

# A Project Report for

## Optimized Selection of Food Delivery Points for Nomgogo across Penn

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### Introduction

Mathematical Optimization or Mathematical Programming is a sub-field of Mathematics, Computer Science and Operations Research that aims at finding the most feasible solution to an engineering, science or business problem based on certain constraints. Optimization techniques have been prevalent since the early days of Greek civilization and are still being used every day by small and large businesses and numerous engineering and technology companies.<sup>[1]</sup> The beauty of optimization tools and techniques lies in their ability to provide us the best possible solution to a problem under limitations on the resources we have, while also allowing us to gain useful insights into our solutions by answering "what-ifs" to the problem. Based on how our problem is mathematically modelled, optimization problems are classified into two major types: Linear Programming (LP) problems and Non-linear Programming (NLP) problems. Mathematically speaking, LP problems consist of linear objective function(s) and linear constraint equations whereas NLP problems consist of nonlinear objectives function(s) and linear or nonlinear constraints. There is another class of optimization problems called Integer Programming (IP) problems. IP problems are formulated in a similar way as LP problems with the only difference that the variables in an IP problem take on integer values only instead of real values In this project, we have addressed a business problem by developing a mathematical model of its objectives and its requirements. We have tried to solve the problem using the optimization tools at our dispense and generate valuable insights from the solutions.

#### Statement of Problem

Nomgogo is a food and meal delivery venture founded by Chen Feng (MBA'18) (CEO) and a group of students of the Wharton Business School, University of Pennsylvania. Nomgogo serves as a medium between consumers and restaurants, wherein they take online orders through their website or mobile application and provide fresh meals on campus at a fixed delivery point. At present, the venture is at a premature stage however the business shows a rise of 400% since the last two months.<sup>[2][3]</sup> Currently, Nomgogo's main objective is to expand Nomgogo spots or 'Nomspots' all over Penn. 'Nomspots' are the fixed delivery points where in consumers can collect their order.

We would like to work with the venture to create an optimization model for their business requirements, wherein we would like to determine the minimum number of 'Nomspots' they need to open in order to increase their customer base in an optimal fashion. The mathematical model

will contain a minimization optimization objective concerning these 'Nomspots', and will cater to constraints on number of 'Nomspots' to be opened and distance constraints (as Penn is spread out across a large area).

### Objectives of the Problem

- 1. To retrieve data about potential locations on Penn campus, distance between these locations and the normal time required to travel the distance between them.
- 2. To create a mathematical model for the business problem in terms of decision variables, parameters and constraints.
- 3. To use Integer Programming formulations and software packages to obtain optimal solution for the requirements, i.e., create a potential 'Nomspots'.
- 4. To run sensitivity analysis test using LINDO in order to understand the effect of different parameters on optimal food delivery and to get the insights for future operations.

### Approach Used for Finding the Number of 'Nomspots'

The problem statement of finding the minimum number of 'Nomspots' the venture needs to open in order to increase their customer base can be viewed as a Set Cover problem. [4] In a set cover problem, we are given a ground set of n elements,  $\mathcal{E} = \{e_1, e_2, ...., e_n\}$  and a collection of m subsets of  $\mathcal{E}$ ,  $\mathcal{E} = \{S_1, S_2, ...., S_m\}$  and a non-negative weight function cost:  $\mathcal{E} \to \mathbb{Q}^2$ . The goal is then to find a minimum weight collection of subsets that covers all elements in  $\mathcal{E}$ . We want to find a set cover  $\mathcal{C}$  that minimizes  $\sum_{S_j \in \mathcal{C}} w_j$  subject to  $\bigcup_{S_j \in \mathcal{C}} S_j = \mathcal{E}$ . If  $w_j = 1 \,\forall j$ , then the problem is called an unweighted set cover problem. In this project, we use the unweighted version of the set cover problem.

