

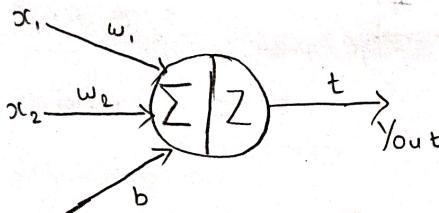
# Assignment

## Perceptron Models

i) Develop a perceptron for the following logic functions with bipolar inputs and targets (initial weights are 0, learning rate = 1, & threshold = 1)

i) OR    ii) NAND    iii) NOR    iv) NOT

ii) OR



Given,

$$w_1 = w_2 = b = 0, \quad d = 1, \quad \theta = 1$$

| $x_1$ | $x_2$ | $t = x_1 \text{ OR } x_2$ | $y_{\text{out}}$ |
|-------|-------|---------------------------|------------------|
| 1     | 1     | 1                         | ?                |
| 1     | -1    | 1                         | ?                |
| -1    | 1     | 1                         | ?                |
| 1     | -1    | -1                        | ?                |

$$\text{Activation function } y_{\text{out}} = f(y_{\text{in}}) = \begin{cases} 1 & \text{if } y_{\text{in}} \geq 1 \\ -1 & \text{if } y_{\text{in}} < 1 \end{cases}$$

0 For first input

$$\begin{array}{lll} x_1 = 1 & x_2 = 1 & t = 1 \\ w_1 = 0 & w_2 = 0 & b = 0 \end{array}$$

$$\begin{aligned} & \text{Initial state: } w_1 = 0, w_2 = 0, b = 0 \\ & \text{Input: } x_1 = 1, x_2 = 1 \\ & \text{Summing junction: } z = w_1 x_1 + w_2 x_2 + b = 1 \\ & \text{Activation function: } y_{\text{out}} = f(z) = 1 \end{aligned}$$

$$y_{in} = x_1 w_1 + x_2 w_2 + b$$

$$y_{in} = 1 \times 0 + 1 \times 0 + 0$$

$$y_{in} = 0$$

$$y_{out} = f(y_{in})$$

$$y_{out} = f(0) = -1$$

$$\boxed{t=1 \neq y_{out} = -1}$$

Update the weight

$$\text{i)} w_1(n) = w_1(0) + \alpha t x_1, \quad \text{ii)} w_2(n) = w_2(0) + \alpha t x_2$$

$$w_1(n) = 0 + 1 \times 1 \times 1$$

$$w_1(n) = 1$$

$$w_2(n) = 0 + 1 \times 1 \times 1$$

$$w_2(n) = 1$$

$$b(n) = b(0) + \alpha t$$

$$= 0 + 1 \times 1$$

$$b(n) = 1$$

$$x_1 = 1 \text{ if } x_2 = 1$$

$$w_1 = 1 \text{ if } w_2 = 1 \quad b = 1$$

$$y_{in} = x_1 w_1 + x_2 w_2 + b$$

$$= 1 \times 1 + 1 \times 1 + 1 \times 1$$

$$y_{in} = 3$$

$$y_{out} = f(y_{in}) = f(3) \quad (\text{target} = 1) \quad (t=3 \text{ target})$$

$$y_{out} = 1$$

$$(\text{target} = 1) == (y_{out} = 1)$$

② For 2nd let of inputs  $t=1$

$$x_1 = 1 \quad x_2 = -1 \quad t = 1 \quad d + w_1 x_1 + w_2 x_2 = \text{target}$$

