CheggSolutions - Thegdp

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## Population Growth Modeling

**Sub-Subject: Calculus**

**Topic: Population Growth Modeling**

Given the differential equation, $\frac{dP(t)}{dt} = kP(t)$, the solution $P(t) = b e^{kt} - a$, and a specific solution $P(14) = 7,600,000$, determine the number of years it will take for the population to reach 14,700,000 people.

### Step 1: Initial Condition Calculation

**Given:**

$$P(t) = 5.3 \times 10^6 \cdot e^{0.027(t - 1974)}$$

The initial population $P(14)$ is given by:

$$P(14) = 7,600,000$$

Since:

$$P(t) = 5.3 \times 10^6 \cdot e^{0.027(t - 1974)}$$

Therefore:

$$5.3 \times 10^6 \cdot e^{0.027(14 - 1974)} = 7,600,000$$

**Supporting Statement:** The initial population $P(14)$ is set as the base to find other specific values derived from the growth model.

**Explanation:**

The expression $5.3 \times 10^6 \cdot e^{0.027(t-1974)}$ models the population growth; substituting $t = 14$ allows solving for parameters using the given initial conditions.

### Step 2: Solving for Population at a Future Time

**Given $P(t) = 14,700,000$, the equation:**

$$14,700,000 = 5.3 \times 10^6 \cdot e^{0.027(t-1974)}$$

Rearranging the equation to isolate $e^{0.027(t-1974)}$:

$$e^{0.027(t-1974)} = \frac{14,700,000}{5.3 \times 10^6}$$

Simplify the ratio:

$$e^{0.027(t-1974)} = \frac{14.7}{5.3} = 2.77358$$

**Supporting Statement:** This step rearranges the exponential equation for solving the unknown future time.

**Explanation:**

Isolating the exponential term allows setting the equation in a form that opens it to applying logarithmic transformations.

### Step 3: Applying Logarithms to Solve for $t$

Applying the natural logarithm to both sides:

$$\ln \left( e^{0.027(t-1974)} \right) = \ln (2.77358)$$

Utilize the property $\ln(e^x) = x$:

$$0.027(t-1974) = \ln (2.77358)$$

Solving for $t$:

$$0.027(t-1974) = 1.0202$$

$$t-1974 = \frac{1.0202}{0.027}$$

$$t - 1974 = 37.785$$

$$t = 1974 + 37.785$$

$$t \approx 2011.785$$
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Supporting Statement: Application of log arithmetic isolates the variable `t` and thereby solves for it numerically.

Explanation:

By taking natural logarithms, the exponent is isolated and shifted to finding `t` by simplifying the transformed linear equation.

Final Solution:

The population will reach 14,700,000 approximately in the year `2011.785`, which translates to around late 2011 or early 2012.

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