### Mathematical Model Development

In this project, we consider 18 locations on Penn's campus as potential 'Nomspots' based on the fact that there are a considerable number of students at or around these buildings at any given time during the day. [4] These 18 buildings (along with their abbreviations used in this project) are:

- 1. John M. Huntsman Hall (JMHH)
- 2. 1920 Commons (1920)
- 3. Graduate School of Education (GSE)
- 4. School of Social Policy & Practice (SP2)
- 5. Annenberg School for Communication (ANNS)
- 6. Steinberg Hall-Deitrich Hall (SHDH)
- 7. Van Pelt Library (VPelt)
- 8. School of Design (Design)
- 9. School of Engineering & Applied Science (SEAS)
- 10. David Rittenshouse Laboratories (DRLB)
- 11. Singh Center for Nanotechnology (Singh)
- 12. Education Commons (EdCom)
- 13. Chemistry Laboratories: 1973 Wing (Chem)
- 14. Penn Law School (Law)
- 15. Hospital of the University of Pennsylvania (Hosp)

- 16. Children's Hospital of Philadelphia (CHOP)
- 17. School of Nursing (Nursing)
- 18. Penn Museum (Museum)

We used Google Maps to compute the distances (in miles) of these buildings from each other. We set a threshold distance of 0.2 miles and say that if two buildings are within this threshold, i.e., the distance between the two buildings is less than or equal to this threshold, then it is possible for these two buildings to form a network and the student can commute between these two buildings to pick up the orders without any potential time loss. The table of the distances and the corresponding binary selection table can be found below as Table 1 and Table 2 respectively.

	Variable Name	JMHH	1920	GSE	SP2	AnnS	SHDH	Vpelt	Design	SEAS	DRL	Singh	Ed Com	Chem	Law	Hosp UP	СНОР	Nursing	Museum
JMHH	X1		0.1	0.1	0.05	0.15	0.15	х	X	Х	Х	Х	Х	Х	Х	Х	X	Х	Х
1920	X2	0.1		0.2	0.1	Х	0.2	Х	Х	Х	Х	Х	Х	Х	Х	Х	X	Х	Х
GSE	ХЗ	0.1	0.2		0.1	0.1	0.2	х	X	Х	Х	Х	Х	Х	Х	Х	Х	Х	Х
SP2	X4	0.05	0.1	0.1		0.1	0.1	Х	Х	Х	Х	Х	Х	Х	Х	Х	X	Х	Х
AnnS	X5	0.15	Х	0.1	0.1		0.1	0.2	X	X	Х	Х	Х	Х	х	Х	X	X	Х
SHDH	X6	0.15	0.2	0.2	0.1	0.1		0.15	Х	Х	Х	Х	Х	Х	Х	Х	X	Х	Х
Vpelt	X7	Х	Х	Х	х	0.2	0.15		0.1	0.2	Х	Х	Х	Х	0.2	х	X	Х	х
Design	X8	Х	Х	X	Х	Х	Х	0.1		0.1	0.2	Х	Х	0.15	0.2	Х	X	Х	Х
SEAS	X9	Х	Х	X	х	Х	Х	0.2	0.1		0.05	0.1	0.2	0.2	0.2	X	X	Х	х
DRL	X10	Х	Х	X	Х	Х	Х	Х	0.2	0.05		0.1	0.1	0.2	Х	Х	X	X	Х
Singh	X11	X	X	X	X	X	Х	Х	Х	0.1	0.1		0.2	X	Х	X	X	X	Х
Edu Comm	X12	Х	Х	X	Х	X	Х	Х	Х	0.2	0.1	0.2		Х	Х	Х	X	X	Х
Chem	X13	X	X	Х	X	X	Х	Х	0.15	0.2	0.2	Х	X		Х	0.05	0.2	X	0.2
Law	X14	X	X	X	Х	Х	Х	0.2	0.2	0.2	Х	Х	X	X		Х	X	X	X
Hosp UP	X15	X	X	Х	X	X	X	X	X	X	Х	X	Х	0.05	X		0.1	X	0.2
CHOP	X16	Х	Х	X	X	X	Х	X	X	X	X	Х	Х	0.2	Х	0.1		0.2	0.2
Nursing	X17	X	X	X	X	X	X	X	X	X	X	X	X	X	Х	X	0.2		X
Museum	X18	X	X	X	X	X	Х	Х	X	X	X	Х	Х	0.2	Х	0.2	0.2	X	

