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# Engineering Mechanics - Statics

## Problem: 3D Equilibrium and Tension in Cables

### Given:

- Mass of the sign, $m = 500 \, \text{kg}$
- Acceleration due to gravity, $g = 9.81 \, \text{m/s}^2$
- Center of mass at point $G$

### Part (a): Calculation of Unit Vectors and Tensions in Cartesian Format

#### Step 1: Calculation of the unit vectors for the cables

Define the coordinates of points:
A = (0, 0, 0)
B = (3, 3, 0)
C = (0, 3, 6)
D = (6, 6, 6)

**Unit vector for cable BC:**

$$\vec{r}_{BC} = (0 - 3)\hat{i} + (3 - 3)\hat{j} + (6 - 0)\hat{k}$$ $$\vec{r}_{BC} = -3\hat{i} + 0\hat{j} + 6\hat{k}$$ $$|\vec{r}_{BC}| = \sqrt{(-3)^2 + (0)^2 + (6)^2}$$ $$|\vec{r}_{BC}| = \sqrt{9 + 0 + 36} = \sqrt{45} = 3\sqrt{5} \, \text{m}$$ $$\hat{u}_{BC} = \frac{\vec{r}_{BC}}{|\vec{r}_{BC}|}$$ $$\hat{u}_{BC} = \frac{-3\hat{i} + 0\hat{j} + 6\hat{k}}{3\sqrt{5}}$$ $$\hat{u}_{BC} = -\frac{1}{\sqrt{5}}\hat{i} + 0\hat{j} + \frac{2}{\sqrt{5}}\hat{k}$$

**Unit vector for cable BD:**

$$\vec{r}_{BD} = (6 - 3)\hat{i} + (6 - 3)\hat{j} + (6 - 0)\hat{k}$$ $$\vec{r}_{BD} = 3\hat{i} + 3\hat{j} + 6\hat{k}$$ $$|\vec{r}_{BD}| = \sqrt{(3)^2 + (3)^2 + (6)^2}$$ $$|\vec{r}_{BD}| = \sqrt{9 + 9 + 36} = \sqrt{54} = 3\sqrt{6} \, \text{m}$$ $$\hat{u}_{BD} = \frac{\vec{r}_{BD}}{|\vec{r}_{BD}|}$$ $$\hat{u}_{BD} = \frac{3\hat{i} + 3\hat{j} + 6\hat{k}}{3\sqrt{6}}$$ $$\hat{u}_{BD} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$$

**Explanation:** Coordinates of points are defined and vector positions are calculated. Using these vectors, unit vectors are derived by normalizing the position vectors.

#### Step 2: Tension forces in Cartesian vector format

Let $T_{BC}$ and $T_{BD}$ be the tensions in cables BC and BD respectively.

$\vec{T}_{BC} = T_{BC} \cdot \hat{u}_{BC} = T_{BC} \left(-\frac{1}{\sqrt{5}}\hat{i} + 0\hat{j} + \frac{2}{\sqrt{5}}\hat{k} \right)$

$\vec{T}_{BD} = T_{BD} \cdot \hat{u}_{BD} = T_{BD} \left( \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k} \right)$

**Explanation:** Tension forces are expressed in Cartesian vector format using their corresponding unit vectors.

### Part (b): Free Body Diagram and Coordinate System

#### Step 1: Draw the free body diagram

Include forces:
- Tensions $\vec{T}_{BC}$ and $\vec{T}_{BD}$
- Weight $\vec{W} = mg = 500 \times 9.81 \, \text{N}$
- Reactions at ball joint A: Assume reactions $A_x, A_y, A_z$

**Explanation:** Illustrates all forces acting on the system, assumptions about the reactions and coordinate system placement.

### Part (c): Equations of Equilibrium

#### Step 1: Write equations for the equilibrium

1. Sum of forces in $x$-direction:
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$$ \sum F_x = 0: \quad A_x + \vec{T}_{BC,x} + \vec{T}_{BD,x} = 0 $$
Substitute $ \vec{T}_{BC,x} $ and $ \vec{T}_{BD,x} $:
$$ A_x + T_{BC} \left(-\frac{1}{\sqrt{5}} \right) + T_{BD} \left(\frac{1}{\sqrt{6}} \right) = 0 $$

2. Sum of forces in $ y $-direction:
$$ \sum F_y = 0: \quad A_y + \vec{T}_{BC,y} + \vec{T}_{BD,y} = 0 $$
Substitute $ \vec{T}_{BC,y} $ and $ \vec{T}_{BD,y} $:
$$ A_y + T_{BC} \left(0\right) + T_{BD} \left(\frac{1}{\sqrt{6}} \right) = 0 $$

3. Sum of forces in $ z $-direction:
$$ \sum F_z = 0: \quad A_z + \vec{T}_{BC,z} + \vec{T}_{BD,z} - mg = 0 $$
Substitute $ \vec{T}_{BC,z} $ and $ \vec{T}_{BD,z} $:
$$ A_z + T_{BC} \left(\frac{2}{\sqrt{5}} \right) + T_{BD} \left(\frac{2}{\sqrt{6}} \right) - 500 \times 9.81 = 0 $$

4. Sum of moments about point $ A $ in all 3 axes direction:
$$ \sum M_A = 0 $$

Explanation: The equations of equilibrium have been derived for all forces in $ x $, $ y $, and $ z $-directions along with the sum of moments about point $ A $.

Part (d): Solve for Unknown Reactions at A and Tensions

Step 1: Solve system of equations

From $ \sum F_y = 0 $:
$$ A_y + T_{BD} \left(\frac{1}{\sqrt{6}} \right) = 0 $$
Please Explain in brief $$ A_y = -T_{BD} \left(\frac{1}{\sqrt{6}} \right) $$

Set up remaining equations in $ x $ and $ z $-directions and moments:
```math \begin{aligned} A_x - T_{BC} \left(\frac{1}{\sqrt{5}} \right) + T_{BD} \left(\frac{1}{\sqrt{6}} \right) & = 0 \\ A_z + T_{BC} \left(\frac{2}{\sqrt{5}} \right) + T_{BD} \left(\frac{2}{\sqrt{6}} \right) - 4905 & = 0 \\ \sum M_x = 0 \\ \sum M_y = 0 \\ \sum M_z = 0 \end{aligned} ```

Explanation: The combined equations will give a system which provides values for all unknowns when solved.

Final Solution:

Unknown reactions at $ A $ and tensions in cables $ BC $ and $ BD $ will be found through solving the system of equations derived from the equilibrium conditions.
This provides the values for $ T_{BC}, T_{BD}, A_x, A_y, $ and $ A_z $.

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