$$w_1 = 1 \quad w_2 = 1 \quad b = 1 \quad 1 + 1 \cdot 1 + 1 \cdot (-1) = \dots$$

$$y_{in} = x_1 w_1 + x_2 w_2 + b \quad 1 + (-1) + 1 = 1 \text{ (target)}$$

$$= 1 \times 1 + (-1) \times 1 + 1 \quad 1 + (-1) + 1 = 1 \text{ (target)}$$

$$y_{in} = 1$$

$$y_{out} = f(y_{in}) = f(1) \quad ((1 < 3) \Rightarrow (1 = \text{target}))$$

$$y_{out} = 1$$

$$(\text{target} = 1) == (y_{out} = 1)$$

③ For 3rd let of inputs

$$x_1 = -1 \quad x_2 = 1 \quad t = 1$$

$$w_1 = 1 \quad w_2 = 1 \quad b = 1$$

$$y_{in} = x_1 w_1 + x_2 w_2 + b$$

$$= (-1) \times 1 + 1 \times 1 + 1$$

$$y_{in} = 1$$

$$y_{out} = f(y_{in}) = f(1) \quad y_{out} = 1$$

$$(\text{target } t = 1) == (\text{yout} = 1)$$

For 4<sup>th</sup> set of inputs

$$x_1 = -1 \quad x_2 = -1 \quad \alpha t = -1$$

$$\omega_1 = 1 \quad \omega_2 = 1 \quad b = 1$$

$$y_{in} = x_1\omega_1 + x_2\omega_2 + b$$

$$= (-1)x_1 + (-1)x_2 + 1$$

$$y_{in} = -1$$

$$y_{out} = f(y_{in}) = f(-1) = -1$$

$$(\text{target } t = -1) == (\text{yout} = -1)$$

For the linearly class - unlabelled class

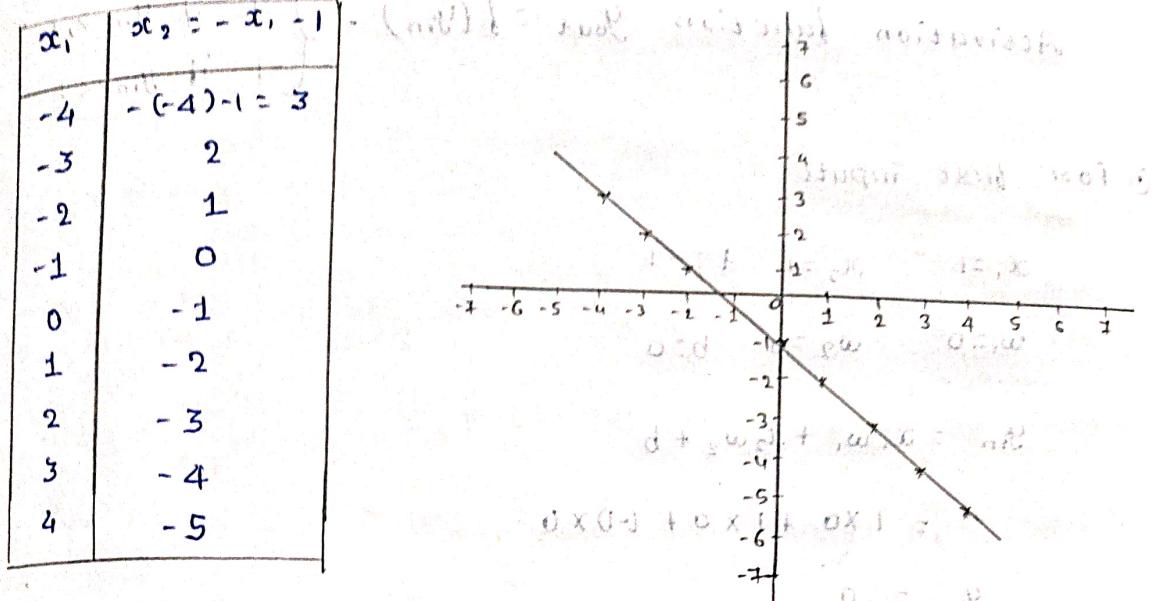
$$\omega_1 x_1 + \omega_2 x_2 + b = 0$$

$$\omega_1 = 1 \quad \omega_2 = 1 \quad b = 1$$

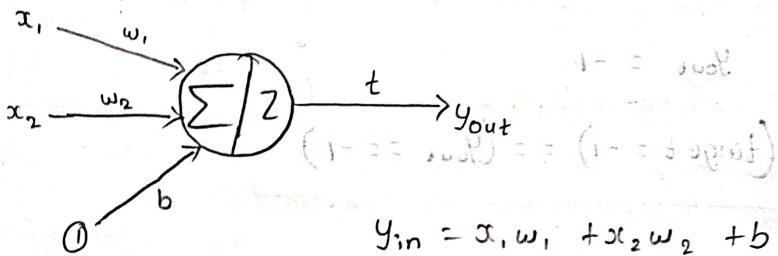
$$x_1 x_1 + x_2 x_2 + 1 = 0$$

$$x_1 = -x_2 - 1$$

| $x_1$ | $x_2$ | $t = x_1 \text{ OR } x_2$ | $y_{out}$ |
|-------|-------|---------------------------|-----------|
| 1     | 1     | 1                         | 1         |
| 1     | -1    | 1                         | 1         |
| -1    | 1     | 1                         | 1         |
| -1    | -1    | -1                        | -1        |



