Supplementary Notes

Practical Quantum Computation using Qiskit and IBMQ

Introduction

These notes contain further definitions related to multi-qubit states and operations that is intended to be a continuation of the material covered during the course. This is intended to bridge the gap between the class lecture slides and other standard texts and research papers.

The notes start with the definition of two-qubit states using tensor products and will set up a convention to deal with general n-qubit states and gates.

1 Tensor products and multi-qubit states

Consider two qubits in states $|\psi\rangle = a|0\rangle + b|1\rangle$ and $|\phi\rangle = c|0\rangle + d|1\rangle$ respectively, the combined two-qubit state is defined by the ordered tensor product.

$$\begin{aligned} |\psi\rangle \otimes |\phi\rangle &= (a|0\rangle + b|1\rangle) \otimes (c|0\rangle + d|1\rangle) \\ &= (ac|0\rangle|0\rangle + ad|0\rangle|1\rangle + bc|1\rangle|0\rangle + bd|1\rangle|1\rangle) \\ &= (ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle) \end{aligned} \tag{1}$$

Here the shorthand notation $|i\rangle \otimes |j\rangle \equiv |i\rangle|j\rangle$ where, $i, j \in \{0, 1\}$ are used to define the single qubit computational basis. The vectors $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ defined using the above shorthand define the two-qubit computational basis.

It can be verified that as long as $|\psi\rangle$ and $|\phi\rangle$ are normalized, the vector $|\psi\rangle \otimes |\phi\rangle$ is also normalized.

1.1 Boolean Strings and Decimals

The computational basis for a system of n-qubits is given by the tensor products of n single qubit computational basis vectors, $\{|x_0\rangle \otimes |x_1\rangle \otimes |x_2\rangle \cdots |x_{n-2}\rangle \otimes |x_{n-1}\rangle : x_i \in \{0, 1\}\}$. The basis vectors are of the form $\underbrace{|0\rangle|0\rangle \cdots |0\rangle}_{n \text{ times}}$ to $\underbrace{|1\rangle|1\rangle \cdots |1\rangle}_{n \text{ times}}$ and there are 2^n such vectors.

These basis vectors can therefore be represented by Boolean strings of length n. The computational basis for n-qubits is therefore given by $\{|\mathbf{x}\rangle : \mathbf{x} \in \{0,1\}^n\}$. It is also possible to represent these states by converting the Boolean strings to their decimal values. The same basis may therefore be represented as $\{|\mathbf{i}\rangle : \mathbf{i} \in \mathbb{Z}, \mathbf{0} \leqslant \mathbf{i} \leqslant \mathbf{2^n - 1}\}$.

Therefore, for the case of $n = 2$, the four basis vectors are represented as Boolean Strings, $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ or as the decimal numbers corresponding to the same strings, $\{|\mathbf{0}\rangle, |\mathbf{1}\rangle, |\mathbf{2}\rangle, |\mathbf{3}\rangle\}$. These notations extend the classical notation for bits to define the basis vectors for quantum bits.

1.2 The conjugate complex and inner products

The state represented by in Equation 1 can be represented as a column vector as,

$$|\psi\rangle \otimes |\phi\rangle \equiv |\psi\phi\rangle = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} \tag{2}$$

The conjugate complex (Hermitian conjugate) of this column vector gives the bra vector

$$\begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix}^{\dagger} = \begin{pmatrix} \overline{ac} & \overline{ad} & \overline{bc} & \overline{bd} \end{pmatrix} = \begin{pmatrix} \overline{a} & \overline{b} \end{pmatrix} \otimes \begin{pmatrix} \overline{c} & \overline{d} \end{pmatrix} = \langle\psi| \otimes \langle\phi| \tag{3}$$

The relationship between ket and bra vectors of tensor product states can be shown as follows,

$$(|\psi\rangle \otimes |\phi\rangle)^{\dagger} = \langle\psi| \otimes \langle\phi| \tag{4}$$

Consequently, the normalization criteria for the state vector represented in Equation 1 can be written as,

$$\langle\psi\phi|\psi\phi\rangle = (\langle\psi| \otimes \langle\phi|)(|\psi\rangle \otimes |\phi\rangle) = \langle\psi|\psi\rangle \langle\phi|\phi\rangle = 1 \cdot 1 = 1 \tag{5}$$

