Fundamentals of Machine Learning

Assignment - 6

2. Dataset given below has two input variables (x_1 & x_2) and one output variable (y). Apply logistic regression model to make predictions after two epochs for each data point. Use 0.5 as the learning rate.

Input		Actual Output
x_1	x_2	y
2.7810836	2.550537003	0
8.675418651	-0.242068654	1

i) Initialize weights for Epoch 1

we start by initializing the weights as zero

$w_0 = 0$ $w_1 = 0$ $w_2 = 0$ $x_1 = 2.7810836$ $x_2 = 2.550537003$

learning rate $\alpha = 0.5$

$y = 0$

Compute the Linear combination

$$x = w_0 + w_1 x_1 + w_2 x_2$$

$$= 0 + 0.(2.7810836) + 0(2.550537003)$$

$$x = 0$$

Sigmoid function

$$t = \frac{1}{1+e^{-x}} = \frac{1}{1+e^{-0}} = 0.5$$

ii) Calculating the new co-efficient values

$$w_{(new)} = w_{(old)} + \alpha\, t\, (y-t).(1-t)\, x_i$$

$$w_0 = 0 + 0.5 \times 0.5\, (0-0.5).\,(1-0.5) \times 1$$

$$\boxed{w_0 = -0.0625}$$

$$w_1 = 0 + (0.5 \times 0.5)\, (0-0.5).\,(1-0.5) \times 2.781036$$

$$\boxed{w_1 = -0.1738}$$

$w_2 = 0 + (0.5 \times 0.5)\ (0 - 0.5)\ (1 - 0.5) \times 2.550537003$

$\boxed{w_2 = -0.1594}$

iii) Linear Combination of weighted sum

$x = w_0 + w_1 X_1 + w_2 X_2$

$= (-0.0625) + (-0.1738)(2.781036) + (-0.1594)(2.550537003)$

$\boxed{x = -0.9524}$

iv) Apply to sigmoid function

$$t = \frac{1}{1 + e^{-x}}$$

$$= \frac{1}{1 + e^{-(-0.9524)}} = \underline{0.2784}$$

Epoch 2

Initialize weights. here weights are updated

$w_0 = -0.0625$ $w_1 = -0.1738$ $w_2 = -0.1594$ $x = -0.9524$

$t = 0.2784$

ii) Calculating the new co-efficient values

$w_{(new)} = w_{(old)} + \alpha t\ (y - t).(1 - t)\ x_i$

$w_{0new} = \cancel{(-0.0625) + (0.5 \times 0.5)\ (0 - 0.5)\ (1 - 0.5) \times 1}$

$w_{0new} =$

$~~w_{i\,new} = w_{i\,old} + \alpha t (y-t)(1-t) x_i~~$

$~~w_{i\,new} = (-0.1738) + (0.5 \times 0.5)(0-0.5)(1-0.5) \times~~$

$$w_{0\,new} = (-0.0625) + (0.5 \times 0.2784)(0-0.2784)(1-0.2784) \times 1$$

$$\boxed{w_{0\,new} = -0.0904}$$

$$w_{1\,new} = w_{1\,old} + \alpha t (y-t)(1-t) x_i$$

$$w_{1\,new} = (-0.1738) + (0.5 \times 0.2784)(0-0.2784)(1-0.2784) \times$$

2.7810836

$$\boxed{w_{1\,new} = -0.2515}$$

$$w_{2\,new} = (-0.1594) + (0.5 \times 0.2784)(0-0.2784)(1-0.2784) \times$$

(2.550537003)

$$\boxed{w_{2\,new} = -0.2307}$$

iii) Linear combination of weighted sum

$$X = w_0 + w_1 X_1 + w_2 X_2$$

$$= (-0.0904) + (-0.2515)(2.7810836) + (-0.2307)(2.550537003)$$

$$\boxed{X = -1.3782}$$

iv) Apply to sigmoid function

$$t = \frac{1}{1+e^{-x}} = \frac{1}{1+e^{-(-1.3782)}} = \underline{\underline{0.201}}$$

After two Epochs the loss is getting saturated & we can classify to zero category

