QUANTUM CHECKERS

Two qubits are prepared in the singlet state, $|sing\rangle = \frac{1}{\sqrt{2}}(|ud\rangle - |du\rangle)$, they are both placed at point $(0, 0)$, and then they are allowed to propagate freely on the grid of points $\{-2, -1, 0, 1, 2\}^2$, that wraps onto itself, so that, for instance, $(-2, 1)$ is to the right of $(2, 1)$ and $(0, -2)$ is above $(0, 2)$ etc. Time is measured in increments of N^{-1}, and we can choose to make a measurement after every k such increments.

Each time we measure the qubits, they are localized somewhere on the grid, not necessarily together. The measurements don't observe the qubits' spins, only their locations on the grid. Every time the two qubits find themselves in locations that agree in only one coordinate, they pass through a CNOT gate, controlled by the one with the more positive value of the other coordinate, while every other time one is chosen randomly with equal probability and is passed through a Hadamard gate, while the other is left alone.

Your first task is to simulate this evolution and create a visual animation of it. Once this is accomplished, you can then proceed to play the following game.

Two players take turns choosing to apply one of the Pauli matrices to the qubit that was previously left alone, whenever the two qubits agree on one but not both of their coordinates. They may still choose to leave it alone. If they choose to apply a Pauli matrix, then they measure the resulting spin. That player then proceeds to gain or lose the measured value of the spin ($+1$ or -1). Otherwise they pass the turn to the other player.

Your second task is to design a strategy that maximizes the points you accumulate after time T.

Clarification: Note that, in order to measure the propagation of the two qubits in our grid, we have discretized time. This approximation to the Schrödinger equation has some peculiar consequences. First of all, it creates parity constraints for the particle paths. For example, each particle has nonzero probability of returning to its starting point only at even time steps. In fact, the probability computations based on Feynman's 'sum over histories' that we have seen make sense only for even step sizes, because of this parity constraint. Secondly, while we have disregarded some constants that take care of the units, in order to use this scheme to approximate the value of the wave function at some time t, we need to write $t = \frac{2k}{N}$ and let both $N \longrightarrow \infty$ and $k \longrightarrow \infty$ while maintaining their fixed ratio. Realize that the k asymptotic entails repeated summations over the prior lattice points the particle has visited...