The above equation gives the rule for evaluating the inner products of tensor product states. The inner product is performed only between corresponding vectors. The same rule can be used to verify that the elements of the multi-qubit computational basis indeed orthonormal, for example:

$$\langle 01|11\rangle = (\langle 0| \otimes \langle 1|)(|1\rangle \otimes |1\rangle) = \langle 0|1\rangle \langle 1|1\rangle = 0 \cdot 1 = 0 \tag{6}$$

another example,

$$\langle 01|{+}1\rangle = (\langle 0| \otimes \langle 1|)(|+\rangle \otimes |1\rangle) = \langle 0|+\rangle \langle 1|1\rangle = \frac{1}{\sqrt{2}} \cdot 1 = \frac{1}{\sqrt{2}} \tag{7}$$

and for more than two qubits,

$$\langle 010|101\rangle = (\langle 0| \otimes \langle 1| \otimes \langle 0|)(|1\rangle \otimes |0\rangle \otimes |1\rangle) = \langle 0|1\rangle \langle 1|0\rangle \langle 0|1\rangle = 0 \cdot 0 \cdot 0 = 0 \tag{8}$$

These rules allow the conversion of vector multiplication in innner products into multiplication between complex numbers as long as the single qubit inner products are known.

2 Qubit Transformations

Single qubit operators were originally defined in terms of the outer products $|0\rangle\langle 0|$, $|0\rangle\langle 1|$, $|1\rangle\langle 0|$ and $|1\rangle\langle 1|$. These quantities are evaluated as,

$$\begin{aligned}
|0\rangle\langle 0| &= \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\\
|0\rangle\langle 1| &= \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0 & 1\end{pmatrix} = \begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}\\
|1\rangle\langle 0| &= \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1 & 0\end{pmatrix} = \begin{pmatrix}0 & 0\\1 & 0\end{pmatrix}\\
|1\rangle\langle 1| &= \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0 & 1\end{pmatrix} = \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}
\end{aligned} \tag{9}$$

The Z gate, $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ can be represented as $|0\rangle\langle 0| - |1\rangle\langle 1|$. The Hadamard gate $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$ can be represented $\frac{1}{\sqrt{2}}(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|)$

The action of operators on the state is obtained through matrix multiplication (as seen in the various quiz questions). It is also possible to represent these operations in the bra-ket notation. Consider the Hadamard gate is applied to the initial state $|0\rangle$

$$\begin{aligned}
H|0\rangle &\equiv \frac{1}{\sqrt{2}}(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|)|0\rangle\\
&= \frac{1}{\sqrt{2}}(|0\rangle\langle 0|0\rangle + |0\rangle\langle 1|0\rangle + |1\rangle\langle 0|0\rangle - |1\rangle\langle 1|0\rangle)\\
&= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle
\end{aligned} \tag{10}$$

2.1 From outer products to inner products

Consider a linear transformation $A = \begin{pmatrix} A_{00} & A_{01} \\ A_{10} & A_{11} \end{pmatrix}$ acting on the basis vectors. The result can be evaluated using matrix multiplication.

$$\begin{aligned} A\left|0\right\rangle &= \begin{pmatrix} A_{00} \\ A_{10} \end{pmatrix} \\ &= A_{00}\left|0\right\rangle + A_{10}\left|1\right\rangle \\ \text{and} & \\ A\left|1\right\rangle &= \begin{pmatrix} A_{01} \\ A_{11} \end{pmatrix} \\ &= A_{01}\left|0\right\rangle + A_{11}\left|1\right\rangle \end{aligned} \tag{11}$$

It should be noted that the action, $A\left|0\right\rangle$ is a vector that has been represented in the computational basis. The values of the matrix elements can be evaluated using the projections of the actions on to the computational basis.

$$\langle 0|A|0\rangle = A_{00}\ ,\ \langle 1|A|0\rangle = A_{10}\ ,\ \langle 0|A|1\rangle = A_{01} \text{ and } \langle 1|A|1\rangle = A_{11} \tag{12}$$

The matrix elements of the operator A can now be represented in terms of the actions of A on the computational basis $\{\left|0\right\rangle, \left|1\right\rangle\}$.

$$A = \begin{pmatrix} \langle 0|A|0\rangle & \langle 0|A|1\rangle \\ \langle 1|A|0\rangle & \langle 1|A|1\rangle \end{pmatrix} \tag{13}$$