Table 1: Distance Table

	Variable	JMHH	1920	GSE	SP2	AnnS	SHDH	Vpelt	Design	SEAS	DRL	Singh	Ed Com	Chem	Law	Hosp UP	СНОР	Nursing	Museum
JMHH	X1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
1920	X2	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
GSE	Х3	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
SP2	X4	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
AnnS	X5	1	1	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
SHDH	Х6	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
Vpelt	X7	X	0	0	0	1	1	1	1	1	0	0	0	0	1	0	0	0	0
Design	X8	X	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
SEAS	Х9	X	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
DRL	X10	X	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
Singh	X11	X	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0
Edu Comm	X12	X	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0
Chem	X13	X	0	0	0	0	0	0	1	1	1	0	0	1	0	1	1	0	1
Law	X14	X	0	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	0
Hosp UP	X15	X	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	1
CHOP	X16	X	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1
Nursing	X17	X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
Museum	X18	X	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	1

Table 2: Selection Table

The black boxes in Table 1 correspond to the distance of a building from itself which is of course zero. The X's represent that the buildings in the row and the corresponding column are more than 0.2 miles apart. In Table 2, the pink boxes correspond to whether a building is accessible from itself, which it obviously is and hence, all such values are 1. The distance entries from Table 1 are changed to binary values 0 and 1, indicating whether the buildings in a given and the corresponding column are accessible to each other or not. As per Nomgogo's requirement, there should be three primary delivery locations across Penn, where the restaurant/catering service personnel can drop off meals. The sections marked in yellow, blue and green correspond to the network of buildings from which the delivery points will be chosen. The campus map with the buildings marked with stars and the network of buildings shown by bold lines is shown below.

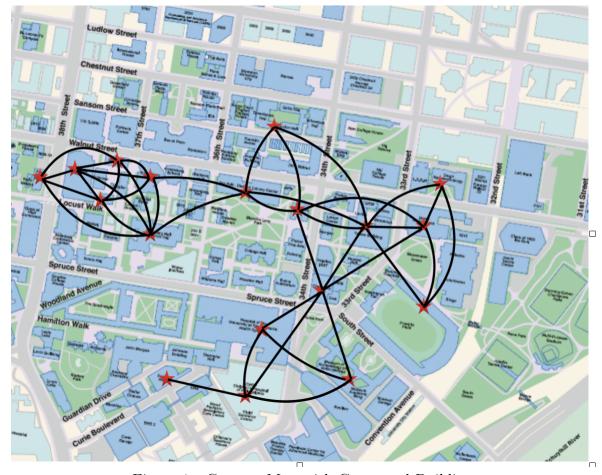


Figure 1 : Campus Map with Connected Buildings

Now, according to the set cover approach mentioned in the previous section, we can say that set  $\mathcal{E} = \mathcal{S} = \text{set}$  of the 18 buildings = {JMHH, 1920, GSE, SP2, ANNS, SHDH, VPelt, Design, SEAS, DRLB, Singh, EdCom, Chem, Law, Hosp, CHOP, Nursing, Museum}. Let  $x_i|x_i = \{0,1\} \ \forall \ i \in \{1, 2, ..., 18\}$  be the decision variable associated with each of the respective building in the set  $\mathcal{S}$ . So, if  $x_i$  is 0 for building i, it is not selected as a 'Nomspot' and if  $x_i$  is 1 for building i, it is selected as a 'Nomspot'. The constraints of the model say that every building in the set  $\mathcal{S}$  should be connected to at least one building which is a 'Nomspot', so that its students are not left out of pickup options even though the building itself is not a 'Nomspot'. Also, currently JMHH already serves as a 'Nomspot' and so,  $x_1 = 1$ . Then, the IP model is expressed mathematically as:

