Tutorial 4

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Ques 1 $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n)$

$a \geq 1 \quad b > 1$

on comparing

$a = 3, \; b = 2, \; f(n) = n^2$

Now $c = \log_b^a = \log_2^3 = 1.584$

$n^c = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$T(n) = \Theta(n^2)$

Ques 2 $T(n) = 4T(n/2) + n^2$

$a \geq 1, \; b > 1$

$a = 4, \; b = 2, \; f(n) = n^2$

$c = \log_2^4 = 2$

$n^c = n^2 = f(n)$

$\therefore T(n) = \Theta(n^2 \log n)$

Ques 3 $T(n) = T(n/2) + 2^n$

$a = 1 \quad b = 2 \quad f(n) = 2^n$

$c = \log_b^a = \log_2^1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \Theta(2^n)$

Ques 4 $T(n) = 2^n T(n/2) + n^n$

Here master's theorem can't be applied as 'a' must be constant

Ques 5 $T(n) = 16\,T(n/4) + n$

$a = 16, \; b = 4$

$f(n) = n$

$c = \log_4^{16} = \log_4^{4^2} = 2$

$n^c = n^2$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

Ques 6 $T(n) = 2T(n/2) + n \log n$

$a = 2 \quad b = 2$

$f(n) = n \log n$

$c = \log_2^2 = 1$

$n^c = n$

since $n \log n > n$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(n \log n)$

Ques 7 $T(n) = 2T(n/2) + n/\log n$

$a = 2 \quad b = 2 \quad f(n) = n/\log n$

$c = \log_2^2 = 1$

$n^c = n$

Since $n/\log n < n$

$\therefore T(n) = \Theta(n)$

Ques 8 $T(n) = 2T(n/4) + n^{0.51}$

$a = 2 \quad b = 4 \quad f(n) = n^{0.51}$

$c = \log_b^a = \log_4^2 = 0.5$

$n^c = n^{0.5}$

Since $n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \Theta(n^{0.51})$

Ques 9. $T(n) = 0.5\,T(n/2) + 1/n$

$a = 0.5, \; b = 2$

$\therefore$ A/c to Masters theorem $a \geq 1$ but here a is 0.5 so M.T cannot be applied.

Ques 10. $T(n) = 16\,T(n/4) + n!$

$a = 16 \quad b = 4 \quad f(n) = n!$

$c = \log_b^a \Rightarrow \log_4^{16} = 2$

$n^c = n^2$

as $n! > n^2$

$\therefore T(n) = \Theta(n!)$

Q11 $4T(n/2)+\log n$

$a=4, b=2, f(n)=\log n$

$c=\log_b^a = \log_2^4 = 2$

$n^c = n^2$ and $f(n)=\log n$

$\log n < n^2$

$T(n) = \Theta(n^2)$

Q12 $T(n) = \text{sqrt}(n)\,T(n/2)+\log n$

here MT cannot be applied as 'a' must be constant

Q13 $T(n) = 3T(n/2)+n$

$a=3 \quad b=2 \quad f(n)=n$

$c=\log_b^a = \log_2^3 = 1.5849$

$\therefore n < n^{1.5849}$

$f(n) < n^c$

$T(n) = \Theta(n^{1.5849})$

Q14 $T(n) = 3T(n/3)+\text{sqrt}(n)$

$a=3, b=3, c=\log_3^3 = 1$

$n^c = n$

As $\text{sqrt}(n) < n$

$\therefore f(n) < n^c$

$T(n) = \Theta(n)$

Q15 $T(n) = 4T(n/2)+cn$

$a=4, b=2$

$c=\log_b^a = \log_2^4 = 2$

$n^c = n^2$

$cn < n^2$ (for any constant)

$\therefore T(n) = \Theta(n^2)$

Q16 $T(n) = 3T(n/4)+n\log n$

$a=3, b=4 \quad f(n)=n\log n$

$c=\log_b^a = \log_4^3 = 0.792$

$n^c = n^{0.792}$

$n^{0.792} < n\log n$

$T(n) = \Theta(n\log n)$

Q17 $T(n) = 3T(n/3)+n/2$

$a=3, b=3$

$c=\log_b^a = \log_3^3 = 1$

$f(n) = n/2$

$\therefore n^c = n^1 = n$

as $n/2 < n$

$T(n) = \Theta(n)$

Q18 $T(n) = 6T(n/3)+n^2\log n$

$a=6 \quad b=3$

$c=\log_b^a = \log_3^6 = 1.6309$

$n^c = n^{1.6309}$

as $n^{1.6309} < n^2\log n$

$T(n) = \Theta(n^2\log n)$

Q19 $T(n) = 4T(n/2)+n\log n$

$a=4 \quad b=2 \quad f(n)=n\log n$

$c=\log_2^4 = 2$

$n^c = n^2 > n/\log n$

$T(n) = \Theta(n^2)$

Q20 $T(n) = 64T(n/8)-n^2\log n$

MT can't be applied here as $f(n)$ is -ve

Q21 $T(n) = 7T(n/3)+n^2$

$a=7, b=3 \quad f(n)=n^2$

$c=\log_b^a = \log_3^7 = 1.7712$

$n^c = n^{1.7712} < n^2$

$\therefore T(n) = \Theta(n^2)$

Q22 $T(n) = T(n/2)+n(2-\cos n)$

$a=1 \quad b=2$

$c=\log_b^a = \log_2^1 = 0$

$n^c = n^0 = 1$

$n(2-\cos n) > n^c$

$\therefore T(n) = \Theta(n(2-\cos n))$