2.2 From one to many

The above definitions can be readily extended from the single qubit operators to multi-qubit operators. Consider a two-qubit linear transformation M, this is given by a 4×4 matrix.

$$M = \begin{pmatrix} M_{00} & M_{01} & M_{02} & M_{03} \\ M_{10} & M_{11} & M_{12} & M_{13} \\ M_{20} & M_{21} & M_{22} & M_{23} \\ M_{30} & M_{31} & M_{32} & M_{33} \end{pmatrix} \tag{14}$$

The matrix elements of this operator can be represented in terms of the action on the two-qubit computational basis $\{\left|\mathbf{0}\right\rangle, \left|\mathbf{1}\right\rangle, \left|\mathbf{2}\right\rangle, \left|\mathbf{3}\right\rangle\}$, represented here in the decimal form.

$$M = \begin{pmatrix} \langle \mathbf{0}|M|\mathbf{0}\rangle & \langle \mathbf{0}|M|\mathbf{1}\rangle & \langle \mathbf{0}|M|\mathbf{2}\rangle & \langle \mathbf{0}|M|\mathbf{3}\rangle \\ \langle \mathbf{1}|M|\mathbf{0}\rangle & \langle \mathbf{1}|M|\mathbf{1}\rangle & \langle \mathbf{1}|M|\mathbf{2}\rangle & \langle \mathbf{1}|M|\mathbf{3}\rangle \\ \langle \mathbf{2}|M|\mathbf{0}\rangle & \langle \mathbf{2}|M|\mathbf{1}\rangle & \langle \mathbf{2}|M|\mathbf{2}\rangle & \langle \mathbf{2}|M|\mathbf{3}\rangle \\ \langle \mathbf{3}|M|\mathbf{0}\rangle & \langle \mathbf{3}|M|\mathbf{1}\rangle & \langle \mathbf{3}|M|\mathbf{2}\rangle & \langle \mathbf{3}|M|\mathbf{3}\rangle \end{pmatrix} \tag{15}$$

Using the above expression, one can easily evaluate the matrix forms of operators such as CNOT and SWAP. The same method can be used on the three-qubit operators to evaluate the matrix form of the CCX (Toffoli) gate.

Circuits to Matrices : It is also possible to define the three-qubit version of a two-qubit operator when tensor product expansion is not possible. This method will also work for tensor product operators. Consider the following quantum circuit involving a CNOT_2^0 operation.

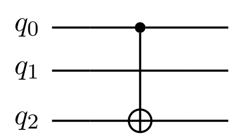

Figure 1: The CNOT gate applied on a three-qubit system

The actions of this circuit on the three-qubit computational basis are given by,

$$\begin{aligned} &\text{CNOT}_2^0\,|000\rangle \;=\; |000\rangle \;,\; \text{CNOT}_2^0\,|001\rangle \;=\; |001\rangle \;,\; \text{CNOT}_2^0\,|010\rangle \;=\; |010\rangle \\ &\text{CNOT}_2^0\,|011\rangle \;=\; |011\rangle \;,\; \text{CNOT}_2^0\,|100\rangle \;=\; |101\rangle \;,\; \text{CNOT}_2^0\,|101\rangle \;=\; |100\rangle \\ &\text{CNOT}_2^0\,|110\rangle \;=\; |111\rangle \text{ and } \text{CNOT}_2^0\,|111\rangle \;=\; |110\rangle \end{aligned} \tag{16}$$

The matrix corresponding to this circuit can be evaluated using the above actions and is given by,

$$\text{CNOT}_2^0 = \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0 \end{pmatrix} \tag{17}$$

like the two-qubit CNOT gate or the Toffoli gate, this matrix cannot be separated into a tensor product of smaller matrices.

3 Unitary Transformations and Orthonormal Bases

As discussed in the course, the qubit transformation or gate operations U considered in quantum computation are unitary transformations. The unitary property is defined by

$$UU^\dagger = U^\dagger U = I \tag{18}$$

where, $U^\dagger$ is the transposed conjugate (or adjoint) of the operator U and I is an identity operator of appropriate dimensions. On a side note, the terms operator, operation and transformation are used interchangeably in the notes here.

The reason that only unitary transformations are considered is due to the fact that any transformation of qubit states over time can be represented by unitary transformations. It should also be noted that the unitary transformations are defined to act on any arbitrary multi-qubit state.

