation:

```
X(s) = 1/(s+1) + 1/(s+1)^2
```

**Step 2:**

Simplify:

```
X(s) = 1/(s+1) + 1/(s+1)^2
```

**Step 3:**

Find the inverse Laplace Transform:

```
x(t) = e^(-t) + t e^(-t)
```

**Final Solution:**

```
x(t) = e^(-t) + t e^(-t)
```

## (c)

**Given:**

```
X(s) = 1 / (s^2 + s + 1)
```

**Step 1:**

Solve for the roots of the denominator:

```
s = -1/2 ± j√3/2
```

**Step 2:**

Find the inverse Laplace Transform:

```
x(t) = (2/√3) e^(-t/2) sin((√3/2) t)
```

**Final Solution:**

```
x(t) = (2/√3) e^(-t/2) sin((√3/2) t)
```

## (d)

**Given:**

```
X(s) = (s+1) / (s(s+4)(s+3)) - e^(-0.5s)
```

**Step 1:**

Express *X(s)* in partial fractions:

```
X(s) = A/s + B/(s+3) + C/(s+4)
```

**Step 2:**

Find constants *A*, *B*, and *C*:

```
A = 1/12, B = 2/3, C = -3/4
```

**Step 3:**

Find the inverse Laplace Transform:

```
x(t) = 1/12 + (2/3) e^(-3t) - (3/4) e^(-4t) - δ(t-0.5)
```

**Final Solution:**

```
x(t) = 1/12 + (2/3) e^(-3t) - (3/4) e^(-4t) - δ(t-0.5)
```