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Mechanical Engineering - Strength of Materials

Torsion in Circular Shafts

Given Data and Introduction

- The radius of the solid shaft (r) = 20 mm = 0.02 m
- Power transmitted by the shaft (P) = 2.5 kW = 2500 W
- Frequency (f) = 50 Hz

The objective is to determine the maximum shear stress (τ_{max}) in the shaft.

Step 1: Calculating the Torque

The relationship between power (P), torque (T), and angular velocity (ω) is given by:

$$P = T \cdot \omega$$

First, calculate the angular velocity (ω) using the frequency (f):

$$\omega = 2 \pi f$$

Substitute $f = 50 \text{ Hz}$:

$$\omega = 2 \pi \times 50$$
$$\omega = 100 \pi \text{ rad/s}$$

Explanation: The angular velocity is calculated from the rotational frequency using the relation $\omega = 2 \pi f$.

Supporting Statement: Calculating ω is essential because it is directly used in the power-torque relationship.

Step 2: Solving for Torque (T)

Using the power-torque relationship:

$$P = T \cdot \omega$$

Rearrange to solve for T:

$$T = \frac{P}{\omega}$$

Substitute $P = 2500 \text{ W}$ and $\omega = 100 \pi \text{ rad/s}$:

$$T = \frac{2500}{100 \pi}$$
$$T = \frac{2500}{314.16}$$
$$T \approx 7.96 \text{ N}\cdot\text{m}$$

Explanation: The torque is obtained by dividing the power by the angular velocity, reflecting the load on the shaft due to angular motion.

Supporting Statement: Torque calculation is critical as it directly relates to the shear stress in the shaft material.

Step 3: Calculating Maximum Shear Stress

The maximum shear stress (τ_{max}) in a solid shaft is given by:

$$\tau_{max} = \frac{T \cdot r}{J}$$

Where:

- J is the polar moment of inertia for a solid circular shaft
- $J = \frac{\pi}{2} r^4$

Substitute $r = 0.02 \text{ m}$:

$$J = \frac{\pi}{2} (0.02)^4$$
$$J = \frac{\pi}{2} \times 1.6 \times 10^{-7}$$
$$J = 8 \times 10^{-8} \pi$$
$$J \approx 2.513 \times 10^{-7} \text{ m}^4$$

Now, substitute $T = 7.96 \text{ N}\cdot\text{m}$, $r = 0.02 \text{ m}$, and $J = 2.513 \times 10^{-7} \text{ m}^4$:

$$\begin{aligned}\tau_{\max} &= \frac{7.96 \times 0.02}{2.513 \times 10^{-7}} \\ \tau_{\max} &= \frac{0.1592}{2.513 \times 10^{-7}} \\ \tau_{\max} &\approx 633,200 \text{ Pa} \\ \tau_{\max} &\approx 633.2 \text{ kPa}\end{aligned}$$

Explanation: Shear stress is derived from the torque and physical dimensions of the shaft using its polar moment of inertia, capturing the resistance of the shaft to torsional stress.

Supporting Statement: The shear stress calculation provides the measure of internal forces acting in a tangential direction, critical for material strength assessments.

Final Solution

The maximum shear stress in the solid shaft is approximately 633.2 kPa