Introduction to SLAM

(Simultaneous Localization And Mapping)

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January 20, 2013

Outline

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Motivation

- **Localization** - Where is the robot?
  Robot estimates its position with respect to the environment (Map provided)
- **Mapping** - What doas the world look like?
  Robot maps the positions of the objects in its environment (Robot position known)
- **SLAM Problem - How to do it simultaneously?**

Motivation

When do we need SLAM?

- When the robot must be truly autonomous
- When there is no prior knowledge about the environment
- When we cannot use external positioning systems (e.g. GPS)
- When the robot needs to know where it is

History

- IEEE Robotics and Automation Conference - San Francisco, 1986
- Some Pioneers
    - Hugh Durrant-Whyte
    - Jim Crowley
    - Peter Cheeseman

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Formulation of the SLAM Problem

Quantities Notation:

- $\mathbf{x}_k$: The state vector discribing the location and the orientation of the vehicle.
- $\mathbf{u}_k$: The control vector, applied at time $k-1$ to drive the vehicle to a state $\mathbf{x}_k$ at time k.
- $\mathbf{m}_i$: A vector describing the location of the i^{th} landmark whose true location is assumed time invariant.
- $\mathbf{z}_k$: An observation taken from the vahicle at time k.

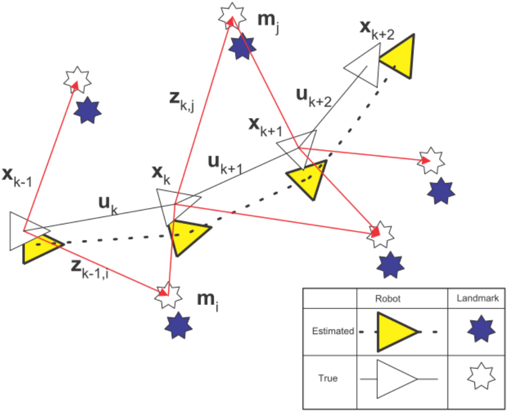
Formulation of the SLAM Problem

Sets Notation:

- $\mathbf{X}_{0:k} = \{\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_k\} = \{\mathbf{X}_{0:k-1}, \mathbf{x}_k\}$: The history of vehicle locations.
- $\mathbf{U}_{1:k} = \{\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_k\} = \{\mathbf{U}_{1:k-1}, \mathbf{u}_k\}$: The history of control inputs.
- $\mathbf{m} = \{\mathbf{m}_1, \mathbf{m}_2, \cdots, \mathbf{m}_n\}$: The set of all landmarks.
- $\mathbf{Z}_{1:k} = \{\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_k\} = \{\mathbf{Z}_{1:k-1}, \mathbf{z}_k\}$: The set of all landmark observations.

Formulation of the SLAM Problem

Graphical representation of the SLAM Problem:



Source: Durrant-Whyte and Bailey, SLAM: Part I (2006)

Probabilistic SLAM

- Probability distribution:

$$P(\mathbf{x}_k, \mathbf{m} \,|\, \mathbf{Z}_{1:k}, \mathbf{U}_{1:k}, \mathbf{x}_0)$$

- Recursive solution:

  Starting with an estimate for the distribution $P(\mathbf{x}_{k-1}, \mathbf{m} \,|\, \mathbf{Z}_{1:k-1}, \mathbf{U}_{1:k-1})$ at time $k-1$, the joint posterior, following a control $\mathbf{u}_k$ and observation $\mathbf{z}_k$, is computed using Bayes Theorem.

- This computation requires a motion model and an observation model.

Probabilistic SLAM

- **Observation model**: $P(\mathbf{z}_k \mid \mathbf{x}_k, \mathbf{m})$
- **Motion model**: $P(\mathbf{x}_k \mid \mathbf{x}_{k-1}, \mathbf{u}_k)$
- **Time-update**:

$$P(\mathbf{x}_k, \mathbf{m} \mid \mathbf{Z}_{1:k-1}, \mathbf{U}_{1:k}, \mathbf{x}_0)$$

$$= \int P(\mathbf{x}_k \mid \mathbf{x}_{k-1}, \mathbf{u}_k) \times P(\mathbf{x}_{k-1}, \mathbf{m} \mid \mathbf{Z}_{1:k-1}, \mathbf{U}_{1:k-1}, \mathbf{x}_0) d\mathbf{x}_{k-1}$$

