

CSCN-8000 Lab-2 Artificial Intelligence Algorithm

Theoretical Part A

Location (L)	Size (S)	Price (Label)
Urban	Large	Expensive
Suburban	Medium	Affordable
Rural	Small	Cheap
Urban	Medium	Affordable
Suburban	Large	Expensive
Rural	Medium	Affordable
Urban	Small	Cheap
Suburban	Small	Cheap
Rural	Large	Expensive
Urban	Large	Expensive

Question

Calculate

$$P(\text{Price} | L = \text{Urban}, S = \text{Medium})$$

Step 1

Identify the type of Naive Bayes we equation which is applicable for the above data.

- The data is not binary. Its columns are having categorical values
- The data is not continous values like numerical values.

Given the above characteristics we can apply MultiNomial Naive Bayes for solving this.

Step 2

Equation for solving

$$P(\text{Price} | L = \text{Urban}, S = \text{Medium}) = \frac{P(\text{Urban} | \text{Price}) * P(\text{Medium} | \text{Price}) * P(\text{Price})}{P(\text{Urban}) * P(\text{Medium})}$$

Step 2.1

Calculating Naive bayes for Price = Expensive

Type	Probability
P(Expensive)	4/10
P(Urban)	4/10
P(Medium)	3/10
P(Urban Expensive)	2/4
P(Medium Expensive)	0

Substituting in the formula above we get

$$P(\text{Expensive} | L = \text{Urban}, S = \text{Medium}) = (2/4 * 0 * 4/10) / (4/10 * 3/10) = 0$$

Step 2.2

Calculating Naive bayes for Price = Affordable

Type	Probability
P(Affordable)	3/10
P(Urban)	4/10
P(Medium)	3/10
P(Urban Affordable)	1/3
P(Medium Affordable)	3/3

Substituting in the formula above we get

$$P(\text{Affordable} | L = \text{Urban}, S = \text{Medium}) = (1/3 * 3/3 * 3/10) / (4/10 * 3/10) = 0.1/0.12 = 10/12 = 0.8333$$

Step 2.2

Calculating Naive bayes for Price = Cheap

Type	Probability
P(Cheap)	3/10
P(Urban)	4/10
P(Medium)	3/10
P(Urban Cheap)	1/3
P(Medium Cheap)	0/3

Substituting in the formula above we get

$$P(\text{Cheap} | L = \text{Urban}, S = \text{Medium}) = 0.33 * 0 * 0.3 / 0.4 * 0.3 = 0$$

Result

Type	Probability
P(Cheap L=Urban, S=Medium)	0
P(Affordable L=Urban, S=Medium)	0.833
P(Expensive L=Urban, S=Medium)	0

So based on Naive Bayes formula we get that If the location is urban and size is medium there is 83.3% chance that it will be affordable.

In order to solve the zero probability issue we can add 1 to all class combinations and then calculate the probability. But in the question we only needed to figure out the max probability class which will still remain to be the same "Affordable" with 83.3% value going down and zero probability for others increasing a bit.

Theoretical Part-B

Question B

$$4X - 3Y + Z = -10$$

$$2X + Y + 3Z = 0$$

$$-1X + 2Y - 5Z = 17$$

Step1

X	Y	Z	CoEff
4	-3	1	-10
2	1	3	0
-1	2	-5	17

Step 1 : 2*R2 - R1 = R2

X	Y	Z	CoEff
4	-3	1	-10
0	5	5	10
-1	2	-5	17

Step 2: $4R_3 + R_1 = R_3$

X	Y	Z	CoEff
4	-3	1	-10
0	5	5	10
0	5	-19	58

Step 3: $R_3 - R_2 = R_3$

X	Y	Z	CoEff
4	-3	1	-10
0	5	5	10
0	0	-24	48

Step 4 : Substitute values in R_3

$$-24Z = 48$$

$$Z = -2$$

Step 5 - Substitute in 2nd equation

$$5Y + 5Z = 10$$

$$5Y - 10 = 10$$

$$5Y = 20$$

$$Y = 4$$

Step 6 - Substitute in 1st equation

$$4X - 3Y + Z = -10$$

$$4X = 3Y - Z - 10$$

$$4X = 12 + 2 - 10$$

$$4X = 4$$

$$X = 1$$

Result

X	Y	Z
1	4	-2

Gaussian Elimination

In order to properly change the matrix to row-echelon form we can divide each row by its leading value after Step 3

X	Y	Z	CoEff
4	-3	1	-10
0	5	5	10
0	0	-24	48

- $R3 \cdot -1/24$
- $R2 \cdot 1/5$
- $R1 \cdot 1/4$

X	Y	Z	CoEff
1	-3/4	1/4	-10/4
0	1	1	2
0	0	1	-2

Now this is in proper row-echelon form. Again Substituting we get the same result.

- $Z = -2$
- $Y = 4$
- $X = 1$