Applied Probability and Statistics

Assignment - 1

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Big Data Analytics

1. A Student taking a test has to select 7 out of 10 questions. How many different choices does she have if

a) there are no other restrictions

$n=10 \quad r=7 \quad {}^{10}C_7 \quad \binom{10}{7} \Rightarrow \frac{10!}{7!(10-3)!} = \frac{10!}{7!(3!)}$

b) She has to answer Exactly 2 of the last 4?

total to answer = 7, from last 4 Questions → ${}^4C_2$

from first 6 Ques → ${}^6C_5$ $\quad \binom{4}{2} \cdot \binom{6}{2}$

c) Answer Exactly 2 of the first 6?

first 6 ques → ${}^6C_2 * {}^4C_5$ → not possible selecting 5 out of 4

$\binom{6}{2} \times \binom{4}{5} = \frac{6!}{2!(4!)} \times 0.$ Question so zero.

d) She has to answer at least 3 of the first 5?

$({}^5C_3 \times {}^5C_4) + ({}^5C_4 \times {}^5C_3) + ({}^5C_5 \times {}^5C_2) \Rightarrow \binom{5}{3} \times \binom{5}{4} + \binom{5}{4}\binom{5}{3} + \binom{5}{5}\binom{5}{2}$

2. If Eight identical blackboards are to be divided among 4 Schools, how many divisions are possible? what if Each school must receive at least one blackboard?

$r = 8$. (Identical) $\quad n = 4$

i) no of divisions are (for formula) $\binom{n+r-1}{r} = \binom{4+8-1}{8} = \binom{11}{8} \quad {}^{11}C_8$

ii) If at least one blackboard. to Each School

$\binom{r-1}{n-1} = \binom{8-1}{4-1} = \binom{7}{3} \quad {}^7C_3$

3. 9 computers are brought in for servicing (and machines are serviced one at a time). Of the 9 computers, 3 are PCs, 4 are Macs, and 2 are Linux Machines. Assume that all Computers of the same type are indistinguishable

(i.e., all the PCs are indistinguishable, all the Macs are indistinguishable, etc.).

a) In how many distinguishable ways can the computers be ordered for servicing.

9 Computers _ _ _ _ _ _ _ _ _ } 3 PCs, 4 Macs, & 2 Linux

$$^{9}C_{3} \times {}^{6}C_{4} \times {}^{2}C_{2} = \frac{9!}{3!\,\cancel{6!}} \times \frac{\cancel{6!}}{\cancel{2!}\,4!} \times \frac{\cancel{2!}}{2!} = \frac{9!}{3!\;4!\;2!}$$

(n distinct object into k unlabeled groups) $\frac{n!}{r_1! \cdot r_2! \cdots r_k!}$

b) In how many distinguishable ways can the computers be ordered if the first 5 machines serviced must include all 4 macs.

$$\rightarrow {}^{5}C_{4} \times {}^{5}C_{3} \times {}^{2}C_{2} = \frac{5!}{\underset{mac}{4!}\;\underset{PC\ lin}{1!}} \times \frac{5!}{3!\;2!} \times \frac{2!}{2!}$$

c) In how many distinguishable ways can the computers be ordered if 2 PCs must be in the first three and 1 PC must be in the last three computers serviced.

$${}^{3}C_{2} \times {}^{3}C_{1} \times {}^{6}C_{4} \times {}^{2}C_{2}$$

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${}^{3}C_{2} \rightarrow$ PCs $\underset{Macs}{{}^{6}C_{4}} \times \underset{Linux}{{}^{2}C_{2}}$ ${}^{3}C_{1}$ - PCs

4. A 3-member project team for performing literature survey, coding, and documentation for a projects has to be selected from a class of 60 students where in each person takes up one role only. How many different choices of teams are possible if

a) There are no restrictions

60 Students $\rightarrow n$

3 members, Literature, coding, documentation $\rightarrow r$

$${}^{n}C_{r} = {}^{60}C_{3} \Rightarrow \binom{60}{3}$$

b) Two of the students will not work together

Total no of selection = no of selections in which 2 students are together + no of selections in which 2 students are not together.

$\left({}^{58}C_1 \times 3!\right) + \left({}^{58}C_3 \times 3!\right)$ 3 ways to assign the roles in team

two already selected — two eliminated.

c) Two of the students will work together or not ~~at~~ at all

$\left({}^{58}C_1 \times 3!\right)$

d) one of the students must be in the team

$\left({}^{59}C_2 \times 3!\right)$ select one already and select from the rest ~~all~~ of all and 3! ways of assign roles.

e) one student can only do coding.