3.1 Single qubit transformations

Consider a gate operation U acting on single qubit state. As discussed in subsection 2.1 the operation U is completely defined by its action on the computational basis $\{|0\rangle, |1\rangle\}$. The actions are defined as follows:

$$\begin{aligned} U|0\rangle &= |u_0\rangle \\ U|1\rangle &= |u_1\rangle \end{aligned} \tag{19}$$

The bra vectors corresponding to the ket vectors $|v_0\rangle$ and $|u_1\rangle$ are defined by

$$\begin{aligned} \langle u_0| &= \langle 0| U^\dagger \\ \langle u_1| &= \langle 1| U^\dagger \end{aligned} \tag{20}$$

The inner products between these vectors can be evaluated using the fact that U is unitary and $U^\dagger U = I$ as given in Equation 18.

$$\begin{aligned} \langle u_0|u_0\rangle &= \langle 0|U^\dagger U|0\rangle = \langle 0|0\rangle = 1 \\ \langle u_1|u_1\rangle &= \langle 1|U^\dagger U|1\rangle = \langle 1|1\rangle = 1 \\ \langle u_0|u_1\rangle &= \langle 0|U^\dagger U|1\rangle = \langle 0|1\rangle = 0 \\ \langle u_1|u_0\rangle &= \langle 1|U^\dagger U|0\rangle = \langle 1|0\rangle = 0 \end{aligned} \tag{21}$$

This implies $\{|u_0\rangle, |u_1\rangle\}$ is an orthonormal basis and since U was assumed to be a general unitary transformation, it can be seen that the action of any unitary operation U on the computational basis forms an orthonormal basis on single qubit states.

It is also possible to prove using similar methods to Section 2.1 that for a given orthonormal basis $\{|v_0\rangle, |v_1\rangle\}$, one can define a unitary transformation V that acts on the computational basis $||0\rangle\rangle |1\rangle$ such that

$$\begin{aligned} V|0\rangle &= |v_0\rangle \\ V|1\rangle &= |v_1\rangle \end{aligned} \tag{22}$$

It is therefore possible to define a unitary transformation corresponding to any given orthonormal basis and it is also possible to generate an orthonormal basis using the action of any unitary transformation on the computational basis.

3.2 Multi-qubit bases and Measurement

It is possible to extend the discussion from the previous section to multi-qubit states too. In this case, the action of an n-qubit unitary operation U on the computational basis $\{|\mathbf{x}\rangle : \mathbf{x} \in \{0,1\}^n\}$ generates an n-qubit orthonormal basis.

Since it is possible to associate a unitary transformation with any orthonormal basis, the operator U generating the basis can be used to label the basis itself. The basis generated by the operation U as the U basis.

The unitary transformations applied in quantum computation can therefore be seen as a series of basis transformations (or rotations). This final result of the computation can be evaluated by performing measurements on this resultant state.

3.2.1 Measuring with respect to rotated bases

The notion that every orthonormal basis can be associated with a unitary transformation can be used to define measurements in the that given basis. Let $\{|u_0\rangle, |u_1\rangle\}$ be single qubit orthonormal basis generated by the unitary transformation U. This shall be referred to as the U basis in the subsequent discussion.

A measurement with respect to this basis yields a resultant state $|u_0\rangle$ or $|u_1\rangle$. For a state $|\psi\rangle$, the probability of a measurement with respect to the U basis yielding the state $|u_0\rangle$ is given by $|\langle u_0|\psi\rangle|^2$ and the probability of the result being $|u_1\rangle$ is given by $|\langle u_1|\psi\rangle|^2$.

Consider the transformation $U^\dagger$ applied to the state $|\psi\rangle$. Measuring this state with respect to the computational basis yields the resultant state $|0\rangle$ or $|1\rangle$ with the probabilities

$$\Pr\{\text{outcome} = |0\rangle\} = \left|\langle 0|U^\dagger|\psi\rangle\right|^2 \text{ and } \Pr\{\text{outcome} = |1\rangle\} = \left|\langle 1|U^\dagger|\psi\rangle\right|^2 \tag{23}$$

Using the definitions in Equation 20, it can be observed that these probabilities are identical to the measurement outcomes $|u_0\rangle$ or $|u_1\rangle$ respectively with respect to the U basis.

This was the concept discussed in the first programming assignment. The probability that the outcome of measuring the state $U^\dagger |\psi\rangle$ in the computational basis yields the result $|0\rangle$ (or $|1\rangle$) is identical to the probability of obtaining the outcome $|u_0\rangle$ (or $|u_1\rangle$) while measuring the state $|\psi\rangle$ with respect to the U basis.

