CheggSolutions - Thegdp

Electrical Engineering

Fourier Transform

Given:

Find the corresponding Fourier transform $F(\omega)$ for $f(t)$.

1. $f(t) = \frac{\sin(10t)}{20t}$
2. $f(t) = \frac{e^{-8t^2}}{\pi}$

Solution:

(a) $f(t) = \frac{\sin(10t)}{20t}$

Step 1: Define the Sinc function

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

This means $\frac{\sin(10t)}{20t} = \frac{10}{20} \cdot \text{sinc} \left(\frac{10t}{\pi} \right) = \frac{1}{2} \cdot \text{sinc} \left(\frac{10t}{\pi} \right)$.

Step 2: Fourier Transform of a General Sinc Function

$$F(\omega) = \frac{1}{|\alpha|} \cdot \text{rect} \left(\frac{\omega}{2\alpha \pi} \right)$$

Where the rectangular function is:

$$\text{rect}(x) = \begin{cases} 1 & \text{if } |x| \leq 1/2 \\ 0 & \text{if } |x| > 1/2 \end{cases}$$

Step 3: Adapt the Fourier Transform for $f(t) = \frac{1}{2} \cdot \text{sinc} \left(\frac{10t}{\pi} \right)$

Substitute $\alpha = \frac{10}{\pi}$ into the Fourier transform formula:

$$F(\omega) = \frac{1}{2 \cdot \left| \frac{10}{\pi} \right|} \cdot \text{rect} \left(\frac{\pi \omega}{20 \pi} \right)$$

Simplify the expression:

$$F(\omega) = \frac{\pi}{20} \cdot \text{rect} \left(\frac{\omega}{20} \right)$$

Final Solution for (a):

$$F(\omega) = \frac{\pi}{20} \cdot \text{rect} \left(\frac{\omega}{20} \right)$$

(b) $f(t) = \frac{e^{-8t^2}}{\pi}$

Step 1: Recognize the Gaussian function

The Fourier Transform of e^{-at^2} where $a > 0$ is:

$$F(\omega) = \sqrt{\frac{\pi}{a}} e^{-\frac{\omega^2}{4a}}$$

Here, $a = 8$. Thus, Fourier Transform of e^{-8t^2}:

$$F(\omega) = \sqrt{\frac{\pi}{8}} e^{-\frac{\omega^2}{32}}$$

Step 2: Include the scaling factor in the given function

The given function introduces an additional scaling factor and the properties of Fourier Transform allow scaling factors to be included without affecting the core transformation:

$F(\omega) = \frac{1}{\pi} \cdot \sqrt{\frac{\pi}{8}} e^{-\frac{\omega^2}{32}}$

Simplify this expression:

$F(\omega) = \frac{1}{\sqrt{8\pi}} e^{-\frac{\omega^2}{32}}$

Final Solution for (b):

$F(\omega) = \frac{1}{\sqrt{8\pi}} e^{-\frac{\omega^2}{32}}$

Solution Summary:

a) $F(\omega) = \frac{\pi}{20} \cdot \text{rect} \left(\frac{\omega}{20} \right)$

b) $F(\omega) = \frac{1}{\sqrt{8\pi}} e^{-\frac{\omega^2}{32}}$

Both solutions derive from known Fourier transform properties for simplified functions.