$\left({}^{59}C_2 \times 2!\right)$ → one position set for coding for 1 student

5) 100 units of stabilizing weights are to be placed into 5 vehicles. Because of different veehicle characteristics, vehicle 1 needs at least 10 units, vehicles 2 and 3 at least 12 each, vehicles 4 and 5 travel in a convoy & they need at least 4 combined. How many distributions of these weight units are feasible?

vehicle 1 ⇒ 10 units
veh 2 ⇒ 12 units
veh 3 → 12 units
veh 4, veh 5 } → 4 units

38 units

with replacement & order doesn't matter

$\binom{n+r-1}{r} = \binom{62+5-1}{4} = \binom{66}{4}$

$= {}^{66}C_4$

Total units = 100
Remaining units = 100 − 38 = 62 units

7) In how many ways can r identical server requests be distributed among n servers so that the ith server receives at least m_i requests, for each $i = 1, 2 \ldots n$? You can assume that $r \geq (m_1 + m_2 + \ldots + m_n)$

$n \rightarrow$ Identical requests servers among $n \rightarrow$ server

$r \geq (m_1 + m_2 + \ldots + m_n)$

if suppose 10 request 2 request given among 3 servers

then $10 - 6 = 4$ rest for 4 is new r^* & how to divide among the servers

Here we have m_n request to be served for n servers so the formula

$$\binom{n+r-1}{r} \Rightarrow \binom{n+r^*-1}{r^*} \quad \binom{n+\sum_{i=1}^{r} m_i - 1}{\sum_{i=1}^{r} M_i}$$

14.

8) Suppose a particle starting from the origin can move only up or down; the binomial option pricing model addresses stock price movements using such an idea. Show that the no of ways the particle can move from the origin to position k in n steps is $\binom{n}{\frac{n+k}{2}}$. Assume than $n+k$ is even.

$n_u \rightarrow$ no of up steps $\quad n_D \rightarrow$ no of steps

$$n_u + \cancel{n_D} = n$$
$$n_u - \cancel{n_D} = k$$
$$2n_u = n+k$$

① $n_u + n_D = n \rightarrow$ total no of steps

② $n_u - n_D = k \rightarrow$ Position of the points

from eq ① & ② upon adding them we get

$2n_u = n+k \Rightarrow n_u = \frac{n+k}{2}$

from eq ① & ② upon substracting then we get

$$2n_D = n - k \Rightarrow n_D = \frac{n-k}{2}$$

for the given graph we have from origin to origin = 6 step

up up down up down down , by this it reach origin

here we have 3 up 3 down from eq ① & ② we

① $\Rightarrow 3+3 = 6 \Rightarrow n$ ② $\Rightarrow 3 - 3 = 0$

$n_u + n_D = n$ $n_u - n_D$ k

from $\binom{n}{\frac{n+k}{2}} \Rightarrow \binom{6}{\frac{6+0}{2}} \Rightarrow \binom{6}{3} \Rightarrow {}^6C_3 \Rightarrow$ 20 ways

11) A total 28% of American males smoke cigarattes, 7% smoke cigars, and 5% smoke both cigars & cigarettes. Let A & B represent the Events that a randomly chosen person is a cigarette smoker and a cigar smoker, respectively. Explain in plain English what the following compound events represent & calculate their probabilities 1) $(A \cup B)^c$ 2) $B \cap A^c$

28% → cigarattes ⇒ 0.28 → $P(A)$

7% → cigars ⇒ 0.07 → $P(B)$

5% → both cigarattes & cigars ⇒ 0.05 ⇒ $P(A \cap B)$

i) $(A \cup B)^c$ → The chosen person will not smoke cigarattes or cigar

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.28 + 0.07 - 0.05 = 0.3$$

for $P(A \cup B)^c = 1 - P(A \cup B)$

$$= 1 - 0.3 = 0.7$$ 70% does not smoke.

i) $(B \cap A^c)$ -> person smoke cigar but not cigaratte.

$P(B) - P(B \cap A) = 0.07 - 0.05 = 0.02$

$P(B \cap A^c) = 0.02$.

12) what is more likely? Provide quantitative support

a) obtaining at least one 6 in 4 rolls of a single die

$P(\text{getting a six on single throw}) = \frac{1}{6}$

$P(\text{not getting six on single throw}) = 1 - \frac{1}{6} = \frac{5}{6}$

P (obtaining at least one 6) one 4 roll dice $= 1 - P$ (obtaining no one 6 in 4 rolls)

$= 1 - \left(\frac{5}{6}\right)^4 \simeq 0.518$

b) obtaining at least one 12 in 24 rolls of a pair of dice

$1 - \left(\frac{35}{36}\right)^{12} \simeq 0.513$

∴ obtaining at least one 6 on 4 roll is more likely.