Therefore, measuring the state $|\psi\rangle$ with respect to the U basis is equivalent to measuring the state $U^\dagger |\psi\rangle$ with respect to the computational basis. Once again, it is emphasised that the single qubit case is considered here for the sake of simplicity and all of the above results are also true for any general multi-qubit state.

4 Quantum Teleportation revisited

Consider Alice and Bob, instead of exchanging random keys using QKD, they try to solve the following problem. Given a single qubit in some unknown state $|\psi\rangle$ Alice has to use this qubit to somehow transfer the state to the qubit present with Bob.

According to the *No Cloning Theorem*, it is impossible to replicate a given qubit in an arbitrary state into another qubit. It is also impossible to know the state of a qubit state using only a single measurement only. As discussed in the lectures, it is possible to transfer the qubit state using an entangled pair of qubits between Alice and Bob.

The entangled pair of qubits (labelled "A" and "B" for Alice and Bob) are in the Bell state $|B_{00}\rangle$ given by

$$|\Phi\rangle_{AB} = (|00\rangle + |11\rangle) \tag{24}$$

In addition to the entangled pair, Alice has the qubit in the state $|\psi\rangle = a\,|0\rangle + b\,|1\rangle$ and is labelled as "0", the combined three-qubit state is given by $|\psi\rangle \otimes |\Phi\rangle_{AB} \equiv |\psi\rangle\,|\Phi\rangle_{AB}$. The process of teleportation involves transferring the state $|\psi\rangle$ from the first qubit to the one present with Bob (qubit "B").

4.1 The teleportation process

The initial state of the three-qubit system is given by,

$$\begin{aligned} |\psi\rangle\,|\Phi\rangle_{AB} &\equiv \frac{1}{\sqrt{2}}\,(a\,|0\rangle + b\,|1\rangle)\,(|00\rangle + |11\rangle) \\ &= \frac{1}{\sqrt{2}}\,(a\,|000\rangle + a\,|011\rangle + b\,|100\rangle + b\,|111\rangle) \end{aligned} \tag{25}$$

The process of quantum teleportation is performed in two stages, the first step involves applying the CNOT gate between qubits "0" and "A" with "0" acting as the control qubit, this is represented in the three-qubit form as $\left(\mathrm{CNOT}^0_A \otimes I\right)$. This is followed by the Hadamard transformation on qubit "0", this is represented in three-qubit form as by

$(H \otimes I \otimes I)$. At the end of this stage, the qubits "0" and "A" are measured with respect to the computational basis.

$$
\begin{aligned}
&\frac{1}{\sqrt{2}}\left(a\left|0\right\rangle + b\left|1\right\rangle\right)\left(\left|00\right\rangle + \left|11\right\rangle\right) \xrightarrow{(\mathrm{CNOT}_A^0 \otimes I)} \frac{1}{\sqrt{2}}\left(a\left|000\right\rangle + a\left|011\right\rangle + b\left|110\right\rangle + b\left|101\right\rangle\right) \\
&\frac{1}{\sqrt{2}}\left(a\left|000\right\rangle + a\left|011\right\rangle + b\left|110\right\rangle + b\left|101\right\rangle\right) = a\left|0\right\rangle\left|B_{00}\right\rangle + b\left|1\right\rangle\left|B_{01}\right\rangle \\
&a\left|0\right\rangle\left|B_{00}\right\rangle + b\left|1\right\rangle\left|B_{01}\right\rangle \xrightarrow{(H \otimes I \otimes I)} a\left|+\right\rangle\left|B_{00}\right\rangle + b\left|-\right\rangle\left|B_{01}\right\rangle
\end{aligned}
\tag{26}
$$

The resultant state $(a\left|+\right\rangle\left|B_{00}\right\rangle + b\left|-\right\rangle\left|B_{01}\right\rangle)$ can be expressed by grouping the first two qubits as follows.