ii) NAND



Given  $\omega_1 = \omega_2 = b = 0$ ,  $\alpha = 1$ ,  $\theta = 1$

| $x_1$ | $x_2$ | $t = x_1 \text{ NAND } x_2$ | $y_{\text{out}} = ?$                |
|-------|-------|-----------------------------|-------------------------------------|
| 1     | 1     | -1                          | $(0 + 0)^{\alpha} = 0^{\alpha} = 0$ |
| 1     | -1    | 1                           | $(0 + 0)^{\alpha} = 0^{\alpha} = 0$ |
| -1    | 1     | 1                           | $(0 + 0)^{\alpha} = 0^{\alpha} = 0$ |
| -1    | -1    | 1                           | $(0 + 0)^{\alpha} = 0^{\alpha} = 0$ |

Activation function  $y_{out} = f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} > 1 \\ -1 & \text{if } y_{in} < 1 \end{cases}$

① For first inputs

$$x_1 = 1 \quad x_2 = 1 \quad t = -1$$

$$w_1 = 0 \quad w_2 = 0 \quad b = 0$$

$$y_{in} = x_1 w_1 + x_2 w_2 + b$$

$$= 1 \times 0 + 1 \times 0 + (-1) \times 0$$

$$y_{in} = 0$$

$$y_{out} = f(y_{in}) = f(0)$$

$$y_{out} = -1$$

$$\boxed{(\text{target } t = -1) == (y_{out} == -1)}$$

② For second inputs:

$$x_1 = 1 \quad x_2 = -1 \quad t = 1$$

$$w_1 = 0 \quad w_2 = 0 \quad b = 0$$

$$y_{in} = x_1 w_1 + x_2 w_2 + b$$

$$= 1 \times 0 + (-1) \times 0 + 0$$

$$y_{in} = 0$$

$$y_{out} = f(y_{in}) = f(0)$$

$$y_{out} = -1$$

target = 1  $\neq$   $y_{out} = -1$

update the weights

i)  $w_1(n) = w_1(0) + \alpha t x_1$

$$w_1(n) = 0 + 1 \times 1 \times 1$$

$w_1(n) = 1$

$b(n) = 1$

ii)  $w_2(n) = w_2(0) + \alpha t x_2$

$$w_2(n) = 0 + 1 \times 1 \times -1$$

$w_2(n) = -1$

$x_1 = 1 \quad x_2 = -1 \quad t = 1$

$w_1 = 1 \quad w_2 = -1 \quad b = 1$

$$y_{in} = x_1 w_1 + x_2 w_2 + b$$

$$(1 \times 1) + (-1 \times -1) + 1$$

$y_{in} = 3$

$y_{out} = f(y_{in}) = 1(3)$

$y_{out} = 1$

$(\text{target} = 1) \neq (\text{yout} = 1)$

For third input!

$x_1 = 0 \quad x_2 = 1 \quad t = 1$

$$x_1 = -1 \quad x_2 = 1 \quad t = 1$$

$$w_1 = 1 \quad w_2 = -1 \quad b = 1$$

$$y_{in} = x_1 w_1 + x_2 w_2 + b$$

$$y_{in} = -1$$

$$y_{out} = \text{sgn}(y_{in}) = -1$$

$$(target = 1) \neq (y_{out} = -1)$$

Update the weights

$$w_1(n) = w_0(n) + \alpha t x_1$$

$$w_1(n) = 1 + 1 \times 1 \times -1$$

$$w_1(n) = 0$$

$$w_2(n) = w_2(0) + \alpha t x_2$$

$$w_2(n) = 0$$

$$b(n) = b_0 + \alpha t$$

$$b = 2$$

| $x_1$ | $x_2$ | $t = 1, \text{NAND } x_2$ | $y_{out}$ |
|-------|-------|---------------------------|-----------|
| 1     | 1     | -1                        | -1        |
| 1     | -1    | 1                         | 1         |
| -1    | 1     | 1                         | 1         |
| -1    | -1    | 1                         | 1         |

