Use of Model Predictive Control (MPC) for Rocket Altitude Correction

Nikhil Peri, Anthony Lin, Manit Ginoya, Paul Buzuloiu
uOttawa Rocketry Team
{nperi104, alin102 mgino015, pbuzu025}@uottawa.ca

***Abstract*—**

KeywordsModel Predictive Control, Aerospace.

I. Introduction

Rockets unlike other aircraft have high speed and dynamic flights, as a result rocket control systems have to be extremely responsive and precise. Classical control systems based on observed sensor feedback would not be able to meet the demands of rocket flight since the latency between plant actuation affecting the the physical world and detecting that change through sensor observations is too slow for such dynamic flight environments. Model Predictive Control (MPC) solves these problems by introducing state estimation. This process involves maintaining a kinetic model of the actuators we are using for altitude control, in this case is a drag model for a drag inducing airbrake system. Using this model we can predict the effect of actuation on the target altitude.

II. System States

In both classical and MPC design it is important to first outline all the system states. For most kinetic systems these are the acceleration, velocity and position of the object being controlled for each axis x, y, z in the state space representation below

$$\dot{p_x}\dot{v_x}\dot{a_x}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_x \\ v_x \\ a_x \end{bmatrix} \quad (1)$$

These vectors need to be projected on a single reference frame, and for the purposes of calculating rocket trajectory after lift-off and maximum altitude above the ground it makes the most sense to use the earth as a reference frame.

We also have the additional rotation around each of the specified axis commonly referred to as pitch roll and yaw

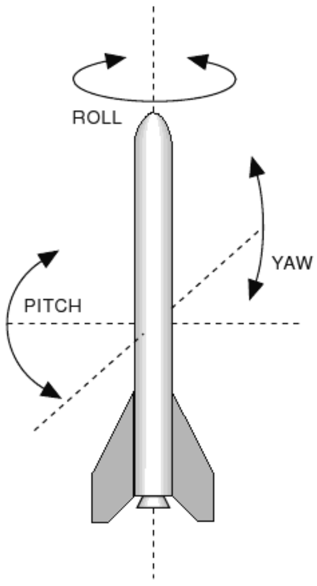


Fig. 1: Rocket Axis of Rotation

For simplicity we can omit the roll since it does not contribute to the heading of the rocket in flight. We can then apply the following reference frame transformation to convert the acceleration experienced by the rocket to the acceleration it is experiencing with reference to the earth frame using the Euclidian Matrix [1]

$$a_x \ a_y \ a_z$$

$$= \begin{bmatrix} cos(\theta_x) & sin(\theta_x)sin(\theta_y) & sin(\theta_x)cos(\theta_y) \\ 0 & cos(\theta_y) & -sin(\theta_y) \\ sin(\theta_y) & cos(\theta_x)sin(\theta_y) & cos(\theta_x)cos(\theta_y) \end{bmatrix} \begin{bmatrix} a'_x \\ a'_y \\ a'_z \end{bmatrix} \quad (2)$$

Where

$$\theta_x$$

is the yaw and

$$\theta_y$$

is the pitch of the rocket and a' is the acceleration of the rocket. from the rockets reference frame. These form the state space for our rocket system and most 3D kinetic systems

III. Airbrake Model

The Airbrake is our primary control surface, it is actuated by a servo that deploys set of fins out of streamlined rocket body. These can be used to increase the drag on the rocket body as a result reducing the velocity and final altitude of the rocket.

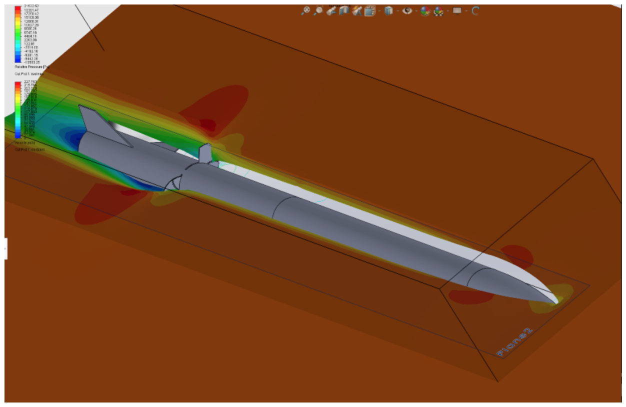


Fig. 2: Airbrake Mechanism

The drag force is a function of the surface area and rocket relative velocity as seen below

$$(v', A) = \frac{1}{2} C_d v'^2 A \rho \tag{3}$$

In order to characterize this system we must determine the drag coefficient of the rocket during airbrake deployment. Due to the simplicity of the Airbrake we can use a basic model for the experimentally determined drag of a flat plate[2] which is

$$C_d = 1.28 \tag{3}$$

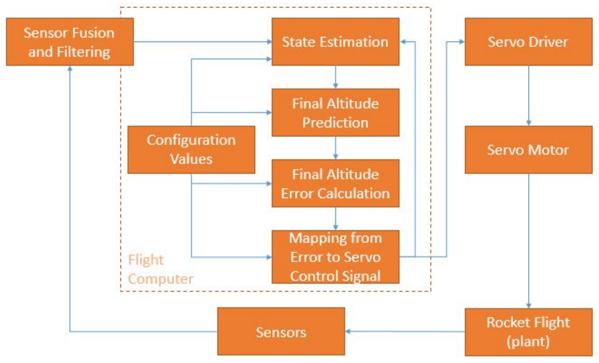
But since this drag model will be used to predict the future state of the system, it is imperative that this model is as representative of the physical world as possible. Using a simplistic model is a good starting point, but using Computational Fluid Dynamics (CFD) simulations or wind tunnel testing.

