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## Subject: Physics

### Topic: Electric Field and Potential

**Problem:**

Determine the electric field in the XY plane due to an infinite line charge along the Z-axis by finding the gradient of the field potential given $\rho = \frac{\lambda}{2 \pi \epsilon_0} \ln\left(\frac{r}{r_0}\right)$.

**Solution:**

##### Step 1: Given and Introduction

Given:
1. The potential ($\rho$) function is $\rho = \frac{\lambda}{2 \pi \epsilon_0} \ln\left(\frac{r}{r_0}\right)$, where $\lambda$ is the line charge density, $\epsilon_0$ is the permittivity of free space, $r$ is the radial distance from the z-axis, and $r_0$ is a reference distance.

> **Explanation:** The problem involves finding the electric field in the XY plane caused by an infinite line charge along the Z-axis using the potential function provided. The electric field ($\mathbf{E}$) is related to the potential ($\rho$) by the relation $\mathbf{E} = -\nabla \rho$.

##### Step 2: Expression for the Gradient of Potential

The electric potential as a function of position is given by: $$\rho = \frac{\lambda}{2 \pi \epsilon_0} \ln\left(\frac{r}{r_0}\right)$$ where $r = \sqrt{x^2 + y^2}$.

The gradient of the potential function in Cartesian coordinates ($x, y$) is: $$\nabla \rho = \frac{\partial \rho}{\partial x} \hat{i} + \frac{\partial \rho}{\partial y} \hat{j}$$

> **Explanation:** This step shows the general form and establishes the relationship between the potential and the coordinates that will be used to find the gradient.

##### Step 3: Calculations of Partial Derivatives

Compute the partial derivatives of the potential $\rho$.
$$\frac{\partial \rho}{\partial x} = \frac{\lambda}{2 \pi \epsilon_0} \frac{1}{r} \frac{\partial r}{\partial x}$$ $$r = \sqrt{x^2 + y^2}$$ $$\frac{\partial r}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}} \Rightarrow \frac{\partial \rho}{\partial x} = \frac{\lambda}{2 \pi \epsilon_0} \frac{x}{x^2 + y^2}$$

Similarly,
$$\frac{\partial \rho}{\partial y} = \frac{\lambda}{2 \pi \epsilon_0} \frac{y}{x^2 + y^2}$$

> **Explanation:** The partial derivatives of $\rho$ with respect to $x$ and $y$ are calculated using the chain rule. $\frac{\partial r}{\partial x}$ is found by differentiating the expression for $r$.

##### Step 4: Full Gradient of the Potential

Using the results from Step 3: $$\nabla \rho = \frac{\lambda}{2 \pi \epsilon_0} \left( \frac{x}{x^2 + y^2} \hat{i} + \frac{y}{x^2 + y^2} \hat{j} \right)$$
The electric field is given by $\mathbf{E} = -\nabla \rho$, hence: $$\mathbf{E} = -\frac{\lambda}{2 \pi \epsilon_0} \left( \frac{x}{x^2 + y^2} \hat{i} + \frac{y}{x^2 + y^2} \hat{j} \right)$$

> **Explanation:** By taking into account the results from the earlier steps, the full gradient is formed. Then using the relation between electric field and potential, the electric field is obtained as the negative gradient.

##### Step 5: Final Solution

The electric field in the XY plane due to an infinite line charge along the Z-axis is:
$$\boxed{\mathbf{E} = -\frac{\lambda}{2 \pi \epsilon_0} \left( \frac{x}{x^2 + y^2} \hat{i} + \frac{y}{x^2 + y^2} \hat{j} \right)}$$

> **Final Explanation:** The negative gradient of the potential function yields the electric field. This conclusion matches the result derived from theoretical considerations of a line charge's electric field.

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