- For unseen data

$$x_1 = -1, x_2 = 1, t = 1$$

$$w_1 = 0, w_2 = 0, b = 2$$

$$y_{in} = w_1 x_1 + w_2 x_2 + b$$

$$y_{in} = 2, y_{out} = \text{sgn}(y_{in}) = 1(2)$$

$$(target = 1) == (y_{out} = 1)$$

For  $x_1=2$  and  $x_2=0$  inputs

$$x_1 = -1, x_2 = -1, t = 1$$

$$w_1 = 0, w_2 = 0, b = 2$$

$$y_{in} = x_1 w_1 + x_2 w_2 + b$$

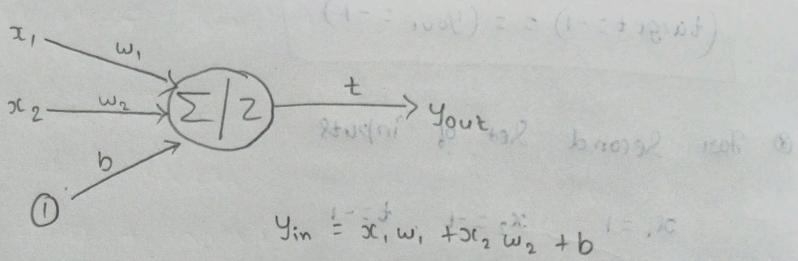
$$y_{in} = (-1 \times 0) + (-1 \times 0) + 2$$

$$y_{in} = 2$$

$$y_{out} = \text{sgn}(y_{in}) = 1(2) = 1$$

$$(target = 1) == (y_{out} = 1)$$

iii) NOR



Given

$$w_1 = w_2 = b = 0, \quad \alpha = 1, \quad \theta = 1, \quad \text{and } w_1, w_2, b \neq 0$$

| $x_1$ | $x_2$ | $t = x_1 \text{ NOR } x_2$ | $y_{\text{out}} = ?$                            |
|-------|-------|----------------------------|---|
| 1     | 1     | -1                         | $0 + 0 \cdot 1 + 0 \cdot 1 = 0 \neq 1$          |
| 1     | -1    | -1                         | $0 + 0 \cdot (-1) + 0 \cdot 1 = 0 \neq 1$       |
| -1    | 1     | -1                         | $0 + 0 \cdot 1 + 0 \cdot (-1) = 0 \neq 1$       |
| -1    | -1    | 1                          | $(1 - z \cdot w_1) = 1 - z \cdot w_2 \cdot x_2$ |

Activation function -  $y_{\text{out}} = f(y_{\text{in}}) = \begin{cases} 1 & \text{if } y_{\text{in}} > 1 \\ -1 & \text{if } y_{\text{in}} < 1 \end{cases}$

① For first set of inputs

$$x_1 = 1, x_2 = 1, t = -1$$

$$w_1 = 0, w_2 = 0, b = 0$$

$$y_{\text{in}} = x_1 w_1 + x_2 w_2 + b$$

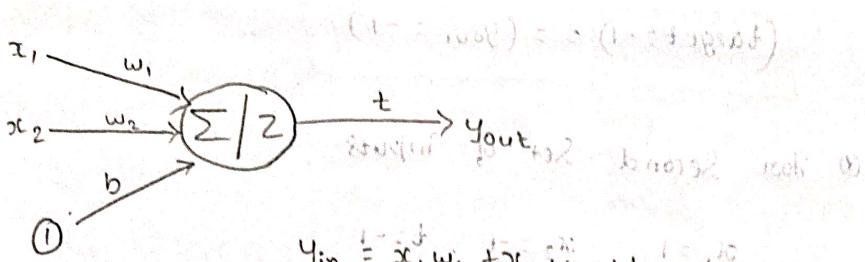
$$= 1 \cdot 0 + 1 \cdot 0 + 0$$

$$y_{\text{in}} = 0$$

$$y_{\text{out}} = f(y_{\text{in}}) = f(0)$$

$$y_{\text{out}} = -1$$

iii) NOR



Given

$$w_1 = w_2 = b = 0, \alpha = 1, \theta = 1$$

| $x_1$ | $x_2$ | $t = x_1 \text{ NOR } x_2$ | $y_{in} = ?$ |
|-------|-------|----------------------------|--------------|
| 1     | 1     | -1                         |              |
| 1     | -1    | -1                         | ?            |
| -1    | 1     | -1                         |              |
| -1    | -1    | 1                          |              |