IV. Model Predictive Control

The complexity of rocket flight partially arises from the numerous states required to fully define an entire set of dynamics. These states include acceleration, velocity, position, orientation, angular speed and angular acceleration of the rocket in all three spatial dimensions. Additionally, the states related to the actuators (in this case, the servo motor controlling the airbrakes) must also be considered. Designing a classical controller or even a digitally implemented classical controller would bring along several inconveniences; for this reason, a model predictive controller was considered and implemented.

The major concept of model predictive control is to maintain a real time simulation; This simulation's role is to produce a prediction of future system states using feedback from sensors and the system's current states. For the case of achieving a particular target altitude by deploying airbrakes, the MPC will produce a predicted final altitude every control loop. Using this predicted final altitude, the error between the predicted final altitude and the desired final altitude will be calculated. The final part of the MPC process is to map the calculated error to a specific (whether heuristic or deterministic) control signal that will be sent to the airbrake actuator.

Figure 3 is the fully defined control system diagram corresponding to the description provided above. Running through the control schema yields the following sequence of events: sensors are read, filtered and fused, states are estimated, trajectory is predicted, final altitude error is determined, control signal is generated, signal is sent to actuator driver, actuation occurs which affects rocket flight.


Sensor Fusion and Filtering
State Estimation
Servo Driver
Final Altitude Prediction
Configuration Values
Servo Motor
Final Altitude Error Calculation
Mapping from Error to Servo Control Signal
Flight Computer
Sensors
Rocket Flight (plant)
Fig. 3: Overall Control Diagram of airbrake system

When comparing figure 3 to a typical control systems diagram, the input to the system seems to be missing; This input (the desired final altitude) is merely hidden away as a part of the digitally implemented configuration values. Some important inferences of the presented control diagram include understanding that the configuration values include all physical rocket parameters, as well as the desired final altitude. Therefore, the presented figure is related more to the physical implementation than to control theory notation.

V. Imeplementation

Here, each block residing within the flight computer as shown in the diagram presented in figure 3 is defined further to provide insight into the implementation of this system.

A. Sensor Fusion

In order to achieve accurate state estimation, which is an essential functional component of the control schema, certain feedback values are required in a non-inertial frame. However the sensors on board the rocket can only measure quantities in an inertial frame. Therefore, the sensor fusion block is necessary to not only filter sensor values but also to acquire accurate feedback values in a non-inertial frame.

To determine what types of filtering and fusion are required, it is helpful to first determine a set of sensors that will be used. Figure 4 depicts the three main modules of sensors being used. They are, shown left to right, Yost Labs 3 space LX Embedded IMU, Adafruit MTK3339 GPS and the MPL3115A2 altimeter.

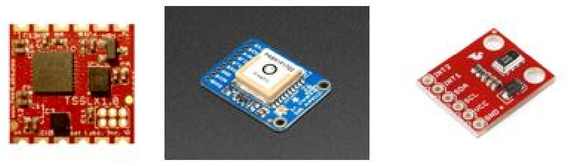


Fig. 4: Image of Sensors Used for System Implementation

These sensors were chosen to help minimize the amount of self developed fusion and filtering. The Yost Labs 3 space LX Embedded IMU has nine degrees of freedom and provides filtered, three dimensional acceleration, speed and position in a non-inertial frame (via proprietary, low level embedded algorithms). Using the data from this sensor module, all required feedback values for state estimation can be satisfied. However to make the state estimation as accurate as possible, the stand alone GPS and altimeter modules will be used to correct IMU drift and perform self-checking.

B. State Estimation

The state estimation block uses the feedback values from sensors (after filtering and fusion) and the previous states of the system to calculate the current state of the system. Since the feedback values already provide measured acceleration, speed and position in a non-inertial frame, they can be used directly to update the states of the system. However,

C. Final Altitude Prediction

Recall that equation **??**eq:3) allows us to model the drag of the rocket given current velocity and Airbrake area. Notice that this is a controllable value based on the airbrake actuation area. The second variable is the velocity of the rocket through the relative airflow, this we require us to apply the inverse of the Euclidian Angle matrix from equation **??**eq:2) to transform the velocity reference frame back to that of the rocket (ie. v')

From here we can compute the drag force and apply a ballistic model of rocketry flight that tells us the maximum altitude of the rocket given its current state and current drag forces. To derive this we must first find the time at which the vertical velocity v_z is zero, by solving the integration of velocity we can get the predicted distance to this point, and as a result the max altitude

(3)

We can use this expression to determine the affect of our air brake deployment on the maximum altitude.

D. Final Altitude Error Calculation

Using the altitude prediction determined above in equation V-D we can compute an error function given by

$$error = target - y_{zmax} \tag{3}$$

E. Mapping from Error to Control Signal

VI. Conclusion

In this brief, an implementation of model predictive control is presented for a custom airbrake system to be deployed on a sounding rocket. The convenience of using MPC for this specific application is shown. Furthermore, and with some modification, it is shown that the implemented MPC can be deployed on cheap electrical hardware.

VII. Acknowledgement

The authors would like to thank the University of Ottawa for continued support of the rocketry team. Additionally, specific thankfulness is issued to professor Habash and the teaching assistants of the ELG 4157 course.

References

[1] S. Mallick, "Home," Jun 2016. [Online]. Available: https://www.learnopencv.com/rotation-matrix-to-euler-angles/

[2] May 2015. [Online]. Available: https://www.grc.nasa.gov/www/k-12/airplane/drageq.html