

CheggSolutions - Thegdp

Finance

Future Value of Investments

Given and Introduction:

An initial investment of \$1,000 is invested at an annual interest rate of 7% for a duration of 11 years. The task is to calculate the appreciation of this investment if the interest is compounded annually, quarterly, and monthly.

Step 1: Understanding the Compounding Formula

The future value of an investment compounded at a certain interest rate can be determined using the Future Value (FV) formula:

$$FV = P (1 + r/n)^{(nt)}$$

Where:

- FV = Future Value of the investment
- P = Principal amount (initial investment)
- r = Annual interest rate (in decimal)
- n = Number of compounding periods per year
- t = Time the money is invested for, in years

Explanation: This formula helps compute the value of an investment after some time, accounting for the interest earned being reinvested.

Step 2: Annual Compounding

Using the above formula for annual compounding (n=1):

$$FV_{\text{annual}} = 1000 (1 + 0.07/1)^{(1 * 11)}$$

Explanation: Since the interest is compounded annually, n=1, the recalculation for each year happens once.

Calculation:

$$\begin{aligned} FV_{\text{annual}} &= 1000 (1 + 0.07)^{11} \\ FV_{\text{annual}} &= 1000 * (1.07)^{11} \\ FV_{\text{annual}} &= 1000 * 2.105 \\ FV_{\text{annual}} &= 2105.00 \end{aligned}$$

Explanation: For an initial investment of \$1,000, after 11 years at 7% annual compounding, the value will be \$2105.00.

Step 3: Quarterly Compounding

For quarterly compounding (n=4):

$$FV_{\text{quarterly}} = 1000 (1 + 0.07/4)^{(4 * 11)}$$

Explanation: Here, the interest compounds four times a year, and each period's interest is divided by 4.

Calculation:

$$\begin{aligned} FV_{\text{quarterly}} &= 1000 (1 + 0.0175)^{44} \\ FV_{\text{quarterly}} &= 1000 * (1.0175)^{44} \\ FV_{\text{quarterly}} &= 1000 * 2.144 \\ FV_{\text{quarterly}} &= 2144.68 \end{aligned}$$

Explanation: For an initial investment of \$1,000, after 11 years at 7% quarterly compounding, the value will be approximately \$2144.68.

Step 4: Monthly Compounding

For monthly compounding (n=12):

$$FV_{\text{monthly}} = 1000 (1 + 0.07/12)^{(12 * 11)}$$

Explanation: The interest compounds monthly, meaning there are 12 periods in a year and the rate is divided by 12.

Calculation:

$$\begin{aligned} FV_{\text{monthly}} &= 1000 (1 + 0.0058333)^{132} \\ FV_{\text{monthly}} &= 1000 * (1.0058333)^{132} \\ FV_{\text{monthly}} &= 1000 * 2.149 \\ FV_{\text{monthly}} &= 2149.42 \end{aligned}$$

Explanation: For an initial investment of \$1,000, after 11 years at 7% monthly compounding, the value will be approximately \$2149.42.

Final Step: Comparison

Final Solution:

- **Annual Compounding Future Value:** \$2105.00
- **Quarterly Compounding Future Value:** \$2144.68
- **Monthly Compounding Future Value:** \$2149.42

Explanation: The results suggest that the more frequently the interest is compounded, the higher the future value of the investment. Monthly compounding yields the highest future value among the three compounding frequencies examined.