Activation function -  $y_{out} = f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \geq 1 \\ -1 & \text{if } y_{in} < 1 \end{cases}$

For first set of inputs

$$x_1 = 1, x_2 = 1, t = -1$$

$$w_1 = 0, w_2 = 0, b = 0$$

$$\begin{aligned} y_{in} &= x_1 w_1 + x_2 w_2 + b \\ &= 1 \times 0 + 1 \times 0 + 0 \end{aligned}$$

$$y_{in} = 0$$

$$y_{out} = f(y_{in}) = f(0)$$

$$y_{out} = -1$$

$$(\text{target} = -1) == (\text{y}_{\text{out}} = -1)$$

④ For second set of inputs

$$x_1 = 1 \quad x_2 = -1 \quad t = -1$$

$$w_1 = 0 \quad w_2 = 0 \quad b = 0$$

$$y_{\text{in}} = x_1 w_1 + x_2 w_2 + b$$

$$= 1 \times 0 + (-1) \times 0 + 0$$

$$y_{\text{in}} = 0$$

$$y_{\text{out}} = f(y_{\text{in}}) = f(0)$$

$$y_{\text{out}} = -1$$

$$(\text{target} = -1) == (\text{y}_{\text{out}} = -1)$$

⑤ For third set of inputs

$$x_1 = -1 \quad x_2 = 1 \quad t = -1$$

$$w_1 = 0 \quad w_2 = 0 \quad b = 0$$

$$y_{\text{in}} = x_1 w_1 + x_2 w_2 + b$$

$$y_{\text{in}} = (-1 \times 0) + (1 \times 0) + 0$$

$$y_{\text{in}} = 0$$

$$y_{\text{out}} = f(y_{\text{in}}) = f(0)$$

$$y_{\text{out}} = -1$$

$$(\text{target} = -1) == (\text{y}_{\text{out}} = -1)$$

④ For fourth set of inputs

$$x_1 = -1 \quad x_2 = -1 \quad t = 1$$
$$w_1 = 0 \quad w_2 = 0 \quad b = 0$$

$$y_{in} = x_1 w_1 + x_2 w_2 + b$$

$$y_{in} = (-1) \times 0 + (-1) \times 0 + 0$$

$$y_{out} = f(y_{in}) = f(0)$$

$$y_{out} = -1$$

$$\boxed{\text{target } t = 1 \neq y_{out} = -1}$$

Update the weights

$$w_1(n) = w_1(0) + \alpha t x_1$$

$$w_2(n) = w_2(0) + \alpha t x_2$$

$$w_1(n) = 0 + 1 \times 1 \times -1$$

$$= 0 + 1 \times 1 \times -1$$

$$w_1(n) = -1$$

$$w_2(n) = -1$$

$$b(n) = b(0) + \alpha t$$

$$y_{in} = x_1 w_1 + x_2 w_2 + b$$

$$b(n) = 0 + 1 \times 1$$

$$y_{in} = (-1 \times -1) + (-1 \times -1) + 1$$

$$y_{in} = 3$$

$$y_{out} = f(y_{in}) = f(3)$$

$$y_{out} = 1$$

$$\text{target } t = 1 == y_{out} = 1$$

For the linear class

$$w_1x_1 + w_2x_2 + b = 0$$

$$w_1 = 1 \quad w_2 = -1 \quad b = 1$$

$$-x_1 - x_2 + 1 = 0$$

$$x_1 + x_2 = 1 \quad 0 + (0 \cdot 1) + 0 \cdot (1) = 1 \text{ is not}$$

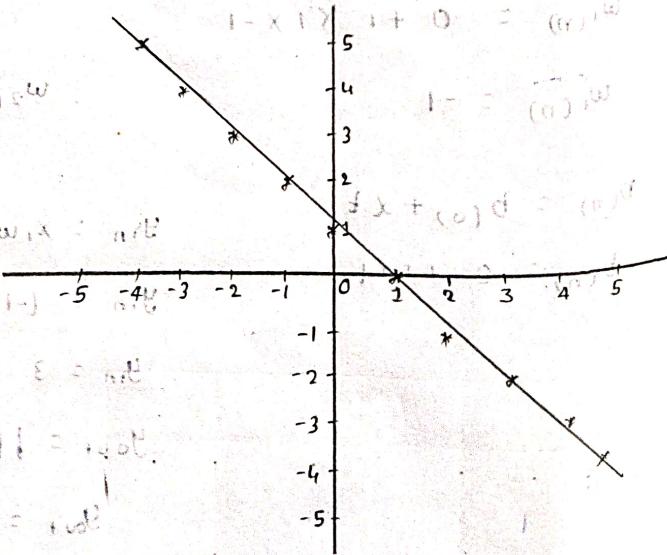
$$x_1 = 1 - x_2 \quad (0)1 = (0 \cdot 1) + 0 \cdot 1 = 0 \text{ is not}$$

