CheggSolutions - Thegdp

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## Differential Equations

### Topic: Total Response for Linear Differential Systems

#### Given Information:

The differential equation is:

$$\frac{d^2}{dt^2} y(t) + 2 \frac{d}{dt} y(t) = \frac{d}{dt} x(t) + x(t)$$

Initial conditions: $y(0) = 2$, $\dot{y}(0) = 1$

Input: $x(t) = u(t)$, where $u(t)$ is the unit step function.

#### Objective:

To find the total response $y(t)$ of the system.

#### Step 1: Transform the Differential Equation using the Laplace Transform

The Laplace transform will convert the differential equation into an algebraic equation. Given the Laplace transforms:

$$\mathcal{L}\left\{ \frac{d^n}{dt^n} f(t) \right\} = s^n F(s) - s^{n-1} f(0) - \ldots - \frac{d^{n-1}}{dt^{n-1}} f(0)$$

Applying the Laplace transform to each term of the differential equation:

$$\mathcal{L}\left\{ \frac{d^2}{dt^2} y(t) \right\} = s^2 Y(s) - s y(0) - \dot{y}(0)$$
$$\mathcal{L}\left\{ 2 \frac{d}{dt} y(t) \right\} = 2 \left( s Y(s) - y(0) \right)$$
$$\mathcal{L}\left\{ \frac{d}{dt} x(t) \right\} = s X(s) - x(0)$$
$$\mathcal{L}\left\{ x(t) \right\} = X(s)$$

Explanation: Transforming the differential equation into an algebraic expression helps to solve for $Y(s)$.

#### Step 2: Apply Initial Conditions and Substitute into the Transformed Equation

Given:

$$\mathcal{L}\{ y(0) \} = 2$$
$$\mathcal{L}\{ \dot{y}(0) \} = 1$$
$$\mathcal{L}\{ x(t) = u(t) \} = \frac{1}{s}$$
$$\mathcal{L}\left\{ \frac{d}{dt} u(t) \right\} = 1$$

Substitute these values:

$$s^2 Y(s) - s \cdot 2 - 1 + 2(s Y(s) - 2) = s \cdot \frac{1}{s} - 1 + \frac{1}{s}$$

Simplify the equation:

$$s^2 Y(s) - 2s - 1 + 2s Y(s) - 4 = 1 - 1 + \frac{1}{s}$$

Combine like terms:

$$(s^2 + 2s) Y(s) - 2s - 5 = \frac{1}{s}$$

Explanation: After substituting initial conditions and simplifying, the transformed equation can be solved for $Y(s)$.

#### Step 3: Solve for $Y(s)$

Rearrange the equation to isolate $Y(s)$:

$$(s^2 + 2s) Y(s) = \frac{1}{s} + 2s + 5$$
$$Y(s) = \frac{\frac{1}{s} + 2s + 5}{s^2 + 2s}$$

Simplify further by factoring and separating terms:

$$Y(s) = \frac{1}{s(s+2)} + \frac{2s}{s(s+2)} + \frac{5}{s(s+2)}$$

Partial fraction decomposition:

$$\frac{1}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$
$$1 = A(s+2) + Bs$$
$$\Rightarrow A = 1, B = -1$$

Similarly decompose the other terms:

$$\frac{2s}{s(s+2)} = \frac{2}{s+2}$$
$$\frac{5}{s(s+2)} = \frac{5}{2} \left( \frac{1}{s} - \frac{1}{s+2} \right)$$

Combine all terms;

$$Y(s) = \frac{1}{s} - \frac{1}{s+2} + \frac{2}{s+2} + \frac{5}{2} \left( \frac{1}{s} - \frac{1}{s+2} \right)$$
$$= \left( \frac{1}{s} + \frac{5}{2s} \right) + \left( \frac{-1 + 2 - 5/2}{s+2} \right)$$
$$= \frac{7}{2s} - \frac{2}{s+2}$$

Explanation: The simplified form of $Y(s)$ in terms of partial fractions is crucial for finding the inverse Laplace Transform.

#### Step 4: Take the Inverse Laplace Transform
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Apply the inverse Laplace transform to find \(y(t) \):

$$
\mathcal{L}^{-1} \left\{ \frac{7}{2s} \right\} = \frac{7}{2} u(t)
$$
$$
\mathcal{L}^{-1} \left\{ \frac{2}{s+2} \right\} = 2 e^{-2t} u(t)
$$

Therefore:

$$
y(t) = \frac{7}{2} u(t) - 2 e^{-2t} u(t)
$$

Total Response:

$$
y(t) = \left(\frac{7}{2} - 2 e^{-2t} \right) u(t)
$$

Explanation: Taking the inverse Laplace provides the time-domain solution for \(y(t) \), representing the total response of the system.

Final Solution:

$$
y(t) = \left(\frac{7}{2} - 2 e^{-2t} \right) u(t)
$$

This is the total response for the given linear differential system with specified initial conditions and input.

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