Minimize  $\sum_{i=1}^{18} x_i$ 

#### such that:

```
\begin{array}{l} x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \geq 1 \text{ (for JMHH, GSE, SP2)} \\ x_1 + x_2 + x_3 + x_4 + x_6 \geq 1 \text{ (for 1920)} \\ x_1 + x_3 + x_4 + x_5 + x_6 \geq 1 \text{ (for ANNS)} \\ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \geq 1 \text{ (for SHDH)} \\ x_5 + x_6 + x_7 + x_8 + x_9 + x_{14} \geq 1 \text{ (for VPelt)} \\ x_7 + x_8 + x_9 + x_{10} + x_{13} + x_{14} \geq 1 \text{ (for Design)} \\ x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} \geq 1 \text{ (for SEAS)} \\ x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} \geq 1 \text{ (for DRLB)} \\ x_9 + x_{10} + x_{11} \geq 1 \text{ (for Singh)} \end{array}
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x_9 + x_{10} + x_{11} + x_{12} \ge 1 (for EdCom)

x_8 + x_9 + x_{10} + x_{13} + x_{15} + x_{16} + x_{18} \ge 1 (for Chem)

x_7 + x_8 + x_9 + x_{14} \ge 1 (for Law)

x_{13} + x_{15} + x_{16} + x_{18} \ge 1 (for Hosp)

x_{13} + x_{15} + x_{16} + x_{17} + x_{18} \ge 1 (for CHOP)

x_{16} + x_{17} \ge 1 (for Nursing)

x_{13} + x_{15} + x_{18} \ge 1 (for Museum)

x_1 = 1 (JMHH already a 'Nomspot') and x_i = \{0, 1\}, \forall i \in \{2, 3, ..., 18\}
```

The implementation of the above model along with the sensitivity analysis report can be found in the Appendix.

### Conclusion

Referring to the output from the LINDO implementation, we get our optimal solution at  $x_1, x_9$  and  $x_16$ , where  $x_1$  is JMHH,  $x_9$  is the SEAS department and  $x_16$  is CHOP.  $x_1$  has been kept 1 in the constraints as JMHH already exists as a *Nomspot*. Furthermore, Nomgogo wanted that *Nomspots* be assigned in such a manner that there exists immediate road access to them. Immediate road access would allow bulk deliveries from restaurants as the vehicle can be parked right outside the building and the delivery can be received by a Nomgogo ambassador. This is true and can be verified from the map shown above in Figure 1. The solution we obtain from our model supports this criteria as well.

### Further Possible Improvements

Nomgogo can try to improve the above model by adding costs in the form of weights (i.e., using a weighted set cover problem) corresponding to student densities at different locations. The problem can be made more granule by integrating the student density and cultural diversity into the cuisine which they prefer the most. This will allow to improve food volumes at each Nomspot and ultimately lead to better customer experience.

#### References

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## Appendix



## LP OPTIMUM FOUND AT STEP 2

## OBJECTIVE FUNCTION VALUE

# 1) 3.000000

VARIABLE X1 X2 X3 X4 X5 X6 X7 X8 X9 X10 X11 X12 X13 X14 X15 X16 X17 X18	VALUE 1.000000 0.000000 0.000000 0.000000 0.000000	REDUCED COST 0.000000 1.000000 1.000000 1.000000 1.000000 0.000000 0.000000 1.000000 1.000000 0.000000 0.000000 0.000000 0.000000
ROW 2) 3) 4) 5) 6) 7) 8) 9) 10) 11) 12) 13) 14) 15) 16)	SLACK OR SURPLUS 0.000000 0.000000 0.000000 0.000000 0.000000	DUAL PRICES 0.000000 0.000000 0.000000 0.000000 0.000000