$$
\begin{aligned}
\left|+\right\rangle\left|B_{00}\right\rangle + b\left|-\right\rangle\left|B_{01}\right\rangle &= \frac{1}{2}\left(a\left(\left|0\right\rangle + \left|1\right\rangle\right)\left(\left|00\right\rangle + \left|11\right\rangle\right) + b\left(\left|0\right\rangle - \left|1\right\rangle\right)\left(\left|01\right\rangle + \left|10\right\rangle\right)\right) \\
&= \frac{1}{2}\left(a\left|000\right\rangle + a\left|011\right\rangle + a\left|100\right\rangle + a\left|111\right\rangle + b\left|001\right\rangle + b\left|010\right\rangle - b\left|101\right\rangle - b\left|110\right\rangle\right) \\
&= \frac{1}{2}\left(\left|00\right\rangle\left(a\left|0\right\rangle + b\left|1\right\rangle\right) + \left|01\right\rangle\left(a\left|1\right\rangle + b\left|0\right\rangle\right) + \left|10\right\rangle\left(a\left|0\right\rangle - b\left|1\right\rangle\right) + \left|11\right\rangle\left(a\left|1\right\rangle - b\left|0\right\rangle\right)\right)
\end{aligned}
\tag{27}
$$

At this point, it should be pointed out that the first stage involved operations and measurements performed only by Alice with the qubits that she possesses. Depending on the outcome of the measurement performed by Alice, the state of the qubit "B" will be different. This determines what operations need to be performed by Bob in order to transfer the state $\left|\psi\right\rangle$ to his qubit.

If the measurement outcome of Alice is the state $\left|00\right\rangle$, the Bob has the state $\left|\psi\right\rangle$ and needs to perform no additional operation. If Alice's outcome is the state $\left|01\right\rangle$, Bob needs to apply the X gate to obtain the state $\left|\psi\right\rangle$. If the outcome is the state $\left|10\right\rangle$, Bob needs to apply the Z gate. If Alice's outcome is the state $\left|11\right\rangle$, Bob has to apply the $X \circ Z$ gate in order to obtain $\left|\psi\right\rangle$.

Thus the state of the qubit "0" present with Alice is transferred to the qubit "B" through operations performed by Alice and Bob on their respective qubits. Quantum teleportation can be performed even if Alice and Bob in different places. The only information required is the outcome of Alice's measurement which needs to be transmitted by Alice to Bob.

4.2 Reinterpreting Quantum teleportation

The transformation involved in quantum teleportation can be represented by the following circuit

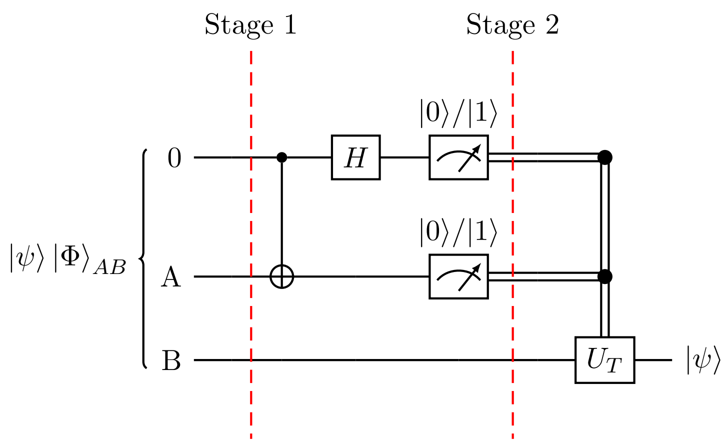

Figure 2: Circuit corresponding to quantum teleportation

The transformation U_T is applied by Bob and as explained before, it depends on the outcome of the measurement performed on Alice's qubits with respect to the computational basis. U_T can either be the identity transformation (I) or the X, Z or $X \circ Z$ gate.

The unitary transformation applied on the Alice's qubits (0 and A) can be written as $\mathrm{CNOT}_A^0 \circ (H \otimes I)$. The Bell states are generated by the unitary transformation $U_B = (H \otimes I) \circ \mathrm{CNOT}_A^0$, the transformation applied in the first stage of teleportation is simply $U_B^\dagger$ followed by a measurement in the computational basis.

This implies that according to Section 3.2.1 the operations performed in the first stage correspond to a measurement in the bell basis. Alice's part in quantum telelportation is therefore referred to as a measurement in the Bell basis or simply a *Bell measurement*.

Quantum teleportation is an example in quantum computation that involves measurements in rotated multi-qubit bases. The same logic can in principle be applied to any algorithm in quantum computation. Solving problems using quantum algorithms typically involves the selection of a basis to measure with respect to and then applying the operation corresponding to the rotated measurement.