

## Discrete Mathematics

### Inductive Proof

#### Given and Introduction:

Given statement:

Proofs are used in this class to prove statements of the form  $\forall n \in D, n \geq a, P(n)$  where  $P(n)$  is a predicate.

#### Step-by-Step Solution:

##### Step 1: Introduction to Inductive Proof

Inductive proof is a mathematical proof technique typically used to prove statements about natural numbers. It involves two main steps:

1. **Base Case:** Prove the statement for the initial value (usually  $n=0$  or  $n=1$ ).
2. **Inductive Step:** Prove that if the statement holds for an arbitrary case  $n = k$ , then it holds for the next case  $n = k+1$ .

Supporting Statement: Inductive proof is a method used to establish that a statement holds for all natural numbers or a specified set with an initial condition and an induction step.

##### Step 2: Identify Valid Domains for Inductive Proof

###### $D = \mathbb{N}$ (Natural Numbers):

- The domain of natural numbers ( $\mathbb{N}$ ) is the standard setting for inductive proofs because they have a well-defined base case (usually starting from  $n=0$  or  $n=1$ ) and can be incremented by 1.

*Explanation:* Natural numbers are used as they start from a base case and can incrementally build upon each successive case.

###### $D = \mathbb{Z}$ (Integers):

- Inductive proofs can also be applied to integers ( $\mathbb{Z}$ ) by starting at a base case and incrementing. For negative values, a similar methodology applies but usually involves proving a statement with decrements.

*Explanation:* Integers extend inductive proofs to negative values and include a zero base case.

###### $D = \mathbb{R}$ (Real Numbers):

- Inductive proofs typically are not applied to real numbers ( $\mathbb{R}$ ) because real numbers are continuous, and induction relies on discrete steps.

*Explanation:* Real numbers do not fit into the traditional increment-by-one property of natural integers.

###### $D = \mathbb{C}$ (Complex Numbers):

- Inductive proofs do not apply to complex numbers ( $\mathbb{C}$ ) because complex numbers do not have a natural ordering or increment structure necessary for induction.

*Explanation:* Complex numbers lack the discrete sequential nature required for induction.

Supporting Statement: Only discrete and ordered sets, which facilitate base cases and incremental steps, are suitable for inductive proofs.

#### Final Selection:

Valid domains for inductive proofs:

- $D = \mathbb{N}$  (Natural Numbers)
- $D = \mathbb{Z}$  (Integers)

#### Conclusion:

From the given options, the valid domains for applying inductive proofs are the set of natural numbers ( $\mathbb{N}$ ) and the set of integers ( $\mathbb{Z}$ ).

Supporting Statement: The selection aligns with the requirement of having a discrete and ordered set to apply the inductive procedure.