13) Data was collected from the residents of a town & displayed as follows

Age (Years)	Income <\$25k	\$25k-\$70k	>70k
<25	952	1,050	53
25-45	456	2055	1570
>45	54	952	1008

a) what fraction of people are less than 25 Years old?

Total no of residents

952 + 1050 + 53 + 456 + 2055 + 1570 + 54 + 952 + 1008 = 8150

i) 952 + 1050 + 53 = 2055 $= \frac{\text{no of ages less than 25}}{\text{total no}} = \frac{2055}{8150}$

= 0.252 % = 25 %

b) what is the probability that a randomly chosen person is more than 25 Years old?.

$1 - P(25) \Rightarrow 1 - 0.25 \Rightarrow 0.75 \Rightarrow 75\%$

c) what fraction of people Earn less than $ 70,000

$952 + 1050 + 456 + 2055 + 54 + 952 \Rightarrow 5,519 \Rightarrow \frac{5519}{8150} \Rightarrow 0.677$

$= 67\%$

d) what is the probability that a randomly chosen person is less than 25 Years old and Earns more than $ 70,000?

$\frac{53}{8150} = 0.0065$

e) what fraction of people among those who Earn less than $25,000 are b/w 25-45 Years old.

952 + 456 + 54 = 1462

$\frac{456}{1462} = 0.311$

f) If the next random person you see happens to be more than 45 Years old, what is the probability that the person earns less than $70,000?

more than > 45 = 54 + 952 + 1008 = 2014

Earns less than $70,000 = 54 + 952 = 1006

$\Rightarrow \frac{1006}{2014} = 0.499$

9. a) write the sample space showing any two outcomes in it clearly. Explain what the outcomes mean

{11111} → All Judges vote correctly

{11100} → Judges ABC vote correctly & Judges DE vote incorrectly.

b) How many outcomes n are there in the sample space?

n = 5 {0, 1} are two outcomes

$\therefore 2^5 = 32$

c) How many outcomes n(E) are there in the event of interest

$n(E) = {}^5C_3 + {}^5C_4 + {}^5C_5 = 16$

d) Explain briefly why or why not the probability of the event of interest can be calculated as n(E)/n

10. a) Suppose the after investigating the broken bottles. Mr. Brown finds 2 tablets that are still intact in the bottle for tablet A. the other 18 tablets are found to be mixed in a pipe. Is it better for him to take one known tablet from the bottle & one from the pile, or take two tablets from the pile? Answer this by calculating the respective probabilities that he will not have any serious health issues for both options

1 mg dosage for both A & b   Type A → 10 tablet Type B → 10 tablet

P (taking tablet A) $= \frac{2}{2} = 1$ (since two tablets are still in tackt)

P (taking tablet B) $= 10/18 = 0.555$

P (taking 2 tablet A) randomly from other pile. $= \frac{8}{18} \times 7/17 = \frac{28}{153}$

P (taking 2 tablet B) $: \frac{10}{18} \times \frac{9}{17} = \frac{45}{153}$

P(taking type A followed type B) $= \frac{8}{18} \times \frac{10}{17} + \frac{10}{18} + \frac{8}{17} = \frac{80}{153}$

b) Suppose that after investigating the broken bottles, Mr. Brown finds that the tablets are all mixed up. What is the probability that he will not have any serious health issues if he randomly picks 2 tablets?

P (picking 2 type A or B) $\Rightarrow \frac{10}{20} \times \frac{9}{19} = \frac{\cancel{45}}{\cancel{19}} \quad \frac{\cancel{90}\,45}{\cancel{380}\,190} =$

P (picking 1 type A & 1 type B) $\Rightarrow \overset{A}{10/20} \times \overset{B}{10/19} + \overset{B}{\frac{10}{20}} \times \overset{A}{\frac{10}{19}}$

$= \frac{100}{380} + \frac{100}{380} = \frac{200}{380} = \frac{100}{190}$

$= 52.63\%$

6) Total players = 11

Forward or midfield = 4

Only defense or midfield = 5

requirements = 3 forward, 4 defense, 3 midfield, 1 Goalie

$$^{2}C_{1} = \begin{bmatrix} {}^{5}C_{3} \times {}^{4}C_{3} \times {}^{3}C_{3} \\ + \\ {}^{5}C_{4} \times {}^{4}C_{3} \times {}^{2}C_{2} \\ + \\ {}^{5}C_{4} \times {}^{4}C_{2} \times {}^{3}C_{3} \end{bmatrix} = 180$$

∴ In 180 ways we can divide the team into 4 groups