## RANGES IN WHICH THE BASIS IS UNCHANGED:

X1 X2 X3 X4 X5 X6 X7 X8 X9 X10 X11 X12 X13 X14 X15 X16 X17 X18	CURRENT COEF 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000	OBJ COEFFICIENT RANGES ALLOWABLE INCREASE INFINITY	ALLOWABLE DECREASE INFINITY 1.000000 1.000000 1.000000 0.000000 0.000000 1.000000 1.000000 1.000000 0.000000 1.000000 1.000000 1.000000 0.000000 0.000000 0.000000
ROW  2 3 4 5 6 7 8 9 10 11 12 13 14 15	CURRENT RHS 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000	RIGHTHAND SIDE RANGES ALLOWABLE INCREASE 0.000000 0.000000 0.000000 0.000000 0.000000	ALLOWABLE DECREASE INFINITY

## THE TABLEAU

ROW (BASIS)  1 ART 2 SLK 2 3 SLK 3 4 SLK 4 5 SLK 5 6 SLK 6 7 SLK 7 8 SLK 8 9 SLK 9 10 SLK 10 11 SLK 11 12 SLK 12 13	X1 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	X2 1.000 -1.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	X3 1.000 -1.000 -1.000 -1.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	X4 1.000 -1.000 -1.000 -1.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	X5 1.000 -1.000 -1.000 -1.000 -1.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	X6 1.000 -1.000 -1.000 -1.000 -1.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
ROW X7  1 0.000 2 0.000 3 0.000 4 0.000 5 -1.000 6 0.000 7 0.000 8 0.000 9 1.000 10 1.000 11 1.000 11 1.000 12 1.000 13 1.000 14 0.000 15 0.000 16 0.000 17 0.000 18 0.000	X8 0.000 0.000 0.000 0.000 0.000 0.000 0.000 1.000 1.000 1.000 0.000 0.000 0.000 0.000	X9 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	X10 1.000 0.000 0.000 0.000 0.000 -1.000 -1.000 -1.000 -1.000 0.000 0.000 0.000	X11 1.000 0.000 0.000 0.000 0.000 0.000 -1.000 -1.000 -1.000 0.000 0.000 0.000 0.000	X12 1.000 0.000 0.000 0.000 0.000 0.000 -1.000 -1.000 -1.000 0.000 0.000 0.000 0.000 0.000	X13 0.000 0.000 0.000 0.000 0.000 -1.000 -1.000 -1.000 0.000 0.000 0.000 0.000 0.000

ROW 1 2 3 4 4 5 6 6 7 8 9 10 11 12 13 14 15 16 17 18	X14 0.000 0.000 0.000 0.000 0.000 0.000 1.000 1.000 1.000 1.000 0.000 0.000	X15 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	X16 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	X17 1.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	X18 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	SLK 2 0.000 1.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	SLK 3 0.000 0.000 1.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
ROW 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	SLK 0.000 0.000 0.000 1.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	SLK 5 0.000 0.000 0.000 1.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	SLK 6 0.000 0.000 0.000 0.000 1.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	SLK 7 0.000 0.000 0.000 0.000 0.000 1.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	SIK 8 0.000 0.000 0.000 0.000 0.000 0.000 1.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	SLK 9 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	SLK 10 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000

ROW 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	SLK 11	SLK 12	SLK 13	SLK 14	SLK 15	SLK 16
	0.000	0.000	1.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	-1.000	0.000	0.000	0.000
	0.000	0.000	-1.000	0.000	0.000	0.000
	0.000	0.000	-1.000	0.000	0.000	0.000
	0.000	0.000	-1.000	0.000	0.000	0.000
	0.000	0.000	-1.000	0.000	0.000	0.000
	0.000	0.000	-1.000	0.000	0.000	0.000
	0.000	0.000	-1.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000
ROW 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	SLK 17 1.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000	-3.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 1.000 0.000 0.000 0.000				