<u>Sample 2</u>:- <u>Epoch 1</u>:-

$w_0 = 0$, $w_1 = 0$, $w_2 = 0$ ~~xo~~ $y = 1$ $x_1 = 8.6754186 51$

$x_2 = -0.2420686549$ learning rate $\alpha = 0.5$

compute the Linear combination

$$X = w_0 + w_1 x_1 + w_2 x_2$$

$$= 0 + 0(8.67541865 1) + 0(-0.2420686549)$$

$$X = 0$$

Sigmoid function

$$t = \frac{1}{1+e^{-x}} = \frac{1}{1+e^{-0}} = 0.5$$

ii) Calculating new co-efficient values

$$w_{new} = w_{old} + \alpha t (y-t)(1-t) x_i$$

$$w_0 = 0 + (0.5 \times 0.5)(1-0.5)(1-0.5)(1)$$

$$w_0 = 0.0625$$

$w_1 = 0 + (0.5 \times 0.5)(1-0.5)(1-0.5)(8.675418651)$

$\boxed{w_1 = 0.5422}$

$w_2 = 0 + (0.5 \times 0.5)(1-0.5)(1-0.5)(-0.2420686549)$

$\boxed{w_2 = -0.0151}$

iii) Linear Combination of weighted sum

$x = w_0 + w_1 x_1 + w_2 x_2$

$= (0.0625) + (0.5422)(8.675418651) + (-0.0151)(-0.2420686549)$

$\boxed{x = 4.7699}$

iv) Apply to Sigmoid function

$$t = \frac{1}{1+e^{-x}} = \frac{1}{1+e^{-(4.7699)}}$$

$\boxed{t = 0.9915}$

Sample 2 Epoch 2:-

i) Initialize weights, here weights are updated

$w_0 = 0.0625$ $\quad w_1 = 0.5422$ $\quad w_2 = -0.0151$ $\quad x = 4.7699$ $\quad t = 0.9915$

ii) Calculating new co-Efficient values

$W_{new} = W_{old} + \alpha t (y-t)(1-t) x_i$

~~$W_0 = 0.0625 + (0.5 \times 0.5)(1-0.5)(1-0.5)(1)$~~

$W_0 = 0.0625 + (0.5 \times 0.9915)(1-0.9915)(1-0.9915)(1)$

$\boxed{W_0 = 0.0625}$

$W_1 = 0.5422 + (0.5 \times 0.9915)(1-0.9915)(1-0.9915)(1)$

$\boxed{W_1 = 0.5422}$

$W_2 = -0.0151 + (0.5 \times 0.9915)(1-0.9915)(1-0.9915)(1)$

$\boxed{W_2 = -0.0150}$

iii) Linear Combination of weighted sum

$X = W_0 + W_1 X_1 + W_2 X_2$

$= (0.0625) + (0.5422)(8.675418651) + (-0.0151)(-0.2420686549)$

$\boxed{X = 4.7699}$

iv) Apply to Sigmoid function

$$t = \frac{1}{1+e^{-X}} = \frac{1}{1+e^{-4.7699}} = \underline{\underline{0.9915}}$$

After two Epochs the loss is getting saturated & we can classify to one (1) category

1. write the step-by-step process used in logistic regression model.

→ Logistic regression is a supervised learning algorithm used for binary classification problems

i) Initialization of weights and Bias

- Logistic regression starts by initializing weights (W) and a bias term (b). these parameters are typically set to small random values or zeros.

- The weights determine the importance of each input feature. while the bias helps shift the decision boundary.

ii) Compute the weighted sum (Linear Combination)

- For each input feature vector $X = [X_1, X_2, \ldots X_n]$, Compute the linear combination using:

$$Z = w_0 + w_1 X_1 + w_2 X_2 + \ldots + W_n X_n$$

iii) Apply the sigmoid function

- Logistic regression uses the sigmoid function to map the linear combination Z into a probability (P) between 0 and 1.

$$P = \frac{1}{1+e^{-z}}$$

- P represents the estimated probability that the output is 1 (true) if $P \geq 0.5$ the model predicts class 1; otherwise, it predicts class 0.

iv) co - Efficient new values for calculating weight

$w_{new} = w_{old} + \alpha (y-t)\, t\, (1-t)\, x_p$

where α -> learning rate

t -> transformed

v) Linear Combination of weighted sum

Each input feature vector $x = [x_1, x_2 \ldots x_n]$

Compute the linear combination using:

$z = w_0 + w_1 x_1 + w_2 x_2 + \ldots + w_n x_n$

vi) Repeat the sigmoid process

$P = \dfrac{1}{1+e^{-z}}$ to map the linear combination z into a probability (P) b/w 0 and 1.

vii) Repeat the Multiple Epochs

The process of computing the linear combination, applying the sigmoid function, calculating the loss and updating the weights is repeated over multiple iterations (epochs). with each epoch the weights become more refined. and the cost gradually decreases.

FML

Assignment - 6 : Logistic Regression

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