| $x_1$ | $x_2$ | $t = x_1 \text{ NOR } x_2$ | $y_{out}$                                    |
|-------|-------|----------------------------|--|
| 1     | 1     | -1                         | 1 = 1 $\neq$ 1 = 1 $\text{not}$              |
| 1     | -1    | -1                         | -1   |
| -1    | 1     | -1                         | -1 $\text{not true}$ $\neq$ 0 $\text{false}$ |
| -1    | -1    | 1                          | 1  |

For unseen data

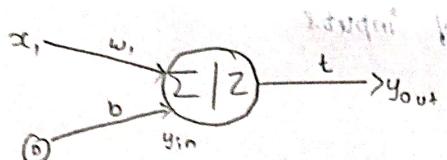
$$1 - x_1 \text{ if } 1 = 0$$

| $x_1$ | $x_2 = 1 - x_1$ |
|-------|-----------------|
| -4    | $1 - (-4) = 5$  |
| -3    | 4               |
| -2    | 3               |
| -1    | 2               |
| 0     | 1               |
| 1     | 0               |
| 2     | 1               |
| 3     | 2               |
| 4     | 3               |



$1 = \text{not}$   $\neq$   $1 = \text{not}$

iv) NOT



$$y_{in} = x \cdot w + b$$

$$y_{out} = f(x \cdot w + b)$$

Given  $w_1 = w_2 = b = 0$ ,  $\alpha = 1$ ,  $\theta = 1$ ,  $f(x) = \text{NOT}$

| $x_i$ | $t = \text{NOT } x_i$ | $y_{out} = ?$ |
|-------|-----------------------|---------------|
| 1     | -1                    |               |
| -1    | 1                     |               |

$$t = \text{NOT } x$$

$$t = \text{not } x$$

$$(0) \oplus (1) = \text{not } x$$

$$t = \text{not } x$$

Activation function

$$y_{out} = f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \geq 1 \\ -1 & \text{if } y_{in} < 1 \end{cases}$$

For the first set of inputs

$$x_1 = 1 \quad t = -1$$

$$w_1 = 0 \quad b = 0$$

$$\begin{aligned} y_{in} &= x_1 \cdot w_1 + b \\ &= 1 \cdot 0 + 0 \end{aligned}$$

$$y_{in} = 0$$

$$(\text{target} = -1) \neq (\text{predicted} = 0)$$

$$y_{out} = f(0) = 0$$

$$y_{out} = -1$$

$$(\text{target} = -1) \neq (y_{out} = -1)$$

② For the second set of inputs

$$x_1 = -1 \quad t = 1$$

$$\omega_1 = 0 \quad b = 0$$

$$y_{in} = x_1 \omega_1 + b$$

$$= -1 \times 0 + 0$$

$$y_{in} = -1$$

$$y_{out} = f(y_{in}) = f(-1)$$

$$y_{out} = -1$$

$$\boxed{(\text{target } t = 1) \neq (y_{out}, N = -1)}$$

Update the weights

$$\omega_{1(n)} = \omega_{(0)} + \alpha t x_1$$

$$= 0 + 1 \times 1 \times -1$$

$$\omega_{1(n)} = -1$$

$$x_1 = -1 \quad t = 1$$

$$\omega_1 = -1 \quad b = 1$$

$$y_{in} = x_1 \omega_1 + b$$

$$= -1 \times -1 + 1$$

$$y_{in} = 2$$

$$y_{out} = f(y_{in}) = f(2)$$

$$y_{out} = 1$$

| $x_1$ | $t = \text{NOT}(x_1)$ | $y_{out}$ |
|-------|-----------------------|-----------|
| 1     | -1                    | -1        |
| -1    | 1                     | 1         |

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