- **Measurement-update**:

$$P(\mathbf{x}_k, \mathbf{m} \mid \mathbf{Z}_{1:k}, \mathbf{U}_{1:k}, \mathbf{x}_0)$$

$$= \frac{P(\mathbf{z}_k \mid \mathbf{x}_k, \mathbf{m}) P(\mathbf{x}_k, \mathbf{m} \mid \mathbf{Z}_{1:k-1}, \mathbf{U}_{1:k}, \mathbf{x}_0)}{P(\mathbf{z}_k \mid \mathbf{Z}_{1:k-1}, \mathbf{U}_{1:k})}$$

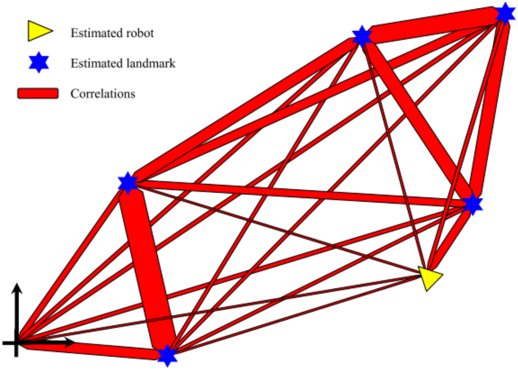
Structure of the Probabilistic SLAM

Key observations:

- Error between estimated and true landmark location is due to error in robot location
- $P(\mathbf{m}_i, \mathbf{m}_j)$ is highly peaked even when $P(\mathbf{m}_i)$ may be quite dispersed
- The joint probability density on all landmarks $P(\mathbf{m})$ becomes monotonically more peaked as more observations are made
- All landmarks are highly correlated

Structure of the Probabilistic SLAM

Correlations between landmarks.



Source: Durrant-Whyte and Bailey, SLAM: Part I (2006)

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Solutions to the SLAM Problem

- Finding an appropriate representation for the observation and motion model
- The two important solution methods:
  - EKF-SLAM
  - FastSLAM

EKF-SLAM

- **Motion model**:

$$P(\mathbf{x}_k \,|\, \mathbf{x}_{k-1}, \mathbf{u}_k) \Longleftrightarrow \mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k) + \mathbf{w}_k$$

- **Observation model**:

$$P(\mathbf{z}_k \,|\, \mathbf{x}_k, \mathbf{m}) \Longleftrightarrow \mathbf{x}(k) = \mathbf{h}(\mathbf{x}_k, \mathbf{m}) + \mathbf{v}_k$$

EKF-SLAM

Four key issues:

- Convergence
- Computational Effort
- Data Association
- Non-linearity

FastSLAM

- The joint SLAM state factored into a vehicle component and a conditional map component

$$P(\mathbf{X}_{0:k}, \mathbf{m} \mid \mathbf{Z}_{1:k}, \mathbf{U}_{1:k}, \mathbf{x}_0)$$

$$= P(\mathbf{m} \mid \mathbf{X}_{0:k}, \mathbf{Z}_{1:k}) P(\mathbf{X}_{0:k} \mid \mathbf{Z}_{1:k}, \mathbf{U}_{1:k}, \mathbf{x}_0)$$

- Key property: **Independant landmarks**

FastSLAM

- The map is represented as a set of independent Gaussians, with linear complexity, rather than a joint map covariance with quadratic complexity

$$P(\mathbf{m} \mid \mathbf{X}_{0:k}, \mathbf{Z}_{1:k}) = \prod_{j}^{M} P(\mathbf{m}_j \mid \mathbf{X}_{0:k}, \mathbf{Z}_{1:k})$$

- Recursive estimation - partical filter for the pose states and EKF for the map states

Outline

Conclusion

- Introduction to the SLAM problem
- Probabilistic SLAM problem
- EKF-SLAM
- FastSLAM

Further Sources

- OpenSLAM - collection of SLAM algotithms http://openslam.org/
- The Mobile Robot Programming Toolkit http://mrpt.org/
- ...

The End

Thank you for your attention!

Any questions?